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Knowledge of Numbers and Geometry of pre-service teachers in the Primary Education Degree

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ABSTRACT

Background: The study of the initial mathematical knowledge of primary education teachers in training is considered important, since this knowledge influences that of the students. **Objectives:** to study the initial knowledge of numbers and geometry of the pre-service teachers. In addition, some of the mistakes they make are analysed. **Design:** to carry out the study, 20 questions released from the TIMSS tests were used, specifically, arithmetic (numbers) and geometry questions were selected. **Setting and Participants** the TIMSS-type test was applied to 97 first-year pre-service teachers. **Data collection and analysis:** This research is quantitative and the sample used in this investigation is a purposive sample, participants answered a questionnaire with questions the knowledge of mathematical content in numbers and geometry. **Results:** the results indicate that, in general, the students present greater difficulties in the geometry questions. Specifically, 36% of students fail geometry, while 14% fail in the case of numbers. In problem-type questions, which belong to the cognitive domains of application and reasoning, a high percentage of errors and blank answers are obtained. Finally, in the study of errors, the difficulties that students have with the decimal numbering system are shown. It is also observed that they present difficulties in the matter of geometric measurements (areas, perimeters and volumes). **Conclusions:** In this research, on the one hand, weaknesses that can be reinforced by means of the mathematics subjects of the Degree have been detected. On the other hand, errors about the decimal numbering system should be a warning indicator to try to improve its understanding during the Primary Education stage, which is when it is introduced.

Keywords: Pre-service teachers; Primary Education; Geometry; Numbers

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Conhecimento dos Números e Geometria dos futuros professores do Ensino Primário

RESUMO

Contexto: O estudo dos conhecimentos matemáticos iniciais dos professores do ensino primário em formação é considerado importante, uma vez que estes conhecimentos influenciam os dos alunos. **Objetivos:** estudar o conhecimento inicial dos números e da geometria dos formação de professores. Além disso, alguns dos erros que cometem são analisados. **Design**: para abordar o estudo, foram selecionadas 20 questões libertadas dos testes TIMSS, especificamente, aritmética (números) e questões de geometria. **Ambiente e participantes:** O teste TIMSS foi aplicado a 97 alunos do primeiro ano do ensino primário. **Coleta e análise de dados**: Esta pesquisa é quantitativa e a amostra utilizada nesta investigação é uma amostra intencional, os participantes responderam a um questionário com questões de conhecimento do conteúdo matemático em números e geometria. **Resultados:** Os resultados indicam que, em geral, os estudantes têm mais dificuldades com questões de geometria. Especificamente, 36% dos estudantes falharam em geometria, enquanto 14% falharam em números. Nas perguntas do tipo problema, que pertencem aos domínios cognitivos de aplicação e raciocínio, obtém-se uma elevada percentagem de erros e respostas em branco. Finalmente, no estudo dos erros, são mostradas as dificuldades que os estudantes têm com o sistema de numeração decimal. Observa-se também que apresentam dificuldades em matéria de medidas geométricas (áreas, perímetros e volumes). **Conclusões:** Nesta investigação, por um lado, foram detectadas fraquezas que podem ser reforçadas através das disciplinas matemáticas do grau. Por outro lado, os erros sobre o sistema de numeração decimal devem ser um indicador de aviso para tentar melhorar a sua compreensão durante a fase do Ensino Primário, quando este é introduzido.

Palavras-chave: Formação de professores; Ensino primário; Geometria; Números

INTRODUCTION

The study of the initial training of Primary school teachers is essential, since the knowledge of the teachers influences that of the students. In fact, there is a series of investigations that evaluate the knowledge of pre-service teachers (Turnuklu & Yesildere, 2007; Muir & Livy, 2012; Livy, Muir, & Maher, 2012; Lacasa and Rodríguez, 2013; Alpízar & Alfaro, 2019). In addition, previous studies have pointed out the importance of analysing the errors and difficulties that future teachers present. Undoubtedly, this analysis is considered the main source for diagnosing learning difficulties and knowing the type of reasoning

used by students (Rico, 1998; Salinas, 2007; Socas, 2007; Brodie, 2014; Utomo et al., 2018).

Besides, an aspect that is considered important in the teaching-learning process is the analysis of errors and difficulties that students present when solving certain mathematics tests. As Socas (2007) comments, research carried out in recent years has shown the importance of focusing not only on the correct responses of students, but also on the mistakes they make. The error will have different origins, but it will always be considered as an inadequate cognitive scheme and not only as a consequence of a lack of knowledge or an absent-mindedness. These errors and difficulties are, at times, so profound that they question the entire teaching-learning process of mathematics (Nortes & Nortes, 2016). Some authors recognize that the development of problems with decimals is an important source of learning difficulties with students and teachers in training (e.g., Stacey et al., 2001; Ubuz & Yayan, 2010). Therefore, it is common for students to make repeated errors in problems with decimals.

It is clear that for a teacher to be successful in the classroom it is necessary that they have didactic knowledge of the area, but without any doubt they must master the content of the subject. It is important that in the teaching process they do not make mistakes since these can be transmitted to their students. Future teachers in their training must know mathematical content in order to develop their role as mathematics teachers (Nortes & Nortes, 2017). Similarly, for Ball, Thames and Phelps (2008), the teacher's own understanding of the content is essential for teaching.

The objective of this work is to study and evaluate the initial knowledge of mathematics that first-year students of the Primary Education Degree possess and detect errors in order to analyse them in detail. The idea is to study the weaknesses that students present and try to reinforce the corresponding contents and procedures during the studies of the Primary Education Degree. Unlike the majority of previous works, which focus on tests of basic competencies corresponding to an autonomous community, in this research an international test was chosen. Specifically, to carry out the research, 20 questions of the TIMSS 2011 test were selected from the content domains of numbers and geometry.

The rest of the article is structured as follows. First, the literature review is presented. The second part presents the methodology that will be followed to achieve the objective. Next, the instrument and the procedure to be followed to carry out the investigation are introduced. After, the results obtained are presented. The article ends with the discussion, together with some conclusions.

LITERATURE REVIEW

In general, pre-service teachers recognize the importance of mathematics, its teaching and the difficulty of becoming a good teacher of the subject and value it in their training at the same level as the other subjects (Ruiz de Gauna, García & Sarasua, 2013). However, previous research has verified that the pre-service teachers have deficiencies in the knowledge acquired in the early stages of study (Salinas, 2007; Barrera, Infante & Liñán, 2013; Nortes & Nortes, 2018). It is important to mention that difficulties and errors in learning mathematics are today a focus of study and research in future teachers (Socas, 2007). Some of this research is discussed below.

The most important international study in pre-service teacher was the TEDM-S (Teacher Education Study in Mathematics) tests. The study was carried out during the years 2006-2010 for pre-service teachers in primary and secondary education training with the participation of 17 countries. Spain participated in the study on the training of primary school teachers and obtained 481 points in mathematical knowledge, below the average (500 points). These results put in evidence the deficiencies in the knowledge of mathematical contents and the knowledge of the didactics of the students to teacher, which puts on alert the preparation of the initial teacher training.

It is important to mention the research by Lacasa and Rodríguez (2013) on the TEDM-S tests, where they indicated that the results are not positive in the tests of mathematical knowledge and mathematics didactics in comparison with neighbouring countries. According to Egido and López (2013), pre-service teachers are installed in compliance with certain minimums, and they do not seem to find sufficient incentives to attract better students or to differentiate themselves from the rest through an improvement in the quality of both their content, as of the practicum.

In the same way, in his research, Salinas (2007) stated that the pre-service teachers do not master the contents related to school mathematics, in the sense of remembering knowledge acquired in the early stages of teaching. Salinas discusses the importance of place value understanding to know and understand our decimal numbering system and operations. Also, the author found that students have knowledge gaps and conceptual errors in mathematical content that should have been acquired in the first years of primary education.

In their study, Turnuklu and Yesildere (2007) investigated the knowledge of mathematics and the knowledge of the teaching of mathematics

of pre-service teachers. The authors found a connection between knowledge of mathematics and knowledge of mathematics teaching. It is suggested that candidates for primary school mathematics teachers should be educated both in the aspects of mathematical knowledge and knowledge of pedagogical content.

Regarding the study of errors and difficulties, Rico (1998) clearly showed that, from their errors, a young person or a child can learn different properties of a concept of which they were not previously aware. By making a mistake, the student expresses the incompleteness of his knowledge and allows his classmates or the teacher to help him complete the additional knowledge or lead him to understand for himself what was wrong. Thus, mistakes can contribute positively to the learning process. It is also necessary to indicate that the errors arise in a consistent conceptual framework, based on previously acquired knowledge.

Livy et al. (2012), in their research, analysed the mathematical concepts of area and perimeter of teachers in training. The authors state that many pre-service teachers in all cohorts have a procedural understanding of area and perimeter, exhibited similar misconceptions to their student counterparts, and had a limited ability to demonstrate examples of the mathematical knowledge required to teach these subjects.

In their research, Nortes and Nortes (2016) analysed errors and difficulties that pre-service teachers have when solving elementary math problems. Concretely, they study how they develop the Mathematics test for entry into the 2013 Body of Primary Teachers of the Community of Madrid, of contents corresponding to 6th grade of Primary. Most of the errors are due to poor learning of facts, skills and previous concepts produced by misused data, lack of verification of the solution and calculation errors. The percentage of error exceeds 50% in all courses, with half of the students failing the test.

In a more recent study, González and Eudave (2018) analysed the common knowledge of mathematical content about fractions and decimals of students for primary school teachers. Among the main results, it stands out that, for the most part, future teachers have greater difficulty in solving problems that involve the use of fractions than those that involve decimals.

Moreover, Utomo et al. (2018) used some questions from TIMSS 2011 (Trends in International Mathematics and Science Study). The TIMSS objective questions were modified to essay questions, to later analyse the types of errors made by the students. The most frequent errors are carelessness in reading and not using all the available data.

METHODOLOGY

Participants

The focus of this research is quantitative. The sample used in this investigation is a purposive sample (Patton, 2002). The population under study corresponds to the first year students of the Primary Education Degree at the Rovira and Virgili University in the 2018/2019 academic year. Specifically, the test was applied to 97 students, representing 71% of the total population enrolled in the first year. For the sample, the test was applied to all students who attended the first day of class. The test was applied to a total of 97 students, representing 71% of the total population enrolled in the first year. In this study, 57% are women and 43% are men. It is important to note that the students of the Primary Education Degree are not undergoing specific training as a mathematics teacher, but that they all receive the same training in this subject, on a mandatory basis.

Participation in this test was optional and completely anonymous. Considering these two aspects, it was not necessary to have the consent of the research participants. Therefore, *Acta Scientiae* is explicitly exempted from comprehensive assistance and eventual compensation for damages caused to any of the research participants.

Instrument

The instrument used to carry out the research gathers information on the knowledge of mathematical content in numbers and geometry. These two content domains were selected since previous studies have pointed out the importance of learning arithmetic (numbers) (Castro, Gorgorió & Prat, 2015) and geometry (Torregrosa, Quesada & Penalva, 2010) in the mathematical training of future teachers.

Measuring teacher knowledge is a complex task and there is little consensus on how it should be done (Ball, Lubienski & Mewborn, 2001). From the beginning, the scheme and procedures of the TIMSS study were consolidated as valid for assessing mathematical knowledge (Gutiérrez et al., 2016). For this reason, and also to consider international statements, the test was prepared from the questions released from TIMSS 2011. Considering the Mathematical Content Domain of Secondary Compulsory Education (ESO), 20 questions were selected: 12 questions from the Number Content Domain (Q1-

Q12), representing the 60% of the test, and 8 questions from the Geometry Content Domain (Q13-Q20), representing the 40% of it. It should be highlighted that eleven questions were posed to force the students to write the process they followed to answer the question. These items will be referred as problem questions (Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q17, Q18, Q19 and Q20). The other questions maintain the structure of the TIMSS test and will be referred as objective questions. This TIMSS type test was used in the investigation of (Segarra & Julià, 2021).

The details of the used test are presented in the Table 1 of Appendix 1.

Procedure

The students who attended the first day of classes were given 40 minutes to answer the 20 questions of the test. Then, we proceeded to generate a database with all the information provided by the test. In order to grade the test, the following details were set: the objective questions are scored 0 or 1, depending on whether they were correct or incorrect. In the case of the problem type questions, the procedure of solving the questions is also taken into account in addition to the answers. Specifically, the score is 0 if both the answer and procedure are incorrect, 0.5 if the answer or procedure is correct, and 1 if both are correct. The test is scored on a scale from 0 up to 10. All calculations of the descriptive and inferential statistics were performed using the R programming language. The graphs were generated through Microsoft Excel and R programming language.

To determine the validity of the test, the Exploratory Factor Analysis method was used. In particular, the Kaiser-Meyer-Olkin (KMO) test and the Bartlett Sphericity test (BTS) (Bartlett, 1950; Kaiser, 1974) (KMO = 0.64, $p < 0.001$) were used. Besides, the KMO test indicates the adequacy of the sample size used. Moreover, Bartlett's sphericity test (BTS) indicates that the correlations between the elements is not an identity matrix. The extracted factors explain 65% of the total variance of the data. The correlations between the corrected elements of the scale vary 0.32 and 0.62. These values indicate that questions should not be deleted and that the test is valid. Additionally, to determine the reliability of the results obtained, the internal consistency was analysed, Cronbach's alpha test was applied (Cronbach, 1951). The alpha coefficient obtained (α-Cronbach) is $\alpha = 0.79$ (acceptable, according to the criteria proposed by George and Mallery (2003).

RESULTS AND ANALYSIS

In this section, the results obtained are analysed in detail, considering different aspects. Specifically:

- In the first part, the results obtained in the test are analysed and interpreted by content domains of numbers and geometry; grading scale for students classified by suspense, approved, notable, and excellent; and for each one of the problem type questions separated into correct, blank and incorrect.
- In a second part, the errors made by students in problem-type questions are analysed in detail. Specifically, general errors and errors about the decimal numbering system are analysed.

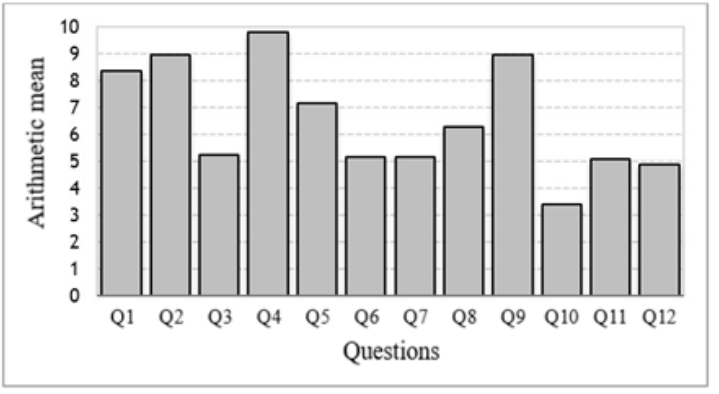
Analysis and interpretation of results

Results by content domain

In the knowledge and skills test used in this research, a global arithmetic mean of 6.14 and a standard deviation of 1.76 were obtained.

Figure 1

Results questions of numbers



This subsection compares results obtained in numbers and geometry content domains. Figure 1 shows the mean obtained in each question of the numbers content domain. If the scores corresponding to the 12 numbers content

questions are taken, a mean of 6.54 and a standard deviation of 1.65 are obtained (the mean is higher than the global one, 6.14).

Figure 1 shows that the question with the highest score is Q4, with a mean of 9.79. The questions with the lowest scores are Q10 and Q12, with a mean of 3.4 and 4.9, respectively.

According to TIMSS 2011 (INEE, 2012b), question Q4 is in the content domain of numbers and the cognitive domain of applying. Within the content domain of numbers, it is associated with the topic of fractions and decimals. The objective of the question is to represent and operate with fractions and decimals, using models (number lines), and to identify and use these representations.

Question Q10 corresponds to the cognitive domain of applying and it is associated with the topics ratio, proportion, and percentage. The goal is to identify and find equivalent ratios, model a given situation using a ratio.

Besides, question P12 is classified within the cognitive domain of reasoning and is on the subject of organization and representation of data. The goal is to read scales and data from tables, pictograms, bar charts, pie charts, and line charts. In addition, this question is associated with intuiting what is the pattern followed by the values represented by the graphs.

Lastly, question Q12 is classified within the cognitive domain of reasoning and is on the topic of organization and representation of data. The goal is to read scales and data from tables, pictograms, bar charts, pie charts, and line charts. Additionally, this question is associated with deducing the pattern that the values represent in the given graphs.

Similarly, Figure 2 illustrates the mean obtained in each of the questions in the geometry content domain. The mean obtained in this domain is 5.54 and the standard deviation is 2.57. Recall that the mean and standard deviation obtained by taking all the questions are 6.14 and 1.76, respectively.

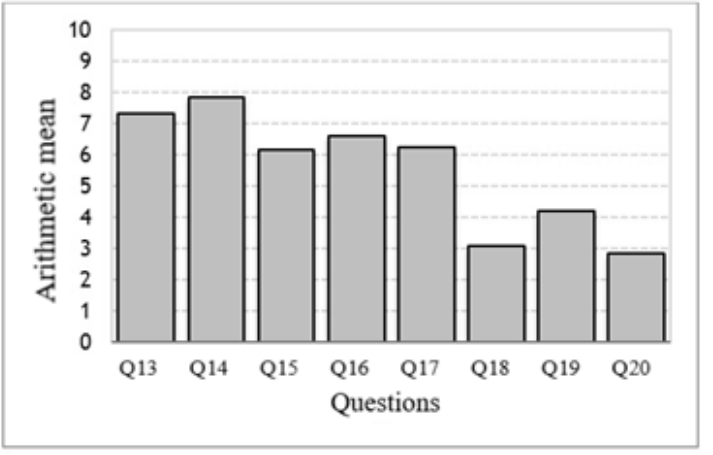
In Figure 2, the question with the highest score is Q14, with an average of 7.84. The questions with the lowest scores are Q18, Q19 and Q20, with a mean of 3.09, 4.23 and 2.84, respectively.

Question Q14 is in the applying cognitive domain. It belongs to the theme of geometric shapes and spatial reasoning. The goal is to recognize geometric properties in two dimensions or three dimensions forms, including linear and rotational symmetry. Both Q18 and Q19 are from the cognitive domain of applying and question Q20 is from reasoning. These questions refer

to the topic of geometric measurements and are associated with the objective of selecting and using appropriate measurement formulas for perimeters, circumferences, areas, surfaces, and volumes.

Figure 2

Results of geometry content domain questions



Score range

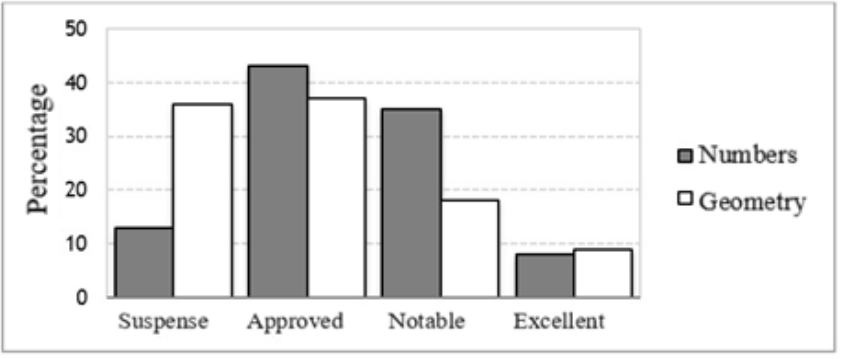
In this subsection, the obtained scores in the test are classified in suspense (0 - 4.9), approved (5 - 6.9), notable (7 - 8.9) and excellent (9 - 10). Figure 3 shows the percentage of pre-service teachers that obtain each of these scales.

As shown in Figure 3, there is a considerable percentage of students at the exceptionally low learning level (suspended): 14% in numbers content and 36% in geometry content, a remarkably higher number.

At the approved level, we have 43% in numbers content compared to 37% in geometry content, and in notable level, we get 35% in numbers content and 18% in geometry content. Notice that at this level there is a considerable difference between the two domains. Finally, the excellent level has 8% in numbers content and 9% in geometry content.

Figure 3

Percentage of pre-teachers obtaining each scale.



Results of correct, blank and erroneous questions

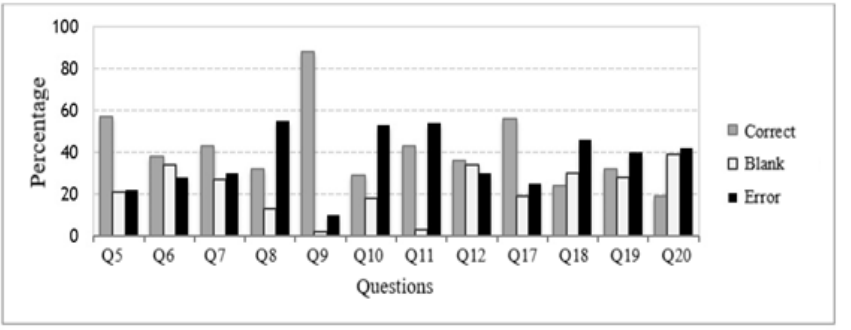
This section analyses the answers that the students have provided in the case of problem type questions. Specifically, Figure 4 shows, for each question, the percentage of students who answered it well, the ones who left it blank and the ones that made mistakes.

In Figure 4, it is observe that the question with the greatest difficulty in the number domain is Q10: only 29% of the students answer it correctly, 53% make some kind of error and 18% of students let it blank. This question belongs to the cognitive domain of applying. In contrast, Q9 is the question with the highest percentage of students who answer it well, with 88% of correct answers, 10% of erroneous answers and only 2% of blank answers. This question is in the cognitive domain of knowledge.

In the case of the geometry content domain, the most difficult question for the students is Q20, in the reasoning cognitive domain: 19% answered it correctly, 42% made mistakes, and 39% of students leave it blank. Question Q17, on the other hand, is the question with the highest percentage of correct answers, with 56% of correct answers, 25% of erroneous answers and 19% of blank answers. This question is in the cognitive domain of knowledge.

Figure 4

Percentage of correct, blank and error answers in each of the questions



Moreover, it can be seen that the percentage of students who did not include the procedure of the problems in the resolution is very high, especially in the following questions: Q6 and Q12 (34%), Q7 (27%), Q18 (30%), Q19 (28%) and Q20 (39%). It is also important to highlight that some questions have a high error rate: Q6 (28%), Q7 and Q12 (30%), Q8 (55%), Q10 (53%), Q11 (54%), Q18 (46%), Q19 (40%) and Q20 (42%). Lastly, notice that the percentage of corrected answers is very low in some questions. Specifically, the ones with less than 40% of corrected answers are the following: Q6 (38%), Q8 (32%), Q10 (29%), Q12 (36%), Q18 (24%), Q19 (32%) and Q20 (19%).

Error analysis

General errors

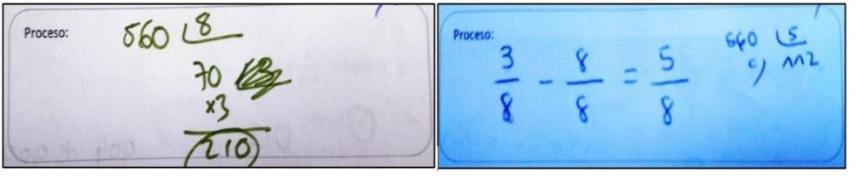
This section studies in detail the errors made by the students in some of the problem type questions. Specifically, to carry out that study, the problem type questions with an arithmetic mean of less than 5 in an interval from 0 to 10 are considered. In particular, in the number content domain, the questions correspond to Q10 and Q12, and in the Geometry content domain, they correspond to Q18, PQ9 and Q20.

Each of the questions and examples of the common mistakes made by the students are shown below. Specifically, for each studied error, the percentage of students who make it is detail.

Q10 (Ana and Jenny divide 560 euros between them. If Jenny gets 3/8 of the money, how many Euros will Ana have? See the complete test in Appendix 1). The error shown in Figure 5 (left) is due to a lack of understanding of the problem and/or incorrect reasoning. The 25% of students who make mistakes in this question present this difficulty. The students calculate the money obtained from Jenny and not from Ana. It is observed that they perform the calculation of 3/8 of 560 obtaining a result of 210. Hence, the process is incomplete since the subtraction of 560-210 = 350 is missing. The 75% of students make errors in which the non-understanding of fractions is identified. See Figure 5 (right) as example. Moreover, it is also detected that the students have not carried out a final check to validate if they have really responded what is requested.

Figure 5

Examples of errors in question P10.



Question Q12 (The graph shows the sales of two types of soda for 4 years. If sales trends continue for the next 10 years, determine the year in which the sales of Guinda Cola will be equal to the sales of Limón Cola. See Appendix 1 for details). In this case, 34% of the students the students leave the question blank; and in 30% of the incorrect answers, the pre-service teachers do not include a correct reasoning. The wrong answers are diverse, most are meaningless and do not provide important information. Therefore, no example was added.

Q18 (The area of a square is 144cm^2. What is the perimeter of the square? See Appendix 1 for details). The 34% of students make mistakes like the one shown in Figure 6 (left). This error occurs because they do not know the formula for the area of the square. Notice that to calculate the length of the side of the square, the students divide by 2 instead of calculating the square root. The value of the side length of the square wrongly calculated by the students is 72cm and the perimeter is 288cm. The 53% of students make errors following

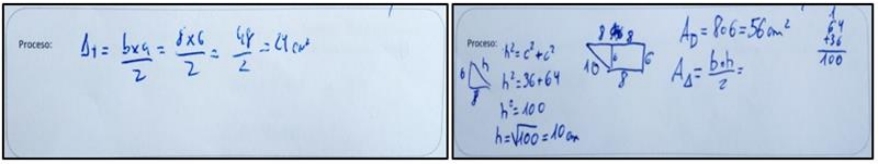
the logic shown in Figure 6 (right). It is observed that they do calculate the square root of 144, but they do not multiply the result by 4, leaving 12 as the result, which represents the length of the side of the square. The remaining 13% of students present remarkably diverse errors, evidencing the lack of knowledge of the formulas and the process to be carried out. It is manifested that they do not have a clear notion of the area nor of the perimeter.

Figure 6

Examples of errors in question P18.

Figure 7

Examples of errors in question Q19.



Q19 (In the Figure 7, what is the area in cm of the shaded region? See Appendix 1 for details). The 30% of the students calculate the area of the unshaded region, as shown in Figure 7 (left). It seems that they do not understand correctly the problem statement or that they do not know how to solve it. The 20% of student try to calculate the shaded area, as shown in Figure 7 (right), but the students do not know the formulas to reach the correct solution.

Q20 (Raúl is packing books in a rectangular box. All books are the same size. What is the largest number of books that will fit in the box? See Appendix 1 for details). Some of the procedures given by the students are shown in Figure 8.

The errors shown in Figure 8 evidence that students do not understand the problem or do not know how to reach the solution. It should be noted that, unlike other questions, this requires an effort of reasoning. A percentage of 39% of the students leave this question blank. Others try to solve it, but few succeed. In fact, in this question the students present a great diversity of errors. For example, in the case of Figure 8 (left), the procedure used by the student shows that he/she is not considering the organization of the books: he leaves spaces in the box. In the procedure shown in Figure 8 (right), the student makes adequate reasoning, but he/she does not correctly master the basic operations and this prevents him/her from reaching the correct solution.

Figure 8

Examples of errors in question Q20

Errors about the decimal number system

This subsection shows examples of errors that students present in considerably basic concepts and procedures. There exist important deficiencies when manipulating the numbers, showing little control over the properties of the decimal number system. Below are some of these errors, grouped by similarity. Figure 9 shows the first group of errors which are due to difficulties in understanding the positional value of the digit.

In the first example (Figure 9 (1)), although whole tens are subtracted, the result does not correspond to whole tens (280 - 70 gives 209). The other three examples correspond to sums with decimal numbers. In Example 2, the student adds a 0 to the tenths place. In Example 3, the student adds the decimal part and the integer part separately. Finally, in Example 4, the student does not add the numbers correctly. Recall that both the positional value of the digits and the decimal numbers are contents that are taught in Primary Education.

Figure 9

Basic errors 1 (positional value of the digits.

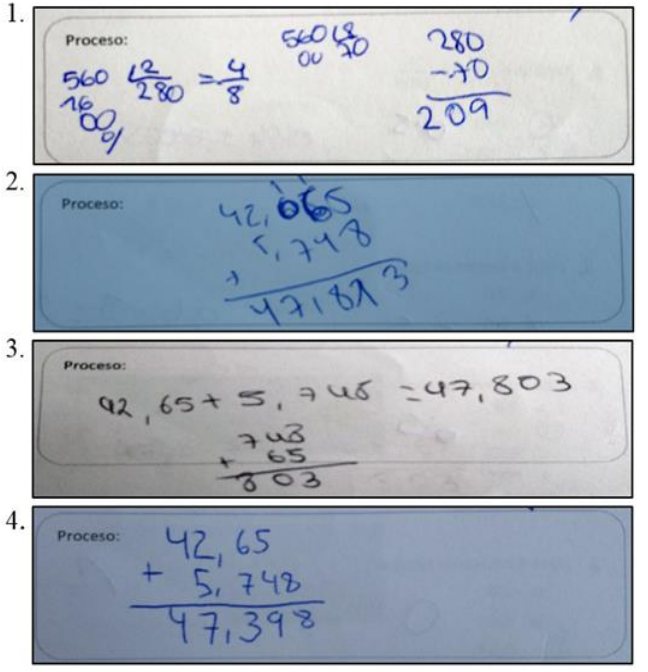


Figure 10

Basic errors 2 (Multiplication tables)

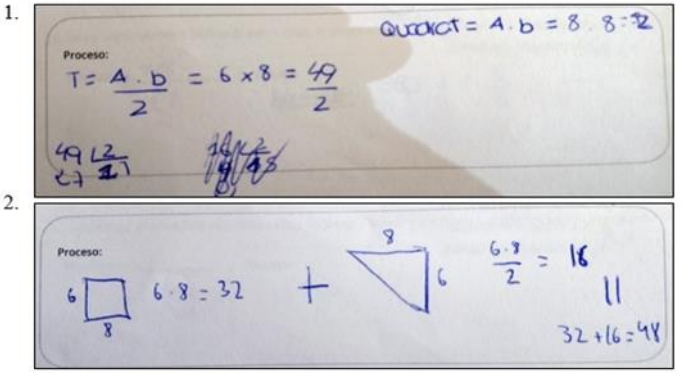
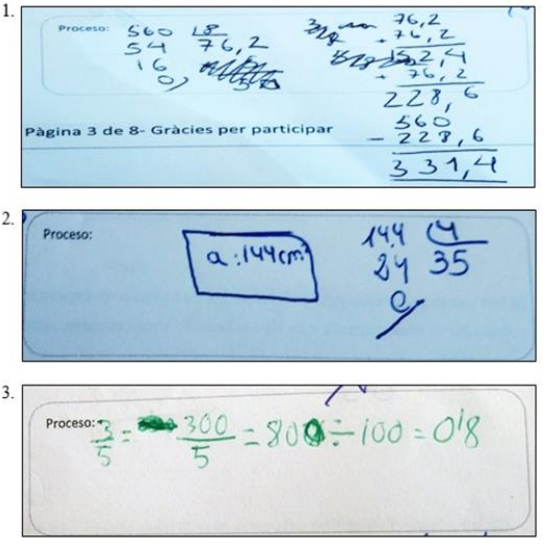


Figure 10 shows examples where students make an error when computing a product between two quantities. It is interesting to highlight the case in which the student responds that 6 × 8 is 49, an odd number. This fact shows that this student does not understand some of the basic properties of numbers (the product of two even numbers cannot result in an odd number). As in the case of the previous basic errors (Figure 10), these types of properties are taught in Primary Education.

Figure 11

Basic errors 3 (Division algorithm)



The examples of errors in the Figure 11 show that students present difficulties in developing the division algorithm (examples 1 and 2) and conceptual errors in the properties of decimal numbers (example 3).

CONCLUSIONS

In this research, the initial knowledge of numbers and geometry possessed by first-year students of the Primary Education Degree was studied. In addition, the errors and difficulties presented by the students were detected and analysed in detail.

The first part, the results were analysed by content domain: in numbers an average of 6.54 is obtained and in 16% of the questions, the average is less than 5. In the case of geometry, the average obtained is 5.54 and in 38% of the questions, the average is less than 5. Subsequently, the level of performance of the students classified as suspense, approved, notable and excellent was analysed. There is a concern about the percentage of students who fail: 14% in numbers and 36% in geometry. In this research it is evidenced that students have greater difficulty in geometry than in numbers. These results are in agreement with those presented in the literature. These results are in agreement with those presented in the literature (Barrera et al., 2013; Nortes 2017).

Next, the results were analysed for each of the problem type questions separated into correct, blank and error. In numbers, in 6 of the 8 questions 50% correct answers are not reached, while in geometry, it is not reached in 3 of the 4 questions. Also, in 3 number questions, more than 50% of the students make some kind of mistake. In 3 of the geometry questions, more than 40% of the students make some kind of mistake. These results are alarming since the test is composed of contents that are learned in Primary Education. This situation is also evidenced in other investigations of pre-service teachers, such as in Muir and Livy (2012), where it is shown that students from grade to teacher start the grade without having the mathematical competencies that they are supposed to have afterwards to attend Obligatory Education.

In the second part, some of the mistakes made by the students in the problem-type questions are analysed in detail. First, the errors made by some students in questions with an arithmetic mean less than 5 are analysed. Students have difficulty remembering the formulas for the area and perimeter of the square and rectangle. Similarly, they also do not know the formula for the volume of a prism. We agree with the research by Livy et al. (2012), where the authors indicate that the students present deficiencies in solving problems of the area and perimeter.

Finally, the errors about the decimal numbering system are analysed. Ma (1999) highlights the concerns about some aspects of the content knowledge practice of teachers, specifically, these concerns extend to the knowledge of decimals. In this investigation it is verified that some students make mistakes in basic operations. Specifically, students make mistakes in addition, multiplication, and the division algorithm. It is important to note that some studies have shown that limited knowledge of the decimal system affects the ability of pre-service teachers to identify errors in students' thinking and apply appropriate teaching approaches (e.g., Maher & Muir, 2011).

The results of this research indicate that the questions with the highest percentage unanswered, those with a mean lower than 5 and those with a high percentage of error are from the cognitive application and reasoning domains. Students better understand the questions corresponding to the cognitive domain of knowledge. In addition, a group of errors is identified that are due to the fact of working exclusively with numbers digit by digit and not considering them globally. In most cases, the errors show a lack of knowledge of the basic properties of the decimal system. In others, it is evident that some algorithms, such as that of division, are forgotten due to lack of use. Multiplication tables are also forgotten.

Another important result that this study highlights are a lack of understanding in some specific topics, in the content domain of numbers: reading and interpreting data from bar graphs and identifying the pattern that the graph follows; and in the subjects of ratio, proportion and percentage. In geometry, in the subject of geometric measurement, the weakness that students present is in the application of formulas for areas, surface and volume.

We think that in order to improve the quality of teacher training and, consequently, the quality of mathematics teaching in the Primary stage, it would be interesting to apply a diagnostic test at the beginning of the course of the first mathematics subject in the Grade Primary education. In fact, it is what has been done in this research. Then it would be necessary to analyze and discuss the errors and difficulties that students present in this test during the subject of teaching and learning mathematics. We agree with Rico (1998) in the fact that a young person can learn from his mistakes, since they allow classmates or the teacher to help him complete the additional knowledge. Based on the errors and difficulties presented by the students in this investigation, it would be necessary to analyse the contents of the subject of teaching and learning mathematics in order to reinforce some contents and include others.

In a future investigation, the test should be applied to students of all Primary Education Degree courses, in order to verify if the difficulties and errors made by students decrease and if their mathematical knowledge improved throughout the courses.

AUTHORS' CONTRIBUTIONS STATEMENTS

JS and CJ actively participated in the development of the theory, the methodology, discussion of the results, reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that supports the results of this study will be made available by the corresponding author J.S., upon reasonable request.

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APPENDIX 1

Table 1

Instrument TIMMS

Q1. Which of these shows how 36 can be expressed as a product of prime factor?

a. $6 \cdot 6$ b. $4 \cdot 9$ c. $4 \cdot 3 \cdot 3$ d. $2 \cdot 2 \cdot 3 \cdot 3$

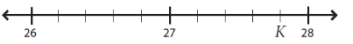
Q2. Which fraction is equivalent to 0.125?

a. $\frac{125}{100}$ b. $\frac{125}{1.000}$ c. $\frac{125}{10.000}$ d. $\frac{125}{100.000}$

Q3. Which shows a correct method for findings $\frac{1}{3} - \frac{1}{4}$?

$\frac{1-1}{4-3}$ b. $\frac{1}{4-3}$ c. $\frac{3-4}{3*4}$ d. $\frac{4-3}{4*3}$

Q4. What number K represents on this number line?



a. 27.4 b. 27.8 c. 27.9 d. 28.2

Q5. Which number is equal to $\frac{3}{5}$?

0.8 b. 0.6 c. 0.53 d. 0.35

Q6. $\frac{4}{100} + \frac{3}{1000}$

a. 0.043 b. 0.1043 c. 0.403 d. 0.43

Q7. The fractions $\frac{4}{14}$ and $\frac{a}{21}$ What is the value of a?

a. a. 6 b. 7 c. 11 d. 14

Q8. A worker cut off $\frac{1}{5}$ of a pipe. The piece he cut off was 3 meters long. How many meters long was the original pipe?

a. 8 b. 12 c. 15 d. 18

Q9. 42.65+5.748=

Answer: _____

Q10. Ann and Jenny divide 560 euros between them. If Jenny gets $\frac{3}{8}$ of money, how many euros will Ann get?
Answer: Answer:_____

Q11. Carla is packing eggs into boxes. Each box holds 6 eggs. She has 94 eggs. What is the smallest

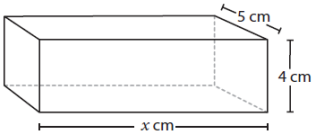
Q12. The graph shows the sales of two types of soft drink over 4 years. If the sales trends continue for the next 10 years, determine the year in which the

number of boxes she needs to pack all the eggs?

Answer:
_____boxes.

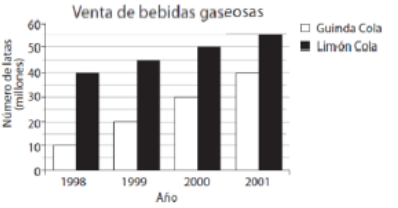
Q13. The length of side of each of the small square represents 1 cm. Draw an isosceles triangle with base of 4 cm and a height of 5 cm.

Q15. The volume of the rectangular box is 200 cm^3. What is the value of x?



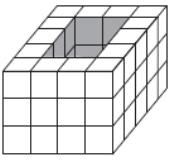
Answer: _____

sales of Cherry Cola will be the same as the sales of Lemon Cola.



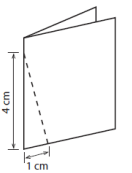
a. 2003 b. 2004 c. 2005 d. 2006

Q14. The figure down shows a shape made up cubes that are all the same size. There is a hole all way through the shape. How many cubes would be needed to fill the hole?



a. 6 b. 12 c. 15 d.18

Q16. A piece of paper in the shape of a rectangle is folded in half as shown in the figure down. It is then cut along the dotted line, and the small piece that is cut is opened. What is the shape of the cut-out figure?



a. an isosceles triangle

b. two isosceles triangles

c. a right triangle

d. an equilateral triangle

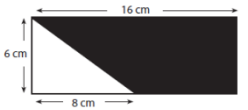
Q17. The perimeter of a square is 36 cm. What is the area of this square?

a. 81 cm^2 b. 36 cm^2 c. 24 cm^2 d. 18 cm^2

Q18. The area of a square is 144 cm^2. What is the perimeter of the square?

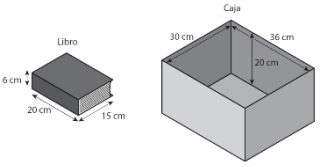
a. 12 cm b. 48 cm c. 288 cm d. 276 cm

Q19. In the figure down, what is the area of the shaded region in cm^2?



a. 24 b. 44 c. 48 d. 72

Q20. Ryan is packing books into a rectangular box. All the books are the same size.



What is the largest number of number of books that will fit inside the box?

Answer: _____

Source: National Institute of Educational Evaluation (INEE, 2012a)