

For an undistorted view of Newton's Second Law

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Received for publication on 8 July 2019. Accepted after review on 25 Oct. 2019.

Designed editor: Renato P. dos Santos

ABSTRACT

Context: In general, teachers and students have distorted views about the functioning of a scientific work. This research uses the episode of the historical construction of the Second Law of Movement and seven distorted views on the subject. **Objectives:** This work aims to exemplify the distorted views students can construct about scientific endeavour if a didactic approach that does not consider the history and philosophy of science is used. **Design:** Seven distorted views regarding scientific work are explained and, subsequently, exemplified from the episode in question, showing how this history is seen, in general, in textbooks, and by most teachers. **Environment and participants:** This is a theoretical analysis, not including participants. The materials used were primary and secondary bibliographic sources. **Data collection and analysis:** Seven distorted views on science, from the episode of the historical construction of the Second Law of Movement, were collected, and a historical-philosophical reflection was carried out on them. **Results:** Following the presentation of the views and examples, criticism and allusion to how this episode should be treated are offered. **Conclusions:** Throughout the text, as well as in its final considerations, relevant aspects to be worked on in the classroom are discussed, for a critical view of the construction of scientific knowledge.

Keywords: Newton's Second Law; Second Law of Movement; History and Philosophy of Science; Newton; Leonhard Euler.

Para uma imagem não deformada da Segunda Lei de Newton

RESUMO

Contexto: Em geral, professores e alunos possuem visões deformadas a respeito de como funciona o trabalho científico. Neste trabalho, foi utilizado o episódio da construção histórica da Segunda Lei do Movimento e sete visões deformadas a respeito do tema. **Objetivos:** Este trabalho tem por objetivo exemplificar visões deformadas que podem ser construídas pelos estudantes a respeito do empreendimento da Ciência, caso seja utilizada uma abordagem didática que não leve em conta a História e Filosofia da Ciência. **Design:** Sete visões deformadas a respeito do trabalho científico são explicadas e, na sequência, exemplificadas a partir do episódio em questão, mostrando a forma como essa história é vista, em geral, nos livros didáticos, e pela maioria dos professores. **Ambiente e participantes:** Esta é uma análise teórica, e, portanto, não conta com participantes. Os materiais utilizados foram fontes bibliográficas primárias e secundárias. **Coleta e análise de**

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dados: Foram coletadas sete visões deformadas sobre a ciência, a partir do episódio da construção histórica da Segunda Lei do Movimento, e realizada uma reflexão histórico-filosófica sobre estas.

Resultados: Na sequência da apresentação das visões e exemplificações, são oferecidas críticas e alusão a como esse episódio deveria ser tratado. **Conclusões:** Ao longo do texto, assim como em suas considerações finais, são discutidos aspectos relevantes a serem trabalhados na sala de aula, para uma visão crítica da construção do conhecimento científico.

Palavras-chave: Segunda Lei de Newton; Segunda Lei do Movimento; História e Filosofia da Ciência; Newton; Leonhard Euler.

INTRODUCTION

A large number of textbooks, both for High School and for Higher Education, describe that the formalism “ $F = ma$ ”¹ corresponds to Newton’s Second Law. However, this expression is neither stated in Newton’s masterpiece *Principia* (Mathematical Principles of Natural Philosophy, 1990), nor does it relate in any way to acceleration or to the rate of change of motion. Thus, if the second law does not deal with such magnitudes used in $F = ma$, would it be correct to state that this is Newton’s Second Law (Pourciau, 2011)?

Newton’s Second Law, as described in the *Principia* is:

“*The change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed*” (Newton, 1990, p. 15-16).

When we talk about change of motion, we refer to a variation of velocity, which would possibly result in a proportional force to velocity, and not proportional to acceleration, as it is used today (Sitko, 2019a). According to Sitko’s analysis (2019a), it can be concluded that, indeed, $F = ma$ was not produced by Newton, but by Leonhard Euler², about sixty years after the law proposed by Newton, due to advancements and developments of the Analytical Mechanics, an inexistent formulation at Newton’s time (Sitko, 2019b).

According to Maronne and Panza (2014) and Sitko (2019b), several mathematicians and physicists influenced the construction of the Newtonian Mechanics, as it is described today. The second law, in special, have undergone great changes, extensions and clarifications since the *Principia*, up to what today we know as $F = ma$. Great part of such changes is owed to Euler, who, apart from the analytical formalism, introduced the Cartesian coordinates (Maronne & Panza, 2014), the use of functions, of variational principles, generalized the concept of force, established an analog for the angular case, and, lastly, generalized the resolution of problems for all kinds of mechanical cases (Sitko, 2019a; 2019b).

In view of such findings, the need to spread a critical view regarding the construction of this knowledge in the school environment is a concern, since many Physics teachers

¹ F is the sum of the forces acting on the body, m is the body mass and a is the acceleration suffered by the body.

² With the contribution from several successor scientists to Newton and contemporaneous to Euler.

have a simplistic view in relation to this historical episode, which may result in distorted understandings by students regarding the scientific endeavor. Besides, the fact that Newton's statement and $F = ma$ do not have a direct relationship (which can be identified from a historical approach) can lead the students to find it more difficult understand mechanics³.

If we observe the disclosure of the historical aspects of Newton's second law, for instance, in most textbooks of High School and Higher Education, as well as in media commentaries on the History of Science, we will see that it is possible to identify elements of a distorted view of the construction of the scientific knowledge taught in the classroom, both in Basic and in Higher Education.

Thus, based on the seven distorted views of Science presented by Gil-Pérez, Montoro, Alís, Cachapuz and Praia (2001), possible distortions in the understanding of the scientific knowledge related to the historical episode of the construction of Newton's Second Law are approached and discussed in this text, as well as how these elements should be treated more properly, regarding the epistemology of science.

DISTORTED VIEWS OF THE SCIENTIFIC WORK IN THE CONSTRUCTION OF NEWTON'S SECOND LAW

Gil-Pérez et al. (2001) wrote about the importance of recognizing teachers' distorted views concerning the scientific work. This article lists seven distorted views with comments on each one of them, aiming to show the most appropriate epistemological view.

This work will present and comment the seven views listed by Gil-Pérez et al. (2001). Next, for a better understanding of the distorted views, and, at the same time, to show how important it is to work the content of Newton's second law using the historical contextualization in the classroom, the episode of the construction of $F = ma$ will be inserted in each one of those views. This analysis may highlight the importance of working on the epistemology of Science in Physics and Science lessons.

1. *The inductivist-empirical and atheoretical conception*: one of the researchers' concern in Education in relation to the issue of Epistemology refers to the predominance of views of an inductivist-empirical character among Physics teachers (Praia, Cachapuz & Gil-Pérez, 2002), who clearly move away from the contemporaneous literature considered fundamental for scientific production and for what the idea of science means. This is evident, as Chalmers (1993) explains, when educators naively believe that scientific theories are proved through observation and experimentation. Others also believe that theories can be figured out through systematic observation, that is, theories are created from experimentation.

³ Six years of developments were required for this relation to be established, and the textbook and the teacher want the student to learn this instantly.

Many students, and also teachers, believe that knowledge is a true copy of nature and that scientific questioning is more like a Baconian process of observing facts than a process of building up theories which cope with experimental observations (Kuhn, 1989). However, if we observe over the history of the development of the scientific thought, it is possible to see that none of its great theories or fundamental principles originated from observation; one observation is already full of hypotheses: it is always an observation in the light of theories.

This kind of conception highlights the neutral role of experimentation and coincides with the idea of “discovery”, publicized in comics and by the media (Lakin & Wellington, 1994; Gil-Pérez et al., 2001).

In the episode of the construction of Newton’s Second Law, it is possible to identify this first distorted conception from the superficial account of the theme brought in textbooks (Chaib & Aguiar, 2016) and by the popular knowledge about the story of the fall of an apple from which Newton constructed mechanics that the teachers accept as true (Martins, 2006). However, knowledge is not built overnight; it took Newton many years to build his mechanics (Martins, 2006). Besides, an observation (or systematic observations) does not prove a theory.

2. *Strict, accurate, infallible view*: the strict, algorithmic, infallible view is largely diffused among Science teachers, according to which strict observations and experiences contribute to the accuracy of the results obtained and are based on evidence. This view reinforces the idea of a scientific method, which is a sequence of steps to be followed. It has a quantitative character and puts aside scientists’ creativity, the tentative character and doubt (Gil-Pérez et al., 2001).

Nonetheless, as Feyerabend advocates in *Contra o método* (1977), (“Against Method”, 1977), science does not work through a single method, but from a methodological pluralism, which he also calls epistemological anarchism. Based on this principle, Feyerabend believes that the violation of the methodological rules is necessary for the advancement of science. If we observe the History of Science in general, from the historical episodes like the one of the construction of the Galilean physics (Feyerabend, 1977), which used advertising as its greatest “scientific” booster, it is possible to observe that the violation of the rules of the scientific method was what actually led to the expansions and replacements of some theories and knowledge.

On the other hand, Kuhn (2007) describes science through times of crisis and revolutions, when paradigms are changed, that is, concepts are expanded and replaced. This can also be observed in episodes such as the establishment of the universal gravitation by Newton, which is treated in Kuhn as the replacement of the Aristotelian paradigm, of a terrestrial world separated from the celestial one, by the Newtonian paradigm, which unified the terrestrial and celestial laws.

Even if these are two different and conflicting epistemological views, it is possible to establish a parallel between them, in which both complement each other in the sense that both agree that science is not accurate, strict or infallible, since, regardless of which

of these views is taken as a basis, both understand science as changeable, either because of the change of a paradigm, or for the methodological anarchism.

Pourciau (2011) comments that there are some manuscripts, found in the *Portsmouth Collection* of the library of Cambridge University, dated from 1692-1693, that show esthetic reviews in Newton's work, in relation to the Second Law. The scientist may have formulated eight different forms for the Second Law! Seven forms are crossed out. The one left is the one mentioned at the beginning of this work.

Then, Newton's tentative character, as well as his doubt in Science can be demonstrated, when he writes at least eight different forms of the Second Law. Some decades afterwards, Euler offered criticism to the Newtonian works, broadened their scope and replaced what could no longer be used. However, even Euler had his own decades of trials and errors, until he was able to reach the fundamental principle of Mechanics.

Thus, we can see that Newton was not infallible, neither was his mechanics, which, taken as an example, was essentially geometrical, which limited it to only some classes of problems⁴, and, depending on their complexity, made the calculations too extensive (Panza, 2002).

Apart from the geometrical method, Newton enunciated laws that expressed relationships between "*objects (or better, their masses), their motions and the forces acting on them and because of them*" (Maronne & Panza, 2014, p.14). When a detail was changed in the problem, it was no longer possible to resume the calculation. The use of an analytical method, which could solve the highest possible number of problems, was necessary. This happened later, with Euler and the use of functions (Hepburn, 2007; Sitko, 2019b). Again, a replacement was necessary, or rather, an extension of what Newton had done.

However, only the change, the replacement of geometric elements to analytical ones, or else, the mathematical formalisms, was not enough for the transformation of the geometric into the analytical method. The use of the differential formalism should allow the expression of the relationship between relative mechanical quantities, that is, it was necessary for the symbols to have a mechanical meaning (Maronne & Panza, 2014). The use of functions and Cartesian coordinates brought by Euler allowed, from then on, mathematical objects to be treated, which generalized the calculations to all classes of problems. Euler brought, thus, conceptual advancements, or else, he showed Newton's fallibility, but all the same, his great contribution.

From this excerpt of the episode, it can be noticed that the construction of Mechanics, even though it was a great achievement, was not immutable or infallible. It relied on Newton's doubts, on Varignon's, Bernoulli's and Hermann's trials, and on trials by others that have not been mentioned here (Sitko, 2019b), with Euler's reformulation and expansion, and from this point on, by many other physicists that helped on what we call Analytical Mechanics today (Truesdell, 1975; Maltese, 1992).

⁴ The extensive deformable and fluid bodies, for instance, were left out.

3. *Cumulative view of linear growth*: It is a view that ignores crises. It is complementary to the strict view, presented in the previous item. It is a simplistic interpretation of the construction of scientific knowledge, which does not show that there are rival theories to a certain piece of knowledge, neither controversies (Gil-Pérez et al., 2001, p.132) regarding its construction. It has the idea that the scientific method always works and in a linear way.

As mentioned in the previous item, Euler was fundamental for the transformation and expansion of Mechanics (Pulte, 2001). The conception brought in textbooks, even in those of Higher Education, is simplistic in the sense that it portrays the construction of Newton's laws as if they had been written by Newton in the *Principia* once and for all, and since 1687 they had not been exceeded or modified (Sitko, 2019a). The collective role and of all periods is ignored, the crises, the developments and ups and downs that made the Mechanics we know today emerge, and which we represent here by Euler's contributions⁵. More importantly, what is also ignored is that $F = ma$ was not written by Newton, but only about 60 years later, by Euler, in his *Découverte d'un nouveau principe de mécanique* (Discovery of a new principle of mechanics 1750). In other words, the crisis, which limited the resolutions of problems and in which Euler had to interfere and exceed, is ignored; this ignorance creates an image of Science with linear growth: Newton thinks about mechanics, an apple falls on his head, and so he devises the laws of motion and universal gravitation (Martins, 2006).

Between Newton and Euler, scientists such as Hooke, Varignon, Hermann and Johann Bernoulli were very close to writing the fundamental principle. Although their conceptions complemented each other, some of them were parallel and non-convergent. From that on, we can see, again, that a linear evolution, without the ups and downs of Science, does not take place. For instance, Euler described an alternative to the Newtonian basis⁶ (Fraser, 1988; Panza, 2003; Sitko, 2019b) while thinking about mechanical problems.

4. *Individualist and elitist view*: It is the conception that Science is made by isolated geniuses, ignoring the role of the collective. The character of human and collective construction of Science is not revealed. It is seen as a male activity. Hesitations and errors are not shown. There is also the other side of this view, which faces the scientific activity as simple and close to common sense (Gil-Pérez et al., 2001, p.133). The main counterpoint of this view would be that it presents a science as a product of a collective.

We can resume history before *Principia* was produced, when Newton studied Huygens' work, *Horologium oscillatorium* (1673), in which Huygens presented three hypotheses regarding the motion of falling bodies. The first hypothesis was about the uniform motion in the absence of gravity (acceleration), and the second one dealt with the motion of bodies where there is gravity (Pourciau, 2011).

⁵ Euler's name is mentioned several times because of his great contribution in the area; however, many other scientists were also part of the collective that built $F = ma$.

⁶ From the variational principles.

Such hypotheses are similar to Newton's First and Second Laws, which demonstrates that Newton got support from Huygens for the production of his Mechanics. Besides, even with the disagreements with Hooke, we cannot deny that he contributed, through his criticism and discussions, to Newton's mechanical thoughts⁷ (Westfall, 1995).

Euler's fundamental role for the emergency of Analytical Mechanics has already been mentioned in the previous items. However, it is important to remember that among the changes carried out by the scientist are the introduction of the Leibnizian formalism, of the Cartesian coordinates, and of the unification of the concept of force. For these changes to happen, support in the studies of Leibniz, Descartes, and Johann Bernoulli (Maltese, 1992; Dias, 2017; Truesdell, 1955) was necessary. What we intend to show with these remarks is that only one scientist does not make Science: a collective is necessary to do so.

After Euler and his studies of Mechanics, which were in accordance with Newton's original method, and established an analytical treatment to any mechanical system, based on one single general equation, the so-called Variational Principle, Lagrange was able to obtain new results for the development of Analytical Mechanics (Pulte, 2001). This shows that Euler was not definitive, either; science is always under construction.

Another aspect to be mentioned in this topic is once again related to hesitations and errors. Newton was insecure and intolerant to criticism, which cost him the non-publication of several works, and which also led him to publish three editions of the *Principia*, either because he found errors in the calculations of the previous versions (Westfall, 1995), or only for esthetics, such as the case of the Second Law's wording (Pourciau, 2011, p. 1017). Hence, Newton would never achieve the fundamental principle on his own; he needed his successors, who would use his work as a basis and would work as a collective to establish a general knowledge in Mechanics.

5. unproblematic, ahistorical, dogmatic view: In this view, ready knowledge is transmitted, without mentioning the problem from which it was conceived. However, knowledge is problematic, it originates from a problem. This is a view built by omission (Gil-Pérez et al., 2001, p. 131). As Bachelard states (1996), "All knowledge is in response to a question".

In this case, the question would be: Why did Newton enunciate the laws of motion? What would be the question that arose such response?

In the decade of 1680, it was already a consensus among physicists that the centripetal force of the planets in relation to the Sun should decrease with the inverse square of their period. The big question at the time was how to reach Kepler's laws from the principles of dynamics, that is, how to demonstrate the laws of planetary motion through the relationship of the inverse square (Westfall, 1995).

⁷ When handling, by mail, the planetary problem which originated the Universal Gravitation. It is important to remember that the three laws of motion are axioms necessary for the enunciation of the Universal Gravitation.

In 1684, Halley visited Newton aiming at questioning him about the trajectories described by the planets and the respective calculations. Newton said that he had already done the calculations before, but had lost them, and that he would send them to Halley soon (however, Newton still had the pieces of paper, and just did not want to give them to Halley, so that the same situation experienced with Hook would not be repeated, in which Hook “stole” his ideas regarding the calculation). When Newton decided to redo the calculations, he realized he had made errors in some diagrams, and thus, did not manage to solve the problem anymore (Westfall, 1995).

Nevertheless, he did not give up the problem, and could resolve it later on, still in the same year, resulting in the treaty *De motu Corporum Liber Secundus*. Even after the delivery of the treaty to Halley, Newton continued to write about the subject, redoing definitions and hypotheses, using Flamsteed’s data (Westfall, 1995) to improve his demonstrations, which would result, in a few months, in the *Principia*. In this material, the laws of motion were a prerequisite of the Universal Gravitation, that is, they were part of Newton’s work, but not its main objective.

Another important question to be asked would be: Why did Euler want to change Newton’s Second Law?

It is important to point out that at first his objective was not to change it without a well-defined purpose, but, once the Newtonian principles were not enough to study the rotation of solid bodies, which were Euler’s object of study (Sitko 2019b; Maltese, 1992), it became necessary to find adequate principles (Euler did not study only Newton, but Varignon, Hermann, and all those who produced works about Mechanics) (Maronne & Panza, 2014). This way, the formulation of $F = ma$ was just a part of his study, and not his central objective. His aim was to build Mechanics valid to all the mechanical cases (Maltese, 1992).

Another factor that must be considered is that science cannot be treated as a dogma, as something that cannot be wrong or changed; science is in constant transformation. As Truesdell (1960; 1968) explains, science is liable to changes; the biggest proof is that the idolized Newton’s Laws were greatly modified, and Euler’s contribution is what expanded the Second Law to what we know as the Fundamental Principle of Motion, or Newton’s Second Law.

6. Exclusively analytical view: This view points out the fragmented division of the studies, and has a simplifying and limited character. There is devaluation of the unification process (Gil-Pérez et al., 2001) and distancing from reality, that is, from the context in which knowledge was produced.

One example cited by Gil-Pérez et al. (2001) is the Newtonian synthesis, which eventually unified the celestial and terrestrial motions, previously refused in Copernicus’s and Galilean’s works; however, such synthesis is not mentioned in the study of Mechanics. It is Newton’s most important contribution, and it is not mentioned.

Apart from that, we can retake the previous item, which shows that Newton’s laws of motion were only part of his masterpiece, and that were described to solve a bigger

problem, which was the attraction between bodies. The laws were, in this case, part of a whole that would be unified in Newton's Mechanics.

It is also possible to discuss about Euler's works for the change of the Second Law being omitted. Euler generalized Newton's work, or, as he himself says in his article (Euler, 1750), formulated a new principle of mechanics. Euler's major role in this endeavor, since he gathered several pieces of knowledge and tools for the production of Analytical Mechanics, is not mentioned today in the studies in the area. He was the one who introduced the differential and integral formalism, the Cartesian coordinates, the use of functions, unified the concepts of forces, which were confusing in Newton; (Sitko, 2019a), and, thus, he was able to generalize the Newtonian idea to a great variety of problems. Without the use of a historically contextualized approach, the construction of $F = ma$ becomes vague and confusing.

Also in this item, the exclusively analytical conception when presenting the formula $F = ma$ without its contextualization in textbooks -which can prevent the students from understanding the content and lead them to having a false image of how such knowledge was produced- can be clearly observed (Sitko, 2019a). This view does not care about how knowledge was produced, only about its product.

7. *Image of socially neutral science*: This view forgets the Science, Technology and Society relations. According to it, scientists are neither neutral, do not take sides, do not have interests, nor are affected by the world around them (Gil-Pérez et al., 2001).

In the first edition of the *Principia*, Newton refers to Hooke, who also postulated principles of planetary attraction, despite their disagreements. After the edition, Hooke informed Halley about his intention to take the credit for the invention of the law of the inverse square (Westfall, 1995). Newton reports his anger through letters sent to Halley, since he believed that such law did not belong to Hooke, as already mentioned in this text. Thus, in the final version of the book, Newton withdrew all the references he had previously made to Hooke.

Therefore, due to their disagreements, even having received contributions from Hooke, Newton omitted them out of anger. Scientists are not neutral, as they are affected by the world around them.

Moreover, there is also a controversy regarding the invention of the Differential and Integral Calculus between Newton and Leibniz (Dhombres, 2014). Because of this feud, Newton would never use Leibniz's notes, which were later used by Euler (according to Sitko 2019b) to formulate the modern form of the Second Law. If scientists were really neutral, maybe Newton would have used them and, maybe, would have got closer to $F = ma$ (for more details on the elements used by Euler for the construction of $F = ma$, see Sitko 2019b).

As to Euler, he was studying the Second Law with the intention of winning a contest of the French Academy of Science and because of all the problems and needs of the naval engineering of that time. The interest was monetary. Nobody goes to a laboratory or

searches about an object without a particular interest, an idea *a priori* of what is desired. Scientists are not neutral, and, as a result, Science is not, either.

Another conception that we can fit in the neutral science is the idea that it is geared on behalf of technology to improve our quality of life. Science is developed from problems and these problems always originate from interests; this process may lead to the development of technologies in favor of humanity, but this does not mean that science works towards this end.

FINAL CONSIDERATIONS

The construction of Newton's Second Law did not take place during the fall of an apple, not even inside Newton's office at a moment of creative insight. Scientific knowledge is produced over time, it relies on the help of a collective, and not on isolated geniuses, and it must face crises, replacements and expansions. After Newton, more than sixty years of research were required for the formulation of "his" law.

Knowledge is not free from values or influences, as it was observed in Hooke and Leibniz's disagreements; it is not infallible, and every production is a response to a question. In Newton's case, the Laws of Motion are part of a work concerned about explaining the motions of the celestial bodies in orbit.

This work was based on the article *Para uma imagem não deformada do trabalho científico* ("For an undistorted view of the scientific work") (Gil-Pérez et al., 2001), using the historical episode of the construction of $F = ma$ to exemplify the distorted views presented in the work by Gil-Pérez et al. 2001. It is intended to show that the teaching of Newton's Laws, more specifically the Second Law, just as other topics, may present distorted views of the scientific work in the face of an unproblematic approach, exclusively analytical, proposed by an infallible genius.

Moreover, the wording of the Second Law, as presented in the *Principia*, and which is practically the same as the one presented in textbooks, followed by the formulation $F = ma$, does not provide a direct relationship, and, for the student to be able to understand the relationship between the original wording and the Eulerian formula, the historical approach is necessary.

Therefore, as Matthews (2015) explains, besides providing a greater meaning in relation to the content at issue, the use of History and Philosophy of Science in the teaching of Science is believed to be able to offer the humanization of the scientific work, bringing Science close to the students, showing that scientists are common people, just as anyone else. Thus, we believe that from historical approaches the students will be more interested in the study of Science, making it easier for them to understand how scientific knowledge is produced.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article, as no new data were created or analysed in this study.

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