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Capturing Strategies and Difficulties in Solving Negative Integers: A Case Study of Instrumental Understanding

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ABSTRACT

Background: Many elementary school students have made mistakes in solving negative integer problems, especially for students with instrumental understanding. Studies that focus on strategies and difficulties faced by students can be a solution to finding student weaknesses so that they achieve a better level of understanding. **Objectives**: The objective of this study was to classify cases of fifth-grade elementary school students with instrumental understanding in solving negative integer tasks with a focus on strategies and difficulties faced with number line models. **Design**: This study uses a case study research design. **Setting and Participants**: Fifth-grade elementary school students who have studied the material on negative integer operations in Sukodono, Indonesia, were included. One of the students with the most varied instrumental understanding and representation strategy was chosen as a case study. **Data collection and analysis**: Qualitative data collection was done by providing a number line model task and interview instructions. Data analysis was performed by comparative analysis, namely, by comparing all data collected, including the transcribed audio and video recordings. **Results**: Researchers found various types of strategies that experienced difficulties to cognitive completion by students with instrumental understanding, along with difficulties in solving number problems. **Conclusions**: The implications of this study are very useful for further research and lifelong learning practices, especially in dealing with elementary school students who

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have difficulty constructing knowledge and the concept of negative integer arithmetic operations.
Keywords: strategy and difficulty; representation; instrumental understanding; negative integer; case study.

Capturando Estratégias e Dificuldades ia Resolução de Números Inteiros Negativos: Um Estudo de Caso de Compreensão Instrumental

RESUMO

Contexto: Até recentemente, muitos alunos do ensino fundamental cometeram erros ao resolver problemas de números inteiros negativos, especialmente para alunos que tinham compreensão instrumental. Estudos que enfocam as estratégias e dificuldades enfrentadas pelos alunos podem ser uma solução para encontrar as fragilidades dos alunos para que eles alcancem um melhor nível de compreensão. **Objetivos**: O objetivo deste estudo foi classificar casos de alunos do 5º ano do ensino fundamental que possuem compreensão instrumental na resolução de tarefas de inteiros negativos com foco nas estratégias e dificuldades enfrentadas com modelos de linha numérica. **Design**: Este estudo usa um projeto de pesquisa de estudo de caso. **Ambiente e participantes**: alunos do 5º ano do ensino fundamental que estudaram o material sobre operações de número inteiro negativo em Sukodono, Indonésia. Um dos alunos que possui as mais variadas estratégias de compreensão e representação instrumental foi escolhido como estudo de caso. **Coleta e análise de dados**: A coleta de dados qualitativos é feita fornecendo uma tarefa de modelo de linha numérica e instruções de entrevista. A análise dos dados foi realizada por meio de análise comparativa, nomeadamente comparando todos os dados recolhidos, incluindo as gravações de áudio e vídeo transcritas. **Resultados**: Os pesquisadores encontraram vários tipos de estratégias que enfrentaram obstáculos para a conclusão cognitiva por alunos com compreensão instrumental, juntamente com dificuldades na resolução de problemas numéricos. **Conclusões**: As implicações deste estudo são muito úteis para futuras pesquisas e práticas de aprendizagem ao longo da vida, especialmente no processo de como lidar com alunos do ensino fundamental que têm dificuldade em construir conhecimento e o conceito de operações aritméticas inteiras negativas.

Palavras-chave: estratégia e dificuldade; representação; compreensão instrumental; número inteiro negativo; estudo de caso.

INTRODUCTION

Elementary school students understand that the concept of negative integers is not as easy as understanding positive integers. According to Bofferding (2014), students' difficulties in solving integer problems are due to their lack of prior knowledge of negative number knowledge. When the first negative integer concept is given, it will seem abstract to students, thereby conflicting with previous knowledge of integer arithmetic operations (Cengiz, Aylar, & Yildiz, 2018). Related to solving problems involving negative integers, Bofferding, Aqazade, and Farmer (2017) stated that students who are not accustomed to using negative numbers or those who rely on the concept of positive integers would often ignore the use of negative signs. Likewise, overgeneralizing often occurs when solving problems involving number operations, based on previous students' experiences in operating positive integers. This results in a misconception, where students think that addition is always greater, while subtraction is always smaller (Whitacre et al., 2012).

Rabin, Fuller, and Harel (2013) said that student misconceptions continued. Students would face many challenges when facing new mathematical ideas that came from previous schemes. Therefore, several studies have tried to bridge students to make understanding the concept of negative integers easier. Approaches include mental models (Bofferding, 2014), the problem sequence (Bishop et al., 2014), problems having different cases (Aqazade, Bofferding, & Farmer, 2017), and the opposite model (Cetin, 2019). However, these studies have not emphasized the difficulties faced by students in solving the representation-based problems used. Tambychik and Meerah (2010) explained the understanding of educators or researchers about students' difficulties in solving problems, as can be seen from the representation of answers, which is very important before developing students' thinking skills.

The number line model provides visual assistance so that students can check the relationships between integers with each other (Kent, 2000; Whitacre et al., 2011). Wessman-Enzinger and Bofferding (2014) stated that number lines were introduced to students to facilitate the transition of their understanding of negative numbers. Therefore, in this study, the number line is used as an intermediary for students to solve integer operation problems. Through the number line, the representation of students in adding and subtracting positive or negative numbers will be observed so that the researchers can classify the types of strategies used and identify the difficulties that occur in students in developing strategies.

The solution to a problem depends on a student's understanding. Skemp (2006) argues for two types of understanding: instrumental and relational understanding. The difference between the two types is the difference in the measure of mathematics and the reasons for the decision-making for why the action is taken. Students who are classified as having instrumental understanding are those who are able to perform mathematical steps, but they do not know why the measuring instrument is used. Conversely, students who are classified as having relational understanding are those who are able to take mathematical steps and know the reasons for doing these actions. Suppose the teacher does not find a solution for why students with instrumental understanding or less relational understanding tend to fail in math problems. In that case, students' critical attitudes in solving problems will be hampered (Anderson, 1996).

Several previous studies by Bofferding et al. (2017) explain the strategies students use when facing problems there are differences. Bishop et al. (2014) characterized students' strategies that were classified as relational understanding through a number line model and a series of open sentences. However, previous studies have not characterized the strategies and difficulties used by elementary school students who have instrumental understanding in solving the problem of addition and subtraction of integers through the number line model.

Problems of the Study

This study classifies the strategies students use with instrumental understanding and the difficulties that occur with negative integers to solve problems through number lines and open sentences. This fact leads the researchers to eliminate gaps that occur through the following research questions:

- What are the strategies used by students with instrumental understanding in solving negative integer arithmetic operations on number lines and open sentences?
- What are the difficulties of students with instrumental understanding in developing problem-solving strategies?

These questions are used so that the results of research findings can contribute to science in determining the strategies used by students who are classified as having poor understanding. These findings are expected to be useful for long-term research on how students understand development or transitioning students with instrumental understanding to use relational understanding through settlement strategies.

METHODOLOGY

Research Design

This study tries to explore the strategic cases of students using instrumental understanding in solving the problem of negative integer operations through number lines and open sentences. It also identifies the difficulties experienced by students in solving these problems because the case study is used as the research design (Cresswell, 2012). The types of cases in this study include intrinsic cases of instrumental students who are prone to cognitive problems, which are interesting cases to study (van de Walle, 1998).

This research has been submitted to the Ethics Committee of the University of Muhammadiyah Sidoarjo and was approved on January 12, 2020, with opinion number 259. This is an ethical institution located in the city of Sidoarjo, East Java, Indonesia. Through the study and evaluation of this research proposal by the Ethics Committee, the research was approved to be carried out by involving students (humans), immediately after the proposed research plan was submitted.

Participants

A total of 23 fifth-grade students in an elementary school in Sukodono, Indonesia, were involved in this study. These students have studied the material on negative integer operations. Among the students, four were classified as having instrumental understanding. A student named Yulia (a pseudonym) was chosen as the research subject because she was the only student who had good initial knowledge of negative numbers and a poor understanding of number lines, but a representation strategy that was generally more varied. However, after confirmation, Yulia could not explain the mathematical arguments in completing the numerical count operation on the number line compared with the other three students. Regarding the involvement of all participants in this study, including Yulia, researchers received ethical clearance from the school committee and ethics committee. The students' guardians or parents were also aware of the clearance.

Data Collection and Instruments

Qualitative data collection was carried out by giving the number line model task instrument and interview instructions to answer the research questions. The line model task in this study consisted of the negative numbers, lift, and open sentence tasks.

Table 1

Indicators and Questions from the Negative Numbers Task

Indicators	Questions
• Finding positive and negative integers on the number line.	• Find the ten numbers before five on the number line below.
• Determining the number of negative numbers that appear on the number line.	• How many negative numbers are there on the number line?
• Comparing the number of integers.	• Which is greater between 0 and −6?
• Summarizing the existence of an integer on the number line.	• What can you deduce from the number line?
• Drawing arithmetic operations on a number line.	• Draw the addition and subtraction of 0 + 2 + 5 − 3 on a number line.
• Describing arithmetic operations of positive integers on a number line, starting at zero.	• How many steps to move from number 0 to number 2? In which direction is it moving?
• Stating the number of steps for the displacement of each jump in the direction of the line number.	• How many steps to move from numbers 2 to 7 and numbers 7 to 4? In which direction is it moving?
• Defining other forms of arithmetic operations with results similar to previous arithmetic operations.	• Are there any other forms of addition and subtraction that have the same result? If there is, write down and draw the number line below. If not, give a reason.
• Drawing arithmetic operations for positive integers on a number line that does not start at zero.	• Draw the addition and subtraction of $2 + 5 - 3 = \square$ on the number line.
• Describing the displacement steps and direction of the number line.	• Describe the steps of displacement and number line's direction.
• Summarizing the number line at this number and the previous number.	• What is the conclusion after solving problems 4 and 5?
• Declaring deficiency must draw a line starting at zero.	• Is it possible to draw the form of addition and subtraction on a number line that is not preceded by the number 0?
• Drawing negative integer arithmetic operations on a number line.	• Draw the addition and subtraction of 3 + 4 − 9 on a number line.
• Explaining understanding after completing arithmetic operations on a number line.	• Try to explain what you understand after seeing the form of addition and subtraction.

Negative Numbers Task

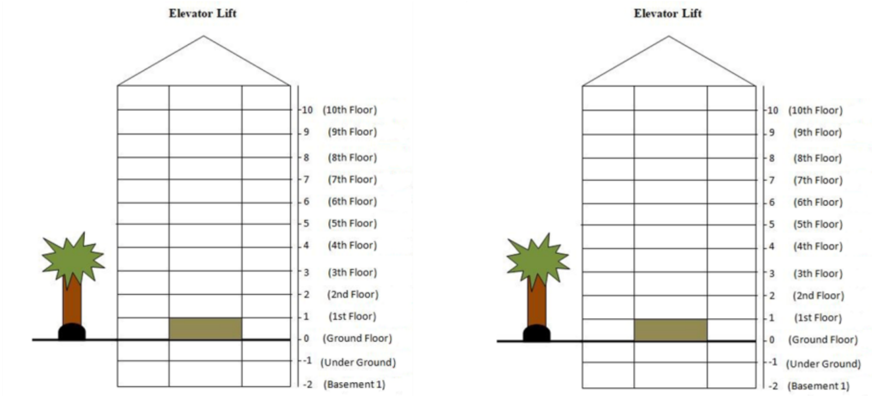
The negative number task aims to task students' initial knowledge of negative numbers and number lines. The results of this task are used as one of the considerations for finding research subjects so that it can be ascertained that the subjects have good initial knowledge. This task was modified by the instrument researcher from Bofferding (2014) in terms of identifying students' knowledge of negative numbers.

Lift Task

The lift task aims to identify student problem solving from negative number operations related to the elevator context. This task is modified by Bofferding (2014), which also trains and cultivates students' instrumental understanding of number lines in everyday contexts.

Figure 1

Descriptions of Lift Task



Rani was at the mall with her brother. They were on the third floor to buy clothes. After that, they went up four floors to buy food. After they ate their fill, they went home and went down nine floors to the parking lot.

Rudi entered the lift on the fifth floor. In the elevator, he met a workmate, who asked Rudi to deliver a letter to Susi on the eighth floor. When Rudi arrived, he found out that Susi was in the parking lot getting something. Rudi went down nine floors to meet Susi.

Table 2

Indicators and Questions of Lift Task

Indicators	Questions
• Illustrating the story's problem in a more contextual situation, namely in an elevator. • Determining the final position according to the problem presented. • The arithmetic operation determines the form of the presented problem process.	• Describe the position of the related problem presented in the lift. • What floor is the parking lot located on? • Determine the arithmetic operations according to the form of the story.
• Illustrating the story's problem in a more contextual situation, namely in an elevator. • Determining the final position of the story subject.	• Describe the position of the subject as the problem presents the story. • On what floor do the two people meet?

Table 3

Indicators and Questions of Open Sentence Task

Indicators	Questions
• Illustrating negative integer operations in the form of an open sentence at the end.	• Illustrate the counting operation $5 - 9 = \square$ $-9 + 6 = \square$
• Illustrating the operation of a negative integer in the form of the initial open sentence.	• Illustrate the calculation operation $\square + 6 = 2$ $\square - 3 = -5$ $\square + 3 = -6$
• Illustrating a negative integer operation in the form of an open sentence in the middle.	• Illustrate the counting operation $7 + \square = 3$ $8 - \square = 11$ $6 + \square = 4$
• Declaring true or false statements of negative integer arithmetic operations.	• Is the calculation operation true or false? • Try to prove the arithmetic operation by drawing the number line. • What is the reason you answered right or wrong? $9 + (-7) = 2$ $9 + -7 = 9 - 7$ $7 - 2 = 7 - -2$

Open Sentence Task

This task aims to classify the strategies used by students with instrumental understanding in completing a series of open sentences in negative integer arithmetic operations. These tasks are formed in two types, e.g., open sentences such as $\square + 6 = 2$ and comparison tasks.

Interview Instructions

Interview instructions were used to explore students' thinking about strategies and instrumental student difficulties in solving number line problems. This interview was conducted two times at different times. This process was also intended as data triangulation to ensure the validity of the data. Students were asked to explain problem-solving steps to see the results of tasks that have been done previously while recording audio and visual of student activities. After completing the entire task and interviewing the researcher screening of the video whether it is clear or not. If some answers are different from the student's results, the researcher will clarify these differences. Questions will be developed according to the answers expressed by students about the problem-solving process carried out by Yulia in making plans and carrying out the analysis of the complete plan.

Data Analysis

All interview data were collected, audio recordings and videos were transcribed, and copies of student work were combined with each transcript. Data analysis procedures involving open, axial, and selective coding processes for continuous and qualitative data involved permanent comparative analysis between each category, and new categories emerged (Cresswell, 2012).

Researchers conducted interviews and compiled transcripts of students' answers. To analyze the interview data and transcripts were reduced to fragments containing explanations of the students' main ideas. Data were coded, sorted, and repeatedly read to answer research questions. After recording, data validity was ensured by determining accurate and complete data collection by managing written tasks and interview transcripts.

RESULTS AND ANALYSES

The results of the study were shown by students' learning on initial knowledge, namely, Yulia's understanding of the context of the situation, strategies, and difficulties in solving problems involving the number line model.

Yulia's Understanding of the Context

Sometimes students still do not know about negative numbers that exist in real situations. Therefore, the next task aims to identify students' understanding of negative numbers in real situations through contextual problem elevators. Students will understand more easily through contextual problems because they are related to everyday life, enabling students to solve problems properly. After the researchers know students' understanding and knowledge, they will be introduced to problems related to everyday life. These cases involve negative integers that contextualize these problems in students' real lives. Such an introduction is represented by an elevator. Students illustrate problems that make up a story into a picture that resembles an elevator. Students shade in the appropriate position on the elevator problem image. Then, students add a curved line next to the picture as a sign of displacement.

Yulia responds to the number line problem, such that when someone uses the elevator and the process moves up, Yulia assumes it involves addition. By contrast, Yulia assumes it involves subtraction when the elevator goes down. Contextual problems are easier to solve for Yulia, and she can also write down the form of arithmetic operations that occur according to the problem presented. This research focuses on how Yulia's strategy in solving problems is related to negative integers by solving using a number line. Questions are designed using open sentences. The researchers hope that they can solve the problem with a strategy that suits the understanding of each student.

The strategy used by Yulia for this problem tends to use signs or lines as the position and direction of the movement in the elevator. Students successfully solve the problem about the story presented, and they can write the form of addition and subtraction according to the problem. When students are able to solve the problem, students should be able to solve the problem of counting negative integers using number lines.

Yulia's Strategies for the In-Line Numbers Problem

This section will explain Yulia as a research subject in detail. Students with knowledge about negative numbers are represented by Yulia. Students can determine 10 numbers under the number 5 in sequence and right on the number line. Students can also name the number of negative numbers that appear on the number line if five numbers are present, namely, −5, −4, −3, −2, and −1. Students are also able to compare between 0 and −6, which is greater than 0 on the grounds that the number 0 has no negative sign. Students conclude that a number further to the right on a number line is greater and vice versa, that a number further to the left is smaller.

The next problem is almost identical to the previous problem. The difference is that in the previous problem, the set of questions before the operation starts with a count of 0, whereas in the next problem, it does not. After completing these questions, students are expected to make conclusions. Students have been able to solve the problem well and are able to conclude that this problem and the previous one have the same result. Students also conclude that the drawing operation on the number line does not have to start with the number 0. This last problem is an evaluation of the previous questions about how students understand the addition and subtraction of negative integers on the number line. Students can answer questions and draw number lines correctly, but when students are asked to write their understanding of the problem, they cannot make conclusions.

Yulia's Strategies and Difficulties in Solving Number Line

In the previous negative number and lift tasks, the students tended to model operations involving negative integers vertically on the number line. Students are asked to illustrate horizontal number lines in the open sentence task. Yulia already understands the problem, as seen from the correct steps to solve it. However, when asked to make addition and subtraction forms that are almost the same as before with the same results, she cannot solve them. This inability is evident in the blank on the student answer sheet. This result is one of the reasons for classifying Yulia as instrumental because she can perform mathematical steps but is unable to connect previous concepts with new concepts. In this case, the researchers deliberately started the operation of counting with the number 0 so that Yulia could compare it with the next question. When interviewing Yulia, the researcher submitted a task sheet that had been done with the previous students in the form of a series of open sentences. Students were interviewed about how and why they solve problems so that the researchers know the strategies used.

The first two problems raised involved the addition and subtraction of negative numbers with open lines as a result of addition or subtraction operations. The first task is task $5 - 9 = \square$ and followed by $-9 + 6 = \square$. Students must complete the task correctly. Yulia's first response resolves $5 - 9 = \square$ by jumping nine times from 5 to the left or toward the smaller number value. The researchers assumed the students used a countdown strategy. Students' understanding of this problem assumes that the reduction will move to the left. If the task before the student moves to the left, and then for the task $-9 + 6 = \square$, Yulia completes the task by jumping to the right of the number -9 jumping six steps. The student assumes that because it is added, it will move to the right. Students' strategies regarding the two problems were the same when students were interviewed a second time. Students use the definition of a number line and the notion of addition as the movement to the right (forward) on the drawn number line and reduced as the movement to the left (backward). Researchers attribute this belief because of addition typically moving to the right. Students' strategies regarding the two problems were the same when students were interviewed a second time. Students use the definition of a number line and the meaning of addition as the movement to the right (forward) on the drawn number line and subtraction as a movement to the left (backward). Students think that because addition occurs, the shift will move to the right.

The second problem lies in the open sentence in the middle of the task such as $7 + \square = 3$, $8 - \square = 11$ and $6 + \square = 4$. Students completed all three tasks correctly. In the fourth task, students were asked to prove the correctness or error of tasks such as $9 + -7 = 2$, $9 + -7 = 9-7$, and $7-2 = 7 - -2$. Students solve the first problem well. The answers are wrong for the second problem, but the strategy is right. For the third problem, students answer wrong.

The third task is $\square + 6 = 2$, $\square - 3 = -5$ and $\square + 3 = -6$. Students must complete the task $\square + 6 = 2$ correctly. Students start the strategy from the results and move back six steps because the open sentence in front of the student's understanding is slightly different from before. When the addition is used, students start from the results of the backward movement. Students continue to find the tasks $\square - 3 = -5$ and $\square + 3 = -6$, difficult. Their answers and strategies remain wrong. Students start from the middle to task $\square - 3 = -5$, from 3, then left to -5. This task aims to find out the results of students calculating the distance between 3 and -5. Students start from the results then add three moves to the right. In this task, students forget the concepts that have been worked on in the point $\square + 6 = 2$. Researchers differentiated Yulia's answers based on the right and wrong strategies when interviewed (Tables 4a and 4b).

Table 4a

Yulia's Correct Strategies and Responses

No	Problems	Strategies	Interview responses
1a	$5-9=\square$	Countdown	• "From the number five, if you lower it to the left, counting from steps five to nine, the result was negative four." Why to the left? "Because if it was reduced to the left." • "5 minus 9" refers to Yulia starting from five then moving left nine steps. The result was -4. Why move left? "Minus."
1b	$-9+6=\square$	Calculating forward	• "From -9 nine plus 6. The result was -3." Why step right? "Because addition." Are you sure you answered correctly? "Certain."
2a	$7+\square=3$	Behind start	• Yulia, when answering "3 plus 7 results in -4," pointed to three then shifted to the left seven steps. Then, the result was -4. Yulia's concept changed at the beginning when she stated that if the addition moved forward. In this case, Yulia actually moved backward. • "3 minus -7 answers 4." Why did you go left? "Due to minus."
2b	$8-\square=11$	Countdown	• "From the number 8, I added 8 from 1, 2, …., 8. The result was -3." Yulia's concept changed at the beginning when she stated that if addition is used, forward movement occurs. However, in this case, Yulia actually moved backward. • "8 minus 7, becomes -3." Why move left? "Due to minus."

2c	$6 + \square = 4$	Behind start	• Yulia, when answering "4 plus 6 results in -2," pointed to the number four then moved left six steps. The result was -2. Yulia's concept changed initially when she stated that if addition is used, forward movement occurs. In this case, she actually moved backward. • "4 minus 6 was -2." Why move left? "Because it was reduced."
3a	$\square + 6 = 2$	Behind start	• "Because the numbers were almost non-existent, the numbers start behind." Yulia pointed to the 2, then moved to the left six steps, finally stopping at -4. Thus, the result was negative four. • "From 2, the result was -4 minus 6." Why was it reduced?
4a	True or false: $9 + -7 = 2$	Behind start	Yulia answered correctly. • From two, Yulia began to draw a line two rungs to the left for nine rungs, ending up at -7. But she was unable to explain how she answered correctly "I didn't know. The grade was calculated beforehand." Yulia answered correctly. "From -7, plus 9, the result was 2." Why did you answer correctly? "Because the result was the same as and because I had calculated it.
4b	True or false: $9 + -7 = 9 – 7$	Calculating forward	Yulia answered incorrectly. • For the Solution $9 + -7$, Yulia answered, "Of -7, the result was 2." Yulia stared at -7 and then progressed nine steps, ending at 2. • 9–7 completion. Here, "nine" refers to Yulia starting at 9, then took seven steps back, ending at two. The answer should be correct because the number line represented both. However, Yulia answered incorrectly. "Because I counted, but I slipped."
		Countdown	Yulia answered incorrectly. • For the solution $9 + -7$, she explained, "Of -7 plus 9, for a result of 2." • The solution was 9–7, making 2.

			Why were line numbers 9 + −7 going to the right and line numbers 9−7 going to the left? "Because 9 + −7 was added, while 9 −7 was subtracted." Why did you answer incorrectly? "Because I counted."

Table 4b

Yulia's Wrong Strategies and Responses

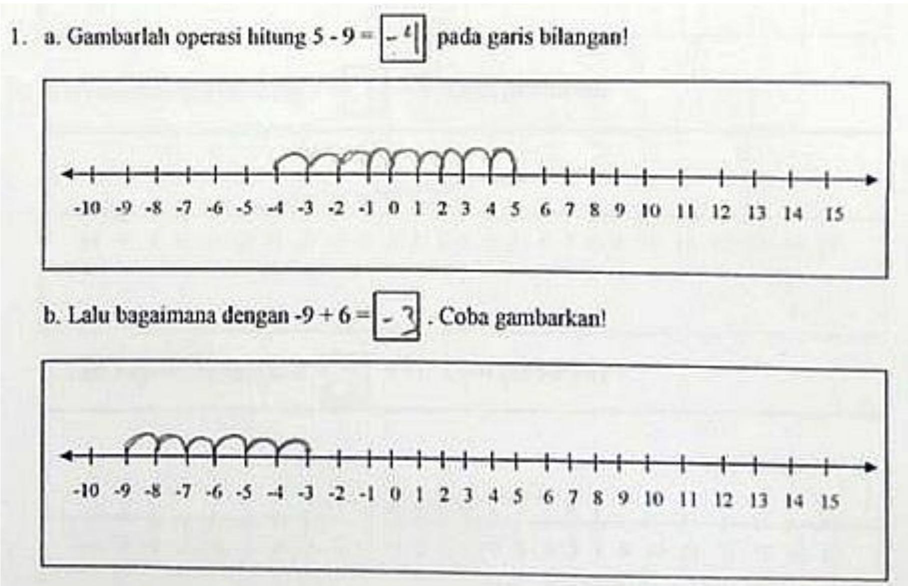
No	Problems	Strategies	Interview responses
3b	$\square - 3 = -5$	Calculating distance	• "I was the beginning of the numbers." Yulia pointed to three and then moved left to -5. "I forgot." Yulia calculated the distance or jumped from three to five negative numbers. The number of jumps was eight. • Yulia forgot where to start. She ranged from -5 plus 8 to 3. Why did you add? "I tried."
3c	$\square + 3 = -6$	Behind start	• "For -6 plus 3, the negative result was 3." Were you sure? "Not sure. It's hard." • "From -6 plus 3, the result was -3." Why did you move right? "Due to plus."
4c	True or False: $7-2 = 7 - -2$	Countdown	Yulia answered true. • For the 7–2 solution, Yulia said, "7 minus 2 results in 5." Yulia pointed to 7 and stepped back two steps. Then, she stopped at 5. For the solution of 7− −2, Yulia pointed to 7 and moved the pencil to 5. She stepped back two steps. "The result is 5, which is the correct answer." Were you sure? "I didn't know, but the result was the same. I took the seconds and I had counted it." Yulia answered correctly.

• For 7−2, Yulia stated, "From 7, I jumped left twice, for a result of 5."
• For 7− −2, Yulia stated, "From 7, I jumped to the left twice, for a result of 5."
How did you answer correctly?
"Because the result was the same and I calculated it."

Yulia suspects that subtraction or addition follows the direction of the shift in the calculation operation steps. This belief may be seen in the strategy used in solving the problems $5 - 9 = \square$ and $-9 + 6 = \square$. Yulia perceives addition as a step forward or a step to the right. By contrast, Yulia assumes that steps are to the left or backward for subtraction. Figure 2 shows the results of Yulia's task.

Figure 2

Yulia's Task Results



To solve problems 1a and 1b, Yulia drew the line from −4 to 5 and −9 to number −3. The researcher interviewed Yulia about the strategies used to clarify the strategy used. To solve 1a, Yulia stepped back (moving to the left) from 5 nine steps, ending at −4. Yulia explained that subtraction moves left or back. The researchers concluded that the strategy used was a countdown. To solve 1b, Yulia stepped forward (moving to the right) from −9 to the right for six steps, ending up at −3. Yulia explained that addition would move to the right. The researchers concluded that the strategy used was calculating forward. In solving problem 2a, namely, $\square + 6 = 2$, Yulia started the solution step from the result or from behind. She started with two then moved to the left six steps because the first number was blank in the sentence. Finally, she stopped at −4. The reason Yulia gave for subtraction is that she was experimenting.

In problems 2a and 2b, Yulia's answers were not correct. In problem 2b, namely, $\square - 3 = -5$, Yulia used the strategy of calculating the distance. Yulia started from the middle, which is 3. Then, she moved left to −5. She calculated the jump distance from 3 to −5. The distance is eight. For problem 2b, namely, $\square + 3 = -6$, Yulia started from behind or from the result. She simply attempted to find the answer by starting from −6 then adding 3. The result is −3. Yulia was not sure of the answer.

The solutions to problems 3a–3c were resolved properly. For the problem $7 + \square = 3$, Yulia started her strategy from the result. She started at 3, then added 7, resulting in −4. She moved to the left in seven steps. Her understanding of addition shifted to the right as the task changed. For the problem $8 - \square = 11$, Yulia counted backward from 8 by eleven steps. The result is −3. For the problem $6 + \square = 4$, Yulia started at 4, then added 6, resulting in –2. Initially, she responded to the way they were added and moved to the left. However, in the interview responses, Yulia explained that she did this because of subtraction, which necessitated moving to the left.

For task 4a, the strategy Yulia used was correct. She answered correctly. She stared from 2, moving the left nine steps, ending up at −7. However, in the second interview, Yulia started with –7, then added 9, ending up at 2. She did not know the reason for the solution in the first interview. Yulia used the counting forward and backward strategies to solve problem 4b, namely, $9 + -7 = 9-7$. She started at −7, moved forward nine steps, ending up at 2. For the counting operation of 9–7, Yulia counted backward from 9 by seven steps, ending up at 2. However, the student still answered that the comparison was wrong, even though the results were stated to be the same. The researcher concluded that the students' answers were wrong, but the strategy used was correct.

Yulia used a countdown strategy to solve problem 4c, namely, $7-2 = 7 - -2$. She started with the "7 minus 2" number strategy. Here, she started from 7, then moved back two steps, resulting in 5. For the counting operation $7 - -2$, Yulia started from 7 and moved back two steps, resulting in 5. The completion strategy used by Yulia lacks correctness. Each of Yulia's difficulties in answering strategy was analyzed, as seen in Table 5.

Table 5

Yulia's Difficulty is Based on the Strategy Used

Strategies	Problems	Difficulties with the strategies used
Countdown	5−9 = □ 8 − □ = 11 True or false: 9 + −7 = 9–7 True or false: 7−2 = 7 − −2	Yulia does not experience problems in tasks where the open sentence is behind. She feels confused about the change in counting operation signs when completing a task where the open sentence is in the middle, resulting in a difference in arguments between the oral statement and the strategy depicted on the number line. For example, Yulia states that for 8 + 11, the addition always moves forward, but she moves the pencil backward. In the comparison problem, Yulia has difficulty counting operations because negative symbols accompany the addition symbol. Yulia ignores one of these symbols. She also has difficulty explaining the reasons for the complete steps.
Count forward	−9 + 6 = □ True or False: 9 + −7 = 9–7	Yulia does not have difficulty on tasks where the open number is behind. The strategies and reasons that she gave were correct. In the comparison problem, Yulia has difficulty counting operations because negative symbols accompany the addition symbol. Yulia ignores one of these symbols. She also has difficulty explaining the reasons for the completion steps.
Behind start	□ + 6 = 2 □ + 3 = −6 7 + □ = 3 6 + □ = 4 True or False: 9 + −7 = 2	Problems arise in tasks where the opening sentence is at the beginning. Yulia has difficulty starting the strategy. She then starts the strategy from the results section. The difficulty also lies in the change in the operation markers due to their changing location. Sometimes Yulia makes the wrong statement, thereby influencing the strategy of providing results.

Calculating distances	$\square - 3 = -5$	The problem that arises lies in the use of count operation marks. These inaccuracies can affect the strategy and result in wrong answers.

Yulia's difficulties are described in each of the strategies she undertook. In the strategy of counting down or counting forward, difficulties occur when solving comparison problems. Students who complete the task have difficulty subtracting or adding with symbols that are close to negative symbols. In the "behind start" strategy, difficulties in beginning the task occur because the open sentence is in the middle. In the strategy of calculating the distance, difficulties occur in the use of operating marks.

Discussion

Students' views about the existence of negative numbers or numbers smaller than zero are different. Students are asked to name ten numbers under five on the number line based on their knowledge, like the initial task given to research subjects. The research subjects can mention regularities well, whereas some of the other students could not. This task uses a number line with 5, 6, 7, 8, 9, and 10. To the left of the number 5, ten squares are left blank by the researcher. Students are asked to identify the numbers in these blanks. Students who do not have good knowledge of negative numbers will name them randomly, even though the number line should also be a position code. According to (Fischer, 2003), representation is automatically associated with left space. The coding and representation of the number line can be adjusted according to the numerical context and the problem.

When students are asked to compare these numbers, the researcher wants to know the extent to which students understand negative numbers. Students are required to compare numbers 0 and −6, which is greater. Some students answer that −6 is greater than 0. However, if the subject has good knowledge and understanding of negative numbers, such comprehension can be seen from their conclusions. Thus, good knowledge and understanding will be the basis for students to further solve problems related to negative numbers. Ask students why and use number line representations will help students make connections between problems, come up with students' answers and strategies, and describe students' thinking (Aqazade et al., 2017).

As regards the elevator problem as a contextual problem, many students do not have direct experience with elevators but are interested in the context of the problem (Yilmaz, Akyuz, & Stephan, 2019). One of the characteristics of a good contextual problem is that it will bring about mathematical interpretation and strategy solutions, representing an informal strategy that serves as development into more formal mathematics (Widjaja, 2013). Through an elevator, in addition to knowing the existence of actual negative numbers, students are also introduced to the use of number lines. These numbers are vertical lines only because of the shape of the elevator. The ability of students to relate mathematics to real life is very important and practical. Such connections are critical because they relate to the introduction of mathematics in everyday life. The introduction must be accurate to develop certain conceptual and mathematics learning (Altay, Yalvaç, & Yeltekin, 2017).

Several implications are related to the findings. First, understanding how the research subjects draw a number line provides a different understanding in compiling and implementing completion plans. The number line that has been made instrumental-only describes a number line that does not emphasize a large number of sequences. The freedom of students to formulate and implement a plan causes differences in the symbols or lines they use when using number lines as a problem-solving tool. Second, different types of problems affect students' understanding of the concept of the material. Such understanding can be observed from the way the students use the solution. Tasks for preliminary studies with tasks to study strategies provide insight into their conceptual understanding. Tasks are designed so that students remain directed at their understanding of using number lines, resulting in incisive insights into students' understanding.

The students who completed the task remembered more formulas in finding results, indicating that they did not know the actual concept of the material. However, with such tasks, students are given more freedom in using strategies according to their respective ways of understanding and thinking. Students' freedom of thought to operate number lines provides insight into the development of their thinking. Thus, the strategy used will be in accordance with their thinking. Students do not have to start from zero in arithmetic operations to solve negative number tasks using number lines.

Although number lines are introduced in the students' textbook, students are indirectly introduced to a set of rules that must be followed to reach the correct answer. Although the research subjects used number lines to solve negative integer problems, it seems that it can resemble the size of a textbook if using number lines. However, students develop very different rules in the process. In using the number line, students are free to start wherever they wish to complete the task being asked, especially in this case, where the arithmetic operation task is in the form of an open sentence. Thus, if students need to complete the task by giving a point at the beginning as the starting point for moving the steps on the number line, then students encounter difficulties if they open a sentence to find the number in front or begin to purposely leave it blank. Therefore, the number line can be used as an appropriate tool so that students can provide strong reasons for negative numbers (Bishop, 2011). Negative integer count operations with number lines and open sentences can be used as a teacher's solution so that students are better able to understand the concept of negative integers. Furthermore, problem-solving strategies that are in accordance with student thinking can be explored. Students' differences in interpreting negative numbers will affect the strategies they use in solving problems. The gap that occurs when introducing negative integers is that students interpret negative numbers in various ways according to their knowledge of numbers, potentially affecting their problem-solving strategies in solving negative integer problems (Aqazade et al., 2017). Students' differences in strategy use depend on their contrasts in analyzing the problem, giving them different benefits in interpreting the problem (Aqazade et al., 2017; Bofferding & Wessman-Enzinger, 2017).

Student difficulties occur because students do not understand the difference between the signs of operation and positive or negative integer signs. Students should understand the problem first when completing the elusive negative integer concept task, even though they have no difficulty placing negative numbers on the number line but have problems comparing values (Cetin, 2019). Giving difficult tasks at the basic education level makes students tired and pessimistic, preventing them from solving problems (Stein & Burchartz, 2006). The difficulty of students in solving these problems arises because of the cognitive barriers; tasks that are too difficult at the beginning can become students' cognitive obstacles. The cognitive barriers to understanding and knowledge that allow students to learn preclude negative integers (Bishop et al., 2014). Sometimes, in the process of perceptual memory, students' perceptions of concepts can be misunderstood, potentially causing cognitive barriers (Fischer, 2003).

CONCLUSIONS

Researchers have found various strategies that examine the instrumental ability of students in linking existing material concepts to new concepts. Students with instrumental understanding remain unable to connect existing concepts to new concepts. Such students are able to solve the problem. However, when asked for an explanation of the reasons for completing the steps, they cannot provide them. Strategies for students with instrumental understanding include counting down, counting forward, calculating distance, and starting backward. Generally, students' difficulties occur because they do not understand the difference between positive or negative operating signs and integer signs and decide the type of strategy used in problem-solving.

AUTHORS' CONTRIBUTIONS STATEMENT

MFA and MDKW conceived the presented idea and research methodology and conducted writing, review, and editing. MF collected the data. HER and NQW conducted data analysis. All authors actively participated in the discussion of the results and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by the corresponding author, MFA, upon reasonable request.

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