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Teacher Action, Student Action and its Connections in Mathematics Classes Planned with Manipulative Materials

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ABSTRACT

Background: most of the publications on teacher education are mainly interested in what teachers should or should not do, emphasizing a normative view of teaching. In this paper we try to move away from that view, trying to describe what teachers and students really do in the classroom. **Objectives**: to characterize the actions of teachers and students, as well as their connections in mathematics classes planned with manipulable materials. **Design**: direct observation of teaching and student actions during classes. **Environment and participants**: a teacher and 30 students, from a 6th grade class at a state school in the state of Paraná. **Data collection and analysis**: data collected through video recordings, audio and field notes and analysed through Content Analysis. **Results**: fourteen categories were found for teaching action and fourteen categories for student actions. As for the connections between teaching and student actions, the results indicate that they can be simple, multiple and/or random and that they can vary according to the time of the class. **Conclusions**: (i) although a class is planned by the teacher, its execution is a joint task between him and the students, that is, if the teaching and student actions are not connected, the class itself is not effective; (ii) didactic approaches that provide a greater amount of teaching actions than traditional classes, may allow more active attitudes of students in relation to their learning, a hypothesis to be further investigated.

Keywords: teacher action; student action; mathematics classes; manipulative materials.

Ação Docente, Ação Discente e suas Conexões em Aulas de Matemática Planejadas com Materiais Manipuláveis

RESUMO

Contexto: grande parte das publicações sobre a formação de professores se interessam sobretudo pelo que os professores deveriam ou não fazer, enfatizando uma visão normativa da

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docência. Neste artigo procuramos nos afastar dessa visão, procurando descrever o que professores e alunos realmente fazem em sala de aula. **Objetivos**: caracterizar as ações docentes e discentes, bem como suas conexões em aulas de Matemática planejadas com materiais manipuláveis. **Design**: observação direta das ações docentes e discentes durante a realização das aulas. **Ambiente e participantes**: um professor e 30 alunos, de uma turma de 6º ano de uma escola da rede estadual de ensino do estado do Paraná. **Coleta e análise de dados**: dados colhidos por meio de gravações em vídeo, áudio e notas de campo e analisados por meio da Análise de Conteúdo. **Resultados**: foram encontradas catorze categorias para a ação docente e catorze categorias para as ações discentes. Quanto às conexões entre as ações docentes e discentes, os resultados apontam que elas podem ser simples, múltiplas e/ou aleatórias e que podem variar conforme o momento da aula. **Conclusões**: (i) embora uma aula seja planejada pelo professor, sua execução é uma tarefa conjunta entre ele e os alunos, ou seja, se as ações docentes e discentes, não se conectam, a aula, propriamente dita, não se efetiva; (ii) as abordagens didáticas que proporcionam uma quantidade maior de ações docentes do que as aulas tradicionais, podem permitir atitudes mais ativas dos estudantes em relação à sua aprendizagem, uma hipótese a ser posteriormente investigada.

Palavras-chave: ação docente; ação discente; aulas de Matemática; materiais manipuláveis.

INTRODUCTION

Studies on teaching are constantly growing and deepening. Researchers have been dedicated to understanding better who the mathematics teachers are, how they think and how such aspect relates to their practice (Ferreira, 2003). It is also known that "we do not start from scratch, the professional development of teachers and the analysis of the processes of learning and teaching have been a constant concern of educational researchers in recent decades" (Marcelo, 2009, p.9).

However, we agree with Tardif & Lessard (2008) when they state that:

> "It seems to us that the first step to be taken to analyze the work of teachers is to make a resolute critique of the normative and moralizing views of teaching, which are primarily interested in what teachers should or should not do, leaving aside what they really are and do" (Tardif & Lessard, 2008, p.36).

Inspired by this quote and considering what Passos (2009) presents us - when, analyzing 32 years of publications in journals in the area of Mathematics Education (1976 to 2007), she concluded that teachers and their education had been the most researched themes in those three decades and that, in most of the investigated articles, their authors highlighted the teachers' "duties"-, we started to question those attributions the author raised (Passos, 2009, p.160): "be this or that"; "do this or that"; "need this or that"; among others; and move our gaze to what the teachers actually do in the classroom.

From this process of reflection on how the teachers act in the classroom, it was possible to verify that in 'traditional' Mathematics classes their actions are characterized as follows: bureaucratic-administrative; wait; explain; write, results presented by Andrade

& Arruda (2017). Advancing in this movement in search of the analysis of this way of acting of the Mathematics teacher, classes planned with games started to be studied. In general, that study showed that teacher actions could be accommodated in two broad categories: that of class management, which concerns "organizing the classroom" and that of managing the subject, that is, "how they plan content to be presented and developed in the classroom" (Dias, Arruda & Passos, 2018, p.10). In the first category Dias, Arruda & Passos (2018) included the following actions: threaten; scold; negotiate; disapprove; and in the second category, ten verbs were highlighted to represent what the teachers did in their classes based on games: thank; argue; comment; check; execute; organize; congratulate; ask; provide; answer.

Considering these results, we began to focus on researching mathematics classes that had been planned and developed with the proposal of using manipulative materials.

In this article we bring the results of the analysis of a class held within this proposition, and which, according to our choice and selection, is a class that represents others that we have followed and that had this planning option.

Thus, we hope, by the end of this document, to have answered two questions: What do teachers do, in fact, in mathematics classes they have planned with the use of manipulative materials and which categories describe their actions? What connections can be established between teacher and student actions in a mathematics class planned and developed in this way?

Following, we bring information about the theoretical references on which we based the investigation; the circumstances of data collection; the procedures for organizing and interpreting the data; the evidence we reached; and, finally, we describe teacher actions, student actions and their connections in such context.

THEORETICAL BACKGROUND

In this section we present briefly some theoretical frameworks that we assume to develop the investigation. Among the topics covered are what we consider as action; how we think about the relationship with knowledge; what we understand by manipulative materials; all of them to elucidate what we seek to do - highlight the teaching and student action and their categorizations and connections in a Mathematics class planned and developed using manipulative materials.

When we access a dictionary, for example, Houaiss (2009), in the first meanings we that "action" can be understood as an "act of acting", "a dynamic process in which there is an agent who does something", "a way of proceeding ". When searching the literature for what is discussed or presented about "action", we find several statements, some of which we highlight: in Aquino (2000) we have a statement by Lahire indicating that human actions are instituted of pure practical sense in everyday situations and rationality regarding what is new; for Weber (1978, p.4), "the action is 'social' insofar

as its subjective meaning takes into account the behavior of others and is thus oriented in its course"; according to Coleman (*apud* Aquino, 2000), the action is designed to enhance utility and most of them can be conceptualized as rational in relation to ends; in this same framework (Aquino, 2000), we are faced with Bourdieu's concept assuming that the action is specified by the *habitus*, which can be understood as the action schemes that permeate the perception of the situation and the appropriate response, highlighting that those actions refer to the actions of individuals inserted in a social group.

Regarding the teacher action, we will consider it generically as the action that the teacher develops in the classroom, with a view to teaching. Similarly, student action would be the action that the student performs in the classroom with a view to learning. Both are described by verbs, as we will see in Tables 2 and 3[1].

For the concept of "relationship with knowledge" we are based on Charlot's theory (2000), agreeing with his statement that people are mobilized for something using the resources they have, and when they move around by mobiles (for that something) we assume that an exchange happens with the world and, according to the author, mobile is understood as "reason to act". Those relationships established with the world can be understood by the relationship with knowledge as a relevant concept, because "the relationship with knowledge is a form of the relationship with the world: this is the basic proposition" (Charlot, 2000, p.77).

As we intend to categorize the action of teachers and students, the world that interests us is the world of school, with its characteristics. To understand that world, we must consider the subject in its relationship with learning. According to Charlot (2000), the relationship with knowledge is the set of the relations that an individual establishes with "learning" and the knowledge. In this way, we can include that, besides the granting of relationships with "learning", which, in our research, was determined by student action, there are also the relationships with "teaching", represented by teacher action, through action verbs that compose the categorization.

With regard to the choice of classes, in which manipulative materials were part of their planning and development, we clarify that we seek inspiration in Kilpatrik (1996), when he indicates several thematic trends in Mathematics Education together with a list of international lines of research in the same field that can be used in the Mathematics teaching and learning process. In this article we emphasize the use of manipulative materials in the Mathematics teaching.

Murari (2011, p.193) emphasizes that "when using didactic material, it is absolutely necessary to be careful to analyze whether it satisfies and provides the achievement of one of the main objectives of the teaching and learning process, which is the understanding of the concepts studied". Therefore, it is necessary for teachers to pay attention to the real meaning that each material can offer to students, being able to provide a vision of

[1] It is clear that not all teachers' actions are directly linked to teaching and not all students' actions are aimed directly at learning, as we will have the opportunity to show in this article.

education, of Mathematics and of the world, and explain a unique pedagogical proposal, without being linked solely to a playful function.

The manipulative resources or didactic materials play different roles in the Mathematics classroom, interfering in teaching and learning, and the objectives expected by the teachers need to be pointed out when they want to use them.

According to Carvalho & Pereira (2007, p.3),

> "Assuming this type of methodology requires a great effort on the part of the teachers to research and adapt materials for the content on which they want to work. Furthermore, when the teacher takes on this new teaching concept, they must be prepared to face problems, such as: the duration of activities and greater interaction between students".

Complementing the discussion on the use of manipulative materials in Mathematics teaching, Grando (2015, p.395) states that:

> "[...] it is necessary to understand that the use of manipulative materials allows students a visualization and a possibility to represent mathematical relationships we as teachers sometimes want students to understand. Its use is not justified just because it involves students and motivates them to learn, but it mobilizes them to establish relationships, observe regularities and patterns, think mathematically".

Having delimited our understanding of some concepts and definitions important for the development of our investigation, we now proceed to clarify the methodological procedures adopted.

METHODOLOGY

The school under analysis is inserted in the state education system in Paraná, Brazil. We collected the data with 30 students from a 6th-grade class aged 11 in average. The class was developed in the Mathematics laboratory. The class teacher had a 2-year experience, has a degree in Mathematics and is a specialist in Mathematics Education, with both qualifications from the same state university in Paraná. To maintain the anonymity of those surveyed, the teacher will be called P and the students will be assigned codes A1 to A30. All procedures related to ethical care were taken, with the insertion of a project linked to Plataforma Brasil and the number referring to the approval, according

to the Ethics Committee of the research in progress, a requirement currently imposed for research involving human beings[2].

As we aim to categorize the teacher and student actions and their possible connections, data records were performed in two ways. A video camera focused on what the teacher did during the lesson, capturing his actions in detail. Although the students' actions were partially captured by the video (the speeches), it was necessary to complement the students' data through notes in a field notebook.

To organize all that information - the transcription of the class, whose theme was symmetry, and the field notes - we prepared a Table with six columns subdivided into characteristic moments of the development of the class. Next, we bring an example of such organization in Table 1, as the 50-minute class generated a Table with approximately 50 pages, which can be accessed in its entirety in Dias (2018).

The six columns that make up the Table organize, in this order, from the first to the sixth, the following descriptions: transcription of the teacher's speeches; speechless teacher's actions; teacher action categories; transcription of students' speeches and actions; researchers' comments; connection between actions. We also highlight that this Table brings four moments of class organization, named as follows: (i) Preliminary actions; (ii) Task; (iii) Theory; (iv) Conclusive actions. We also indicate that Task (ii) had an internal subdivision, as described in Table 1. Finally, we justify that the disclosure of a structure for the class is due to the fact that innumerable classes were analyzed, and the emergence of a structure was always considered, as these characteristics contribute to the organization of the data and its interpretation, as it will be possible to verify as we present the results.

[2] The number of the CEP's substantiated review is 1.666.360. CAAE: 57663716.9.0000.5231.

Table 1
Example of data organization (class taught in 2016)[3]

Transcription of the teacher's speeches	Silent teacher's actions	Teacher action categories	Transcript of students' speeches and actions	Researcher's comments	Connections between actions	
					Teacher	Students
Preliminary Actions (i)						
		Time to get to the classroom. Time to displace to the laboratory.		Arrival at the Mathematics laboratory.	Move.	Move.
Task (ii) - preparation						
	Waits for the students to sit down.	Wait for students to sit down.	They talk.		Wait.	Talk.
A1, come here, at this table. For now, if...		Organize students into groups. Modify the layout of a group.	They organize themselves in groups. They complain.	Modifies the arrangement of a group.	Organize.	Complain.
Task (ii) - performance: step 1 (folding the sulphite sheet)						
Look, take your paper sheet and fold it in half, just right, end to end.		Starts explaining the activity. He responds according to the students' bad behavior.	They talk. They try to guess what will be done. They talk. They fold the sheet.	Teacher shakes his head in disagreement.	Explain.	Comment. Execute.
Task (ii) - performance: step 2 (draw half a heart)						
Oh, so you're going to take the sheet and make a...	Teacher goes to the board to show the drawing that will be done on the sheet.	Goes on explaining. He gives an example on the board of what is students should do.	Students make fun (they say wow).	The whiteboard pen does not work.	Explain.	Play.
	Teacher draws with a chalk on the blackboard.	He gives examples on the board.			Write.	Execute.

[3] Due to the extension of the original document, we inform that we bring some fragments in Table 1 that exemplify the original table.

Transcription of the teacher's speeches	Silent teacher's actions	Teacher action categories	Transcript of students' speeches and actions	Researcher's comments	Connections between actions	
					Teacher	Students
Here is a heart, you will only make half of it here. Oh, but very correctly, pay attention.		Goes on explaining.	Student gets up and borrows a pencil from the teacher.		Explain.	Move. Request. Ask questions.
Task (ii) - performance: step 3 (cut the heart)						
Now you are going to cut this drawing.		Goes on explaining.				Collaborate.
			Let me cut yours.	Student asks if he can cut the teacher's drawing.	Explain.	Execute.
			Teacher, is it like that?	Student indicates with her fingers how she intends to cut.		Ask questions.
Task (ii) - performance: step 4 (paste the figure in the notebook)						
After cutting, you put the date, this title and paste the cut-out figure below.	Teacher indicates on the board what students should do in their respective notebooks.		They talk.		Explain.	Talk.
If you need glue, I have it here.		Offers materials if students do not have them.			Provide.	Comment.
			Teacher, my heart will not fit in my notebook.			
	Teacher walks around the room, watching.	Walks around the room.	Students work and talk a lot.		Supervise.	Execute. Talk.
			What day is today?			Ask questions.
Task (ii) - performance: step 5 (plot the axis of symmetry)						
I'm coming. Who called me?		He addresses students.				
			Student made a very big heart that does not fit in the notebook.		Ask questions.	Execute.

Transcription of the teacher's speeches	Silent teacher's actions	Teacher action categories	Transcript of students' speeches and actions	Researcher's comments	Connections between actions	
					Teacher	Students
He reduces it a little.		Asks the student to change the size of what has been done.			Request.	Execute.
			Theory (iii)			
And in nature?		Asks more questions.				
			The butterfly.			
What else, huh, A8 and A11?			Students are talking.		Scold.	Talk.
						Talk.
So, was it clear to you what this axis of symmetry would be?		Asks if the students understood.			Ask questions.	Request.
						Answer.
			Conclusive actions (iv)			
		Time to Move back to the classroom.			Move.	Move.

We have adopted the Content Analysis procedures to arrive at these results considering that

> "[...] content analysis is not a simple technique, but it can be considered a varied methodology and being constantly under review. In this sense, it is understood that content analysis makes it possible to meet the numerous needs of researchers involved in the analysis of communication data, especially those aimed at a qualitative approach" (Moraes, 1999, p.30).

And yet, "in a way, Content Analysis is a personal interpretation on the part of the researcher regarding his/her perception of the data. Neutral reading is not possible" (Moraes, 1999, p.24).

RESULTS AND ANALYSES

From the investigative movement carried out, we elaborated fourteen categories for teachers' action and fourteen categories for students' action, which are related and described in Table 2 and Table 3, respectively. As can be seen, we used verbs - which for us represent actions - for the nomination of categories.

Before proceeding to the reading and comments after Table 2, we clarify that the categories of teacher actions were elaborated from the organizational process of the data. As previously indicated: we transcribed, in a first moment, what the teacher said, fragmenting each speech before an action performed by him (these lines - transcriptions - and these actions - described by us - are exemplified in Table 1, columns 1 and 2) and all this information can be accessed in Dias (2018); in a second moment, we turned our attention to the interpretation of the records organized in these first two columns, of this analytical movement, we aimed to find a verb that represented such situations, it is they that we insert descriptively in column 3 of Table 1 and that we systematize more compactly in the initial column of Table 2.

We understand that such categories indicate an action on the part of the subject, which for the teacher was assumed to be relations with teaching and for the student as relations with learning.

Table 2
Categories of teacher actions for the class under analysis

Categories of teacher actions	Category description
1. Threaten.	Threaten to send the student out of class: the student should return to the classroom; go to coordination; not go to the lab anymore.
2. Argue.	This category includes the argumentation with the student: regarding the loan of materials, that the borrowed materials are not new; the choice of a figure that represents a heart; about the glue-soiled hand.
3. Scold.	The action of scolding is composed of the secondary action of scolding the students: those who disturb the class; talk in parallel to each other; play by hitting something with the rulers that had been borrowed.
4. Move.	This category refers to the displacement of the teacher before or after class.
5. Write.	The description of the write category is restricted to writing on the blackboard: the theory or to demonstrate and/or exemplify the task.
6. Wait.	The wait action involves several sub-actions, such as waiting for students: copying, answering, sitting, finishing, being silent, taking the pencil.
7. Explain.	The explain category can be represented by explanations regarding the task and its stages and the content of the class.
8. Organize.	Organizing refers to organization into groups: separation into groups; the change in the arrangement of groups; guides where students who are late are supposed to sit.
9. Request.	The request action involves several sub-actions, such as asking: help from students to deliver materials that will be used in the task; that students copy; students' attention; for students to take the pencil; for students to reinforce the folding of the paper sheet; that the student changes the size of what was done; that students share the materials borrowed; that students wait; that students put the material away; to keep the room clean; that students do not touch items in the laboratory; that students copy after pasting.
10.Ask questions.	This category involves several sub-sections, such as asking: whether students completed the steps of the task; why a student is standing; what the date is; whether students are listening; whether teacher can continue; whether they understood; about the content.
11. Provide.	The provide action is related to the provision of materials to students to perform the task.
12. Disapprove.	The disapproval action refers to the student's action.
13. Answer.	This category involves several sub-actions, for example, answering to questions such as: that students will return to the laboratory another day, provided they behave; a waiting request; questions with a subject outside the class topic; questions according to the topic of the lesson.
14. Supervise.	This category is related to group supervision.

Again, we highlight how the interpretations were carried out so that we could elaborate Table 3. Following the same organizational pattern, we transcribe the speeches and actions of the students, in this case jointly, as exemplified in column 4 of Table 1. At

a later time, we resume these descriptions reinterpreting them (by revisiting the video and the field notebook), which allowed us to elaborate column 5 (researcher's comments), which we can call an analytical advance that led us to the construction of column 6 (see Table 1) and the systematization of Table 3, which considered, in column 6, only its " Students" tab.

And therefore, for students, they were defined as relationships with learning.

Table 3
Categories of student action for the class under analysis

Categories of student actions	**Category description**
1. Play.	Playing refers to those moments when students perform some action that they consider fun, such as: playing; singing; switching places; standing up; disrupting the course of the class; trying to change chairs.
2. Call the teacher.	The call action refers to the action of calling the teacher.
3. Collaborate.	This category refers to collaboration: between students performing the task and the teacher delivering materials.
4. Comment.	The comment category involves several sub-actions, such as commenting among students or with the teacher: about the task; about the content; about another subjects.
5. Communicate.	In this action, students limit themselves to communicating to the teacher the end of the task the teacher asked them to perform.
6. Talk.	This category refers to conversations between students or the teacher about subjects not directly related to the class, with each other, turning off the fan, the number of sheets in the notebook, what they are going to do on Saturday.
7. Copy.	The copy action is restricted to copying what is on the blackboard.
8. Move.	This category refers to displacement inside or outside the room/laboratory, for example, with students addressing the teacher, getting up from their desks and going to the trash, to borrow materials.
9. Execute.	The execute category can be represented by what students do, exclusively, in relation to the stages of the task.
10. Request.	The asking action involves several sub-actions, such as asking the teacher or students: to leave the front of the blackboard, to wait, to explain again, to borrow materials.
11. Ask questions.	The ask questions category involves several sub-sections of questions to the teacher, such as questions about the task, the theory, the environment, other subjects.
12. Complain.	Complaining involves some sub-actions, such as: complaining that they will have to copy, that there is no material available for everyone, the places they will have to sit, the places their colleagues are sitting.
13. Answer.	The answer action can be represented by students' answers to the teacher in relation to subjects about: the task, the theory, the environment, other subjects.
14. Value.	The value category expressly, implicitly or explicitly, the student's appreciation of the class, represented in the form of praise or moaning.

When we allocate those actions in the four moments when the class was analytically structured, we arrived at the result shown in the Table below.

Table 4
Moments and categories of teacher and student action in the class under analysis

Moments	Categories of teacher actions	Categories of student action
(i) Preliminary actions.	Move.	Move.
II. Task.	Threaten; argue; scold; write; wait; explain; organize; request; ask questions; provide; disapprove; answer; supervise.	Play, call for the teacher, collaborate, comment, communicate, talk, move, execute, request, ask questions, complain, answer, value.
(iii) Theory.	Threaten; scold; write; wait; explain; request; ask questions; answer.	Play, comment, communicate, talk, copy, move, request, ask questions, complain, answer, value.
(iv) Conclusive actions.	Move.	Move.

The actions of the teacher and students were concentrated in the moments (ii) and (iii) in the class, as we already imagined. It is evident, in the action categories related to teaching, that the moment (ii) - which involved the preparation, explanation and execution of the task - included almost all the actions categorized in the class (thirteen actions). However, moment (iii), called theory, when the content was exposed by the teacher, had less diversity of actions (only eight of them).

With regard to the categories of action related to learning, that is, manifested by the students, we noticed that the moment (ii) - referring to the preparation, explanation and execution of the task - included 93% of the actions categorized in the class (thirteen of them). The only category not contained during the class related to the task was the copy category, an indispensable action in a traditional lecture, and it is understandable that it did not occur during the class aimed at developing the symmetrical figure. As for the moment attributed to the theory (iii) - the exposure of the content - eleven actions took place, leaving only three of them to be evidenced: calling the teacher, collaborating and executing. Our conclusion is that the collaborate and execute categories proved to be directly related to the task; the disappearance of the action call the teacher is also justified, because the disclosure of the content does not encourage this form of request. For the students, the move category was the only one that appeared in the four moments of the class - preliminary actions, task, theory, conclusive actions.

Our attention now turns to column 6 of Table 1, presented in the section dedicated to clarifying the methodological position taken in the investigation.

The result of the data organization in this column (which has already been interpreted) generated Table 5, inserted below. We highlight the connections between the actions of the teacher and that of the students, all organized according to the four structural

moments of the class under research, remembering that moment (ii) was subdivided into preparation and five stages.

Table 5
Connection between the actions of the teacher and the students in the class under analysis

(i) Preliminary actions	
Teacher	**Students**
Move.	Move.
(ii) Task - preparation	
Teacher	**Students**
Threaten.	Talk.
Argue.	Comment.
Wait.	Talk.
Organize.	Ask questions; Complain.
Request.	Collaborate; Talk; Ask questions.
Provide.	Talk; Ask questions.
Disapprove.	Play.
Answer.	Collaborate; Comment; Talk; Ask questions.
(ii) Task - performance: step 1 (folding the sulphite paper sheet)	
Teacher	**Students**
Wait.	Execute.
Explain.	Comment; Execute.
Request.	Ask questions.
Ask questions.	Talk; Execute; Request; Answer.
Answer.	Execute; Ask questions.
(ii) Task - performance: step 2 (draw half a heart)	
Teacher	**Students**
Threaten.	Play.
Argue.	Execute.
Write.	Execute.
Explain.	Play; Talk; Move; Execute; Request; Ask questions.
Organize.	Execute.
Request.	Execute.
Ask questions.	Answer.
Answer.	Execute; Request; Ask questions.
Supervise.	Communicate.
(ii) Task - performance: step 3 (cut the heart)	
Teacher	**Students**
Write.	Ask questions; Answer.
Explain.	Talk; Collaborate; Execute; Request; Ask questions.
Ask questions.	Talk; Answer.
Provide.	Comment; Communicate.
Answer.	Execute; Ask questions.

(ii) Task - performance: step 4 (paste the figure in the notebook)	
Teacher	**Students**
Argue.	Play; Execute; Complain.
Scold.	Answer.
Call the roll.	Comment; Move; Request.
Explain.	Execute.
Request.	Play; Talk; Move; Ask questions.
Ask questions.	Execute; Answer.
Provide.	Comment; Move; Execute; Ask questions.
Answer.	Collaborate; Talk; Execute; Request; Complain.
Supervise.	Execute; Talk; Ask questions.
(ii) Task - performance: step 5 (plot the axis of symmetry)	
Teacher	**Students**
Scold.	Play; Execute; Talk; Ask questions.
Explain.	Execute; Ask questions.
Request.	Talk; Execute.
Ask questions.	Execute; Answer.
Provide.	Talk.
Answer.	Call the teacher; Ask questions.
(iii) Theory	
Teacher	**Students**
Scold.	Play; Talk; Move.
Write.	Play; Talk; Comment; Copy; Request; Ask questions.
Wait.	Call the teacher; Comment; Talk; Copy; Move; Request; Ask questions; Answer; Value.
Explain.	Comment; Copy; Move; Request; Ask questions.
Request.	Collaborate; Copy; Move; Request; Ask questions; Complain; Answer.
Ask questions.	Talk; Copy; Move; Request; Ask questions; Answer.
Answer.	Play; Talk; Copy; Move.
(iv) Conclusive actions	
Teacher	**Students**
Move.	Move.

It is evident, from Table 5, that there are actions of the teacher that are fully connected with that of the students, as is the case of the moments (i) and (iv) - when they move. However, we also perceive that some of the teacher's actions reveal several 'acts of the students'. For example, at moment (ii), in step 2 (drawing half a heart), while the teacher explains, the students play, talk, move, execute what was requested, request, ask questions; and at moment (iii), when the symmetry content is presented, all teacher's actions are connected to more than one of the students' action, especially the waiting action, which involves 9 students' actions: while the teacher waits, students: call the teacher, comment, talk, copy, move, request, ask questions, answer and value.

We ended this section by making a comparison between the teacher's and students' actions found in previous research, in which the Mathematics classes were planned according to different approaches (Table 6):

Table 6
Similarities and differences between categories in Mathematics classes

Planning types	Categories of teacher actions	Categories of students actions
Traditional - (Andrade & Arruda, 2017).	Bureaucratic-administrative. Write; Wait; Explain.	Not yet researched.
Use of games - (Dias, 2018).	Write; Wait; Explain. Threaten; Argue; Scold; Move; Organize; Request; Ask questions; Provide; Answer. Thank; Comment; Check; Execute; Negotiate; Congratulate.	Not yet researched.
Use of manipulative materials.	Write; Wait; Explain. Threaten; Argue; Scold; Move; Organize; Request; Ask questions; Provide; Answer. Fail; Supervise.	Play; Call the teacher; Collaborate; Comment; Communicate; Talk; Copy; Move; Execute; Request; Ask questions; Complain; Answer; Value.

We see in Table 6 that traditional teachers' actions of waiting, writing and explaining are also present in classes planned with games and manipulative materials. However, the number of the teachers' actions is much higher in those two approaches than in traditional classes. The categories of teachers' actions, threatening, arguing, scolding, moving, organizing, requesting, asking questions, providing and answering were found both in classes planned with games and in classes planned with manipulative materials. The differences between those two didactic approaches are due to the categories thanking, commenting, checking, executing, negotiating, congratulating, present only in classes planned with games and the categories disapprove and supervise, found only in classes planned with manipulative materials.

The data indicate that the teacher' action of supervising may be typical of classes in which the student is asked to do some activity. In the case of the class analyzed and whose results are presented in this article, supervision took place at different times when the task was being performed (ii), more specifically, during drawing of the half heart; when using scissors to cut the heart; at the time when the students were engaged in pasting the heart. Regarding the disapproving action, it also occurred during the performance of the task (ii), when a student anticipates performing the procedures without the teacher having indicated what was to be done.

CONCLUSIONS

At this point, we return to our two research questions, seeking to answer them based on the data presented in the previous section.

The answer to the first question we asked (What does the teachers actually do in mathematics classes that have been planned and developed using manipulative materials and which categories describe their actions?) is answered in the fourth row of Table 6. It is necessary to clarify that these were the teacher's actions found in that specific class planned with those materials by that teacher with that group of students and at that level of education. We have at least five variables that can determine the categories of action found in the data presented here (class, material, teacher, group, level of education), which prevents us from generalizing. However, we think that the categories of teacher (and student) actions should not vary so much if the materials used and the level of education are maintained. But this hypothesis can only be verified by collecting new data.

To answer the second question - What connections can be established between the actions of the teacher and the students in a mathematics class that has been planned and developed using manipulative materials? - we were led to develop categories of actions for students. Referring back to Table 5, presented in the previous section of this article, we can identify three types of connections between teacher and student action: *simple*, *multiple* and *random*. We also observed that these types of connections varied according to the time of the class and/or its stage.

In the *simple* connection, the actions seem to be 'attuned', that is, one action of the teacher corresponds to a single action of the student. For example, at moment (ii) task - performance, step 2 (drawing half a heart), the teacher asked, and they answered.

In the *multiple* connection, to a single teacher action, several student actions can correspond. Examples: at moment (ii) task - performance, step 1 (folding the A4 paper sheet), the teacher asks and the students talk, execute, request or answer; at moment (iii) theory, the teacher requests and the students collaborate, copy, move, request, ask questions, complain or answer. We see that most connections were of this type.

Finally, we have the connections we call *random*, like playing and talking. Such student actions are not 'attuned' to the teaching action, that is, they are not direct responses to the teacher's demands, but they can occur despite the demands.

Such considerations can lead us to some situations experienced in the school routine; although planned it's the execution of a class is a teamwork that involves the teacher and the students. As Tardif and Lessard say, "a class is a kind of project or program to be carried out jointly" (Tardif & Lessard, 2008, p.250). In other words, if the actions of teachers and students are not connected, the class itself does not take place. This allows us to highlight the need to present such disconnections to teachers in initial training and to teachers in the process of continuous training, so they can develop a perceptive state regarding the planned and the joint realization.

Another contribution that we think possible to be highlighted is the diversification in teachers' actions: classes planned according to some Mathematics Education trends can provide a much greater amount of teachers' actions, compared to traditional lectures. Our research results also indicate that this didactic approach can allow more active attitudes of students towards their learning, a hypothesis to be further investigated.

Finally, we inform you that the investigations on teacher actions, students actions and their connections are also being extended to other disciplines, such as Chemistry, Physics and Biology.

AUTHORS 'CONTRIBUTIONS STATEMENTS

M. P. D. collected the data. S. M. A. was responsible for supervising the research. M. M. P. was responsible for the methodology used in the research. All authors discussed the results and contributed to the final version of the manuscript.

DATA AVAILABILITY STATEMENT

Readers can access all data and analysis at the following link http://www.bibliotecadigital.uel.br/document/?code=vtls000217592

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