ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6412

What Geometry for teaching? Appropriations of Experts on pedagogical studies mission in Europe, late 19th century

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Received for publication 25 Jan 2021. Accepted after review 19 Apr. 2021
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: The training of teachers who teach mathematics is surrounded by knowledge that changes over time. These changes are for several moments mobilized by specialists, in this context, the official documents are possible historical sources for reading and analyzing changes. **Objective:** To analyze the systematizations of a "Geometry for teaching" put into circulation for the training of teachers, at the end of the 19th century, in Rio de Janeiro, at this time the Republic, by professors Amélia Fernandes da Costa, Luiz Augusto dos Reis and Manoel José Pereira Frazão, appointed by the republican power for the mission of pedagogical studies in Europe. **Data collection and analysis:** Data were collected from the travel records of commissioners, and also from publications in educational, analysed from the perspective of Cultural History. **Design:** Use for the study the official reports found by the documents and Revista Pedagogica, printed for teachers, these documents, acquired or accumulated by the specialists in question, were solved and problematized following the following investigation steps: recompilation of teaching experiences; comparative analysis of teachers' knowledge; analysis of systematization and use of knowledge as knowledge (Valente, 2018; 2020). **Setting and Participant**s: It is a historiographical study in documents. **Results:** An analysis of the sources revealed that the performance of the documents, as well as their active participation in political issues and primary education, the recognition of peers, a call from the State for the pedagogical trip and the consequent responsibility to produce an official report, placed them in the category of *Experts*, thus allowing us to qualify them and put into circulation a proposal for a new "Geometry for teaching", characteristic of intuitive times, and against the previous proposal. **Conclusions:** The "Geometry for teaching" put into circulation, should be mobilized by teachers in close relationship with Design and Handicrafts and was not structured as a subject in primary school.

Keywords: Geometry; Knowledge for use; *Experts*; Pedagogical Mission; International Circulation.

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Que Geometria *"para"* ensinar? Apropriações de Experts em missão de estudos pedagógicos na Europa, final do século XIX

RESUMO

Contexto: A formação de professores que ensinam Matemática está circundada de saberes que se transformam ao longo do tempo. Estas mudanças são por diversos momentos mobilizadas por *Experts*, neste contexto, os documentos oficiais são fontes históricas possíveis para leitura e análises destas modificações. **Objetivo:** Analisar as sistematizações de uma "Geometria *para* ensinar" colocada em circulação para a formação de professores, no final do século XIX, no Rio de Janeiro, neste tempo capital da República, pelos professores Amélia Fernandes da Costa, Luiz Augusto dos Reis e Manoel José Pereira Frazão, nomeados pelo poder republicano para missão de estudos pedagógicos na Europa. **Coleta e análise de dados:** Os dados foram coletados nos registros de viagem dos comissionados, e também em publicações em periódicos pedagógicos, analisados na ótica da História Cultural. **Design:** Utiliza-se para o estudo os relatórios oficiais produzidos pelos docentes e a *Revista Pedagógica*, impressos direcionados aos professores, estes documentos, produzidos ou apropriados pelos *Experts* em questão, foram analisados e problematizados seguindo as seguintes etapas de investigação: recompilação de experiências docentes; análise comparativa de seus conhecimentos; análise da sistematização e utilização dos conhecimentos como saberes. Ambiente e participantes: **Ambiente e Participantes:** Trata-se de um estudo historiográfico em documentos. **Resultados:** A análise das fontes mostrou que o desempenho dos docentes, bem como suas participações ativas nas questões políticas e da instrução primária, o reconhecimento dos pares, a convocação do Estado para a viagem pedagógica e a consequente responsabilidade de produzir um relatório oficial, os enquadrou na categoria de *Experts*, permitindo-nos assim qualificá-los. Colocaram em circulação uma proposta de uma nova "Geometria *para* ensinar", característica dos tempos intuitivos, e em contramão da proposta anterior. **Conclusões:** A "Geometria *para* ensinar" colocada em circulação, deveria ser mobilizada pelos professores em estreita relação com o Desenho e os Trabalhos Manuais e não estava estruturada como uma matéria na escola primária.

Palavras-chave: Geometria; Saberes *para* ensinar; *Experts*; Missão Pedagógica; Circulação Internacional.

INTRODUCTION

The end of the 19th century was a critical period for Brazilian education, times of intense changes and debates, with many updates being introduced in the educational field through, among other forms, international pedagogical trips, i.e., study missions abroad. Those missions were one of the main ways of circulating ideas, a time of appreciation and spread of educational proposals

from several parts of the world (Matasci, 2015).

Moreover, in the last decades of the 19th century, the intuitive teaching method[1], which was part of several educational reforms in the country, gained a reputation. Thus, as it happened in several other countries worldwide, Brazil was inserted in the "internationalisation" moment of the pedagogical discourse.

This study aims to analyse the systematisation of geometric know-how[2] for teacher education put into circulation in Rio de Janeiro, capital of the Republic at the time, by teachers Amélia Fernandes da Costa, Luiz Augusto dos Reis, and Manoel José Pereira Frazão, who, appointed by the Republican power, had recently arrived from a pedagogical mission in Europe,. The question we pose is: what *geometry to teach* did the teachers propose and put into circulation?

To answer that question, the following documentary sources were investigated: *Revista Pedagógica*[3] and the *official pedagogical mission reports* the teachers prepared. Those documents alone, however, do not constitute history and need to be problematised. For this, as proposed by Valente (2018; 2020), the following stages of investigation followed: first, the *recompilation of teaching experiences*, when we selected and separated both information about the pedagogical work registered in the teachers' reports and the excerpts published in *Revista Pedagógica*. After this collection of scattered knowledge, we made a *comparative analysis of the teachers' knowledge*. Here we selected data gathered in the previous stage, refining the collection and enabling us to reveal trends in the settlement of proposals and pedagogical consensus. After these two steps, we *analysed the systematisation* made by Amélia, Luiz Reis, and Frazão *and the use of knowledge as know-how*. In this last stage of the methodological path, we allowed the reading of the information about the teaching experiences, transforming them into teachers' professional know-how, according to the theoretical basis of the research.

[1]Three meanings encompassed the intuitive method: the realisation of objects to understand the abstract; teaching through the senses (see, observe, touch objects) and application of the knowledge to nature, industry, and the student's daily life (Valdemarin, 2004).

[2]Geometric know-how in school culture means geometry arranged for teaching and teacher education in the early school years under different school disciplines (Conceição, 2019).

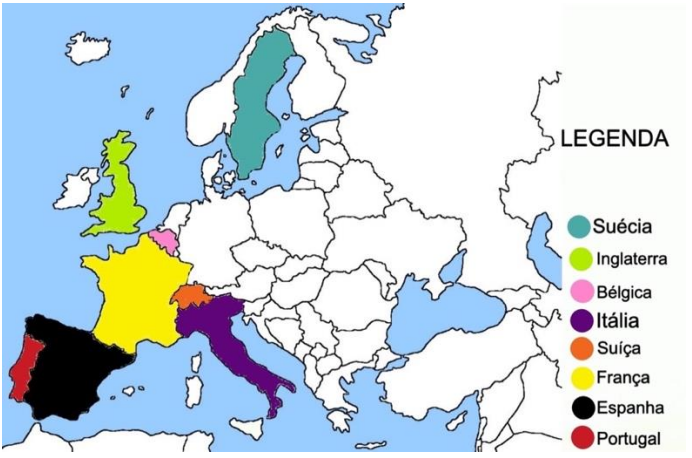
[3]*Revista Pedagógica* was constituted from the National Education Project put into practice in the Republic and was in full operation from 1890 to 1896. It presented international models of instruction and was committed to readers in the dissemination of successful models, practices, and experiences from abroad, which allows us to infer that the journal represents a space for the circulation of international ideas. Finally, it was one of the channels through which Rio de Janeiro had contact with educational updates at the time (Gondra, 1997).

EXPERTS OF THE LATE 19TH CENTURY – THE AGENTS OF KNOW-HOW CIRCULATION FOR TEACHER EDUCATION

The teachers who have their appropriations studied in this research, Luiz Augusto dos Reis, Manoel José Pereira Frazão, and Amélia Fernandes da Costa, was the first Brazilian primary school teachers' official Republican delegation on a pedagogical mission abroad. They travelled through Sweden, England, Belgium, Italy, Switzerland, France, Spain, and Portugal.

Figure 1

Map of countries visited (Conceição, 2019)



When examining *Revista Pedagógica*, we saw, according to the excerpt below, the emphasis given to the three teachers, recognising the work and *expertise* of each one that the teachers from Rio de Janeiro and nationwide would appropriate. As the Federal District then, Rio de Janeiro was the country's model for education and teaching.

Probably the schools where teachers Amélia Fernandes da Costa, Manoel José Pereira Frazão, and Luiz Augusto dos Reis worked began that year to work in conditions that allowed the best teaching methods and processes that had been studied by those distinguished colleagues on their recent and

profitable tour of Europe. From the material and pedagogical point of view, they served as a model for our public and private teaching (Pedagogium, 1895, p. 291).

Regarding the teachers' international pedagogical missions or trips, the literature of the history of education makes us understand that they were common in the late 19th century, as it was the way the country entered in contact with updates in education (Gondra, 1997). Matasci (2015) considers those actions particularly relevant in the construction of modern school systems in the 19th century, both in France and in other Western countries. Those are complex contexts, of many changes and intensified connections between the countries worldwide, period of the first globalisation, with repercussions in social life and education realms, closely associated with the construction of national identities.

During the period, pedagogical journeys sought to come closer to educational policies, aiming at innovative and successful pedagogical practices. According to the studies, the strategy was almost global. Several countries were interested in sending educators abroad for this purpose (Mignot & Gondra, 2007).

The selection of this group of teachers involved in the investigation is justified by the outstanding position they achieved in public education due to their *expertise*. This enabled us to find significant written production on educational proposals from abroad that circulated in the city of Rio de Janeiro in the second half of the 19th century.

The professional *expertise* of teachers who lived in the late 19th century was directly interconnected to each one's skills, and the one who held the necessary know-how to perform tasks he/she had been assigned was called an *Expert*. Thus, *expertise* is "an instance, first recognised as legitimate, attributed to one or more specialists – supposedly distinguished by their knowledge, attitudes, and experiences to examine a situation, to evaluate a phenomenon, to verify facts" (Hofstetter & Valente, 2017, p. 57).

In this perspective, we share Morais's hypothesis (2017, p. 18), considering the "education *expert* as a vector of objectification of know-how in the professional field, in education and teaching." And we understand objectification or, in other words, objectified know-how, defined as

> propositional statements, subject to objects of social judgment that will give you a record of truth or effectiveness. They can even be considered as follows: on the one hand, it formalises a

> representation of the real (it says something about reality); on the other hand, it enunciates a correspondence, a *link* between this representation and the object represented (the notion of truth and the affirmation of this correspondence) (Barbier, 2014, p. 9).

Professional know-how is defined as the one that personalises the teaching professional. In this study, the teacher who teaches geometric know-how, i.e., the know-how that is necessary to mobilise their pedagogical actions. We will consider the know-how produced, systematised, and institutionalised that will, over time, be objectified, becoming a reference in teacher professionalisation (Valente, 2016).

We also understand that the know-how of the teaching craft is configured in a professional culture. This culture is formalised in several ways, such as school disciplines taken in their initial education, in improvement courses, among others; it results from processes of objectification of know-how over time and, thus, the teacher's work is architected as professional work,

> [...] it is possible to affirm that the teacher is a teaching professional because he has the knowledge of what and how to teach someone. Their work is specific because it consists of the systematisation of know-how that concerns the erudite, non-popular culture – linked to science, art, philosophy – as opposed to those of daily and spontaneous order. It is a work carried out intentionally through appropriating specific knowledge that requires specialised and judicious education. It is a complex task that involves rigorous mastery of the technical and didactic fields and a constant stance of questioning its action (Cericato, 2016, p. 278).

The professional teaching know-how of the teacher who teaches mathematics has been the focus in many current studies. Unlike the studies of André (2011), Gatti (2014), Xavier (2014), and Cericato (2016), which value subjectivity in the analysis of teacher education processes, we proposed here to understand geometric knowledge going through history, throughout the late 19th century, identifying processes of *systematisation*, of how this know-how is organised, institutionalised (how it is in the educational action), *objectification*, and how it *circulates*, how it is carried out.

Thus, we understand that the professional knowledge of the teacher who teaches geometry is represented by the articulation of two types of know-

how, which are part of the professional know-how. On the one hand, the *know-how for teaching* constitutes the professional field, in which the reference is the professional *expertise*, the teacher's work tools. And, on the other hand, *the know-how to teach* is the know-how emanating from the disciplinary fields of reference produced by university disciplines and are the object of the teacher's work (Hofstetter & Valente, 2017).

Based on the choice announced before, it is possible to explain the professional know-how of the teaching professional's very *expertise*. Thus, addressing the elements of this professional knowledge, prescribed in the travel reports of the "*1891 Commission*" and the registers in the *Pedagogical Journal*, leads us to realise that *the know-how for teaching* stands out, which are historically present in teacher education, giving them the necessary subsidies for the professional practice of teaching.

It is noteworthy that investigating the *know-how for teaching* is not only studying a teaching method. At some point in the analysis, it can even be configured as such, but it is not a rule. For example, the pedagogical wave in force within the framework of this research, the intuitive method, transformed the way of teaching the contents. Thus, a new teaching method was developed, created exclusively within the scope of school culture, and not restricted only by methodologies.

In this discussion emerges the character of the *Expert*, the one who actually participated in teaching and teacher education in the period, as a relevant agent in the dissemination and implementation of educational changes. The experts are directly linked to public policies and actions of the State, such as Luiz Reis, Amélia, and Frazão, who the government summoned to carry out a pedagogical mission, requiring them to give a response to society by producing an official report.

Experts should not be confused with "intellectuals," a category used in several studies in the history of education, who,

> when taken as an object of study, are expected to answer current questions to demonstrate the permanence of specific questions or solutions, to treat them as "myths" or "heroes" who were out of their time for anticipating issues that would be a consensus later. Thus, "uprooted," they are called to reaffirm an idea or as an authority carrying indisputable arguments, often detached from the social context in which their works and

> actions has been formatted (Machado, Dorigão, & Coelho, 2016, p. 176).

Otherwise, Sartre (1965, p. 14-15) clarifies that the intellectual is "someone who, having acquired some notoriety for works that depend on intelligence, abuse this notoriety to leave his domain and criticise society and the powers established in the name of a global and dogmatic conception of man." Thus, the intellectuals go beyond their field of competence to discuss issues they are not *Experts* about - but believe to be, i.e., "intellectuals are people who deal with something that does not concern them; they are intrusive individuals, curious by nature, who go beyond their occupation, due to their investigative and critical attitude" (Machado, Dorigão, & Coelho, 2016, p. 180).

Another perspective is identified in the work of Jean-François Sirinelli (2003). In discussing the notion of intellectual, he understands it by two definitions, "a broad and sociocultural definition, encompassing cultural creators and mediators, and a narrower one, based on the notion of engagement" (Sirinelli, 2003, p. 242). The author's conception, in the first, includes, for example, journalists, writers, teachers, and other professionals. In the second, the group is composed of individuals with some influence on the public scenario, in search of defence that causes some kind of social impact, for example, which requires, in the second case, a speciality of know-how, as well as some recognition. However, this meaning is not detached from the other since they are socio-cultural.

Thus, the group of intellectuals is composed of "people capable of addressing different fields," regardless of the area of knowledge or a pre-established mission (Burke, 2003). They can be understood as "scholars who take a stand" on specific public issues (Burke, 2005).

On the contrary, the *Experts*, among several attributions and characteristics, differ from the intellectuals because of the call by the State to solve a practical problem based on their professional *expertise* and the response to this call. "A demand from the State will imply the convening of expertise, whose subjects are the *Experts*. This call is triggered by the recognition of the community to which this instance is affiliated" (Morais, 2019, p. 10). In our case, the government's invitation to systematise teaching and educational proposals through a commissioned official report, which was required from the pedagogical mission, unlike what would be required from intellectuals who, for their notoriety in a given field, are free to produce, i.e. they are not obliged to produce a response a State call, but to put their intellect into practice.

> From its most elementary conception, in the constitution of the field "education sciences," the "*education experts*" are subjects whose political stance is legitimised by the production of know-how to meet a practical demand of those who recognise them as such, the State. Such know-how, elaborated taking into account the initial expertise, experiences, and know-how of the expert or group of experts, results in new know-how in response to the call of the State (Morais, 2019, p. 12).

Thus, we can conclude that every *Expert* is also an intellectual, but not every intellectual is an *Expert*. "Certainly, there are similarities, especially regarding the processes and dynamics of evolution of the two fields, the "intellectual field" and the field "education sciences," driven by those two "subjects," the "intellectuals" and the "experts". However, the references adopted describe different purposes" (Morais, 2019, p. 11).

By understanding the commission's qualification as *Education Experts*, we can get to know them more closely. Despite having in common their nationality and historical time, the teachers, from the late 19th-century Rio de Janeiro, entered the teaching profession through different paths. Luiz Reis's education was in practice within primary schools, where he worked as an assistant, becoming a full teacher in 1873 (Schueler, 2002). Amélia graduated from Escola Normal da Corte, adjunct to the primary public schools of the Court in 1855, and became a full teacher in 1877 (Santos, 2019), while Frazão graduated with a higher education degree in Mathematics and Natural Sciences from the Military Academy (Schueler, 2002).

In those years, teachers were significantly involved in political and public education issues. Luiz Augusto dos Reis was secretary of the Permanent Executive Committee of the Teachers, editor-in-chief of the journal *O Ensino Primário* (1884), poet and journalist and Manoel José Pereira Frazão was an editor of the publications *A Verdadeira Instrução Pública* (1872-1873) and *Escola* (1877-1878), author of didactic works, manifestos, articles for the press, member of the Council of Public Instruction of the Court and responsible, after his return from Europe, for building the handicraft programmes in Rio de Janeiro. Amelia, also involved with public education, published several works, one of them entitled "lições de cousas" (lessons of things), distributed and widely disseminated in the newspapers of the time (Biblioteca Nacional, 1897).

Given the above, we can observe how these individuals, who belonged to a distinguished group of the second half of the 19th century, kept participating actively in the educational scenario of the time, also writing books and didactic

materials, participating in various written productions such as memories, reports on schools and practices, letters, correspondence, petitions, chronicles, prose and verse texts, among others. This fact revealed how they handled several practices and shared the codes of literate culture.

The teachers refer us to a profile of active and participating people in the context in which they were inserted. After all, the press, pedagogical magazines, and official reports not only contributed to the debate on education but also, and mainly, disputed ideas, opinions, and political proposals for society (Borges & Teixeira, 2004).

Another relevant aspect is their performance as primary teachers, which agrees with Hofstetter and Valente's (2017, p. 56) conceptualisation of an *Expert*. They state that in the late 19th century, the first experts were associated with a figure of good people, *teachers*, who worked within the school system.

As mentioned, *Revista Pedagógica* was one of the main channels of circulation of pedagogical proposals for teaching, and the constant production of the teachers investigated in this important journal is worth mentioning.

Table 1

Publications in Revista Pedagógica. (Conceição, 2019)

AUTHOR	Frequency
No reference of author	7
CARVALHO, Felisberto de	6
KIDDLE, H; HARRISON, T e CALKINS, N. A.	6
VERÍSSIMO, José	6
COSTA, Amelia Fernandes da	**5**
VIEIRA, Joaquim José Menezes	5
KOPKE, João	4
REIS, Luiz Augusto dos	**4**
Extraído da Eschola Pública – SP	3
FRAZÃO, Manoel José Pereira	**3**
MACIEL, Maximino de A.	3
COELHO, F. Adolfo	2
DORDAL, Ramon Roca	2
LEAL, Presciliano	2
LUIZ, Alambary	2

MAGALHÃES, Valentim	2
SCHMITT, E.	2

As exemplified in table 1, Amélia, Luiz Reis, and Frazão have a considerable number of publications in the *Pedagogy* section of the journal, present in all volumes, that "reflects a concern with what should be taught and with the very way of teaching" (Gondra, 1997, p. 383) indicating their presence in the circulation of pedagogical proposals for teaching. An example is this excerpt from *Revista Pedagógica*:

> When we announce the opening of the exhibition of books and other objects offered to this Pedagogium by the industrious professor Luiz dos Reis, back from his trip to Portugal, Spain, France, and Belgium; we will give the respective list to facilitate our readers to examine such an important collection (*Pedagogium*, 1891, p. 58).

Another important fact is their political action in favour of teachers' interests. Frazão, author of *Cartas do Professor da Roça* (1863) (letters from a field teacher) and signatory of *the Manifesto dos Professores Públicos* (1871), belonged to the Council of Public Instruction, from where he left to go to the pedagogical mission in Europe. He also produced materials and books for primary schools and, in 1863, presented the "*Postilas de Artimética*" (arithmetic handouts) to the Public Instruction Council. Years later, he published the "*Postillas de Grammatica Portugueza"* (Portuguese grammar handouts) and "*Noções de Geographia do Brasil para uso da mocidade brasileira,"* (notions of Brazil's geography to be used by the Brazilian youth), all approved for primary school use. Luiz Augusto dos Reis collaborated as a journalist for the publication of articles that criticised the imperial education in several journals, such as *O Lorense and Echo Municipal de São Paulo*, *O* Estandarte, *A Revista do Brasil* and *O Ensino Primário* (Schueler, 2002).

Thus, considering our initial hypothesis, we maintain that those teachers belonged to a group of *Experts* of the time. They stood out both for their written production (didactic works, books, press articles, memories, etc.) and for their intense participation in political-educational and social groups. There was also the call by the State conferring them a mission, which required them official report feedback and constant publications of excerpts from their reports in the pedagogical press, which can be understood as a recognition of their peers in relation to their craft and the first steps towards the objectification process.

When producing works, books, official pedagogical mission reports, articles for specialised journals, when participating in political activities and public functions, those educators contributed to a process of objectification of know-how in the educational field. Thus, the committee teachers can be considered members of a specific group of education *Experts*. Those educators

> participated actively in educational debates since the early 1870s. Concerned about their professional status, the material conditions of public education and the serious problems of their time – such as discipline, the intellectual elevation of students in public schools, and the formation of the citizen – teachers met in associations, founding newspapers and pedagogical magazines to defend popular education and instruction, besides giving opinions on the paths and destinies of the *nation* (Shueler, 2008, p. 12).

Through the pedagogical press, by preparing books and participating in political debates, Luiz Reis, Frazão, and Amélia acted as *Experts* in education, involved with primary education, the teaching profession, and Rio de Janeiro.

WHICH GEOMETRY "FOR" TEACHING?

Knowing how to read, write and count, historically, was the basis of school education. However, besides these, geometric know-how also comprised curriculum programmes for teaching and teacher education in the late 19th century worldwide. From this perspective, history of mathematics education researchers have been doing significant research on how geometry has been constituted as school know-how, historically.

Although the researchers investigate the geometric know-how that was being constituted in Brazil, international production cannot be left out of this process, as researchers, lawmakers, and teachers appropriated the international discourse through various vehicles such as pedagogical journals, textbooks, translations of materials, and reports of pedagogical missions. Thus, we can understand that local stories are intertwined with contacts, circulation of ideas, meetings, and other forms of exchanges (Chartier, 2009).

Studies in agreement, mostly produced by the history of education researchers, conclude that it is necessary to understand the outside world and international circulation. Thus, the transit of researchers and ideas worldwide

in the late 19th century provided approximations and appropriations (Vidal, 2009).

The analyses in the reports consisted primarily of verifying how geometry behaved in European primary schools the Brazilian teachers visited. The table below provides an overview of how geometry was inserted in the different countries. In advance, one of the first results obtained from the analysis of the distribution of geometry in the programmes is that it was not grouped under a single heading. Several times, we found registers by Frazão, Amélia, and Luiz Reis on the mobilisation of geometric knowledge in other school disciplines such as Drawing and Crafts, which allows us to infer that geometry was not consolidated as a specific subject of primary school at the end of the 19th century. The teaching of this know-how in the nations they visited was comprised of:

Table 2

Geometry in European Programmes (Conceição, 2019)

Country	Primary School Programme Disciplines[4]
Italy	1) Reading; 2) Accounting; 3) Writing; 4) Notions of Things; 5) Metric System; 6) Drawing Geometry is inserted in the classes after primary elementary school.
Switzerland	1) Mother tongue; 2) Arithmetic; 3) German; 4) Geometry; 5) Geography; 6) History; 7) Drawing; 8) Calligraphy; 9) Gymnastics and Games; 10) Singing; 11) Handicraft. From the third class, geometry is included.
England	1) Reading; 2) Writing; 3) Accounting; 4) Drawing; 5) Handicraft Each school is responsible for geometry teaching, designated as a special subject.
Sweden	

[4] This is considered the initial period of the primary study named in most European countries as *Primary Elementary*. Subjects of the complementary primary education, middle primary education, and higher primary education, which follow the primary elementary school studies, are not considered.

	1) Religion and Morality; 2) National Language; 3) Writing; 4) Arithmetic; 5) Geography; 6) History; 7) *Slodj*; 8) Singing and Vocal Music; 9) Drawing; 11) Gymnastics. Geometry is added in the two last classes.
Belgium	1) Reading; 2) Writing; 3) Calculus (Mathematics); 4) Metric System; 5) French-speaking Elements; 6) Flemish-speaking Elements; 7) Geography; 8) History; 9) Drawing Elements; 10) Singing; 11) Gymnastics; 12) Handicraft; 13) Geometric Shapes
France	1) Moral and Civic Instruction; 2) Reading and Writing; 3) National Language; 4) Calculus and the Metric System; 5) History and Geography; 6) Lessons of Things; 7) Drawing Elements; 8) Handicraft; 9) Gymnastics. Geometry appears in upper primary instruction
Portugal	1) Reading; 2) Writing; 3) Four Operations on Integers and Fractions; 4) Portuguese Grammar Elements; 5) Principles of the Decimal Metric System; 6) Principles of Drawing; 7) Christian Doctrine.
Spain	1) Reading and Writing; 2) Arithmetic; 3) Spanish; 4) Metric System; 5) Drawing; 6) Handicraft; 7) History and Geography; 8) Gymnastics.

The teaching objectives of the subjects listed in the table were very similar to each other in the European countries visited and were based on the "development of the child in the set of its faculties [...] and the incessant encouragement of spontaneity of thought" (Reis, 1892, p. 381), marks of the then-popular international pedagogical movement, the intuitive method. The teaching of things linked to life, objects, and facts present in children's daily lives are some of the propositions that stood out. The introduction of didactic objects were indispensable elements to form ideas due to their playful and disciplinary character: a new component of the classroom that guides the path of knowledge by bringing together students and the teacher. The method also brings "the possibility of standardising reasoning, ways of thinking, crystallising a form of appropriation of external things in a process that is directed by the teacher, the representative in that situation of the legacy of

previous generations, including with their values and prejudices" (Valdemarin, 2004, p. 176).

Thus, the teaching of the disciplines that made up the programme of each country

> [...] could not be presented by the teacher solely in lecture form or by simple readings. They should be taught by intuitive exercises that constantly involved the child's activity. The child should be placed, as much as possible, in the presence of the things that make the object of the lesson. The child observes and analyses them, guided by the teacher (Reis, 1892, p. 381).

As mentioned, in most countries, geometry was not included in the primary school curriculum as a specific subject. So, for example, it was mobilised in Belgium as one of the axes of teaching the "*mathematicas*,"

> [...] teaching mathematics will be based on *geometry* and mental calculation. Geometry is taught by the study of geometric bodies; the students observe them, and the teacher makes them understand the terms, teaches them the notions of quantities, of their comparison, of proportion, of symmetry, of equivalence. The most important theorems are demonstrated by intuitive exercises.
>
> The metric system is taught by the actual experience of the measurements. The students calculate the surfaces and volumes by measuring the dimensions and not by operating on imaginary data (Reis, 1892, p. 382).

As in Belgium, Spain, Italy, England, Sweden, France, and Portugal, the non-existence of a subject called geometry did not mean that this know-how was not proposed to children. In these countries, geometric know-how in primary school was rooted in subjects such as geometric forms and more strongly in Handicraft and Drawing.

Teacher Amelia's register reiterates our analysis,

> [...] the geometry taught to children in Italy has a special beauty. I visited schools that had drawing rooms with beautiful paintings and handicraft workshops with several well-constructed shapes that allow elementary school students to observe, compare, and experience geometry (Costa, 1891, p. 165).

Thus, even without being a separate subject, geometry was part of the teaching proposal of the various European countries. The reports and representations show their strong relationship with Drawing and Handicraft and reveal explicit marks of the proposition of the intuitive method, as the teachers suggested in the activities of observation, comparison, and actual experience reported. In other words, the proposal *for teaching* was linked, among other things, to a strong incentive to practical activities.

Still taking the Belgian case, for example, for the teaching of geometry in the "*mathematicas*," Reis underscores that the teaching of geometric models was done very carefully so that the teacher should obey six steps. This set of procedures *for* teaching was the object of study by Conceição and Leme da Silva (2019), where evidence of professional know-how for teaching was identified in the set of steps. Teaching is mobilised by the action of the teacher who accompanies the students in the observation, analysis, and comparison of geometric shapes. The proposal was linked to the idea that observation should not be restricted to sight, emphasising the need to touch, manage, build geometric shapes. Particularly, students' active practice in the production and manipulation of objects is encouraged.

The six steps for teaching geometric models highlight the articulation of teaching professional know-how, *the know-how to teach*, with connection in the sciences, with the disciplinary field of know-how and, thus, composing the teacher's work object; and *the know-how for teaching*, linked to the tools mobilised by the pedagogical field in the practice of the teaching profession.

The same movement was happening in France. There were guidelines for teachers on what geometric know-how to mobilise and how to mobilise them in primary school

> Simple exercises to make recognise and designate the most elementary regular figures: square, rectangle, circle. Different kinds of angle. The idea of the three-dimensions. Notions about solids through embossed models. Frequent measurement exercises to compare quantities in plain sight: approximate assessment of distances and their evaluation in metric measurements (Reis, 1891, p. 202).

It seems that the *know-how to teach* is oriented to teachers with a strong connection to the *know-how for teaching*, in six steps. The observation was indicated not only by sight, but by touch and muscular sense, besides the constructions by the Froebel method (first step); the association of geometric

models with common objects (second step); applications to crafts (third step); description of figures aloud (fourth step); the teaching of geometric figures without formalisations, definitions, theorems, but with the superposition of figures with the help of handicraft through cutting (fifth step), and geometric dictations, where students construct the figures being dictated freehand and check their measurements (sixth step). In France, the movement was quite similar: to compare, check distances, recognise, and give names to the geometric figures, always in plain sight.

We can say that the formal character of concepts and definitions, linked to the reference science, in this case, geometry, is minimised when compared to the experience proposed: exploration and investigation of properties in the constructed models. In this regard, from the circulating proposal of the three *Experts*, it is clear that "the essential element in the primary teacher is not the depth of knowledge, but the gift of transmitting" (Frazão, 1892, p. 164).

The traveller's speech can be read not as a devaluation of *the know-how to teach* but as a proposal for the circulation of new professional know-how for the primary teacher, i.e., *the know-how for teaching*, the tools mobilised in the pedagogical act, should be emphasised in this teacher. They should be seen predominantly in primary teachers, a professional characteristic of their own. Still in Frazão's words, "[...] teachers make a mistake when they say that I lessen the importance of primary education. What I wanted was to make it freer and more independent of secondary education" (Frazão, 1892, p. 67).

In any case, we identify elements put into circulation that value aspects of know-how *to* teach and *for* teaching linked to the understanding that

> on the one hand, the know-how constituting the professional field, in which the reference is the professional expertise (professional know-how or know-how for teaching) and, on the other, the know-how emanating from the disciplinary fields of reference produced by university disciplines (disciplinary know-how or know-how concerning the know-how for teaching) are articulated (Borer, 2017, p. 42).

Representations indicate that the teaching of geometry in primary classes was based on the study of geometric bodies, and the teaching processes were given by the intuitive action of the child directed by the teacher, who makes them observe the properties of the geometric figures based on models: "the students observe them, and the teacher makes them understand the terms,

teaches them the notions of quantities, of their comparison, of proportion, of symmetry, of equivalence. The most important theorems are demonstrated by intuitive exercises" (Reis, 1892, p. 382).

The teacher also affirmed, from his visit to schools of France, that for the teaching of geometric models, the shapes should be

> taught especially to exercise the senses and intelligence and to provide students with practical and useful notions. The teacher proceeds, therefore, by observing, analysing, and comparing geometric shapes. [...] Observation cannot be made solely by sight, which, by the effect of perspective, provides only inaccurate notions about the shapes, dimensions, directions, etc., by touch and muscular sense, which rectify the impressions of sight. They must therefore manage and build the shapes by the processes taken from the Froebel method (modelling, rods, folding, cutting, cardboard, drawing). Students are exercised in searching for common objects in which each form studied is exactly more or less modified. The applications of geometric models in crafts are indicated as completely as possible. The character of teaching must be essentially practical and intuitive: difficult definitions and scientific demonstrations are not given (Reis, 1891, p. 461).

All teaching starts from observation, management of a spatial geometric figure, and the process in which this teaching is derived follows according to an analytical march, i.e., from the whole to the parts, refers to "the decomposition activity through which we distinguish the different elements that make up a whole" (Trouvé, 2008, p. 20).

Each new element placed to teaching follows this process from the whole to the parts, and from this, new contents are brought to be discussed.

As stated, the intuitive method is the core of the pedagogical changes in the late nineteenth century. It proposes a disruption from the traditional pedagogy, characterised by memorisation, verbalisation, mechanised reproductions, and other traditional forms. In this method of pedagogical renewal, touch, observation, and experimentation play a fundamental role: the objects enable children to play an active role, breaking away from the traditional pedagogy. About the march of teaching, the intuitive method, based on Pestalozzi's guidelines and studies, suggests teaching from the known (the spatial figures) to the unknown (particularities of the flat figures), from

concrete (the models produced) to the abstract (definitions), from the simple to the complex (according to the child's understanding), as put into circulation. There are also guidelines on the know-how that the teacher must have *for teaching*, linked to handicraft, drawing, measurements, etc.

By guiding teaching, the guidelines are used to monitor the Drawing and Handicraft programme, pointing to the direct connection between geometric know-how and these disciplines. Thus, in Drawing and Handicraft, the teacher has the place for teaching geometric knowledge.

Another element to be highlighted in the proposals in circulation is the use of the so-called "geometric dictations," which appear in the reports by Reis (1892) and Costa (1891). Once a week, the students freehand traced geometric figures dictated by their teacher, as Reis exemplifies:

> 1st. The teacher imagines and traces exactly and beforehand a geometric figure relative to the notions taught; 2nd. He slowly dictates each condition of construction of this figure: the position that each point or each line must occupy, the direction of the lines, their extension, opening, direction, the magnitude of the angles, etc.; 3rd. The students freehand trace the figures dictated and quickly; 4th. When the work ends, they verify the figure through the Weyel square gauge[5] (Reis, 1892, p. 463).

This mobilisation allows once again to understand the articulations between the *know-how to teach* and the *know-how for teaching*, so that the focus is on activities, in obtaining knowledge actively. The proposal in circulation indicates that the student does not learn a geometric model through its properties and definitions, but by having touched it, compared to other forms used in their daily lives. After the experiments, when their characteristics and properties are dictated, the child can succeed to construct it, either through drawing or some manual construction.

We also perceived, with the analyses of the proposals in circulation, a new production of *know-how to teach* with an original didactic-pedagogical order, contrasting with the classical organisation of geometry, a reference discipline, from the plane to space, to an order that is proper to intuitive times,

[5] "Weyel's square is a triangular wooden square, of which the two sides of the right angle are divided into decimetres, centimetres, and millimetres. The centre is hollow and forms a protractor. The instrument advantageously replaces a ruler, a square, a double decimetre, and a protractor. The teacher quickly examines the work, with the help of the figure correctly cut into a sheet of strong paper: it is sufficient to extend this mould over the students' tracing" (Reis, 1892, p. 463).

from space to the plane with the senses enabling the production of ideas, starting from concrete and ascending to abstraction.

Luiz Reis, Amélia, and Frazão put into circulation what is expected of the teacher for the teaching of geometry in primary school in the late nineteenth century, a professional know-how in direct harmony with the proposals of the intuitive method, a new way to mobilise know-how. Form not constant only in a school discipline, but concerning all disciplines that mobilise geometric knowledge.

Supported by Hofstetter and Valente's (2017) studies, the examination in the reports allows us to observe the close articulation of *know-how to teach* and *know-how for teaching* in the realisation of this professional know-how related to geometric know-how.

The proposals on the teaching of geometry deal with geometric models. They are based on practical activities and reveal that the teachers need other types of know-how, in addition to know-how derived from geometric contents. Cutting, modelling, cardboard, drawing, measures, and geometric dictation are expressed as "new environments" for the teacher to teach geometry, characterising a "new" *geometry for teaching.* Also, Drawing gains importance as another component necessary for the practice of the teacher that teaches geometry.

FINAL CONSIDERATIONS

The analyses of the proposals in circulation indicate a change in the professional knowledge of the teacher of the primary school (early years) for geometry teaching. Another didactic-pedagogical order is recommended: reversing the gait from plane geometry to spatial geometry. This characteristic of the disciplinary field of geometry becomes imperative in the intuitive proposal, since the concrete element, the element the child knows, prevails.

Therefore, the exploration of solids as a starting point is a constant in the proposals. Changes in the *know-how to teach* are articulated with the mobilisation of *know-how for teaching*. New professional types of know-how are at stake for the teacher that teaches geometry, a new *geometry for teaching*.

The analyses also allow us to reflect that understanding how geometry appears in the reports by Amélia, Luiz Reis, and Frazão. They require that researchers look thoroughly at other subjects, primary school programmes from different countries, since, as already mentioned, the *geometry for teaching* read

by the *Experts* in their foreign experiences was always articulated with other types of know-how. In this way, the *Experts* systemised a *geometry for teaching* directly linked and spread to primary subjects: Drawing and Handicraft.

DATA AVAILABILITY STATEMENT

The data extracted from the documents that support the investigation are available in the Digital Content Repository of the Research Group on the History of Matehematics Education (Ghemat – Brasil). http://repositorio.ufsc.br/handle/123456789/1769.

REFERENCES

André, M. E. D. A. (2011). Pesquisas sobre formação de professors: tensões e perspectivas do campo. In Fontoura, H. A., & Silva, M. (Eds.), *Formação de professores, culturas: desafios à Pós-Graduação em Educação em suas múltiplas dimensões*. (pp. 24-36). E-book online. Encontro de pesquisa em educação da região sudeste, 10. ANPED Nacional.

Barbie, J. M. (2014). *Savoirs théoriques et savoirs d'action.* Presses Universitaires de France.

Biblioteca Nacional. (1897). Gazeta de Notícias. Hemeroteca Digital. http://bndigital.bn.gov.br/hemeroteca-digital/

Borer V. (2017). Saberes: uma questão crucial para a institucionalização da formação de professores. In Hofstetter, R. & Valente, W. R. *Saberes em (trans)formação: tema central da formação de professores*. (pp. 173-200). Livraria da Física.

Borges, A., & Teixeira, J. (2004). Homogeneizando mestres: positividades e efeitos das Conferências Pedagógicas da Corte Imperial (1872-1889). *Anais do III Congresso Brasileiro de História da Educação*. PUC Paraná. Sociedade Brasileira de História da Educação.

Burke, P. (2003). *Uma história social do conhecimento*. Zahar

Burke, P. (2005). *O que é História Cultural?* 2. ed. Zahar

Cericato, I. L. (2016). A profissão docente em análise no Brasil: uma revisão bibliográfica. *Revista Brasileira de Estudos Pedagógicos*, *97*(246), 273-289.

https://www.scielo.br/j/rbeped/a/ZGXLgG4kzTjqx5bqcc9pshS/?format=pdf&lang=pt

Chartier, R. (2009). *A história ou a leitura do tempo.* Autêntica.

Conceição, G. L. & Leme da Silva, M. C. (2019). Saberes profissionais do professor que ensina geometria: propostas belgas no relatório de Luiz Reis (1892). *História da Educação*, *23*(1), 1-23. http://doi.org/10.1590/2236-3459/87519

Conceição, G. L. (2019). *Experts em Educação: circulação e sistematização de saberes geométricos para formação de professors (Rio de Janeiro, final do século XIX)*. Tese de doutorado, Universidade Federal de São Paulo, Guarulhos, SP, Brasil. https://repositorio.ufsc.br/handle/123456789/201374

Costa, A. F. (1891). Relatório trimestral da professora Amélia F. Costa. *Revista Pedagógica*, *II*, 1-23.

Frazão, M. J. P. (1892). *O ensino público primário da Itália, Suíça, Suécia, Bélgica, Inglaterra e França*. Rio de Janeiro: Gazeta de Notícias.

Gatti, B. A. (2014). Formação inicial de professores para a educação básica: pesquisas e políticas educacionais. *Estudos em Avaliação Educacional*, *25*(57), 24-54. https://doi.org/10.18222/eae255720142823

Gondra, J. G. (1997). O veículo de circulação da pedagogia oficiail da República: a Revista Pedagogica. *Revista Brasileira de Estudos Pedagógicos*, *22*(188), 374-395. https://doi.org/10.24109/2176-6681.rbep.78i188-89-90.1061

Hofstetter, R. & Valente, W. R. (2017). *Saberes em (trans)formação: tema central da formação de professores*. Livraria da Física.

Machado, M. C. G., Dorigão, A. M., & Coelho, G. F. (2016). As pesquisas com intelectuais em história da educação: um campo profícuo. *Revista HISTEDBR*, *67*(16), 175-188. https://doi.org/10.20396/rho.v16i67.8645233

Matasci, D. (2015). *L'école républicaine et l'étranger. Une histoire internacionale des réformes scolaires en France 1870-1914*. Ens. Éditions.

Mignot, A. C. V. & Gondra, J. G. (2007). *Viagens Pedagógicas*. Cortez.

Morais, R. S. (2017). Experts em educação e a produção de saberes no campo pedagógico. *Rematec*, *12*(26), 61-70. http://www.rematec.net.br/index.php/rematec/article/view/110

Morais, R. S. (2019). Intelectual? No, expert. *Acta Scientiae*, *21*(especial), 3-12. https://doi.org/10.17648/acta.scientiae.v21issEid5169

Pedagogium. (1890-1896). *Revista Pedagogica*, Tomos I-IX. Distrito Federal: 1890-1896.

Reis, L. A. (1891) Relatório do professor Luiz A. dos Reis. *Revista Pedagógica*, II, 277-279; 355-427.

Reis, L. A. (1892). *O ensino público primário em Portugal, Espanha, França e Bélgica.* Imprensa Nacional.

Santos, H. H. M. (2019). Amélia Fernandes da Costa: trajetória biográfico-profissional. *Revista Brasileira de Pesquisa (Auto)Biográfica*, *4*(11), 642-661. https://www.revistas.uneb.br/index.php/rbpab/article/view/5891/pdf

Sartre, J. P. (1965). *Em defesa dos intelectuais.* Ática.

Schueler, A. F. M. (2002). *Culturas escolares e experiências docents na cidade do Rio de Janeiro (1854-1889).* Tese de doutorado, Universidade Federal Fluminense, Nitéroi, Rio de Janeiro, RJ, Brasil.

Schueler, A. F. M. (2008). Ensaios de História social da educação: escolas primárias e professors na corte imperial. *Momento – Diálogos em Educação*, *18*(1), 11-33. https://www.seer.furg.br/momento/article/view/748

Sirinelli, J. F. (2003). Os intelectuais. In: Remond, R. *Por uma história política*. Fundação Getúlio Vargas.

Trouvé, A. (2008). *La notion de savoir elémentaire à l'école.* L'Harmattan.

Valdemarin, V. T. (2004). *Estudando as lições de coisas: análise dos fundamentos filosóficos do Método de Ensino intuitivo*. Autores Associados.

Valente, W. R. (2016). A matemática nos primeiros anos escolares: elementos ou rudimentos? *História da Educação*, *20*(49), 33-47. https://www.scielo.br/j/heduc/a/LRM9YrG6jhsNkSTDnn4rqVr/?format=pdf&lang=pt

Valente, W. R. (2018). Processos de investigação histórica da constituição do saber professional do professor que ensina matemática. *Acta Scientiae*, *20*(3), 377-185. https://doi.org/10.17648/acta.scientiae.v20iss3id3906

Valente, W. R. (2020). A pesquisa sobre história do saber profissional do professor que ensina matemática: interrogações metodológicas. *Revista Paradigma (Maracay)*, *XLI*, 900-911. https://repositorio.ufsc.br/handle/123456789/209268

Vidal, D. G. (2009). Cultura e práticas escolares como objeto de pesquisa em História da Educação. In Yazbeck, D. C. & Rocha, M. B. M. *Cultura e história da educação: intelectuais, legislação, cultura escolar e imprensa*. UFJF.

Xavier, L. N. (2014). A construção social e histórica da profissão docente: uma síntese necessária. *Revista Brasileira de Educação*, *19*(59), 827-849. https://www.scielo.br/j/rbedu/a/nPMpCfpNpMQjnNxnzJMmkQP/?format=pdf