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Students' Thinking Process When Using Abductive Reasoning in Problem Solving

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ABSTRACT

Background: Abductive reasoning is the process of making conjectures to explain surprising observations. Although this conjecture is not certain to be true, in solving a problem, this reasoning is very helpful to determine the best solution strategy. **Objectives**: The study aims to investigate whether all types of abductive reasoning lead to the formation of new schemes. **Design**: This research used a qualitative approach with a descriptive exploratory design. **Setting and Participants**: A total of 41 students of the research degree in mathematics education programme were involved in solving a task. Then, eight of them were chosen for an in-depth interview, representing the undercoded and overcoded abductive reasoning types. **Data collection and analysis**: The data collected were the results of the students' works and task-based interviews. Piaget's schema theory was used to analyse students' thinking processes using abductive reasoning. The analysis was carried out at all steps of problem solving, namely understanding the problem, devising a plan, carrying out the plan, and looking back. **Results**: Those who carried out overcoded abductive reasoning used this reasoning to determine problem solving strategies. Meanwhile, those who carried out undercoded abductive reasoning used it to determine problem solving strategies as well as to form new schemes. **Conclusions**: The results showed that students who did abductive reasoning did not always produce new schemes. This study also notes that the truth value of answers from the application of abductive reasoning in problem solving was open and the importance of the look back step to perform accommodation.

Keywords: thinking process, abductive, reasoning, problem solving, schema.

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Processo de pensamento dos alunos ao usar o raciocínio abdutivo na solução de problemas

RESUMO

Contexto: O raciocínio abdutivo é o processo de fazer conjecturas para explicar observações surpreendentes. Embora essa conjectura possa não ser verdadeira, na solução de um problema, esse raciocínio é muito útil para determinar a melhor estratégia de solução. **Objetivos**: O estudo tem como objetivo investigar se todos os tipos de raciocínio abdutivo levam à formação de novos esquemas. **Design**: Esta pesquisa utilizou uma abordagem qualitativa, com desenho exploratório descritivo. **Ambiente e participantes**: Um total de 41 alunos do programa de bacharelado em matemática estava envolvido na solução de uma tarefa e, em seguida, oito deles foram escolhidos para uma entrevista em profundidade, representando os tipos de raciocínio abdutivo com código insuficiente e com código excessivo. **Coleta e análise de dados**: os dados coletados foram os resultados dos trabalhos dos alunos e entrevistas baseadas em tarefas. A teoria de esquemas de Piaget foi usada para analisar os processos de pensamento dos alunos usando o raciocínio abdutivo. A análise foi realizada em todas as etapas da resolução de problemas, como a compreensão do problema, a elaboração de um plano, a execução do plano e a retrospectiva. **Resultados**: Aqueles que executaram o raciocínio abdutivo com código excessivo usaram esse raciocínio para determinar estratégias de solução de problemas. Enquanto isso, aqueles que executavam o raciocínio abdutivo com código insuficiente usavam-no para determinar estratégias de solução de problemas, bem como para formar novos esquemas. **Conclusões**: Os resultados mostraram que os estudantes que fizeram raciocínio abdutivo nem sempre produziram novos esquemas. Este estudo também observa que o valor verdadeiro das respostas da aplicação do raciocínio abdutivo na solução de problemas estava aberto e a importância da retrospectiva para realizar a acomodação.

Palavras-chave: processo de pensamento, abdução, raciocínio, resolução de problemas, esquema.

INTRODUCTION

Problem solving ability is one part of higher-order thinking skills (HOTS) which is very useful for students to have to face their real life's challenges. Yet, the importance of this ability has not been accompanied by findings in reality. This is revealed from the many studies that show the low problem solving abilities of students (Apriyani et al., 2019; Hadi et al., 2018; Mairing, 2017; Munawaroh & Fathani, 2019; Rismen *et al.*, 2020; Rostika & Junita, 2017; Sapitri et al., 2019; Ali Shodikin, 2016). Cifarelli (2016) shows the important role of abductive reasoning in solving mathematical problems.

Norton (2008) has also shown the role of abductive reasoning in solving problems and building new schemes on students' cognitive processes. By building new schemes, students' knowledge will further develop, and the learning process will occur.

Many researchers assume that abductive reasoning takes many roles in the development of science. These roles include building hypotheses (Kwon et al., 2006), generalizing models (Park & Lee, 2016), supporting the induction process (Rivera & Becker, 2007), increasing reasoning ability (Shodikin, 2017), generating new ideas (O'Reilly, 2016), building new schemes (Norton, 2008), solving mathematical problems (Cifarelli, 2016), being the main trigger for mathematical inquiry (Park & Lee, 2018), making claims about the validity of questions (Wu et al., 2016), and diagnosing medical errors (Velázquez-Quesada et al., 2013). Meanwhile, abductive reasoning itself is conjectural reasoning, whose opinions or conclusions are obtained based on incomplete information, where the conjecture itself is characterised as explicit statements that may be "right or wrong" (Norton, 2008). Hence, the conclusion is only a hypothesis, the best guess, based on the knowledge and evidence provided at the time.

Starting with the classification of abductive reasoning by Eco (1983) in the form of overcoded, undercoded, and creative, it needs to be questioned whether all types of reasoning lead to the formation of a new scheme as Norton (2008) research, which used Pierce's general abductive reasoning logic. Eco (1983) explains that in creative abductive reasoning, the rules used to predict a case or fact from observations do not yet exist and need to be created. This implies that if someone uses creative abductive reasoning, it will form a new rule outside the general rules that already exist. In the cognitive structure, the rules that a person already has can be understood as an old scheme that a person has. While the new rules, which are of course different from the previous ones, are a different component of the scheme that can add or change the existing schemes. Adding or changing this scheme will result in a new scheme. Therefore, it has become certain that if a person makes a new rule, he/she certainly involves the formation of a new scheme in the thinking process. This claim corroborates that an investigation of the types of undercoded and overcoded abductive reasoning is sufficient to identify whether all types of abductive lead to the formation of a new scheme or not.

This research focused on the thinking process of students who use the undercoded and overcoded type of abductive reasoning in solving mathematical problems seen from Piaget's schema theory, namely the assimilation process and the accommodation process. The purpose of this study is to investigate whether all types of abductive reasoning lead to the formation of new schemes.

THEORETICAL BACKGROUND

Abductive reasoning is usually understood as the process of seeking an explanation for surprising observations by making a conjecture (Gabbay & Kruse, 2000; Magnani, 2009; Pedemonte, 2007). With abductive reasoning, a person generates new hypotheses to explain the shocking facts that are being considered (Fann, 1970). This hypothesis represents a plausible initial explanation, which is the best explanation in the situation, and is provisional, in the sense that it is open for further exploration. Peirce (1958), as the founder of this reasoning, explicitly states that the form of concluding abductive reasoning is as follows:

The surprising fact, C, is observed;

But if A were true, C would be a matter of course;

Hence, there is reason to suspect that A is true.

The conclusion obtained in this reasoning is considered to be the best explanation in a given context based on plausibility criteria, rather than the probability criteria in deductive conclusions, or the probabilities in the inductive conclusions (Walton, 2014). For example, a doctor finds symptoms of fever and shortness of breath in a patient who has a history of travel to an area infected with the coronavirus. As is generally known, the symptoms exhibited by people with COVID-19 caused by the coronavirus are fever and shortness of breath. This doctor suspects that this patient might be infected with the coronavirus. The doctor's conclusion is still in the form of allegations that need to be followed up through laboratory examinations. Another example that is being discussed hotly in various countries is the search for a drug formula to prevent and treat COVID-19 disease. Facts show that COVID-19 is caused by a virus and symptoms that appear in sufferers include fever and shortness of breath. Considering the causes of the disease and the symptoms that emerge, some researchers suspect that effective drugs to deal with this virus are lopinavir and ritonavir, which are antivirals used to treat SARS and MERS, or chloroquine which is a malaria drug, or remdesivir which is a drug being studied to treat ebola, or some other medicine formula. Thus, some researchers conclude there is reason to suspect that the use of lopinavir and ritonavir are effective for treating COVID-19 caused by viruses and have symptoms in sufferers of fever and shortness of breath. Similar to the first example, the conclusions drawn by some researchers are in the form of allegations that need clinical trials to see the truth of the conclusions drawn. The conclusion of abductive reasoning is also defeasible or can be canceled (Delrieux, 2004). This means that the conclusions can be withdrawn if further investigation of the facts in the case was discussed and showed that other alternative explanations are "better".

As time goes on, the understanding of abductive reasoning is also growing. Eco (1983) shows that the rules needed in abductive reasoning do not always clearly exist and then identify three types of abductive reasoning: overcoded, undercoded, and creative. Overcoded abduction occurs when the arguer only knows one rule that can be used to explain observations. Undercoded abduction occurs when the arguer knows more than one rule that can be used to explain observations. Meanwhile, creative abduction occurs when the arguer knows that the rules that can be used do not yet exist and the arguer constructs the rules him/herself. Pedemonte (2007) and Pedemonte and Reid (2011) combine the concepts of Pierce and Eco's abductive reasoning into the Toulmin argumentation model to distinguish the three types of abductive reasoning that Eco raised. Pedemonte and Reid (2011) introduced the term 'failed undercoded abduction' to explain some cases of students who failed to find the rules used to justify facts. In this case, a student is not able to choose the rules until someone else (the teacher) tells him what rules to choose. Conner et al. (2014), using Toulmin's argumentation model, emphasise abductive, deductive, inductive, and analogical reasoning. Furthermore, Velázquez-Quesada et al. (2013) explain that abductive reasoning is an activity that follows the phase of recognising the existence of abductive problems; identifying candidates for solutions; selecting 'the best' solutions, and assimilating the chosen solution.

Related to cognitive processes, research on the role of abductive reasoning in cognitive processes has also begun to be sought by many researchers. Norton (2008), by combining the concepts of Pierce's abductive reasoning and Piaget's schema theory, has shown the role of abductive reasoning in solving problems and building new schemes in the context of fractions. Magnani (2015, 2016) built an eco-cognitive model on abductive reasoning and illustrated the importance of analytical methods in argumentation. Cognitive processes themselves occur because of the nature of a person who adapts to the stimulus he/she gets from his/her scheme (mental structure). The cognitive process is assimilation, if the stimulus interpretation uses an existing scheme and is accommodation, if the stimulus interpretation needs to build a new scheme (Piaget, 1950). With the formation of a new scheme, the goal of achieving cognitive balance will be realised. Cognitive processes can also be understood as thinking processes. The thinking process is a mental activity in the form of matching, combining, exchanging, and sorting concepts, perceptions, and previous experiences that are used to help formulate or solve problem, make decisions, and gain understanding (Ruggiero, 2012). A person's thinking process is influenced by the analogy that is built (Shodikin et al., 2019).The ability to think of someone is very influential in the ability to make decisions and solve problems (Viandari, 2013). Therefore, observing a person's thinking process is very important in solving mathematical problems (Sudirman et al., 2015). Observing the thinking process of someone who experiences abductive reasoning in problem solving is included in this case.

METHODOLOGY

This research used a qualitative approach with a descriptive exploratory design. This design was chosen because the researcher wanted to obtain authentic, deep, and detailed data about the thinking processes of students who experienced abductive reasoning in solving problems. Through a qualitative approach, all facts, both in spoken and written forms gotten from observable sources and other related documents were explained as they were, then reviewed and presented as concisely as possible to answer the research questions.

The subjects of this study were 41 students (14 males and 27 females) from 3 different classes at one private university in Lamongan, Indonesia. Participants involved in this study were mathematics education students who were or already taking the capita selecta mathematics course for high school. This subject was chosen because in this course the students are provided with basic skills to solve mathematical problems at the high school level, including problems in the form of mathematical modelling. Besides, they already have previous experience with all the material taught in high school.

The researchers have developed a modelling problem related to the systems of linear equations in two variables (SLETV) and proportion material. The material was chosen because a lot of content can be developed into a contextual mathematical problem. Also, the procedures used to solve problems may vary, enabling subjects to develop their abductive reasoning. The questions developed is as follows:

> A pond will be full within 4 hours if it is filled with water with 2 large pumps and 1 small pump simultaneously. In the same way, the pool will be full within 4 hours if it is filled with water with 1 large pump and 3 small pumps. How much time does it take to fill the same pool, if 4 big pumps and 4 small pumps are used together?

This problem can be solved by using the SLETV or inverse proportion approach. Also, it can be solved by the water debit concept approach. This problem links the relationship between the number of pumps used to fill a pool with the total time needed to fill the pool fully. Two initial conditions indicate the use of two types of pumps (large and small) with the time required. Next, students are asked to calculate the time for other conditions given. Based on this information, a claim can be made that (1) a large pump can fill more water than a small pump in the same time; (2) the time taken by a large pump to fill the pool fully is shorter than that of a small pump; and (3) the more pumps used the shorter the time needed, assuming that the same type of pump is used.

For the task, students were given 40 minutes. Then, based on the variation of answers, the uniqueness of their answers, and their communication skills, seven students were selected for an in-depth interview to explore their thinking process when solving the problem given. The unique answers chosen by the researchers were the answers that used non-proportional methods to solve the problem of proportions, the use of unusual proportional relationships, and the mismatch between questions and answers were written. On average, researchers needed about 20-30 minutes to interview a student. After that, we analysed the results of the interviews to classify whether the students used abductive reasoning to solve the given problem, what kind of abductive reasoning, and how the thinking process resolved the problem.

In this study, the data was obtained from the students’ works and task-based interviews. The data were collected to describe the students’ thinking processes when doing abductive reasoning in solving mathematical problems. The students’ works and interviews were conducted in Indonesian. The interviews were then transcribed and translated into English without changing the conversation content. Based on the interviews, they were grouped to see whether the subjects carried out abductive reasoning or not. The first and foremost thing to identify whether someone is doing the abductive reasoning is by recognising the existence of an abductive problem. In this case, someone acknowledges that there is a mismatch of information obtained from the results of his/her observations with his/her prior knowledge, giving rise to oddities, surprises, or doubts. Then, people make guesses about reasonable ways to solve problems and implement them. The number of reasonable ways a person has is a consideration for grouping them into the types of abductive reasoning that is carried out. This grouping was based on the indicators of abductive reasoning that researchers developed from the phase of abductive reasoning by Velázquez-Quesada et al. (2013), which is shown in Table 1.

Table 1

Indicators of abductive reasoning activities

Abductive reasoning activities	**Indicators**
Recognizing the existence of an abductive problem	Acknowledging the incompatibility of information obtained with the prior knowledge (there are doubts, surprises, oddities).
Identifying candidates for solutions	a. Mentioning the alleged alternative solutions that can explain plausible solutions and which may be taken to answer the problem based on experience.
	b. Able to explain the mismatch of information obtained
Selecting 'the best' solutions	a. Choosing a particular solution from the provided alternatives solutions
	b. Explaining the reason why choosing that solution as the best solution
Assimilating those chosen	Implementing the chosen solution to overcome the problem

These indicators were used to capture the occurrence of abductive reasoning by the students in solving the given problem. Abductive reasoning that occurred was further classified by its type, namely, overcoded and undercoded.

Then, the researchers analysed the thinking processes of students who performed abductive reasoning based on the Piaget thinking process framework as shown in Table 2.

Table 2

Indicators of thinking processes

Thinking process	Indicators
Assimilation	Integrating perception, concept, or a new experience into an existing scheme in mind
Accommodation	a. Modifying an existing scheme to match a given stimulus b. Form a new scheme that corresponds to the given stimulus

The analysis process was done by following the Polya problem solving stages as shown in Table 3. It aims to see the problem solving process of the students.

Table 3

Indicators of problem solving stages (Polya, 1973)

Problem solving stage	**Indicators**
Understand the	The subject can understand what is known and asked

problem	in the given problem
Devise a plan	The subject can determine ways/methods/formulas that can be used to solve the given problem.
Carry out the plan	The subject can use ways/formulas/methods that have been planned to solve the problem given
Look back	The subject corrects the answers that were given to ensure the correctness of the answers

Based on the collected data, by taking into account the variations of answers, uniqueness of answers, and communication skills, from 41 research subjects, seven students were chosen to take part in in-depth interviews. Furthermore, researchers chose two of the seven students to take part in the interview to discuss the thinking process that occurred. The reason for choosing these two subjects was based on the representation of the type of abductive reasoning that was done and the representation of the thinking processes that occurred in other subjects. Furthermore, these two subjects were given the codes S1 and S2.

RESULTS AND ANALISES

The Thinking Process of Students Who Perfomed Overcoded Abduction

In the process of understanding the problem, as in the first stage of problem solving, S1 feels that he has never studied the material being questioned in the test. Then, S1 compares the information obtained from the question with his prior knowledge. S1 found an oddity (surprise) by a mismatch of information on the problem to be solved with what was known beforehand. It showed that S1 recognizes the existence of abductive problems as the initial stage of abductive reasoning. To solve the problem, S1 identifies the information from the question to find a suitable way to solve and how a mathematical model can represent the problem situation given. From this stage, S1 assumes that there are two variables to be represented, namely the large pumps and the small pumps. These two variables were used by S1 to try to connect the problem in this test with the problem in the systems of linear equations in two variables (SLETV). Dialog 1 noticed this stage.

Dialog 1:

R: “Have you ever studied this material?”

S1: “Not yet. That's just what I thought. I suppose the large pump as “x”, and the small pump as “y”.”

R: “What material do you think is appropriate for this problem?”

S1: “This one is a system of linear equations.”

R: “Why do you consider this to be a problem in a system of linear equations?”

S1: “Because I found two variables, x and y. Then I look for the y by using elimination, then later I substitute it, that is. So I find the x and y values.”

R: “So, it means that this problem relates to SLETV material, huh?”

S1: “Yes, it does.”

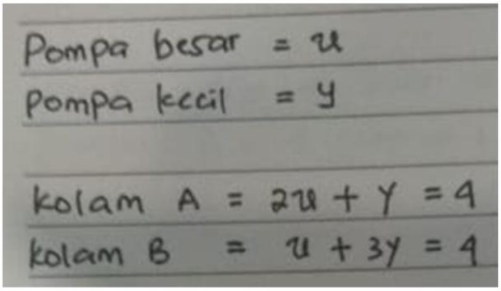
R: “Not the others?”

S1: “No.”

Dialog 1 noted that S1 did the two initial stages of problem solving, namely understanding the problem and making up a solution. S1 then adjusted the thinking process by giving the name for each variable, i.e., x for the large pump and y for the small pump which then is linked to the previously owned scheme which is the SLETV problem. In this case, S1 has carried out the assimilation process. Dialog 1 also noticed that in making a problem solving plan, S1 chose SLETV as the best way to represent the problem situation because he thought that if he found 2 variables, then he met the SLETV problem. This process indicated that S1 was doing assimilation. It can be seen in Figure 1.

Figure 1.

Mathematical modeling by S1



Pompa besar = x
Pompa kecil = y

kolam A = 2x + y = 4
kolam B = x + 3y = 4

Translate:
Large pump = x
Small pump = y

Pool A = 2x + y = 4
Pool B = x + 3y = 4

To construct the mathematical modelling, S1 did the abductive reasoning that can be stated as follows:

> *There is information about the problem that surprised the subject, but this problem contains two variables;*
>
> *If this problem is truly a SLETV problem, then this problem contains two variables;*
>
> *So, there is a reason to suppose that this problem is truly a SLETV problem.*

In this case, S1 chose a SLETV as the most possible way to explain the problem, then the mathematical model that was built was adjusted to the form of a mathematical model in a SLETV. Likewise in determining the method of solving the problem used is the most likely the method used in solving the SLETV problem, namely by substitution and elimination. It can be seen from the following Dialog 2.

> *Dialog 2:*
>
> *R: "What is your reason to choose this method?"*
>
> *S1: "Because I've divided into x and y. So, the most possible way to find x and y is to use elimination and substitution."*
>
> *R: "So, the most possible method is to use elimination and substitution, right?"*
>
> *S1: "Yes."*

Dialog 2 told that the selection of solution was based on the scheme that S1 had before, which is a SLETV using the method of elimination and substitution. In this case, S1 has carried out the assimilation process.

In the third stage, specifically, implementing the plan, S1, by the elimination and substitution method got 9.6 hours as the final result of the time needed to fill the pool with four large pumps and four small pumps as can be seen in Figure 2. The interview showed that S1 did not have another alternative method that could be used to explain the problem given. It aims to see more the types of abductive reasoning done by S1. It can be seen from the following Dialog 3.

Dialog 3:

R: "Do you think any other method to solve this problem?"

S1: "I don't know, I haven't tried the other method. Because it was just elimination and substitution that I just tried."

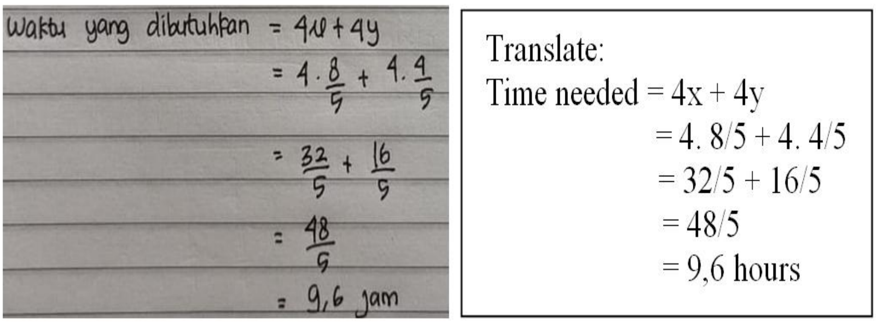
R: "You do not know the other method?"

S1: "No, I don't. I only know this method."

In this case, S1 experienced overcoded abduction because he only knows one rule that can be used to explain the observations which, in this case, are the SLETV rules as a strategy to solve the problems.

Figure 2

S1's answer



In the stage of looking back, S1 initially admitted not validating the answers shortly after completing the problem. In this case, S1 did not realise that the answer he got was the wrong answer. It can easily be said that this answer is the wrong answer just by looking at the final result and what is known, and emphasises the claim that "the more pumps used the shorter the time required". Furthermore, S1 is welcome to look back at the answer and he considers that the answer is correct. This can be seen from the following Dialog 4.

Dialog 4:

R: "Have you double-checked your answer?"

S1: "Not yet."

R: "Please, double-check."

S1: (Double-checking). "If I write x = 8/5 and y = 4/5 to equation 1, the result is correct, 4."

R: "So, is your answer correct?"

S1: Yes."

Dialogue 4 indicated that the process of looking back of S1 was limited to the validation of the finding from the mathematical model, not to the validation of the real problem being asked. The assumption of a correct answer was obtained only by substitution of the finding into one equation without reconsidering what was known from the real problem. In this case, the minimum process of looking back made it did not lead to new cognitive conflicts that allow for accommodation and the formation of new schemes. In Dialog 5 it is shown that S1 only realised his mistake after getting stimulus from the researcher.

Dialog 5:

R: "What do you think, if we pump more, the time will ....?"

S1: "Be less"

R: "Now, check your answer!"

S1: "My finding showed that the more pumps, the more time is needed."

R: "Why?"

S1: (Double-checking the answer). "Oh, my bad, it must be less time."

Based on Dialog 5, it seems that S1 has just realized that the answer is not by his initial understanding of the comparison of the number of pumps used to the time needed after being given a stimulus by researchers. S1 was trapped in the procedural process in solving the SLETV problem and did partial evaluations.

The Thinking Process of Students Who Performed Undercoded Abduction

The presentation of the analysis of students who did undercoded abductive reasoning was carried out in three episodes. Each episode represents one answer generated by the student. The order of episode numbering and data presentation is based on the thinking process carried out by the subjects when solving problems, not on the appearance of data at the time of the interview. The order of the interview is actually episode 2, episode 1, and episode 3.

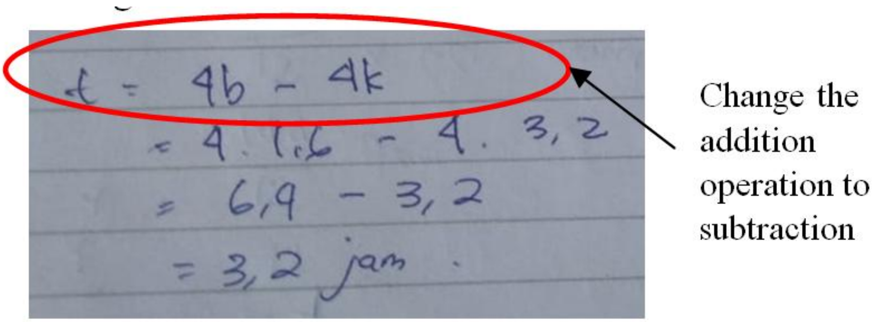
> *Episode 1*
>
> In the process of understanding the problem, S2 initially assumed that this problem was only the SLETV problem and worked on a thinking structure similar to S1. This is seen in Dialog 6 below.
>
> *Dialog 6:*
>
> *R: "Why do you think that this method will solve the problem?"*
>
> *S2: "I initially do not use this method. When I calculate, the time found is not shorter. Then, I rethink."*
>
> *R: "What did you use first?"*
>
> *S2: "The first value 'b' and 'k' was found in the same way. But for those which were asked directly I added, I did not make '1/t'. The point is the first one uses a comparative value, but the second one, inverse proportions."*

Based on Dialogue 6, as in the case of S1, abductive reasoning played a role in determining mathematical objects or procedures that were suitable for describing problem situations and methods that could be used to solve problems. However, in this case, the mathematical object or procedure was still in the form of conjecture. Abductive reasoning that occurred in S2 episodes was the same as the abductive reasoning that occurred in S1.

In determining the value of “t” (time), it should be obtained from the equation “t = 4b + 4k” as done by S1. However, S2 did a different thing. S2 did not use the "4b + 4k" addition operation, but replaced the “4b - 4k” subtraction operation. This was due to fulfilling his understanding of the problem given that “the more pumps used the shorter the time needed to fill a pond”. If he continued to use the addition operation, you would get the value of “t”, which was longer. This result was contrary to his understanding. Then, S2 modified his scheme to obtain a value by changing operations on the scheme from addition operations to subtraction. This can be seen in Figure 5.

Figure 3

S2’s answer in Episode 1



S2 changed this operation to obtain a faster time value compared to the conditions known in the problem. In this case, S2 had carried out the accommodation process (modifying existing schemes) in the thinking process.

Furthermore, in the stage of looking back, S2 thought that the answer was wrong, because the difference of time obtained was only a little compared to the time under known conditions, even though the difference in the use of the pump was large. This results in disequilibrium in S2 cognition. This is seen in Dialog 7 below.

Dialog 7:

R: “You think that your first method is wrong?”

S2: “Yes, because there is only a little time difference. If it is used 1 large pump and 3 small pumps, it only takes 4 hours, *but with 4 big pumps and 4 small pumps, why it only reduced a little?"*

In Dialogue 7, the disequilibrium forced S2 to consider alternative ways to explain the structure of the problem. Gathering new facts that this problem was also related to volume and time leads to the assumption that this problem was also related to the problem of water debit. Furthermore, S2 modified the scheme by combining the concepts of the SLETV and water debit as explained in episode 2. In this case, it can be seen that the stage of looking back at the problem solving stage becomes an important stage which is a turning point for someone to modify their scheme based on the results obtained.

Episode 2

In the process of understanding the problem, after reading the questions, S2 realised that he had studied the material given. He remembered what he knows and compares it with what information was given in the problem. S2 found the similarity of information provided, but there was also a discrepancy with what was known beforehand. This can be seen from the following Dialog 8.

Dialog 8:

R: "Have you learned about this material?"

S2: "Usually it is only one variable, Sir. For example, the first one uses a large pump, the second one uses a small pump. And if you use both, how do you do that? Then if this one directly uses two large pump variables and a small pump together. That's the difference. I have done this kind before."

Based on Dialog 8, S2 realised that the structure of the problem faced was more complex than the structure of thinking he had. In this case, S2 got surprising information which was an indication of abductive problems. To adjust to the stimulus in the form of incompatibility of the structure of the problem with the scheme owned, S2 tried to form a new scheme that matches the given stimulus. In this case, S2 did the accommodation process in the thinking process.

During the stage of making a completion plan, S2 experienced a disequilibrium about what needs to be done first. S2 looked for the facts of the problem and the knowledge he already had to identify possible solutions for the problem. S2 obtained several facts that led him to the assumption that the problem was related to the SLETV and water debit material he had learned. This can be seen from the following Dialog 9.

> *Dialog 9:*
>
> *R: "So, you have learned about this material?"*
>
> *S2: "I relate this with water debit material, Sir. The formula is the volume divided by the time."*
>
> *R: "What about the material?"*
>
> *S2: "I am not sure, Sir, perhaps SLETV."*
>
> *R: "What did you remember about SLETV?"*
>
> *S2: "A system means there is more than one equation, and then there are two variables."*
>
> *R: "So, in your understanding, the problem has two variables, too?"*
>
> *S2: "It is similar sir. It is usually likes that. I got confused, Sir."*

Based on Dialog 9, the finding of more than one equation and the existence of two variables in this test item was used by S2 as the basis for assuming that this problem was a problem of the systems of linear equation in two variables (SLETV). In addition, because it was related to volume and time, S2 suspected that this problem was also a matter of water debit. These claims were used by S2 to form a new scheme that matches with the stimulus provided. The reasons that occurred in episode 2 was as follows:

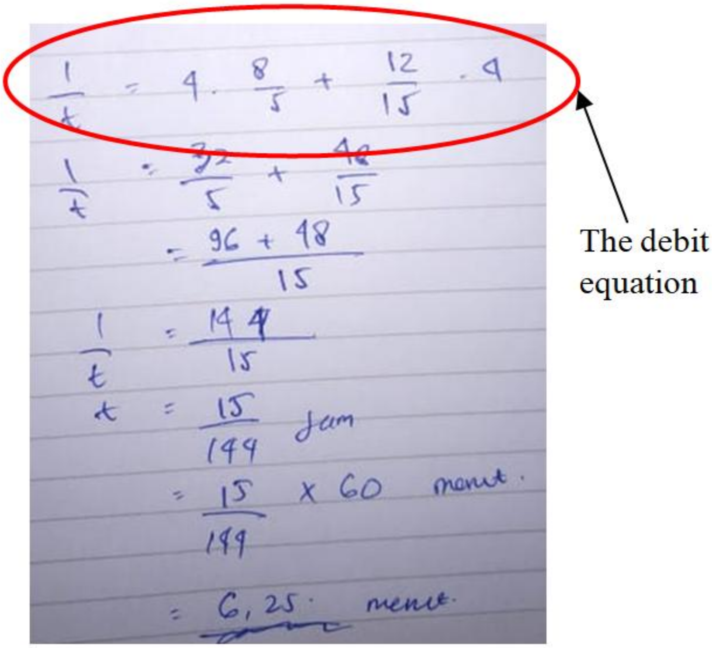
> *Information from the questions contains two variables and is related to volume and time;*
>
> *If this problem is truly a SLETV problem, then this problem contains two variables;*
>
> *If this problem is truly a water debit problem, then this problem is related to volume and time;*
>
> *So, there is reason to suppose that is true this problem is a combination of SLETV and debit problems.*

In this case, it appears how abductive reasoning plays a role in encouraging the formation of new schemes in the stages of making problem solving plans, especially in determining mathematical objects or procedures that are suitable for describing problem situations.

At the stage of carrying out the plan, S2 resolved this problem by finding out the value of variables “b” and “k” using the elimination and substitution. These “b” and “k” values were obtained from the equation system: “2b + k = 4” and “b + 3k = 4”, which represent the two conditions in the problem. Furthermore, these values were used to calculate the time required by four large pumps and four small pumps to fill the pond with the water debit equation as shown in Figure 4.

Figure 4

S2’s answer in Episode 2



It can be seen from Figure 4 that to determine the time (t), S2 used a comparison on the water debit problem, where the debit (d) is equal to the volume (V) per unit time (t) or can be expressed as d = V / t. In this case, S2 considered the volume to be “1” so that the water debit equation is obtained in Figure 4 in the first row. In this way, S2 got a shorter time value than the previous results.

At the looking back stage, S2 thought that his answer was correct that shown in Dialog 10.

Dialog 10:

R: “Are you sure that the method you used is correct?”

S2: “Yes.”

R: “Why do you think this is correct whencompared to the previous one?’

S2: “Because the results are logic, and the time gotten is reduced.”

R: “What’s your logic?”

S2: “Because I relate it with the Water debit case, the volume divided by time.”

From Dialogue 10, S2 was sure that his answer was correct because the time he got was less than the time known. Besides, the scheme modifications made also appear to increase S2”s confidence in the correctness of the answers.

The S2 s answer in episode 2 is still wrong. Then, the researcher gave scaffolding through some questions as shown in Dialog 11.

Dialog 11:

R: ‘Please see and compare the value of ‘b’ and ‘k’!”

S2: “What do you mean, Sir?”

R: “Which one is faster to fill, the big or small pump?”

S2: “The big pump”

R: “So, the value of b compare to k, must be greater or smaller?”

S2: “Smaller”

R: “Check your answer!”

S2: “Oh, God, it’s greater. It must be wrong.”

R: "Why do you think so?"

*S2: "Time needed for the big pump should have been shorter. (*thinking). But, wait, in this equation, I regard this as Water debit, Sir, so b is the speed, not time. So, the speed of pump b is 8/5 per hour. Why I change so easily, huff?!"*

R: "Are you sure?"

S2: "Not really, Sir."

S2 supposes "b" as the duration of one large pump in filling the pond fully and "k" as the duration of one small pump in filling the pond fully. Based on Dialogue 11, after scaffolding, S2 obtained the fact that the results obtained did not match with his understanding. This gives rise to confusion and uncertainty about the answer. This fact causes a new disequilibrium in S2 cognition. These results force him to think harder to find reasons to justify the answer, but to no avail.

Episode 3

In episode 3, at the stage of making a settlement plan, the impasse in seeking justification for previous answers led S2 to use other problem solving strategies. By using the basis that "the more pumps used, the shorter the time required," S2 assumed that this problem can be solved using an inverse proportion. It can be seen in Dialog 12.

Dialog 12

R: "How can you explain this method?"

S2: "Using an inverse proportion."

R: "Why do you do so?"

S2: "Because the more pumps used the shorter the time required."

Based on Dialog 12, the process of making an assumption is a process of abductive reasoning. Abductive reasoning that occurs is:

Information on the question "the more pumps used, the shorter the time required";

If this problem is true inverse proportion, there is an inverse relationship between the value of one variable and the value of another variable;

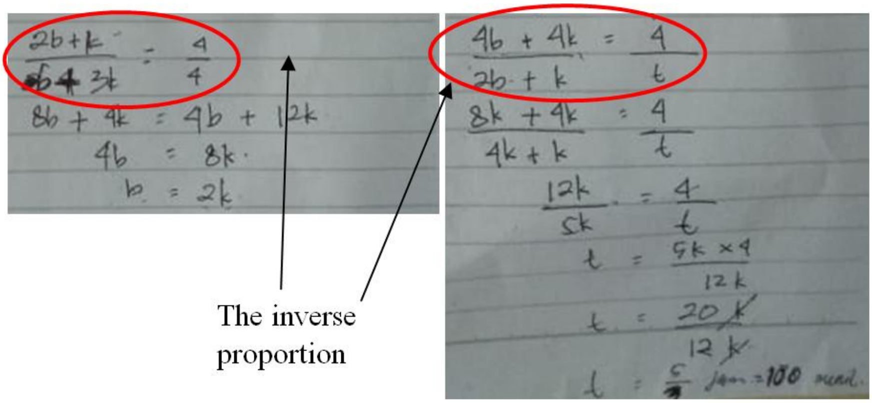
So, there is reason to suppose that this problem is indeed an inverse proportion.

Abductive reasoning which was done by S2 did not cause a modification of the scheme by combining the concept of inverse proportion with other concepts as happened in episode 2. It means that S2 chose to use another scheme rather than modifying an existing scheme as the problem solving strategy. This showed that the abductive reasoning that occurred is the type of undercoded because the choice of strategy (solution) was taken from several alternative strategies.

At the stage of implementing the plan, S2 compared the values of “b” and “k” by utilising the first 2 conditions. This got the value of "b = 2k". This relationship was then used by S2 to calculate the value of “t” by stating the condition “4b + 4k” in the form of the variable “k”. This was shown in Figure 5.

Figure 5

S2’s answer in Episode 3



The problem solving strategy shown in Figure 5 led S2 to the correct answer. This result also fulfills all conditions given to the questions and fulfills the initial understanding of the S2 after a look back phase.

Someone will always tend to maintain balance and adapt to their environment, including their logic. If there is an external stimulus, there will be an imbalance in one”s cognition. Subanji (2015) states that this curiosity shows a cognitive imbalance called disequilibration. To achieve balance again, there was an adaptation process. In the process of adaptation, a person experiences two cognitive processes, namely assimilation and accommodation. These two processes are different based on the use of the scheme they have. When dealing with a scheme, if the perceived results do not match the expected results, people may experience disturbances, which can result in modifications to the scheme (Steffe, 1991; Steffe & Thompson, 2000). In an effort to achieve the desired balance, a person will use all the potential he/she has including the ability of reasoning. In problem solving, students are faced with challenges that lead to curiosity to solve. This challenge will be the motivation for anyone to go into a balance called equilibrium.

In solving mathematical problems, different abductive reasoning will lead to different cognitive processes. Based on the case of S1, he experienced overcoded abductive reasoning in solving problems because he only has one alternative solution to explain the problem situation. Consequently, the construction of problems built up by this type of abductive reasoning will be limited only to the schemes he has or the knowledge he has known before. Meanwhile, someone who uses overcoded abductive reasoning often makes guesses that are automatic or semi-automatic. It makes this reasoning less likely to support productive mathematical inquiry (Bellucci, 2018; Eco, 1983; Park & Lee, 2018). Therefore, someone who experiences overcoded abductive reasoning will tend to do the process of assimilation that does not produce a new scheme. These results have answered doubts that not all types of abductive reasoning always lead to the formation of schemes.

However, it is possible for someone who experiences overcoded abductive reasoning to carry out the accommodation process, especially in the operations in the scheme. As in the case of S2 episode 1, if someone stops at the first look back stage and is satisfied with the results of the episode, then overcoded abductive reasoning may also occur. Surely, with a condition that the person only knows one rule to be used to explain observations as a strategy for solving problems. An accommodation that occurs is limited to operations that exist in such schemes known as functional accommodation (Norton, 2008; Steffe, 1991; Steffe & Thompson, 2000).

Meanwhile, in the undercoded abductive reasoning, as in the case of S2 episode 2, the dissatisfaction of the results obtained in episode 1 caused disequilibrium, which led to the formation of a new scheme by combining two contexts, i.e., the systems of linear equation in two variables (SLETV), which is the result of episode 1 abductive reasoning and the problem of water debit, which is the result of abductive reasoning after paying attention to the facts of the problem. S2 realises that the problems faced are more complex than has been previously imagined. The structure of the problem is more complex than the structure of thinking that someone has will encourage him/her to form a new scheme. Initially, S2 experienced difficulties in the construction process due to problems in the process of assimilation or accommodation. To perform assimilation, there is no suitable scheme according to the problem at hand. However, for accommodations, i.e., amending old schemes or forming new schemes, S2 is still experiencing difficulties because he does not yet have enough schemes that can be used to create new schemes. In this case, the process of breaking down the problems into smaller parts is needed. The process of breaking down complex problem structures into smaller parts is called an analytic process. From the analytic process of this problem, S2 gained a new fact that this problem was related to the volume and time that prompted him to perform abductive reasoning which resulted in the suspicion that this problem was also a matter of water debit. Glasersfeld (2001) explains that abductive reasoning as accommodation that helps stimulate and compose new actions and appear in accommodation action schemes at the sensorimotor level as well as in the next level of concrete and formal mental operations. Furthermore, the problem that has been decomposed is used for restructuring, linking between thinking components and forming new, more complex schemes, namely the merging of the SLETV and water debit concepts. This is where the accommodation process is called metamorphic accommodation (Norton, 2008; Steffe, 1991; Steffe & Thompson, 2000). One might also choose to use other schemes in dealing with disequilibrium caused by dissatisfaction with the results obtained before rather than modifying the existing scheme. This happens to S2 episode 3, in which he chooses to change the problem solving strategy by looking for another strategy rather than modifying the scheme used earlier in episode 1 or episode 2. In this case, it means that S2 has revised its abductive reasoning. Park and Kim (2017) and Park and Lee (2018) told that the revision of abductive reasoning also occurs in sample generalisation and modelling. The result may be a modification to the recognition template and is called generalising assimilation, even though it is also an accommodation (Norton, 2008; Steffe, 2002).

One interesting point to highlight is the overcoded abductive reasoning done by S1 that leads to incorrect answers. It cannot be concluded, though, that certain types of abductive reasoning will tend to lead to wrong results. Overcoded abductive reasoning also has the opportunity to lead someone to the correct answer, for example, in the study of Pedemonte and Reid (2011). Therefore, as the conclusions generated by abductive reasoning, the results by using this reasoning, in solving problems are also open, which may be right or wrong.

Another important thing that cannot be left out is the importance of the look back stage that contributed to the formation of new schemes in determining problem solving strategies. This stage encourages the modification of previous abductive reasoning. This is seen in the move between episode 1 to episode 2 and episode 2 to episode 3 by S2. This modification of abductive reasoning occurs because the problem solver was dissatisfied with the results obtained from the application of the strategy chosen previously. These results also show the defeasible nature of the conclusions obtained from abductive reasoning that can be withdrawn if further investigation provides a better alternative explanation. Velázquez-Quesada et al. (2013) added that abductive solutions must be integrated into segmented information to produce knowledge. This knowledge is then taken into consideration in the selection of alternative problem solving strategies. The look back stage itself is not only limited to the internal problem solver process to investigate because of its desires but also includes processes that are caused by external stimuli, such as scaffolding and teacher intervention. As the strategy changes made by S2 in period 2 to period 3 show how scaffolding led him to modify his abductive reasoning to obtain a better problem solving strategy. Furthermore, Pedemonte and Reid (2011) show that the teacher"s role is also very important in helping students choose rules that are useful for solving problems. Here, teacher intervention changes from undercoded abductive reasoning to overcoded abductive reasoning.

CONCLUSIONS

Three important points to conclude in this research are, first, different abductive reasoning leads to different cognitive processes. Related to forming schemes, overcoded abductive reasoning tends not to produce new schemes, undercoded abductive reasoning tends to produce new schemes, and creative abductive reasoning certainly produces new schemes. Second, the answers to the application of abductive reasoning in solving problems are open, as the nature of conclusions obtained from abductive reasoning, which may be right or wrong. This confirms that the use of certain types of abductive reasoning does not necessarily lead to the correct answer. Third, the important role of the look back stage in building new schemes and modifying abductive reasoning in determining problem solving strategies.

AUTHORS' CONTRIBUTIONS STATEMENTS

A.S. and P. formulated the presented idea and developed the theory. A.S. and S.B. collected and analysed the data. S.D. adapted the methodology and composed the instrument. All authors actively participated in the discussion of the results, reviewed, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting this study will be made available by the corresponding author (SB), upon reasonable request.

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