Mathematical Modelling in Mathematics Education as a way to develop critical consciousness: A theoretical essay

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ABSTRACT

According to Paulo Freire, teaching and learning should involve research, trial and error, curiosity, satisfaction, and pleasure. As proposed by Dionísio Burak, Mathematical Modelling in Mathematics Education starts from the interest and curiosity of the students who choose a theme (or a problem) to investigate during the mathematics class. To do this investigation the students research about a theme, collect some data and organize it. Then they formulate the hypothesis and use mathematical knowledge as a tool to solve problems, making predictions and decisions. Hence, it is possible to find some similarities between Paulo Freire's ideas and Mathematical Modelling, as argued by Burak. Therefore, in this paper, we have presented a theoretical essay based on literature reviews whose goal was to draft some relations between Freire's concepts and Mathematical Modelling as proposed by Burak. Finally, we have shown a frame that relates the main actions performed by students during each step of mathematical modelling activity with the characteristics of critical consciousness as proposed by Freire.

Keywords: Mathematics Education. Mathematical Modelling steps. Critical consciousness. Dionísio Burak. Paulo Freire.

Modelagem Matemática na Educação Matemática como um caminho para desenvolver a consciência crítica: um ensaio teórico

RESUMO

Segundo Paulo Freire, ensinar e aprender deve envolver pesquisa, tentativa e erro, curiosidade, satisfação e prazer. Modelagem Matemática na Educação Matemática, como proposto por Dionísio Burak, parte do interesse dos estudantes, que escolhem um tema (ou problemas) para investigar nas aulas de matemática. Para realizar a investigação os estudantes fazem pesquisas sobre o tema, coletam e organizam dados, formulam hipóteses e usam o conhecimento matemático como uma ferramenta para resolver problemas, fazer predições e/ou tomar decisões. Sendo assim, é possível

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traçar similaridades entre as ideias de Freire e a Modelagem Matemática conforme apontada por Burak. Portanto, nesse artigo, apresentamos um ensaio teórico baseado numa revisão de literatura, cujo objetivo é traçar relações entre alguns conceitos propostos por Freire e a Modelagem Matemática conforme propõe Burak. Por fim, apresentamos um quadro relacionando as principais ações que os estudantes fazem em cada uma das etapas de uma atividade de modelagem matemática com as características da consciência crítica proposta por Freire.

Palavras-chave: Educação Matemática. Etapas da Modelagem Matemática. Consciência crítica. Dionísio Burak. Paulo Freire.

INTRODUCTION

In the context of Brazilian schools, there are challenges that do not help in teachers work, for example: increasing population of students; social, cultural, religious and ethnical differences in the same classroom; and students with learning disabilities. The teacher must choose an appropriate teaching methodology to overcome these challenges and allows everyone to learn.

According to Freire (1979), for students to learn, they must apprehend the need to learn and then prepare themselves for it. Moreover, the learning occurs only with a critical and motivated debate.

There are researchers who have advocated that Mathematical Modelling allows a critical and motivated debate in Mathematics classroom. However, many authors use different arguments, different ways to organize and conduct a mathematical modelling activity. As a result, there are different Mathematical Modelling approaches (BARBOSA, 2006; KAISER; SRIRAMAN, 2006).

Brazilian researchers like Almeida (2003), Barbosa (2006), Jacobini and Wodewotzki (2006) and Araújo (2009) have been arguing about Modelling[1] in different approaches, and they are following the Critical Mathematics Education perspective, as proposed by Skovsmose (2001).

Skovsmose carries Paulo Freire's educational perspective to Mathematics (ARAÚJO, 2009). For this reason, some studies about Mathematical Modelling present parallels between Modelling on Critical Mathematics Education perspective and Paulo Freire's concepts. For example, Malheiros (2012) presented a study sign how ideas and assumptions about the Thematic Research, as proposed by Paulo Freire, converge on Modelling. She studied the concepts from the socio-critical perspective, based in Critical Mathematics Education.

Burak (2010) uses *Mathematics Education* rather than *Critical Mathematics Education*. For him, the critical idea is implicit in his Mathematics Education view. Even Burak (2010) has been arguing that we can use Modelling to teach in a critical way. Till now, enough research has not been done for finding similarities between Freire's concepts and Burak's proposal.

[1] We will use only Modelling instead of Mathematical Modelling in order to avoid repetition.

For this reason, our goal is to draft convergences between Paulo Freire's concepts and Mathematical Modelling in Mathematics Education, as proposed by Burak (1992; 2010). Primarily we are arguing about Modelling as a way of developing a critical consciousness.

In order to do this, we decided for a theoretical essay, a study that emphasizes on personal interpretation (SEVERINO, 2000). According to Meneghetti (2011), traditional methods of data collection are not required to do an essay. Therefore, our first approach to the theme was to do literature reviews. In the next two sections, we have described some of Paulo Freire's concepts and Mathematical Modelling in Mathematics Education as Burak (2010) has been arguing, respectively.

Consequently, we have drafted the convergences between Paulo Freire's concepts and Modelling, highlighting usual actions the students do during a mathematical modelling activity and the characteristics of critical consciousness they can develop.

REVIEWING SOME OF PAULO FREIRE'S CONCEPTS

Paulo Freire has presented several concepts, which have been generating constantly debates. Forner (2005) highlights some of them, as unfinished human beings, different consciousness levels and dialogue. In this section, we approach these concepts, but we begin with Paulo Freire's idea about education.

All Paulo Freire's concepts are interrelated. For example, for him, education is related to how he understands the world (reality), the man and the relation between both of them.

According to Freire (1967), the reality exists independent of men, and the man can recognize it and relate to it. Besides, for this author, the man can think about reality, act and transform it.

> Human relationship with the world are plural in nature. Whether facing widely different challenges of the environment or the same challenge, men are not limited to a single reaction pattern. They organize themselves, choose the best response, test themselves, act, and change in the very act of responding. They do all this consciously, as one uses a tool to deal with a problem. (FREIRE; 2005, p.3)

Men do these actions because, to Freire (2005), we are unfinished human beings who are in constant evolution. "Men exist in time. They are inside. They are outside. They inherit. They incorporate. They modify. Men are not imprisoned within a permanent 'today'; they emerge, and become temporalized" (FREIRE, 2005, p.3-4).

Freire (1979) propounds the education as a way to transform reality, allowing the men to become better person. Further, for a man to become a better person he should not be alone; he needs support of other men who want to become better too.

We need the education in making decisions, for political and social responsibilities. The education is aimed to overcome the disbelief in students. The education must promote discussion about all kinds of problems, such as problems in the country, world, professions and democracy (FREIRE, 1967). Therefore, the education is not a way to adapt the student to reality; it is a way to think. This concept of education provides the formation of autonomous man.

Freire (1967) defends that the education is supported by dialogue. Dialogue, for him, is a horizontal relationship between two persons who are engaged in a joint quest. "It is nourished from love, humility, hope, faith and trust" (FREIRE, 2005, p.40). From this dialogue, a student emerges who can think and discuss his life problems. Besides dialogue, Paulo Freire proposes that the education must happen from the students' context.

To Paulo Freire the education should allow the students to see the reality in a critical way. The education should stimulate a critical consciousness instead of naive consciousness (FREIRE, 2005).

According to Freire (2005, p.14), naive consciousness

> […] is characterized by an over-simplification of problems; by a nostalgia for the past; by underestimation of the common man; by a strong tendency to gregariousness; by a lack of interest in investigation, accompanied by an accentuated taste for fanciful explanations; by fragility of argument; by a strongly emotional style; by the practice of polemics rather than dialogue; by magical explanations.

Freire (2005, p.14) highlights: "If this consciousness does not progress to the stage of *critical transitivity*, it may be deflected by sectarian irrationality into fanaticism". Therefore, it is essential to overcome the naive consciousness and transform it into a critical consciousness. The critical consciousness is characterized by (FREIRE, 2005):

- in-depth interpretation of problem;
- substituting magical explanations for casual principles;
- testing of one's "findings" and by openness to revision;
- attempting to avoid distortion when perceiving problems and to avoid preconceived notions when analyzing them;
- refusing to transfer responsibility;
- rejecting passive positions;
- soundness of argumentation;
- practicing of dialogue rather than polemics;

- receptivity to the new for reasons beyond mere novelty and by the good sense not to reject the old just because it is old – accepting what is valid in both old and new.

Considering Freire's (1979) idea, understanding is a result of process that requires effort, recreation and research. These are some characteristics of the educational practice, which aims to develop the critical consciousness aforementioned. According to Freire (2001), teaching should move towards the authentic and critical thinking unlike imposition of formulas.

Unfortunately, it is common that the Mathematics teachers speak their ideas and impose rules and formulas. In other words, Mathematics teachers usually do not let students think, act, create, invent and reinvent. Freire (1979) has called this idea of education as "banking education". In the banking education, the student is considered empty and the teacher makes "deposits" of concepts and procedures on him/her. For Freire (1979), the student repeats what teacher says in the classroom and it turns him into an object.

Based on Paulo Freire ideas, we believe in an education that transforms people. This way, we see Mathematical Modelling, as proposed by Burak (2010), an opportunity for this. Next, we present Burak's perspective about Mathematical Modelling in Mathematics Education.

MATHEMATICAL MODELLING IN MATHEMATICS EDUCATION: BURAK'S PERSPECTIVE

Looking at the literature on Modelling, we found different ways to organize and conduct a mathematical modelling activity. Six different approach arises in current debate about Mathematical Modelling (KAISER; SRIRAMAN, 2006). There are researchers who defend a realistic or applied modelling approach, where aims are to solve real world problems, understanding of the real world and promotion of modelling competencies. The second approach is a contextual modelling whose central aim is to solve the word problems. There are those who argue about Modelling in an educational way, with a pedagogical aims. Similar to educational approach, there is socio-critical modelling view, but the pedagogical goal involves the critical understanding of the surrounding world. Some researchers study about epistemological or theoretical modelling, whose aim is the promotion of theory development. Finally, there is a cognitive modelling approach, whose aims are analyzing and understanding of cognitive modelling processes and promotion of mathematical thinking processes.

Barbosa (2006) has highlighted that the social-critical modelling approach prevails in Brazil. The socio-critical approach "emphasizes the role of mathematics in society and claims the necessity to support critical thinking about the role of mathematics in society, about the role of and nature of mathematical models and the function of mathematical modelling in society" (KAISER; SRIRAMAN 2006, p.306). An example of this is the

Barbosa (2006, p.294) idea: "In specific terms, I have established the boundaries of modelling as a learning milieu where students are invited to take a problem and investigate it with reference to reality via mathematics".

Modelling linked to socio-critical approach is a way to explore the role of Mathematics in society through modelling activities (ALMEIDA; SILVA, 2010). Usually, when someone links Modelling to socio-critical approach, he/she does this based on Critical Mathematics Education perspective.

Skovsmose (2001) is one of the main defenders of Critical Mathematics Education. For him, this perspective is not only about learning the mathematics concepts, but also about the citizen formation in order to understand and discuss social, political and economic problems. In other words, Critical Mathematics Education perspective intents the critical participation of the students in the society. According to Araújo (2009), Skovsmose brought the educational perspective of Paulo Freire to Mathematics.

Burak (2010a) uses only *Mathematics Education* because, for him, the critical idea is implicit in his Mathematics Education view. For Burak (2010a), in Mathematics Education, Education is a noun, a word used to identify any class of people, place, or things, while Mathematics is an adjective, added to a noun to modify or describe it. In addition, he presents Mathematics Education as a Human and Social Science, and it is composed by different areas of knowledge that supports the Education: Psychology, Sociology, Philosophy and Anthropology. For this reason, Burak (2010a) has been arguing for a vision of Mathematics Education related with the kind of citizen we desire to form. In his words: "we desire a citizen who develops autonomy, who is: critical, able to work in groups, capable of make decisions in ordinary situations, family life, professional life or citizenship[2]" (BURAK, 2010a, p.17).

Therefore, there are some similarities between views of Mathematics Education as proposed by Skovsmose and as proposed by Burak. For example, both of them defend the Mathematics Education as a way to teach mathematics while preparing the students to be critical and able to think about real problems using mathematics concepts.

According to Rius (1989), there are many views about Mathematics Education and each one has a different approach. In this paper, we decided to use Mathematics Education as proposed by Burak (2010a) and we have presented Mathematical Modelling in Mathematics Education as argued by Burak (2010a).

Burak (1992) has been arguing about Modelling as a set of procedures whose goal is to explain, mathematically, the phenomenon that is present in the daily life of human, helping him/her to do predictions and make decisions[3]. Besides, he states that mathematical

[2] Translated from: "desejamos um cidadão que desenvolva a autonomia, que seja: crítico, capaz de trabalhar em grupo, capaz de tomar decisões diante das situações do cotidiano, da sua vida familiar, da sua vida profissional, ou de sua condição de cidadão".

[3] In Portuguese Burak (1992, p.62) says: "Modelagem Matemática constitui-se em um conjunto de procedimentos cujo objetivo é construir um paralelo para tentar explicar, matematicamente, os fenômenos presentes no cotidiano do ser humano, ajudando-o a fazer predições e tomar decisões".

modelling activities in classroom are based on two premises (BURAK, 2010a). First of them, the modelling activity starts from the interest of the students group. The Psychology explains that many of our actions are motivated by interest and this interest generates involvement and sustains procedures performed during the activity. The second premise is that data collection should occur where the interest of the group is. It can be understood as an ethnography research and consequently teaching and learning fit into a broader context.

In order to promote the mathematical modelling activities in classroom, Burak (2010a; 2010b) suggests five steps, which do not necessarily occur linearly and separately. These steps are: 1) choice of the theme; 2) exploratory research; 3) issue(s) identification; 4) issue(s) resolution and study of mathematical subjects in theme context; and 5) critical review of the issue's solution. He proposed these steps after he had some experience in elementary school (BURAK, 2010b).

In the first step, *choice of the theme*, students should choose an interesting theme for them, following the first premise. Students choosing themselves a topic to study turn them into a "researcher" rather than being a "follower". In other words, it allows them to develop autonomy, initiative and freedom to conjecture, "competencies required forming a citizen which will enable them to make difference in their communities[4]" (BURAK; ARAGÃO, 2012, p.90).

Exploratory research is the second step. Following the second premise – data should be collected where the interest of the group is – the second step enables students' critical and mindful formation, someone with attitude towards investigation. For searching data and information about the theme, students need to organize themselves how they should proceed, which information they need, to whom and what they should ask for. Learning how to organize and interpret these data is meaningful for students (BURAK, 2010a).

As a sequence of exploratory research, some questions and inquiries are being established and students headed to the third step as suggested by Burak (2010a) – *issue(s) identification*. In this third step, the teacher must help the students to identify issues. Besides, being encouraged by the teacher, students begin to understand that they can investigate their reality.

According to Burak (2010a), in the fourth step – *issue(s) resolution and study of mathematical subjects in theme context* – the Mathematics becomes meaningful and students check and test mathematical discoveries and learn the need to use this knowledge. Solving real problems using Mathematics highlights that this Science can be used to understand reality. If the students do not know the necessary mathematics subjects to solve the problem(s), teachers can use this moment to teach them.

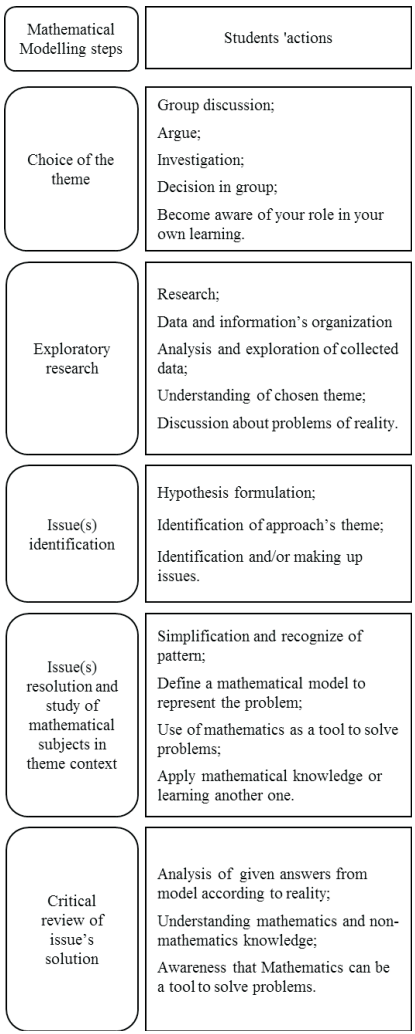
The fifth and last step – *critical review of issue's solution* – is the moment for everybody to be together, discuss and analyse the solutions found. It is important that students and teacher discuss about decisions made in previous steps and also to discuss

[4] Translated from: "competências necessárias à formação de um cidadão capaz de fazer a diferença em sua comunidade".

about relation and repercussion between these decisions and solutions found. This fifth step enables the deepening of mathematical and non-mathematical aspects worked during the development of modelling activity. We can highlight mathematical aspects as mathematical language, coherence and logical consistency of the solution. Regarding to non-mathematical aspects, we can highlight the environmental, social, cultural and anthropological aspects.

To summarize, we did a draft (Figure 1) to illustrate students' principal actions in each step of Modelling in Mathematics Education proposed by Burak (2010a; 2010b).

FIGURE 1 – Students' principal actions in each step of Mathematical Modelling in Mathematics Education.



Source: Authors, 2016.

Considering steps description of Modelling in Mathematics Education as Burak (2010) proposed, we realize this teaching methodology as a way to teach Mathematics and, in the same time, to teach the students to interpret the world, making them able to make decisions and be critical about their problems.

In conclusion, we can recognise some similarities between Paulo Freire's concepts and Modelling in Mathematics Education as presented by Dionísio Burak, as both of them argue about a critical formation beyond mathematics formation. In the next chapter, we have drafted some parallels of these similarities.

BURAK'S MODELLING PERSPECTIVE AND FREIRE'S CONCEPTS: SOME SIMILAR POINTS

Mathematical educators express several concerns about teaching and learning Mathematics through studies and publications. Many of these publications are about relations between mathematics that students learn in school and their life outside it, that is, in the world. Starting from idea that the man is in the world and with the world (FREIRE; 1967, 1979), learning should be related with the world, with a reality beyond school.

Burak (2010a) warns of changes in the world, for example, the expansion of technologies. Therefore, challenges posed in 21st century are different from those of 20th century. These challenges include education. According to Burak (2010a, p.17), the "kind of 'man' we intend to form to confront the challenges of the 21st century is influenced by the way we choose to teach and our expectations from teaching[5]". Therefore, teachers need to choose teaching methodologies considering the 21st century challenges.

In this context, Burak (2010a; 2010b) has been arguing about Modelling in Mathematics Education as a teaching methodology that allows students to use mathematical concepts as a tool to understand the world and make decisions about their lives. In other words, Modelling in Mathematics Education could turn students in a critical and autonomous citizen, who are able to modify their reality.

In addition, Modelling in Mathematics Education, as argued by Burak (2010a), has an interface with Freire's concept of education. We can realize this interface when Freire (2005, p.30) speaks:

> The education our situation demanded would enable men to discuss courageously the problems of their context – and to intervene in that context; it would warn men of the dangers of the time and offer them the confidence and the strength to confront those dangers instead of surrendering their sense of self through submission on the decisions of others. By predisposing men to reevaluate constantly, to perceive themselves in dialectical relationship with their social reality, that education could help men to assume an increasingly critical attitude toward the world and so to transform it.

[5] Translated from: "tipo de 'homem' que se pretende formar para enfrentar os desafios do século XXI é uma questão que tem a ver com a forma de se ensinar e com o que se quer com essa a forma de se ensinar".

Other interface between Burak's Modelling perspective and Freire's concepts is the dialogue. Teachers give liberty to students to choose a theme for start modelling activity which highlights the dialogue between teacher, and students. It allows them to discuss their problems (outside school) in the class, as mentioned by Freire (1967). Considering that the education happens through dialogue, teacher can learn while teaching and students can teach while learning, because everybody has knowledge and experience that can contribute to the formation of the other (FREIRE, 1967).

Exploratory research enables the formation of a more critical and observant student, who acts like a researcher. Using Freire's expression, students can develop autonomy, because, in a free way, they articulate the research done before, analyse data, establish hypothesis and make decisions. In addition, the students take part and discuss about reality problems, as proposed by Freire (1967).

Besides, in a mathematical modelling activity, the students follow the five steps as suggested by Burak (2010a; 2010b) and each step enables them to learn mathematics and stimulates the development of critical consciousness (FREIRE, 1979, 2005).

In the first step of Mathematical Modelling in Mathematics Education, choosing the theme allows the students develop the first characteristic of critical consciousness pointed by Freire, i.e., they do not get satisfied with appearances, because they desire in-depth interpretation of the problem. Besides, the students must make a dialogue to agree about the theme. In this case, others characteristic of critical consciousness can be developed, as practice of dialogue rather than polemics, soundness of argumentation, and reject passive positions.

Doing exploratory research, the second step from Modelling in Mathematics Education, allows the students to develop research skills. This research should be done for all pairs in a group together, that will make them able to recognize the mutability of reality in order to substitute magical explanations for casual principles, reject passive positions, refusing to transfer responsibility and in-depth interpretation of problem.

In the third step, the students should define which problem they will investigate, revealing that they do not get satisfied with appearances and desire an in-depth theme analysis for inquiring and investigating the reality. Nevertheless, to do this, they must make a dialogue again. If someone does not agree with other members of the group than it is necessary to have soundness of argumentation and practice of dialogues rather than polemics to decide which problem will be investigated.

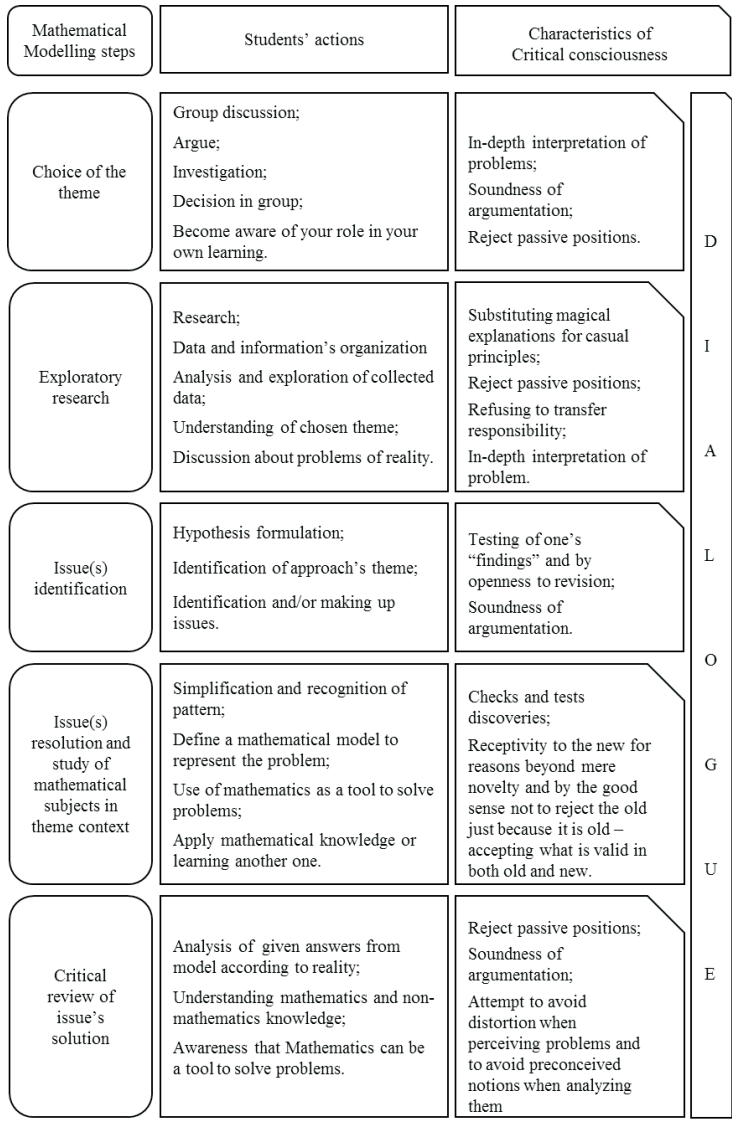
To study and answer the problem – fourth step – the students use the concepts from different knowledge areas, for example, Mathematics, History, and Politics. They check and test of one's "findings" by openness to revision. Besides, they must be open to the new concepts for reasons beyond mere novelty and not to reject the old just because it is old. To accept what is valid in both old and new.

In the fifth and last step from Modelling in Mathematics Education, the students critically analyse the answers and this enable them to develop characteristics of critical consciousness presented by Freire (2005), such as: reject a passive position; soundness of

argumentation; practice of dialogue rather than polemics; attempt to avoid distortion when perceiving problems and to avoid preconceived notions when analysing problems.

To summarise, we did this chart (Figure 2) to illustrate the characteristics of critical consciousness that the students can develop in each step of Mathematical Modelling.

FIGURE 2 – Characteristics of critical consciousness which can be develop in each steps of Modelling.



Source: Authors, 2016.

FINAL CONSIDERATIONS

In this paper, we showed some of the similarities between Modelling in Mathematics Education, as Burak (1992, 2010a, 2010b) has been arguing, and Paulo Freire's concepts.

There are different approaches for understanding and practicing Modelling. These different practices are supported by each researcher to understand the Education, mainly, Mathematics Education.

We have discussed about two different ways to understand Mathematics Education. Skovsmose (2001) has been using *Critical Mathematics Education* to emphasize the idea that Mathematics Education must take part in efforts to educate students to be critical and engaged citizens. Burak (2010b) understands *Mathematics Education* as a Human and Social Science. In his view, the critical idea is implicit because Mathematics Education is composed by different areas of knowledge that supports the Education: Psychology, Sociology, Philosophy and Anthropology. These areas of knowledge highlights how, when, why and for whom we teach something.

There is no right and wrong when we discuss about Mathematics Education, because there are different views. Based on our previous experiences, in this paper we used Mathematics Education as proposed by Burak (2010a, 2010b).

Following his Mathematics Education perspective, in this essay we have argued about Modelling. In order to promote the Mathematical Modelling in classroom, Burak (2010a) suggested five steps. Each step stimulates the development of some abilities, as thinking, reflecting, observing, making strategy, and others. These abilities are not stimulated when the student is passive and reproductive. Using Freire's concept, it is called "banking education".

In general, we can find some relationship between Mathematical Modelling in Mathematics Education as defended by Burak (2010a) and Freire's concepts when we say that a mathematical modelling activity starts with the students choosing a theme; the students looking for solutions to solve the problems related with the theme through dialogues; all the solutions found are discussed and analysed in a critical way by everybody. Therefore, the Mathematical Modelling in Mathematics Education is a possibility for the teacher to disentangle banking education and propitiate the critical education.

To summarize, Mathematical Modelling in Mathematics Education transforms the classroom in a place full of dialogues and discussions. Besides, the Mathematics can be used as a tool to understand and analyse real problems, make decisions and, if possible, modify the reality. Therefore, Mathematical Modelling in Mathematics Education can develop a critical citizen and stimulate the development of critical consciousness. In other words, Mathematical Modelling in Mathematics Education is a possibility for the students to overcome a naive consciousness, as mentioned by Paulo Freire.

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