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Characterisation of Probability Learning in a Rural Environment with the Realistic Mathematics Education

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ABSTRACT

Background: As statistics has pragmatic effects in other sectors, this work argues for proposing teaching-learning strategies based on real situations. **Objectives:** Characterise probability learning in a rural environment with realistic mathematics education. **Design:** Of a qualitative nature with an action-participatory research design, through five methodological phases. **Setting and participants:** Students from rural areas of the department of Córdoba-Colombia, the mathematics education teacher of that department, and the research group. **Data collection and analysis:** The qualitative data were grouped into categories of realistic mathematics education**. Results:** The research revealed that the students present interpretive, theoretical, and algorithmic difficulties. It also highlighted that realistic mathematics education reflects human activity in the statistical learning process. **Conclusions:** The contextualised problems allow students to reach meaningful knowledge. Besides, the teacher cooperates in formalising learning due to its structuring, systematisation, and regularisation under the realistic mathematics education criteria.

Keywords: Context; Statistical education; Realistic mathematics; Random thinking.

Caracterización del aprendizaje de la probabilidad en un entorno rural con la Educación Matemática Realista

RESUMEN

Antecedentes: La Estadística presenta efectos pragmáticos en otros sectores, por lo tanto, se hace necesario proponer estrategias de enseñanza-aprendizaje basadas

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en situaciones reales. Esta investigación estima la Educación Matemática Realista como estrategia de construcción del conocimiento probabilístico, a través de situaciones contextualizadas. **Objetivos:** Caracterizar el aprendizaje de la probabilidad en un entorno rural con la Educación Matemática Realista. **Diseño:** De carácter cualitativo con un diseño de investigación acción-participativa, a través de cinco fases metodológicas. **Contexto y participantes:** Estudiantes rurales del departamento de Córdoba-Colombia, la docente de educación matemática del respectivo departamento y el grupo investigador. **Recopilación y análisis de datos:** Los datos cualitativos se agruparon en categorías de la Educación Matemática Realista**. Resultados:** Se encontró que los estudiantes presentan dificultades interpretativas, teóricas y algorítmicas. También, se destaca que la Educación Matemática realista refleja la actividad humana en el proceso de aprendizaje estadístico. **Conclusiones:** Se concluyó que, los problemas contextualizados permiten alcanzar un conocimiento con sentido y, el docente coopera en la formalización de dicho aprendizaje, debido a la estructuración, sistematización y regularización bajo los criterios de la Educación Matemática Realista.

Palabras claves: Contexto; Formación estadística; Matemática realista; Pensamiento aleatorio.

Caracterização da probabilidade de aprendizagem em um ambiente rural com Educação Matemática Realística

RESUMO

Antecedentes: A estatística tem efeitos pragmáticos em outros setores, portanto, é necessário propor estratégias de ensino-aprendizagem baseadas em situações reais. Esta pesquisa estima a Educação Matemática Realista como estratégia para a construção do conhecimento probabilístico, por meio de situações contextualizadas**. Objetivos:** Caracterizar a aprendizagem de probabilidade em meio rural com Educação Matemática Realista. **Desenho:** De natureza qualitativa com desenho de pesquisa-ação-participativo, por meio de cinco fases metodológicas. **Contexto e participantes**: Estudantes rurais do departamento de Córdoba-Colômbia, o professor de educação matemática do respectivo departamento e o grupo de pesquisa**. Coleta e análise de dados:** Os dados qualitativos foram agrupados em categorias de Educação Matemática Realista. **Resultados:** Constatou-se que os alunos apresentam dificuldades interpretativas, teóricas e algorítmicas. Além disso, destaca-se que a Educação Matemática realista reflete a atividade humana no processo de aprendizagem estatística. **Conclusões:** Concluiu-se que os problemas contextualizados permitem alcançar um conhecimento significativo e o professor colabora na formalização dessa aprendizagem, devido à estruturação, sistematização e regularização sob os critérios da Educação Matemática Realista**.**

Palavras-chave: Contexto; Treinamento estatístico; Matemática realista; Pensamento aleatório.

INTRODUCTION

Over the years, many countries have focused on statistics education, which has been gradually incorporated into programmes related to mathematics teaching. In this sense, probability is one of the most relevant concepts in this field because it allows cognitive and logical components to be related to discerning non-deterministic events (Estrada & Batanero, 2019). Morales (2018) reveals that there is currently little statistical culture because of the lack of consistency of the content in the development of mathematical thinking.

It is noteworthy that primary school students have little random reasoning, which owes to the traditional methodology used in schools, i.e., in the process of teaching probability, schools place a strong emphasis on the Laplace mathematical algorithm: the ratio between the number of favourable cases by the number of possible cases, a problem that, according to Isaza (2020), lies in the use of formulas without the students understanding their application in the real context.

This is misaligned with Andrade's (2019) proposition, as he affirms that probabilistic is one of the educational branches that cooperates in the development of skills to observe, identify, and evaluate with great precision the success or failure of non-deterministic phenomena. It also allows uncertainty to be written in mathematical terms, making it highly relevant today. Thus, Díaz-Levicoy et al. (2019) maintain that supporting the random learning process with the resolution of contextual problems equips students with skills and trains them to face the random needs of their context and similar environments.

In that order, students from rural schools find it difficult to develop random thinking, because most teachers do not implement didactic strategies that integrate the contents of the random component with the context in which the contents can be applied (Tachie & Molepo, 2019), which means that students in those areas do not have a good understanding for making significant decisions in their environment.

In this sense, realistic mathematics education (RME) facilitates the construction of learning by using the environment, ensuring that the connection between the context and mathematical knowledge allows meaningful learning (Julie, 2018).

Now, in terms of probability learning, it is acceptable to reflect that field fifth-grade students do not visualise the use of probability learning in their environment, which brings the following questions: How does realistic mathematics strengthen the learning of probability in the field with fifthgraders? What is the relationship between realistic mathematics and probability learning in schools in rural areas with fifth-grade students?

To answer those questions, the objective of this research work is to characterise probability learning in rural areas, supported by realistic mathematical education.

THEORETICAL BACKGROUND

Taking into account characterizing probabilistic learning from a realistic perspective in a rural context, the use and impact that realistic mathematics education has presented over the years at the international, national and local levels is then made clear in this section. Consecutive to this, a historical journey is presented about the teaching of probability and the different phases that have characterized this trial. Finally, a detailed description of the realistic mathematical education model (RME) is carried out, which will be centred on the six fundamental principles of this model: reality, level, activity, orientation, interactivity and interweaving, which will be outlined throughout this article.

The teaching of probability.

Over the years, different efforts have been made in the educational field to relate the students' environment with the teaching of statistics. The incorporation of the context in the classroom generates critical skills to understand and reason in different entities of uncertainty, since probability is essential to minimize bias in the study of random or ambiguous events. That is why Andrade (2019) states that statistical education must be oriented from experience with an exploratory and investigative value.

However, globalization itself brings with it the need to relate the context to the educational field. So much so that this insertion in the classroom requires students to solve problem situations in various contexts. In this way, it should be noted that the relationship between the environment and the teaching of probability depends on the teacher's abilities to develop learning processes that are different from the traditional one (Gal, 2005).

In this way, various investigations have mediated this area of knowledge from a realistic perspective.

Thus, for example, the research carried out by Alsina (2011) shows that statistics is of great importance in education because it allows objective analysis and evaluation of phenomena and the development of reality. In this sense, it reflects that the teaching of probability arises with the need to accept the abstraction constituted by man in nature. That is why it is considered one of the essential branches of mathematics education. Apart from combining logical thinking with cognitive thinking in the different fields of human activity, it is also considered as a base content to face the society predetermined by chance.

Likewise, in the research carried out by Isaza (2020), the different contributions of the context are evidenced: immediate, situational and sociocultural in the learning of the concept of probability, concluding a high index in the motivation and interest that the contextualized classes develop, where the Students find an applicability in the context, in the media and in decision making where uncertainty is most relevant.

In the same way, in Andrade's research (2019), the study of random thinking skills was also carried out, through games of chance. He punctually emphasizes that statistical training at the national level is governed by the curricular guidelines of the MEN, being its insertion in the educational field from the first school years in order to develop intuitions to quantitatively manipulate uncertainty and common sense.

However, despite the fact that statistical education has an incidence in various daily contexts, most of this training is isolated from the students' environment. This is, that the environment-education relationship is not promoted and that generally results in meaningless learning, which is far from the citizen that is intended to be formed in today's society. And more knowing that reality allows generating robust knowledge through modeling everyday situations related to probabilistic content, which in many cases is not evident by students through a traditional class. And it is interesting, because of the baggage that it offers to link random knowledge with reality.

Realistic math education.

In this order of ideas, the Theory of realistic mathematical education (RME) is proposed, whose framework does not only include the relationship of abstract ideas with the context, but also in the previous, rational, logical and creative knowledge of the students (Rasmussen and Blumenfeld, 2007). In addition, to be focused on the teachings of mathematics. So, this work is relevant, since it allows mathematical learning to start from a contextualized problem, whether generated or imagined. That is, this theory of education allows students to generate opportunities to explore or achieve robust knowledge from their everyday environment.

Now, an exemplification of the objective of this theory and its usefulness in the educational field, is referenced by Palinussa et al., (2017) as the specialty that requires the teaching-learning process in the integration of the triad reality, imagination and experience. in order to reverberate the usefulness of these in the context. However, the knowledge of contextualized integration in mathematics education is not an instructional knowledge specialized in the teacher, but rather the expertise of directing the debate generated by the students.

Within this order of ideas, it should be noted that EMR is represented as a construct of six principles. Denoted this a model that requires the reformulation of the methodology developed in pedagogical practices in relation to mathematics education. Specifically, the ideal people to develop the teaching of mathematics must have both mathematical and mathematical didactic domain. And they are explicit guidelines, given the theoretical clarity that this model shows with respect to the line of mathematics education (Fredriksen, 2021).

In other words, from the point of view of Alsina and Salgado (2018), the EMR theory is known as a broad and detailed model that seeks to translate context problems into mathematical terms, which is why the teaching of mathematics , under this model it becomes a human action which seeks for students to develop and build their own knowledge. Being one of the most important components because the development is focused on the activities of the subject, leaving aside the homogeneous, unilateral and traditional classes where the teacher transmits a theme, becoming a collective search. Therefore, the teacher is a guide and a facilitator in the transformation.

Now, therefore, the doctrines that support this didactic theory will be described, characterized by Freudenthal (1977) as conceptual instruments, which are perceived as principles of EMR.

Principle of reality, corresponds to everything that the subject can think. Thus, for example, there is a close relationship with intellectual activity when imagining, forming ideas or making representations in the mind (Wijdeveld, 1980). In this order of ideas, it is important that the knowledge of the mathematics teacher about the students' environment is robust, so that its insertion in the learning of mathematics is significant.

In that order, Van den Heuvel-Panhuizen and Wijers (2005) argue that the word real or realistic comes from the Dutch word realisren, which means to imagine. Essentially, reality is not only imagining a real world, but accepting what intuition characterizes as real in a certain event.

However, it should be noted that this section results from an experimental pattern related to the observed frequencies. That is why Bressan et al., (2016) point out that the mathematical concept is acquired through observations, reflections or experiences acquired through human activity. And that are derived from previous research or knowledge that are consistent with their training (Sepriyanti and putri 2018).

Based on what has been proposed, this principle focuses on the various situations that can occur in mental activity, such as: representation, recognition, verification and that serve as a generating base to clarify and order contextualized situations (Palinussa et al., 2017).

Level principle, corresponds to the logical and concrete knowledge to learn mathematics. Within this framework, it happens that the mathematical learning process must be gradual. That is why he highlights the self-development of patterns in learning. Now, Gravemeijer and Doorma (1999) specified four levels inherent to this principle, which are: situational level, which refers to contextualized questions; referential level refers to the identification of the model, denoting the relationship of the situational components with mental activities; the general level refers to thinking in a decontextualized way about the previously raised questions; the formal level indicates reasoning mathematically. The previous levels allow the use of mathematics as an instrument to solve problems in a realistic context (Spriyanti and Putri, 2018).

Due to the focus of this level on gradually developing the learning process. It is essential to include mathematization within the EMR in accordance with the needs raised at the aforementioned levels, describing them as follows:

Horizontal mathematization is one which allows students to mathematize their reality based on intuition, informal knowledge, common sense and observation to identify and describe the mathematics inherent in their environment.

Vertical mathematization refers to mathematizing mathematics, that is, it allows formal knowledge to be reinvented by making use of algorithmic forms or symbols. In effect, mathematics itself is taken as an object of study. In effect, mathematics itself is taken as an object of study, which becomes more mathematical. Thus, achieving a level of formalization (Gallego and Pérez, 2013).

Activity principle holds that mathematics should be an activity, where knowledge should not be transmitted as a finished product, but a process in which students self-construct their own knowledge. Thus, this principle offers a broad overview regarding modeling in the educational field of mathematics (Julie, 2018).

The construction of mathematical models to solve context problems, even from similar contexts, originate from the existing relationship of previous ideas with disciplinary knowledge. And that, due to the applicative character from the mathematics approach, as well as from the environment, it allows the self-construction of knowledge from the solution of contextualized problems that are linked to mathematical knowledge. Therefore, said learning must be inherent in the students' environment, in accordance with the objectives of the class (Nuraida et al., 2019).

Principle of orientation determines the position of the ideal agents to guide the learning process. Thus, they are responsible for observing and understanding the process of students, in order to organize, structure and systematize the different ideas in the development of learning.

This section matters, and for many reasons, because here the strategies, resources and methodologies used by the teacher to direct a learning process that involves the context of the students begin to be developed. And that it contributes in the search for the various mathematical models until finally reaching the formalization of mathematical knowledge (Alsina, 2011).

In this order of ideas, Freudenthal (1977) states that guided reinvention is the process of reinventing the various models, abstract ideas or structuring the questions of the environment. This means that Castiglione (2015) characterizes the teacher as a mathematical psychologist, who instead of answering concerns is the expert to generate questions, in order to generate reflection in the students. In this sense, the teacher takes an active role, but not fixed.

On the other hand, this principle seeks to organize, structure and systematize the ideas of the students, in such a way that the changes of levels during the learning process are evidenced. In general, the teaching role is well defined, since they are the mediators for students to achieve significant and robust self-learning (Julie 2018).

Principle of interactivity, corresponds to the effect of communication and cooperation that it presents in the formalization of mathematical learning.

Now, it is convenient to define that the interactivity generated by the teacher and the students generates reflection, promoting a debate. In this sense, the participation of each student cooperates in the construction or reinvention of the various mathematical models. And that due to the influential character, the levels of understanding are achieved with heterogeneous groups (students with different levels of mathematical skill). Since they are supports in the development of abilities and skills in the process of mathematical learning. The foregoing positions the student as the inventors and researchers of their own knowledge (Fredriksen, 2021).

Principle of interweaving, mediated by the simultaneous teaching of the different branches of mathematics (algebra, calculus, geometry, etc.), favoring the strong interweaving of mathematical skills and activities that generate a formal and broad understanding of mathematics (Freudenthal, 1977). Now, the real and close context of the students allows to develop general math skills, establishing skills and connection to solve the context problem according to the structure of the mind, likewise the EMR allows to train a holistic student, the which will interrelate the different mathematical branches with any similar context problem (Alsina and Salgado, 2018).

METHODOLOGY

This research is qualitative, determined by the characterisation, based on subjectivity, elements such as imagination, experience, and probability learning in a context with students in rural areas (Márquez-Mosquera & Olea-Isaza, 2020). The research design is action-participatory. It seeks to link the subjects in the learning process to interpret and solve random problems in the rural context. The population under study in this research were the students of a private field institution, in Córdoba-Colombia, with a low socioeconomic stratum. The sample selection was made among fifth-grade students aged between 10 and 12 years, with good, regular, and low academic levels. An intentional non-probabilistic sampling allowed selecting the most suitable students, according to the researcher's perspective. Regarding the methodology, the study was adapted to the phases proposed by Sampieri (2018), as cited below:

1. Identify the problem, i.e., know the problems, difficulties, obstacles, and needs to be faced.

2. Design and develop techniques and instruments to collect information.
3. Carry out the previous phase.
4. Analyse the results across the RME categories to minimise bias.
5. Characterise the skills, under the *principles of reality, level, activity, guidance, interactivity, and intertwinement*, that allowed the development of probability learning in the field, supported by RME.

The classroom intervention time was three months, with students' direct participation.

Techniques and instruments

The observation allowed us to register the entire process of probability learning in a field diary: opinions, participation, and events that occurred in the classes.

The survey was designed, structured, and systematised with eight questions, which were the resolution of problems inherent to the rural environment. This was in a diagnostic and individual way to recognise, identify, and describe the participants' cognitive skills.

The questionnaire was structured by eight questions, which were written under the RME design, so that students, through context, imagination, experience, and informal knowledge, could develop a robust and formal knowledge about probability. The process was guided by the researcher and the teacher and applied individually and collectively to generate and consolidate mathematical models.

The unstructured interview was applied to the teacher through seven questions, whose purpose was to give relevance to realistic mathematics. In this sense, the relationship of pedagogical practice with statistical topics and the rural context was identified.

The instruments in this research were validated by the Delphi method, a technique designed, planned, and structured through questionnaires to experts, for a gradual deliberation, in which the researchers explore the experts' opinions (Cruz-Ramirez, 2019; Conde-Carmona & Padilla-Escorcia, 2021). In this case, the experts had Spanish, Mexican, and Colombian nationality, and they clearly examined the outline, number of questions, suitability for the audience, and clarity of the content of the introduction. Initially, the experts' opinions did not converge due to problems such as the wording, few questions, inconsistencies in the introduction of each technique, among others. After that, the researchers accepted the experts' suggestions and improved the inputs, and, in effect, in the second interaction, agreed with the categories of each instrument.

In this sense, the valuations are made clear from an illustrative framework shown in Table 1.

Table 1.

Final version of the Delphi method

	MA	BA	A	PA	NA	Addition	Average per row	N-pm
1	0	0.15	0.15	0.15	0.15	0.6	0.103	0.320
2	0	0.08	0.15	0.15	0.15	0.125	0.092	0.38
3	0.68	0.77	0.77	0.77	0.77	3.76	0.740	-0.32
4	0.68	0.77	0.77	0.77	0.77	3.76	0.740	-0.32
5	0.68	0.77	0.77	0.77	0.77	3.76	0.740	-0.32
6	0	0.14	0.14	0.14	0.14	3.76	0.740	0.319
Addition	2.04	2.68	2.75	2.75	2.75			
Cut points	0.47	0.50	0.51	0.51	0.51			

Table 2.

Tabla estadística de la validación de los instrumentos

Cuestionario diagnóstico	Cuestionario desarrollado con la EMR	Entrevista no estructurada.
MA-70	MA-69	MA-34

Note: MA: Very adequate. BA: Fairly adequate. A: Adequate. PA: Little adequate. NA: Not adequate.

From the final application of the Delphi method, we concluded that the mean of each of the validations is located in very adequate (MA), showing that these inputs are suitable and reliable.

RESULTS AND ANALYSIS

The reality principle.

This criterion delved into questions such as: If ten different animals are introduced into a domestic poultry pen, and one is chosen at random. Would you know which animal would come out? Why? We could see that despite the fact that the students experience the researcher's questions in their environment, the solutions given in the diagnostic test show that they do not have the theoretical and algorithmic skills to face those problems from a random perspective.

Furthermore, the participants showed difficulties in interpreting the questions because they could not relate imagination to prior knowledge, which is why the solutions issued are misaligned with their statistical education, in particular, with probability, which, in turn, is related to the fact that the tutor, the participants' teacher, maintains that statistical education, in particular, probabilistic education, is a branch of mathematics that provides interpretive, logical, and algorithmic skills to analyse phenomena, make judgments, and make decisions in the real world.

However, in her pedagogical practice, the teacher does not consider the environment for those statistical topics because she has little expertise and experience in teaching them.

However, it is striking that in the sections supported by RME, the students established a connection between the researcher's question and situations they had already experienced. For example, students (E) answered the question, "Would you know if it will rain on the fourth Monday?", as shown below:

> "If it has already rained three Mondays in a row, I don't think it will rain on the fourth". E1
>
> "I haven't seen rain on four Mondays in a row". E2
>
> "The rainy season does not last a month". E3

> “I don’t know what might happen tomorrow, for example, I have no idea if my mom will scold me tomorrow”. E4
>
> “I will look at the news forecast that Monday”. E5

In this sense, each student’s intervention was supported by the relationship between logic and imagination, which is consistent with the observed frequencies of similar events in its environment. In this way, it is interesting that each participant’s intervention was focused on interpreting problems from a real perspective, establishing a connection between imagination, the environment, and problem solving.

The level principle.

In this category, it was interesting that, in the diagnostic section, the participants always sought to reach numerical solutions in problem solving, leaving aside the reflection of that process. Based on this principle of the RME, some situations were formulated: A doctor arrived at the town hospital to attend the deliveries of pregnant women. However, 12 of the 20 women admitted for childbirth had complications in the procedure due to a lack of instruments. Lady is about to have her baby; she is undecided whether to travel to a nearby city or have it in the municipal hospital because of many complications. What would you recommend: should Lady travel or have the baby at the municipal hospital? Why?

The questioning caused the participants to issue pragmatic answers. There were positive results in problems that did not require vertical mathematisation. Indeed, the students were able to imagine each exposed problem with their living environment; however, they did not obtain the logarithmic ability to mathematise them vertically, which is why they did not justify the answers to each question.

Still, in the learning process supported by the RME, the participants began solving the problems by imagining and comparing real situations in the context with the questions, and, as they progressed, they identified the mathematical models. In other words, they began to write the questions mathematically, so it was interesting to observe that as the development of an exercise advanced, they brought questions that generated contradictory ideas when thinking out of context, until the point in which the participants presented difficulties to take mathematics itself as the object of study and delve even deeper into it.

The activity principle.

Concerning this category, we observed that in the diagnostic section, the participants did not lead the learning process; on the contrary, they expected the researcher to explain the topic, formulating several questions during the section, which is proof of their passive stance in the development of learning (Julie, 2018). On the other hand, the teacher stated that statistical education, especially probabilistic education, is not relevant in the mathematical content of the institution. Consequently, those topics are very little addressed and deepened in the classes.

Likewise, she reveals that she places a strong emphasis on conceptual and procedural mastery; therefore, its applicability is conditioned on random problems posed in textbooks. Now, in the interview with the teacher, questions such as the following were explored: How do you think students see statistics and what attitude do they have towards this topic? To which the teacher (D) replied:

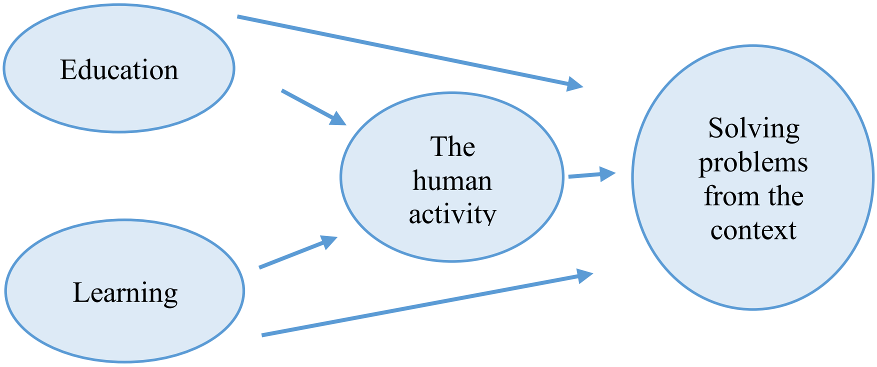
> "Students often see statistics education as a game of numbers, of combinations, since it is a new subject for them, they see it that way". D

From the above, we assume that the teacher identifies that the context is important for learning probability, given that it influences many sectors of real life. However, she presents a contrast in her pedagogical practice because she maintains that she must be subordinated to textbooks by the institution's policies.

On the one hand, probability learning under the RME left aside the conception of teaching as a transfer of concepts applied to games of chance, and perceived knowledge as the consequence of the needs resulting from human activities, as seen in figure 4, which relates the process of activity based on the realistic mathematics education.

Figure 1

The activity process based on realistic mathematics.



In this sense, the characterisation of learning and the teaching of probability were based as a human activity for problem solving. In this way, the research group did not introduce any plan, aiming to develop self-learning.

The guidance principle.

In this category, the importance of the teacher in the learning process was confirmed once again. In mathematics education, the teacher is catalogued as a guide to formal knowledge. We should remark that the research advisors were the research group together with the teacher. In this way, the diagnostic section was done individually to identify the probabilistic abilities in relation to their context. Thus, the advisors could learn about the students' difficulties and abilities to overcome, strengthen, and perfect them in the activity with the RME.

The teacher emphasises that the statistical content follows the conventional didactics because she lacks expertise. She mentions she does not know the different mathematics education theories; therefore, from her pedagogical practice, she does not plan a process that can be approached from the needs of the context (Sugilar et al., 2019).

Table 3.

Teacher's posture within the framework of realistic mathematics.

Reflection	Coordination	Conduct	Guide or Lead	Suitable
In this category, the teacher observes and identifies the participants' skills in the learning process.	Essentially, here the teacher coordinates the intervention of each student to evidence the mathematisation changes in the learning process.	In this section, students' notions are confronted, given that the process is directed from the particular and the individual to formalise knowledge from a general and collective perspective.	Depending on this category, the teacher moderates the debates generated in the classroom, which motivates participation.	This is one of the most important components of the teaching role because it is the suitable agent to formalise the knowledge acquired by students. Moreover, this solves the concerns generated

In this sense, the advisors took an active but not fixed role. They were the suitable people to observe, understand, and improve the learning process of probability. Therefore, they organised, structured, and systematised the students' ideas; in effect, they directed the characterisation so that the changes of levels in the process were evident.

The interactivity principle.

In this category, the importance of communication in educational spaces was redefined. In the diagnostic section, we placed a strong emphasis on the participants' communication with the context because one of the purposes here was to identify the relationship and the random knowledge the participants had with their environment (Isaza, 2020). In the interview with the teacher, we asked questions, such as: From your teaching experience, have you used any didactic strategy that could allow students to generate self-learning, relying on communicative relationships with each other and with you? Which one?

The teacher highlighted once again that she is not an expert in the didactics of mathematics education. However, her pedagogical practice encourages debates, which are conducted under communication. In the section supported by the RME, communication was confirmed as one of the most important components of the learning process (Freudenthal, 1977).

In this way, we must emphasise that the student-student interaction dominated the learning process since the participants predominated in the characterisation, as they were the protagonists, as shown in Figure 5.

Regarding the above, it is evident that communication starts from the dialogue that each student has with the contextualised problems. Then, according to the mental activities, communication is strengthened through ideas, which is why student-student and student-teacher interaction lead to reflection through the confrontation of ideas. We should also note that this category cooperated in minimising the difficulties presented in the heterogeneous group (participants with different mathematical skills), which is why the participants are positioned as the inventors of their knowledge. Regarding the vertical interaction, i.e., the advisors' interaction with the whole class allowed changes of levels, likewise allowed them to organise, structure, and guide each student's ideas (Márquez-Mosquera & Olea-Isaza, 2020).

The intertwinement principle.

In this category, due to the objective of this research, we did not teach the different branches of mathematics (algebra, calculus, geometry, statistics, etc.) simultaneously. It is relevant to highlight that the topic under study admitted the intertwining of other mathematical contents. In this sense, it was interesting that the participants presented inconveniences in the learning process when trying to link previous knowledge; for this reason, some of them confounded possibility with probability (Julie, 2018).

However, the teacher mentioned that in her pedagogical practice, she demonstrated that the students relate the learning of fractions with the definition of probability, which disagrees with the students' position when developing the activities because they showed they were confused about the mathematical operation relating the problem data.

So, there is no doubt that the learning of probability still maintains the stamp of rationality in many parts of the world. Latin America is not out of that, where we simplify the complexity to a single calculation and contextualise the uncertainty.

The RME is a mathematics education theory that enables in different levels of schooling to formalise mathematical knowledge from contextualised problems. However, in rural areas, learning this statistical topic was challenging since it is not considered relevant education.

In this way, this research presents analogies with similar studies. Isaza's (2020) research assures that. Her work verified that the participants imagined situations already experienced with the problems exposed in the sections, but they did not establish a connection with the disciplinary knowledge.

What is described above is believed to be evident because the teacher reveals in her interview that, in her pedagogical practice, she limits the applicability of these contents to games of chance and relies on textbooks. Her attitude disagrees with the research study by Vásquez and Alsina (2017), because they argue that the situations of the textbooks should be modified to the needs of the students.

On the other hand, from Julie's (2018) point of view, the learning process with contextualised problem solving allows students to go beyond repeating algorithms and probabilistic concepts, and becomes an investigative instrument to help them understand random events objectively.

The results of this research confirmed once again how important it is to relate probabilistic knowledge to the environmental aspect, as it provided the opportunity to describe, solve, and make judgments in a state of uncertainty (Méndez-Parra et al., 2021).

It was interesting to realise that the participants acquired formal knowledge through the debate that unfolded from individual and collective reflection. In turn, the advisors played an important role because they conducted the process so that the changes in levels in the development of random learning were evident.

The above does not agree with Revina and Leung's (2019) research, which revealed that the teacher provided predetermined training, and that did not allow the participants to develop their own methods, i.e., they followed the standard instructions. As a closure to this section, we should emphasise that the characterisation product was built from the student-student interactions. Student-teacher interactions and vertical interaction also cooperated in the organisation and structuring of the process.

CONCLUSIONS

The research found that although the students from the rural context participating in the study imagine the problems of the environment, they do not have the mathematical skills to face the random problems, which highlights the little conceptual mastery and disarticulation of the environment in the statistical learning process. This indicates significant contrasts between the notion of statistical education and the teacher's pedagogical practice.

Furthermore, it is important to highlight that the RME is a theory that helped increase and consolidate the participants' mathematisation categories, generating formal knowledge from the resolution of contextualised problems. In turn, this reveals that the probabilistic education must be related to the environment, as the participants reflect on different strategies to solve the problems, which invigorates their random thinking individually.

To conclude, the research underscored one of the most critical components of education, the advisors, who are the suitable people to assist in students' creating or discovering knowledge autonomously.

AUTHORSHIP CONTRIBUTION STATEMENTS

RJCC and CMR conceived the idea presented. IAPE developed the theory. RJCC adapted the methodology to this context, CMP created the models, carried out the activities, and collected the data. RJCC and CMP analysed the data. All the authors actively participated in the discussion of the results, reviewed, and approved the final version of the paper.

DATA AVAILABILITY STATEMENT

Data supporting the results of this research will be made available by all authors (CMP, RJCC, and IAPE), upon reasonable request.

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