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Teachers' Interactions with Curriculum Materials in Mathematics Education

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ABSTRACT

From the 1990s, teachers have been provided with a considerable number of materials produced and distributed by different governments to develop a mathematics curriculum to perform as curriculum implementers and promote the mathematical reform of different teaching systems. These resources have been researching tools. However, the types of use that teachers make of them are still little explored. In this article, we present the results of a study that aimed to understand the relationship between teacher-curriculum materials in the area of mathematics education, which takes discussions about teaching competencies of curriculum design as theoretical contributions. The research analysed a research report, and meta-analysis was the methodology adopted. The results indicate that affordances and constraints qualify the materials and potentiate the agency and its displacement, both for teachers and for materials, thus imparting different interactions between these two agents of curriculum development in mathematics.

Keywords: Curriculum Materials. Teacher-Curriculum Materials Relationship. Mathematics Curriculum. Mathematics Education.

Interações de Professores com Materiais Curriculares em Educação Matemática

RESUMO

A partir dos anos 1990 tem sido oportunizado aos professores um número considerável de materiais para desenvolver o currículo de Matemática, produzidos e distribuídos pelos diferentes governos, atuando como implementadores dos currículos e promovendo a reforma matemática dos diferentes sistemas de ensino. Esses recursos têm sido instrumentos de pesquisa, porém os tipos de uso que professores fazem deles ainda são pouco explorados. Neste artigo, apresentamos os resultados de um estudo que teve por objetivo compreender a relação professor-materiais curriculares na área de Educação Matemática, tomando discussões sobre competências docentes de *design* curricular como aportes teóricos. O material analisado constituiu-se de um relatório de pesquisa, tomada a metanálise como opção metodológica. Os resultados indicam que as *affordances*

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e restrições qualificam os materiais e potencializam a agência e seu deslocamento, ora para os professores, ora para os materiais, imprimindo, assim, diferentes interações entre esses dois agentes do desenvolvimento curricular em Matemática.

Palavras-chave: Materiais Curriculares. Relação Professor-Materiais Curriculares. Currículos de Matemática. Educação Matemática.

TO START THE DIALOGUE

In Brazil, although there is public policy for the purchase and distribution of textbooks for public schools in basic education, it has become increasingly common for state and municipal education departments to produce their own curriculum materials, usually in the form of booklets.

These materials have become a resource that translates prescriptions into learning situations (Gimeno Sacristán, 2013). They aim at implementing and supporting the development of the curriculum of their education networks; that is, these materials are designed to promote the mathematical reform of the proponent teaching systems (Remillard, 2005).

In the academic field, mathematics curriculum materials have been objects of investigation. Its pedagogical-methodological or conceptual characteristics are the focus of analysis of researchers such as Paula (2009), Camargo Junior (2010), Rodrigues, E. (2011), Campos (2011) e Rodrigues, W. (2011). However, the study of teachers' use of these materials has been limited to a few research works, such as Lima's (2014) and Pacheco's (2015).

In 2011 and 2012 a research project was developed, aiming to investigate how teachers who teach mathematics evaluated the use of curriculum materials produced by a Secretary of Education. This project was carried out by two universities with the consent of the Secretary of Education and involved researchers and teachers who taught mathematics from the 1st to the 9th year of elementary education.

In this article, we present a meta-analysis of the report derived from this research. Our objective is to understand the relationship that teachers establish with curriculum materials in the area of Mathematics Education. We chose this project for two reasons: the first one refers to its scope, its objective, its involvement with teachers, and its originality – in the sense that it is unique in the Brazilian scenario; the second reason is that one of the authors of this article were involved as researchers in its development. Thus, in taking the report as an object of study, our perspective will be guided by the question: *what can we learn from the interaction between teachers and mathematics curriculum materials?*

CURRICULUM MATERIALS AS A FOCUS OF RESEARCH IN MATHEMATICS EDUCATION

In the North American context, Remillard (1999, 2005, 2012) considers both the *Curriculum and Evaluation Standards for School Mathematics,* published in 1989 by the National Council of Teachers of Mathematics (NCTM) and the adoption of a single curriculum by school districts, in response to the failure of schools to raise levels of learning, as factors for the production and adoption of curriculum materials as a central strategy to improve students' mathematical performance.

From then on, researchers began to focus their attention and produce studies on curriculum materials and the use that the teachers who teach mathematics have made of these resources (Fan, 2013). These studies have provided contributions on how teachers use the materials and on teachers-curriculum material relationship. This relationship and uses are complex and intertwined with other teaching practices, where the use of curriculum material is understood as "how individual teachers interact with, draw on, refer to, and are influenced by material resources designed to guide instruction" (Remillard, 2005, p.212).

Although research on mathematics curriculum materials, especially on how teachers relate with these resources, has been done for approximately three decades (Fan, 2013), Davis, Palincsar, Arias, Bismack, Marulis & Iwashyna (2014) consider that little is known about this relationship to enhance teaching and learning.

Remillard (2005) mapped about 70 studies on the use of mathematics curriculum materials centred on the interaction between teachers and resources in the North American context. By analysing those works, the author identified four different types of curriculum usage and, consequently, curriculum materials: following or subverting, drawing on, interpreting and participating with.

In his master's research, Aguiar (2014) discussed each of these perspectives under a sociological approach and, by referring to the constructs of Basil Bernstein's Code Theory, proposed the term "recontextualising the text" as a fifth way to understand how teachers use curriculum materials.

Remillard (2005) and Aguiar (2014) helped us to observe that there are different interactions established between teachers who teach mathematics and curriculum materials. It is from the teaching experience, knowledge, beliefs and conceptions, and institutional conditions of implementation that these relationships are established. However, in addition to observing different practices, it is necessary to understand how this interaction between teachers and curriculum materials take place.

CONCEPTUAL FRAMEWORK FOR THE ANALYSIS OF TEACHER-CURRICULUM MATERIALS INTERACTION

Gimeno Sacristán (1997) helps us understand that curriculum materials not only present contents and ways of approaching, but also communicate conceptions of the mathematics curriculum that they intend to implement. These conceptions express values, beliefs, ideologies, power and control of a cultural group about the education of children, youth, adults and the elderly. Thus, the teacher is the one who interprets the different representations of curriculum materials when developing learning situations.

In this sense, Brown (2002, 2009) considers that understanding the teachers' use of curriculum materials requires explaining these representations, in addition to those referring to concepts and actions, trying to identify how teachers perceive and interpret these representations and how they can influence over the pedagogical practice. In the interaction with different materials, teachers are planning agents of the teaching and learning processes.

Curriculum development involves planning, interpretation and intervention practices in the curriculum materials available to teachers. This implies considering that teachers use these resources from guidelines, didactic and methodological choices, theorisations and ideologies underlying the curriculum materials. However, as curriculum-producing agents, teachers also intervene, adapting and improvising materials in response to students' learning needs. Thus, curriculum materials influence on pedagogical practice and teachers influence on the practice of these resources.

Considering this dynamic, Brown (2002, 2009) argues that interactions between teachers and curriculum materials can be understood in terms of different degrees in which these resources are appropriated: offloading, adapting and improvising.

These three degrees do not happen in isolation, often the actions of offloading, adapting, or improvising intertwine in a dynamic process. However, there is no process standing out, in the sense that either would better enhance curriculum development and student learning. In these three processes, teachers' beliefs, conceptions, knowledge and attitudes about the material itself, to mathematics and the teaching and learning processes are implicit.

To understand what motivates these different ways of interacting with curriculum materials, Brown (2002) proposes a conceptual framework to verify how the characteristics of materials interact with the capacities that teachers mobilise for the interactions.

The author considers that the *Design Capacity for Enactment* (DCE), Figure 1, allows us to grasp different elements of the teacher-curriculum material relationship and represents the different types of interactions that occur between teachers' resources and those of materials.

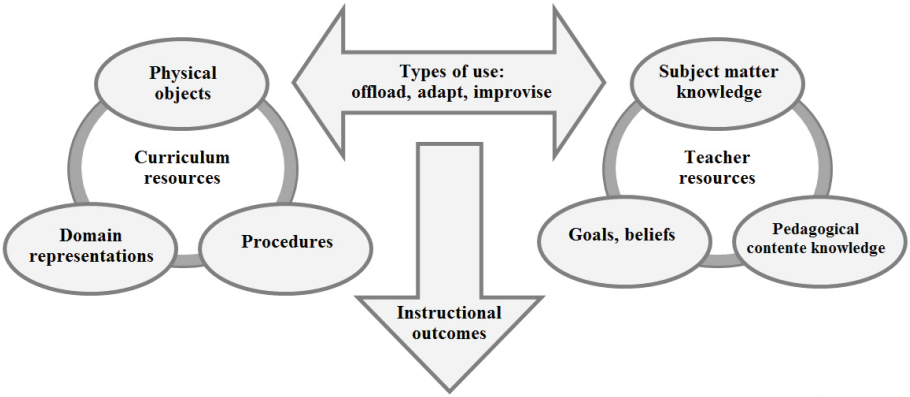

Figure 1. The Design Capacity for Entity Framework – DCE (Brown, 2002, p.91).

In curriculum resources, there are fundamental aspects of materials. "Physical objects" represent the physical characteristics of curriculum materials such as paper quality, illustrations, thickness, number of pages, layout, font size and type of texts. "Procedures" (representations of tasks) represent guidelines for the development of learning situations and include propositions of other materials to extend the resources of the curriculum material, such as software, websites, videos, manipulative artefacts and other tools for consultation. These task representations include procedures for teachers and students.

"Domain representations" refer to concepts and contents. They include criteria for selecting and organising content (Pires, 2000), as well as possible links between the different content blocks; options of contexts (mathematical context, the context of reality or context of other disciplines); and methodological options.

Brown (2002, 2009) argues that, about teachers' resources, these professionals bring at least three different types to the relationship with curriculum materials. "Subject matter knowledge" denotes not only what teachers know about contents and concepts, but also knowledge of facts and concepts related to the content, such as historical and epistemological aspects. They are, therefore, mathematical knowledge mobilised for teaching (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008).

The "pedagogical content knowledge" represents knowledge about modes of teaching, including objectives and purposes of some teaching content. It also includes knowledge regarding: students' hypotheses about content; resources available for teaching and learning processes, such as technological artefacts and data sources; and strategies and methodological aspects for the teaching of selected contents (Shulman, 1986, 1987).

Teachers' attitudes and procedures about the curriculum material or the contents organised and selected for teaching are representations of "goals, beliefs". In this sense, the

teacher's attitude, when related to materials to mediate/promote learning situations, is not limited to their capacity to teach something, but it encompasses their truths, conceptions and visions, based on their experiences, opinions and sociocultural influences, generally indisputable and manifested verbally or by justifying actions (Cury, 1994; Thompson, 1997; Cuadra, Rico & Cano, 2002).

With this conceptual framework, we will present the research project, followed by the analysis based on the analytical and conceptual contributions presented.

RESEARCH SCENARIO

From 2006 to 2012, the Municipal Secretariat of Education of São Paulo (SME-SP) elaborated documents to subsidise schools in the construction of their Political-Pedagogical Projects and to help teachers plan their classes. Among these documents, we highlight *Orientações curriculums e proposição de expectativas de aprendizagem de Matemática para o Ensino Fundamental* (Curriculum guidelines and proposition of Mathematics learning expectations for elementary school) – 1st to 5th year and 6th to 9th year. After the publication of the Curriculum Guidelines, SME-SP invested in the elaboration of material that could implement the curriculum of the municipal education network and subsidise teachers for curriculum development. Among these materials, we highlight *Cadernos de Apoio e Aprendizagem de Matemática (CAA)* (Booklets for Mathematics support and learning) – 1st to 9th grade, in versions for students and teachers. *Cadernos* were distributed to the schools of the municipal teaching network of São Paulo, annually, from 2010 to 2014.

As a consequence of this distribution and aiming to understand which contributions CAA could bring to the innovative pedagogical experiences of the schools from this educational network, in 2011 and 2012 the project "*Avaliação de Professores do Ensino Fundamental da Secretaria Municipal de Educação de São Paulo, em relação a documentos e materiais de apoio à organização curricular na área de Educação Matemática*" (Evaluation elementary education teachers of the Municipal Secretariat of Education of São Paulo, regarding documents and materials supporting the curriculum organization in the area of mathematics education) was developed, inserted in the "Programa de Melhoria do Ensino Público da Fundação de Amparo à Pesquisa do Estado de São Paulo" (Program of improvement of public education of the São Paulo Research Foundation – FAPESP). This project was carried out by the postgraduate programs in the area of Mathematics Education of the Pontifícia Universidade Católica de São Paulo and Universidade Cruzeiro do Sul, with the consent of SME-SP.

The project was coordinated by two researchers and eight collaborator researchers, who coordinated the meetings of 31 teachers in groups corresponding to the years of schooling, 1st to the 9th year, using the methodological strategy of focus groups. Exception made to grade 4th year because the teachers who taught in this school year did not adhere to the research. The meetings were held every fortnight on Saturdays. In total, teachers

participated in 112 hours of discussion and study on themes on mathematics education and analysis of the CAA, published by SME-SP.

At each biweekly meeting, teacher-researchers, grouped by year of schooling, studied subjects related to Mathematics Education; analysed the sequences of activities that make up each of the eight units of the CAA (referring to the year corresponding to the group); and discussed and planned their development in a classroom situation. At the following meeting, they socialised the experiences carried out in the classroom situations with the use of the CAA, followed by discussion, analysis and planning of a new sequence of activities.

At the end of the development of each unit of the CAA, the teachers produced a report describing the accomplishment of the activities. With these reports, the collaborative researcher-collaborator responsible by the group produced a synthesis report on the development of the teaching unit. At the end of the project, the responsible researchers gathered the synthesis reports of each group (1st to 3rd and 5th to 9th years of elementary school) and compiled them in a comprehensive report. This Research Report is the object of our analysis.

THE TEACHER-CURRICULUM MATERIALS RELATIONSHIP: AGENCY, AFFORDANCES AND INTERVENTIONS

Before beginning the analysis, we considered relevant to highlight the methodological choice. Our study consists of producing research on an already existing research, making a "new reading" of what has been reported, systematised and taken as the production of knowledge (Zimmer, 2006) on the relationship between teacher and curriculum materials in the area of mathematics education. Therefore, we chose meta-analysis.

In qualitative research, Bicudo (2014, p.9) considers that meta-analysis is "an investigation that goes beyond those already done", by interpreting the interpretation. Thus, this methodological option allows for theories on already researched themes, expanding and producing new senses and meanings for what was analyzed about a topic of interest of the academic community, which demands new production of knowledge and, consequently, the socialisation of the results.

When we return to our research objective, the meta-analysis will be guided by four categories of analysis that emerged from the reading of the report, which we will explore: agency in materials; affordances and constraints on materials; agency in teachers; and interventions in developing the materials. Rather, it is essential to clarify that in this article, when we use the terms "material" or "curriculum material", we refer to the material *Caderno de Apoio e Aprendizagem de Matemática (CAA),* published by the Municipal Secretariat of Education of São Paulo.

AGENCY IN MATERIALS

In curriculum development, "agency" takes the meaning of decision-making power, a factor that has authority over mathematics and its teaching (McClain, Zhao, Visnovska & Bowen, 2009). Thus, when the agency is in the curriculum material, it determines the curriculum to be carried out. This implies the importance of identifying aspects that characterise it in the material and, therefore, qualify the curriculum and lead it to be implemented.

The contextualised presentation of learning situations is a factor that interferes significantly with teachers' appreciation of the material, making it a potential resource for student learning. Being contextualised means that the material often leads the student to perceive everyday situations they are likely to experience, in which mathematics is present. It also means that the contents are related to other mathematical concepts, in which specific procedures evoke ideas from various themes in the areas of Mathematics. Contextualization gives meaning to what is proposed in the form of contents and, thus, the material acts as a stimulator of learning by showing answers to students' questions, as we can see in the following section:

> "The teachers emphasized that, especially, in the activities proposed in Unit VI, they approach themes close to the children's daily routine, such as daily scenes in which counting is needed, relationships over time (days, weeks and months) and the game of the trail, which proposes an articulated work between the ludic and the mathematics. In this way, the contextualization and the methodological resources suggested in *CAA* were pointed out by the group as factors that influenced in the fulfilment of the learning expectations". (*Research Report, fragment referring to the group of teachers of the 1st year of elementary school*)

The approach of themes that are part of the students' daily life makes it possible for them to mobilise their previous knowledge. A material, to be potential in the teaching and learning processes, needs to present situations that encourage students to use the knowledge they have and to develop hypotheses to explore what is new. Thus, the possibility of moving the previous knowledge potentiates the material, recognising what has already been constructed of learning, taking it as a starting point for the new knowledge.

These two characteristics, contextualization and mobilisation of previous knowledge, are examples of what Brown (2009) calls domain representation. It is the way in which the concepts are approached by the authors of the materials and, consequently, presented to the students to provide learning opportunities. In the author's view, the approach to the content reveals how materials designers conceive mathematical concepts.

There are also examples of domain representation in curriculum material, as described by Brown (2009): the articulation of concepts between different units/chapters;

the adequacy of the program; and the sequencing, degree of complexity and presentation that allow the manifestation of own resolution strategies.

Based on what this author presents, and taking as the object of analysis the Research Report, we understand that the curriculum material becomes a reference and becomes a differential, also, when it facilitates the articulation between different units/chapters in which the various activities mobilise ideas/knowledge acquired in situations that were worked previously. This articulation also occurs with the different blocks of content, which characterises a networked curriculum option. Along with the curriculum option, it allows students to establish different relationships of themes seen in previous units, in addition to getting them to enlarge what was seen. Also, it favours the connections between the different blocks of contents and gives more meaning to the mathematical learning by allowing continuity to the themes proposed.

Another essential aspect that confers agency to the material is the adequacy of the contents to the age range of the students and, consequently, their cognitive abilities to apprehend what is proposed. This adequacy refers to the program (content roll), presentation, sequencing, degree of complexity and way of expressing the own resolution strategies.

As Brown (2002, 2009) proposes in his conceptual framework, the characteristics of curriculum resources related to procedures can be identified as the explanation of what the expectations or learning objectives propose, in which the analysis of the Research Report allows to consider them another factor that attributes agency to curriculum materials. However, the potentiating agent is the articulation between expectations/objectives and the learning situations, i.e., the coherence between prescription and activities. In this case, the articulation is an element of domain representation, assuming a transparency factor of the material that informs the teacher how the contents can be presented to the scope of what it was proposed, which enables a better understanding of the underlying conceptual, didactic and methodological aspects, as we can see in the following section:

> "The teachers of the group agree that the proposed activities put forward by *CAA* meet the expected learning foreseen for the year/grade, and the way in which the activities were presented in the book enabled a growing knowledge building, so that one activity complemented the other, giving the child and also the teacher the clarity of the objectives proposed by the unit, facilitating the understanding of the subject treated, as well as the reflection on the decimal numbering system etc." (*Research Report, fragment referring to the group of teachers of the 3rd year of elementary school*)

When this articulation is explicit, it can awaken in the teacher the sensation that the activities are adequate because they contemplate the foreseen expectations/objectives. Therefore, they will lead the students to build their learning as expected. The expectations/objectives also indicate the role of contents to be worked with the curriculum material,

which can favour the teachers when designing their teaching plan, their classes and managing times and spaces. In this case, the teacher may be led to conceive the curriculum material as a "complete material", in the sense of being well structured, elaborated and, therefore, followed as a guideline and without interventions, which configures the use as "offloading the curriculum" (Remillard, 2005) or as a offload (Brown, 2002).

As for the curriculum in action, the didactic-methodological guidelines, as aspects of procedures of the curriculum resources (Brown, 2009), make of the material a critical instrument for the pedagogical practice. As these guidelines anticipate possible teachers' doubts, they present information regarding conceptual aspects and classroom management, such as presentation of content, an approach that best favours relationships with previous knowledge, student organisation and referral of doubts.

The didactic-methodological guidelines present in curriculum materials, especially those produced for teachers, can also be interpreted as an idea of transparency, informing these professionals, explicitly, what is expected of them, and the students in relation to what is proposed in the materials, providing them with the reasons, assumptions and knowledge needed to select and adapt the different activities (Remillard, 2005).

AFFORDANCES AND CONSTRAINTS OF MATERIALS

In Gibson's (1986) understanding, affordance refers to the possibilities that an object or environment offers to an agent who, in turn, needs to perceive them. In curriculum materials, affordance denotes the possibilities that material allows for its potential use, and the teacher and the students are the agents that perceive them. Thus, affordances are related to the functions and practical aspects of curriculum materials and the perceptions of those who make use of these resources. Constraints, in turn, refer to characteristics that indicate the limitation of these resources.

As for affordances in curriculum material, the composition of how activities are presented and how they interact with students stands out: with a simple approach, in language and vocabulary that students can understand and in everyday contexts, enables articulation with "real situations" and, thus, potentiates the construction of meanings and senses of the topics addressed.

The inference of affordance in a material is also given by the mobilisation of the students' action in solving activities. This action can be described as the possibility of the material to foment in students the elaboration of hypotheses, their testing and validation; the use of their prior knowledge; the use of procedures to solve problems; communication, exchange of knowledge and experiences with colleagues; the socialization of their findings and their problem-solving procedures; and the construction of meanings and senses from collective participation, as the following excerpt illustrates:

> "The socialization of problem-solving procedures, in my own way and my colleague's way, is becoming a facilitator for children to understand that there are

> several ways to solve a problem situation that not only the conventional algorithm, and they began to exchange more their procedures with their colleagues, to express their various hypotheses of resolution without fear of trying, of making mistakes, of solving a problem situation". (*Research Report, fragment referring to the group of teachers of the 3rd year of elementary school*)

The highlighted passage also exemplifies the interaction in mathematics classes as a feature of affordances in the material. In this case, the interaction is fostered by the activities, which request conversations, exchanges of information, analyses of other procedures, justifications, and argumentations for the resolution procedures.

The composition of the activity, i.e., how it was elaborated and how it communicates to the student concepts and mathematical ideas, leading to the mobilisation of different strategies and promoting the interactions in the classroom situations, is an example of representations of curriculum resource domain. As affordances, these representations influence how teachers and students relate to the material and, upon being perceived, re-signify the curriculum and potentiate the construction of senses and meanings regarding mathematical concepts (Brown & Edelson, 2001).

Students' progress is also attributed to the illustrations. This is because, in a curriculum material, images, tables, tables and graphs are figurative elements that help them understand the contents; they are factors that boost reading and that lead students to make sense of the activities and the content involved in them.

In relation to the physical objects of curriculum resources, as Brown (2002, 2009) points out in his conceptual framework, illustrations also include graphic elements such as color, font type and size, line spacing, concise writing, and adequacy of figures and drawings to the age and cognitive range of students, and records. As for the latter, the quality of a material lies in the presentation of different types of records and the alternation between them, such as graphs, tables, diagrams, charts, organisational charts.

By reading and analyzing the Research Report, we identified other aspects that characterize affordances in the material: (a) the organization and sequencing of content, highlighting the links between different units/chapters through activities and underlying content/concepts; (b) the connection between different learning expectations/objectives, which makes the material dynamic and establishes a link between the contents from different blocks; (c) the promotion of teachers' interventions to mediate/promote learning situations; (d) stimulating affectivity in students, which occurs in activities involving familiar situations, places and animals; (e) situations that explore aspects of the city or surrounding areas where the student lives, what arouses the interest to study mathematics associated with these locations, such as public transport route maps, points of interest and places of service to the citizen; (f) the interdisciplinarity and the study of various subjects; (g) organization in network of the contents; (h) guidelines for teachers; (i) knowledge and attitudes promoted by the material, such as preparation of arguments and justifications, the formulation of questions, teamwork and interactions;

(j) conceptual aspects; (k) promotion to reading and writing skills in the reading of texts, image and text production; (l) articulation of the material with other resources such as software, calculators, manipulative materials; (m) promotion of students' development of autonomy, protagonism and authorship; (n) simple writing and well expressed ideas; and (o) methodological axes. This last aspect is highlighted in the evaluation of the teachers participating in the research project, as can be seen in the following section:

> "According to the teachers, Unit 3 presents good learning situations, which enable students to mobilise their previous knowledge, the gathering of hypotheses and the exchange of strategies for the resolution of situations. In this sense, teachers emphasise problem-solving and exploration as methodologies that allow students to interact with each other, with the teacher and with the situations proposed". (*Research Report, fragment referring to the group of teachers of the 6th grade*)

The unit highlighted in the section above is an integral part of the CAA of the 6th year of elementary school. This unit is composed of activities that lead students to represent, compare and order rational numbers and locate them in the numerical line, in their fractional writing; to explore two-dimensional geometric forms, to describe their characteristics and to solve problem situations based on the knowledge of some of their properties; and to perform conversions between some of the most common measurement units of length, mass, capacity, and time to solve problems.

Regarding the constraints in the material, the inadequacy of the activity with the learning expectation/objective or with the student's contextual level may cause limitations to learning. In such cases, the activity elaborated and proposed does not ensure that the prescribed expectations/objectives and the construction of learning will be reached.

This limitation also occurs when activities involve content beyond the students' cognitive level. Likewise, there may be a situation in which the mobilisation of knowledge that is not yet consolidated by them or that is beyond their levels of learning is requested.

> "It was common in the group to comment that the problems concerning the expectation M10 [Producing numerical numbers of known and frequent numbers through the identification of regularities] were elaborated with "high" numbers, so that many children, even understanding the statements, could not solve them as they got lost when counting, since in the previous units the numbers vary between 20 and 40 ". (*Research Report, fragment referring to the group of teachers of the 1st grade*)

The highlighted excerpt refers to numbers written in orders not yet learned/ consolidated. This kind of situation requires teachers to intervene in the students' interaction

with the material, proposing changes and bringing new situations, which requires that the teacher also mobilises his knowledge about the contents addressed. As Brown (2009) points out, these interventions may occur through adaptation or improvisation of the curriculum material, or even through the practice of recontextualization (Aguiar, 2014).

Based on the discussions of the teachers participating in the Research Project, which are textualized in the Report we are analysing, we observe that this fragility is also expressed by the lack of explicitness of a concept, which becomes an essential element for students to understand the activity; or, still, by the reduced number of activities on a particular content/concept. Depending on the expectations to be achieved, it is necessary to increase the number of activities to make it possible.

Brown (2009) explains that as domain representation, the way an activity is elaborated, the type of language, the contexts and the articulation of meanings referring to the new ideas and situations already experienced by the students are what influences teachers' decision making, leading them to use the curriculum material in different ways: reproducing, adapting or improvising in classroom situations.

In reference to Brown and Edelson (2001), Brown (2002, 2009) and Remillard (2005, 2012) on characteristics in curriculum materials that influence their relationship with teachers, we observed in the Research Report that another example of constraint are activities that do not instigate students to mobilize their previous knowledge nor elaborate hypotheses, which can cause disinterest in learning. To these two aspects, we can add the decontextualization, that is, activities that do not establish a connection between the mathematical content and its application in the students' social practices or that do not relate with other concepts and areas of mathematics.

Activities that lack information and data for resolution also limit students' learning, as well as those that do not relate the different contents/concepts addressed in the same unit/chapter, which makes it harder for them to understand and attribute meanings and senses of the themes studied, as the following excerpt illustrates:

> "The activities on pages 90, 91 and 92 take up concepts of face, edge and vertex in a satisfactory way; however the activity decontextualises the sequence of equations of the first degree, since it is a "new" subject for the students, and they were involved with it. They suggest that the sequence with the resumption of concepts of space and shape axis be maintained, but that the concepts of equations of the first degree be contemplated in this activity ". (*Research Report, fragment referring to the group of teachers of the 8th grade)*

The choice by the authors of the material for a particular type of conceptual approach may also restrict students' learning. As an example, the group of teachers of the 6th year considered exploring angle as corner angle in geometric figures, what confers it a static dimension, when the meaning of rotation, as a dynamic dimension, could be worked on.

Although the concepts of affordance and constraint are related to the information of curriculum materials, they are identified by teachers, who mobilise their knowledge, beliefs and conceptions of mathematics and teaching and learning processes (Remillard, 2005, 2012). In this sense, although the discussion focuses on the resources of materials, we cannot dissociate them from the teachers' resources, since these resources interact mutually in the relationship between materials and teachers.

AGENCY IN THE TEACHERS

Teachers are not neutral when developing the curriculum. They are professionals who, through initial and continuous training and their experience, acquire knowledge about the teaching and learning processes and incorporate in them their knowledge about Mathematics. This knowledge, along with its beliefs and conceptions about Mathematics and its teaching, is the foundation of pedagogical practice. In this sense, identifying what motivates the teacher to make the different uses of the curriculum allows us to understand what characterises the agency in this professional.

Thus, when analysing the Research Report in which the discussions of the teachers involved in the project are textualised, we identify as one of the main characteristics the recognition of fragilities when elaborating activities. The fragility here has the connotation of non-consistency with the expectations/objectives prescribed. It is this that leads to the use of the differentiated way of the curriculum material, prevailing what Brown (2009) calls adaptations of what the curriculum proposes.

Identifying activities that mobilise or not expectations/objectives not listed in the material gives authority to the teacher on the curriculum. This requires the exercise of identifying the relationship between expectations/objectives and the activities, as well as the content addressed. This process of identification is related to different characteristics of the teachers' resources, as Brown (2002, 2009) emphasizes in his conceptual framework: knowledge concerning epistemological and conceptual questions of mathematical contents; knowledge regarding the didactic-methodological aspects of the contents; and the involvement of their beliefs, values and knowledge of mathematics, curriculum, school and training.

Still concerning expectations/objectives, it may occur that teachers propose them from what they understand as their students' learning needs or demands, as illustrated in the following excerpt, in which teachers refer to the adequacy of the expectations to the activities of the students' curriculum material:

> "[...] however, in some sequences, the group of teachers added some other expectations beyond what was initially envisaged in the activity". (*Research Report, fragment referring to the group of teachers of the 2nd grade)*

The excerpt also shows knowledge of the curriculum in relation to the contents proposed for a given year and about the skills and abilities to be developed/potentialized

by the students. In the latter case, they recognise that the number of activities is not enough, which may lead teachers to adapt or improvise with curriculum material (Brown & Edelson, 2001).

Teachers have their assumptions about the students' learning path and, anchored in knowledge built from their experiences, analyse the materials and identify constraints. They themselves seek other resources and reshape the activities, proposing another organisation, sequencing, approach and path by the students.

In the analysis of the material and the redirection of the activities, the teachers, in possession of knowledge on didactic-methodological aspects of the teaching of mathematics, make inferences about the use of manipulative resources and materials, so that the students can understand better the content proposed and construct the expected concepts. They modify the structure of the activity, which Remillard (2005) calls subversion of the material. In this case, they are based on their hypotheses about students' learning to propose procedures that facilitate the observation of characteristics and properties, recontextualising what had been proposed (Aguiar, 2014).

Another aspect that confers agency on the teacher is the knowledge that this professional has of mathematics and its teaching, and what makes his/her perceive constraints for the students' learning. In the excerpt below, we observe a teacher's perception of the construction of the concept of a prime number:

> "Regarding the points worthy of complementation, the teacher [...] highlighted the activities related to prime numbers, reporting that it is usual to define prime as any natural number that has only two distinct divisors: the number 1 and itself. For the 6th grade participating teachers, this definition does not make much sense for most students, so they prefer the meaning that considers prime numbers as generators, that is, they form the other numbers through multiplication. This meaning has the advantage of naturally leading to the decomposition of a natural number into prime factors". (*Research Report, fragment referring to the group of teachers of the 6th grade)*

The fact that the teacher presents an agency does not mean that interventions in curriculum material always amplify and enhance the meanings expected by the prescriptions but often develop a practice that contradicts the propositions, as Lima (2014) and Pacheco (2015) concluded in their research works. This can happen because the teacher mobilises his/her knowledge, beliefs and values of mathematics and its teaching to analyse guidelines and forms of presentation and treatment of the contents in the materials and, from this point of view, perceives incoherences and limitations, which results in practices of subversion and infidelity to the curriculum.

> "The teacher [...] says that she almost always explains the contents before developing the activities". (*Research Report, fragment referring to the group of teachers of the 7th grade)*

The above excerpt illustrates that the teachers' conceptions of students' learning and education lead them to misinterpret the theorisations and didactic-methodological aspects underlying the curriculum material. However, what we want to highlight is the teacher as an agent, the one who acts on the material, intervening and resignifying it.

INTERVENTIONS TO DEVELOP MATERIALS

Teachers have as their primary objective to propose activities that lead students to build their learning and develop as human beings to act in society and the world of work. The interventions are carried out aiming at achieving this goal.

As mentioned previously, the constraints identified in the materials are the main responsible for leading teachers to carry out different types of relationship with these resources, as Remillard (2005) and Brown (2002, 2009) highlight. However, other aspects may justify the interventions made by these professionals in curriculum materials.

From reading the Research Report, we observe that the teachers' hypotheses on the learning path, the difficulties and the facilities of the students, are a strong aspect that leads these professionals to organise, select and problematize situations presented in the curriculum materials.

There is the understanding that, in order to enhance interaction with materials and to build learning, students need to have contact with practical activities and, therefore, should be involved in exploration and research, especially in environments outside the school context, and motivated to collect, organize and process information and then make inferences about and from them.

In this perspective, teachers create, participate, adapt or improvise with the materials so that the activities ensure students will achieve the expectations/objectives prescribed. This intervention, then, can be by changing the order of the activities or writing a statement, rewriting it in a less complex language to favour reading and comprehension of what is informed and proposed as a problem; by changing the magnitude of numbers and measures; by including or excluding figurative elements; by expanding or reducing the number of activities; and/or by proposing strategies other than those envisaged.

> "In the divisions, teachers report having used various personal procedures that were gradually being replaced by the schemes of division. These, in turn, have allowed us to understand and make better use of the American algorithm". (*Research Report, fragment referring to the group of teachers of the 5th grade)*

In the passage, what motivated the intervention in the material was the objective of facilitating students' comprehension of a specific procedure. This intervention, then, provides students with a repertoire so that they can better understand what is proposed,

and attribute it more easily sense and meaning. Such decisions, often based on the teacher's knowledge of the students' learning process, show how the concept of agency is in pedagogical practice and how the material can be recontextualised (Aguiar, 2014). In this case, the agency as a competence of authority causes the teacher to be empowered about making decisions when developing the mathematics curriculum.

The interventions made in the curriculum material are also based on conceptual aspects that the teachers think essential to extend the meanings constructed by the students on a specific content.

The interventions are still motivated by the methodological option in the content approach. In this case, for the objectives to be achieved, teachers use strategies other than those possibly proposed in the material, such as technological resources, research, problem-solving, mathematical modelling, history in/of mathematics, projects and social intervention, and use of manipulative materials, as exemplified in the following excerpt:

> "For the sequences of activities involving geometric solids and flat figures, teacher [...] and teacher [...] took the solids to the classroom for the students to manipulate the material. This favoured the students to establish the relationships between the number of sides, vertices, faces and edges with the number of sides of the base polygon and provided indications to the teacher of the students' prior knowledge. (*Research Report, fragment referring to the group of teachers of the 7th grade)*

The development of curriculum material is also altered when the need to get students to mobilise their previous knowledge is perceived. However, teachers are not always explicit about the concept of prior knowledge, proposing situations that serve as underlying knowledge for new learning. In this case, what teachers refer to as prior knowledge, would, in fact, be the prerequisite requirement for carrying out the activities. This type of intervention occurs in the introduction of new content, and there is the understanding that students may not have a good interaction with the material and, consequently, with the concepts addressed in the activities as "they do not have background".

TO END THE ARTICLE AND CONTINUE THE DIALOGUE

Assuming there is no neutrality in curriculum development implies conceiving a mutual relationship between teachers and curriculum materials, in which one influences the practice of the other. As Remillard (2005), Brown (2002, 2009) and Aguiar (2014) explicit, this relation is due to different types of uses.

In this article, when studying this relationship, our objective was to identify and understand how different aspects of the use of curriculum materials emerge, taking as the object of analysis the report of a research project, and being guided by the question: what we can learn from the interaction between mathematics teachers and curriculum materials?

In the study of this type of relationship, the concept of agency is central, being configured as a strong incentive of the interaction between materials and teachers. The agent is the one with the power of decision and authority over mathematics to be taught; therefore, the one who has the agency, the one that determines the different types of uses.

In relation to the teacher, the agency is characterized by the recognition of fragilities in the material; by the identification of the relationship expectation/objectives and activities; by the adequacy of expectations/objectives; by the resizing of activities; by the inference on the use of manipulative resource and materials; and by the knowledge of mathematics and its teaching.

It is also essential to consider that teachers mobilise their beliefs and conceptions about Mathematics, its teaching and the curriculum when they relate to materials and develop learning situations. These beliefs, along with the agency, however, may impede or encourage the use of curriculum materials and new approaches in school Mathematics Education.

By reproducing, adapting, improvising or recontextualising curriculum materials, teachers intervene to a lesser or greater degree. These interventions are justified by the constraints identified in the materials; by the teachers' hypotheses about the learning path, which involves the difficulties and ease of the students; by the contact with practical activities, involving exploration and research; by didactic, methodological and conceptual aspects; by the mobilization of prior knowledge; and by contextualization.

As for the curriculum materials, they are elements that characterise the agency: the contextualization; the mobilisation of prior knowledge; the articulation between expectations/objectives and learning situations; didactic-methodological guidelines; transparency in underlying didactic-methodological issues; and the learning path of the students.

In analysing this relationship, the different types of material use are stimulated by the agency. In this case, it is the displacement of agency – sometimes in materials, sometimes in teachers – that determines the type of relation, as illustrated in Figure 2 below.

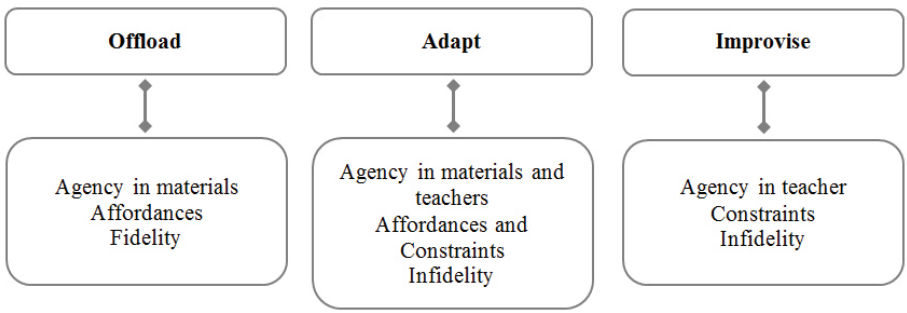

Figure 2. From the teacher-curriculum materials relation: agency, affordances and interventions (Januario, 2017, p.106).

In curriculum development, when we take the conceptual framework presented by Brown (2002, 2009), offload occurs when the agency is in the material, which makes it a complete and potential resource for student learning, so the teacher uses it as a guide and gives fidelity to implementation (Remillard, 2005).

The agency, however, may be in the material and the teacher. In this case, both have the power to decide on the processes of teaching and learning and offload occurs – when interventions are made in what the material proposes –, namely, altering the sequence of activities, proposing another organisation for the classroom, incorporating or excluding activities and guidelines for carrying out an activity. In this type of relationship, fidelity is not possible.

The improvise relationship happens when, in mediating/promoting learning situations, the teacher creates a new situation, having as reference the demands of students' learning. In this case, the teacher has agency about the curriculum material and can collaborate in implementing the materials (Remillard, 2005), recontextualising the activities and social function of the contents addressed in them (Aguiar, 2014).

Still on the curriculum materials, the concepts of affordance and constraint are closely related to the agency, as can be observed in Figure 2. Along with other features already mentioned, it is the affordances that give agency to the material and therefore potentiate it as a curriculum inducer. However, the constraints on the material displace the agency to the teacher and inflict infidelity on the curriculum materials.

As for the *Design Capacity for Enactment* proposed by Brown (2002), the categories agency in materials and affordances and constraints refer to the left side of the conceptual framework, getting connected with the curriculum resources. The categories agency in the teachers and interventions in developing the materials refer to the teachers' resources; therefore, they are located on the right side of the picture. However, these categories are not isolated; there is a dynamization between them and shared characteristics, such as the mobilisation of prior knowledge and the use of contextualisation.

From our analysis of the teacher-curriculum materials relationship, expressed in the Research Report, we perceive that affordances and constraints qualify the materials and potentiate the agency and its displacement, sometimes for teachers and sometimes for materials, thus imparting different interactions between these two agents of curriculum development in mathematics.

Therefore, we can verify the need for urgent studies in this area of research in Mathematics Education, in order to increase knowledge on the relationship between curricula and teachers, especially research that deals with mapping and analyzing theoretical frameworks on affordances, constraints and agencies, bringing contributions to the elaboration of public policies in the production of curriculum materials and for the professional development of teachers.

AUTHORS CONTRIBUTIONS STATEMENTS

G.J. conceived the project and the idea presented, as well as developed the theory, adapted the methodology to the proposal and analysed the data. A.L.M. oversaw the project and the writing of the article. Both authors structured the writing, made adjustments and discussed the results.

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