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Ontosemiotic Analysis of the Use of Multibase Material in Mathematics Textbooks for Primary Education in Chile

Lizzet Morales-Garcia [a]
Danilo Díaz-Levicoy [b]

[a] Universidad Autónoma de Guerrero, Facultad de Matemáticas, Chilpancingo de los Bravo, México.
[b] Universidad Católica del Maule, Facultad de Ciencias Básicas, Departamento de Matemática, Física y Estadística, Talca, Chile

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ABSTRACT

Background: The multibase material is a widely used and recommended resource for mathematics teaching in the first school grades, given its potential in learning numerical concepts. For this reason, we consider analysing the proposal suggested in the textbooks on its use in primary education. **Objective**: To analyse the use of multibase material in the teaching of mathematics in primary education in Chile. **Design**: A qualitative methodology of exploratory-descriptive level is followed. **Setting and Participants**: Sample made up of 12 math textbooks. **Data collection and analysis**: The levels of analysis of mathematical activity and the construct epistemic configuration of primary objects, theoretical and methodological elements of the Onto-Semiotic-Approach (OSA) were used. **Results**: Multibase blocks are used to 1) indicate the cardinality of a set; 2) explain a procedure; 3) explain a concept; 4) represent a number, and 5) compare numbers. **Conclusions**: The analysis of the textbooks allowed us to identify how the multibase material is used in primary education. This information can serve the teachers in service to enhance their use in the instructional process.

Keywords: Multibase blocks; Textbooks; Epistemic configuration; Primary education; Onto-semiotic approach.

Análisis ontosemiótico del uso de material multibase en libros de texto de matemáticas para la Educación Primaria en Chile

RESUMEN

Contexto: El material multibase es un recurso ampliamente utilizado y recomendado para la enseñanza de la Matemática en los primeros grados escolares,

Corresponding author: Lizzet Morales-Garcia. Email: lmgarcia@uagro.mx

dada su potencialidad en el aprendizaje de conceptos numéricos, por tal motivo consideramos analizar la propuesta sugerida en los libros de texto, sobre su uso en Educación Primaria. **Objetivo**: Analizar el uso que tiene el material multibase en la enseñanza de la matemática en Educación Primaria en Chile. **Diseño**: Se sigue una metodología cualitativa de nivel exploratorio-descriptivo. **Entorno y participantes**: Muestra conformada por 12 libros de texto de matemáticas. **Recopilación y análisis de datos**: Se utilizaron los niveles de análisis de la actividad matemática y el constructo configuración epistémica de objetos primarios, elementos teóricos y metodológicos del Enfoque Ontosemiótico (EOS). **Resultados**: Los bloques multibase se utilizan para 1) indicar la cardinalidad de un conjunto; 2) explicar un procedimiento; 3) explicar un concepto; 4) representar un número y 5) comparar números. **Conclusiones**: El análisis de los libros de texto permitió identificar cómo se utiliza el material multibase en Educación Primaria, esta información puede servir a los profesores en servicio para potenciar su empleo en el proceso de instrucción.

Palabras clave: bloques multibase; libros de texto; configuración epistémica; Educación Primaria; enfoque ontosemiótico.

Análise ontossemiótica do uso de material multibase em livros didáticos de matemática para a educação primária no Chile

RESUMO

Contexto: O material multibase é um recurso amplamente utilizado e recomendado para o ensino de Matemática nas séries iniciais do ensino fundamental, dada a sua potencialidade na aprendizagem de conceitos numéricos. Por isso consideramos analisar a proposta sugerida nos livros didáticos, quanto à sua utilização na educação primária. **Objetivo**: Analisar o uso de material multibase no ensino de matemática na educação primária no Chile. **Design**: Segue-se uma metodologia qualitativa de nível exploratório-descritivo. **Ambiente e participantes**: Amostra composta por 12 livros didáticos de matemática. **Coleta e análise de dados**: Foram utilizados os níveis de análise da atividade matemática e o construto da configuração epistêmica dos objetos primários, elementos teóricos e metodológicos da abordagem ontossemiótica. **Resultados**: Os blocos multibase são usados para 1) indicar a cardinalidade de um conjunto; 2) explicar um procedimento; 3) explicar um conceito; 4) representar um número e 5) comparar números. **Conclusões**: A análise dos livros didáticos possibilitou identificar como o material multibase é utilizado na educação primária. Essas informações podem ser utilizadas pelos professores em serviço para potencializar sua utilização no processo instrucional.

Palavras-chave: blocos de múltiplas bases; livros de texto; configuração epistêmica; educação primária; abordagem ontosemiótica.

INTRODUCTION

The textbook is one of the main tools for the teacher during teaching. In this regard, Burgos, Castillo, Beltrán-Pellicer, Giacomone, and Godino (2020) establish that the textbook can be considered a planned instruction process, composed of a sequence of mathematical practices proposed by the author for the study of a particular topic. Another concept is that textbooks can be seen as institutions carrying the meaning of mathematical objects (Godino & Batanero, 1994) constituting one of the basic references for the organisation of a teaching process, hence their importance as an object of continuous review to evaluate their disciplinary and didactic relevance (Konic, Godino, & Rivas, 2010).

In mathematics education, various investigations have been carried out about textbooks and the treatment of different concepts, for example, the measures of central tendency (Díaz-Levicoy, Morales-Garcia, & Rodríguez-Alveal, 2020), the additive structures (Rodríguez-Nieto, Navarro, Castro, & García-González, 2019), analysis of statistical graphs (Díaz-Levicoy, Osorio, Arteaga, & Rodríguez-Alveal, 2018), among other topics. Other studies analyse the representativeness of the meaning of mathematical objects in the texts (e.g., Morales-Garcia & Navarro, in press; Morales-Garcia, Navarro, & Díaz-Levicoy, 2021; Pino-Fan, Castro, Godino, & Font, 2013), as well as the elaboration of a guide for the analysis of textbooks based on didactic suitability criteria (Castillo, Burgos, & Godino, in press). Similarly, studies on textbooks have considered the different educational levels from early childhood education to higher education.

On the other hand, the manipulative material is widely used in primary education, and in textbooks, it is generally included in a section called *cut-out material* (material recortable). However, there is little research that addresses the analysis of manipulative materials proposed in mathematics textbooks (e.g., Ávila, 1996; Cade, 2015; Carvajal, 2004) since, for the most part, teaching proposals that involve their use for teaching mathematical concepts are reported. In accordance with the above considerations, this work aims *to analyse the suggested use of multibase material in mathematics textbooks for primary education in Chile*.

FUNDAMENTALS

Manipulative materials in primary education

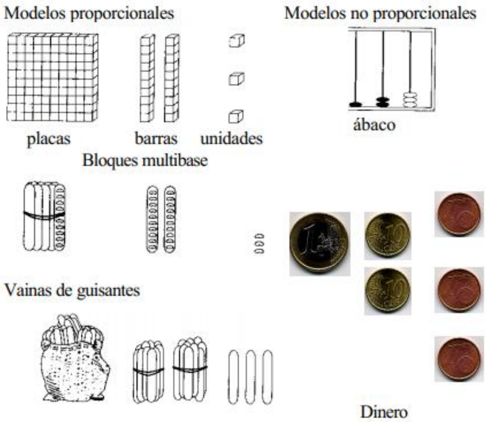
The importance of manipulative material in the classroom lies in the fact that it can serve as a mediator in mathematics teaching, by allowing interaction with the concept studied. According to Godino (2003), any medium or resource used in the teaching and learning processes of mathematics can be considered as didactic material, and can be classified as *study aids* if they are resources that directly support the teacher's function, for example, textbooks; as *(semiotic) instruments for mathematical reasoning* if they are physical objects taken from the environment or specifically prepared, generally recognised with the term manipulative, and are distinguished between tangible manipulatives, where tactile perception is at stake, and graphic-textual-verbal manipulatives, in which visual and auditory perception participate.

Leguizamón, Patiño and Suárez (2015) identified didactic trends of teachers about the use of manipulative material in the classroom, the results indicated that they are used as a motivational tool that allows awakening students' interest in the subject and the mechanisation of concepts. On the other hand, Murillo, Román and Atrio (2016) studied the availability of different mathematics didactic resources in the primary classes of Latin America and their impact on the students' performance. The results showed that in the third degree the resources whose access seems to affect to a greater extent the performance in mathematics are the tangram, the multibase material, the calculator, and the geoboard. While in sixth grade the materials whose availability improves mathematics learning are the workbook, the tangram, the logical blocks, the Cuisenaire rules, and the geoboard.

On the other hand, Cid, Godino, and Batanero (2003) classified the manipulative materials into proportionals and non-proportionals (Figure 1). In the first case, for the base 10 proportionals, the material that represents the ten is ten times larger than the one that represents the unit; the representation of the hundred is ten times larger than the ten, and so on. While the non-proportional materials do not maintain any size relationship between the different parts that compose them.

Figure 1

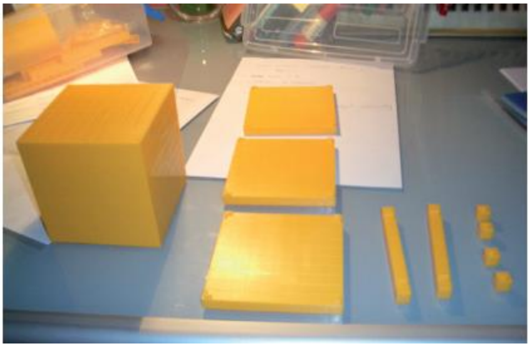
Proportional and non-proportional manipulative material (Cid et al., 2003, p. 219)



There are different manipulative materials that can be used for teaching and learning mathematics, taking into account the content addressed. Specifically, multibase blocks are a resource designed for children to understand the decimal numbering system (SND), "on a specific manipulative basis, based on pieces of a certain size that report on their numerical value" (Alcalde, Pérez, & Lorenzo, 2014, p. 30). The material consists of a series of pieces traditionally made of wood, representing the units of first, second, third, and fourth-order in the SND, i.e., units, tens, hundreds, and units of thousands (Figure 2).

Figure 2

Representation of the number 1, 324 with multibase blocks (Alcalde et al., 2014, p. 32)



In the SND, and in correspondence with what was indicated by Alcalde et al. (2014), each of the pieces is described:

- *1 cm-edge cubes*. They represent the simple or first-order units.
- *Bars*. Rectangular prisms formed by 10 cubes of 1 cm of edge and represent the second-order units or the tens.
- *Plates*. Quadrangular prisms formed by 10 bars, representing the third-order units or hundreds.
- *Blocks*. Cubes made up of 10 plates attached, representing the fourth-order units or units of thousands.

Particularly, the multibase blocks are a proportional manipulative material (Cid et al., 2003), and by their presentation can be classified into *tangible manipulative* (Figure 3), if there is a direct interaction with the pieces, i.e., each piece of the material can be perceived with the touch; *virtual manipulative* (Figure 3a), it interacts with the material by means of a technological resource such as the computer, the tablet or a smartphone, where visual and auditory perception and the *graphic manipulative* (Figure 3b) intervene, in which it interacts with the graphic representation of the material, by means of visual perception.

Figure 3

Types of multibase material

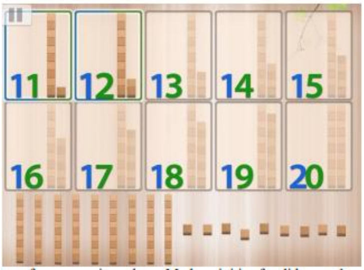


Figure 3a. Virtual manipulative (Tucker, Lommatsch, Moyer-Packenham, Anderson-Pence, & Symanzik, 2017, p. 653)

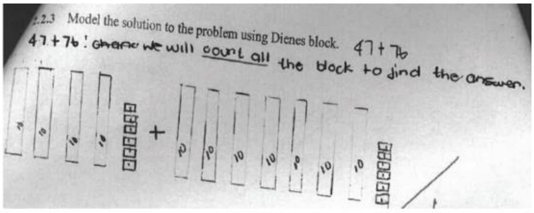


Figure 3b. Graphic manipulative (Ndlovu, & Chiromo, 2019, p. 7)

Regardless of the nature of the material, it can be used in different situations related to the SND, for example, to carry out addition and subtraction operations, to work on double and half concepts, and help solve daily problems that involve operations with natural numbers (Alcalde et al., 2014) to work on concepts such as unit, ten, or hundred, to perform additive decomposition of a number, etc.

Multibase blocks in primary education

Manipulative materials are widely used in primary education and can support students in connecting multiplicative concepts with physical materials (Hurst & Linsell, 2020). Likewise, they are considered an effective teaching approach in the classroom since they improve the students' thinking process in problem solving, for example, in the factorisation of algebraic expressions (Larbi & Okyere, 2016). Knowledge about the proper handling of these materials is important, since, as mentioned by West (2018), resources alone do not provide effective educational experiences unless they are accompanied by adequate teaching, to ensure rich, connected learning experiences.

There is research on the use of multibase blocks in numeracy teaching, for example, Ndlovu and Chiromo (2019) investigated teachers' conception of the use of multibase material for improvement in skills and reasoning skills when modelling the solution of numerical operations. The results indicated that the use of multibase material to express and solve addition and subtraction operations of two numbers evolved gradually from using the blocks to represent numbers to using them to perform addition and subtraction operations and using the associated mathematical language.

On the other hand, Tucker et al. (2017) analysed children's interactions with virtual manipulative material for the teaching of the number sense. Particularly on the interactions with multibase material (dienes blocks), the researchers found that the children improved the learning on number representation with the material, for example, a group of children went from representing two-digit numbers using only cubes (units) to representing the numbers using the bars (tens).

Now, on the proposals and use of manipulative material in textbooks, Ávila (1996), reported the use that teachers in primary education give to free mathematics texts in Mexico, highlighting the significant use of cutout material or manipulative material by teachers. For example, in the first grade, the author identified a high valuation of the manipulable material, which was considered valuable because it develops motor skills, children like it, and it helps to reason, and it collaborates in the constructive process of the child.

Along the same lines, Cade (2015) analysed the impact that the mathematics teaching materials prepared by the teachers of the Mathematics Education Study Group (GEMP) for the Programme for the Integration of Technical Vocational Education to Secondary Education in the Youth and Adult Education Modality (PROEJA) has on students. These materials addressed contents such as the exploration of numbers, shapes, arithmetic operations, measurement of surfaces, among other topics. The results showed the positive impact of the use of teaching materials, for example, the students felt motivated, involved in the class, promoted reflection, etc. However, to successfully develop the work with the different teaching materials, teacher training is essential.

In turn, Carvajal (2004) analysed the school practices of mathematics in the first grade of primary school, with the launch of a new curriculum 1993 reform at this educational level, when new free textbooks were distributed in Mexico, in which more weight was given to the use of didactic material (the official proposal includes a book with cut-out material). The results of the analysis of the practice and the adjustments made by a teacher of this school grade in the 1993-1994 cycle reported that the teacher gives great importance to the use of didactic material, for example, to work the tens by using banknotes of different denominations, to work the count and the sum with the use of a fingers, pillboxes, or beans calculator.

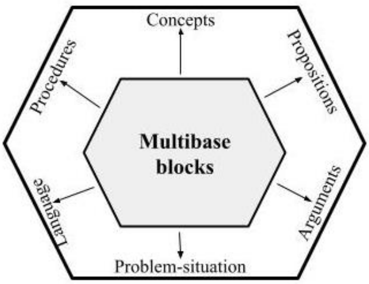
Given the potential of including manipulative material in the mathematics classroom (Leguizamón et al., 2015; Murillo et al., 2016), particularly the introduction of multibase blocks in the teaching of the SND in primary education (Alcalde et al., 2014; Ndlovu & Chiromo, 2019; Tucker et al., 2017), we find it important to analyse the proposed use of this type of material in primary education textbooks in Chile. The above is important, given that it provides information on some elements to enhance its use in teaching-learning and thus guide its use in the classroom.

THEORETICAL ELEMENTS

This research was based on some theoretical references of the Onto-Semiotic Approach (OSA), such as the notion of epistemic configuration (Font & Godino, 2006) formed by the articulation between six primary mathematical objects (problem situations, language, procedures, concepts, propositions, and arguments). Since the focus of attention in this research is the use of multibase blocks in textbooks, the analysis focused on how these primary objects are activated (Figure 4) in the associated mathematical activity, to show the potential that this manipulative material has in primary education. Making the following distinction between the primary objects:

Figure 4

Epistemic configuration



- *Problem-situation*. They are extra-mathematical and intra-mathematical applications of the mathematical object. In this case, we identified whether the problem-situation where the use of multibase blocks is involved is extra-mathematical or intra-mathematical.

- *Language*. They are terms expressions, notations, graphs in their various registers (written, oral, gestural, etc.). We identify the representations promoted with the use of multibase blocks, for example, the symbolic representation of three-digit numbers.
- *Procedures*. They are the algorithms, operations, calculation techniques used in mathematical activity. For example, the procedure for performing the addition of two-digit numbers with multibase blocks.
- *Concepts*. Entered by definitions or descriptions. In this case, those concepts promoted in the solving of tasks with multibase blocks, for example, unit, ten, or hundred.
- *Propositions*. They are statements about concepts used in solving the task. For example, the relationship between units, tens or hundreds and their representation with multibase material.
- *Arguments*. They are statements used to validate or explain the propositions and procedures used to solve a task. For example, when it is necessary to justify the procedure for performing the additive decomposition of a three-digit number with multibase blocks.

METHODOLOGY

The research is qualitative, exploratory-descriptive, and aims to analyse tasks involving multibase blocks in mathematics textbooks for primary education in Chile. For this, the following research phases were considered: 1) identification of tasks where multibase blocks are used; 2) analysis of the tasks and 3) identification of the primary objects involved in each use of the multibase material.

Task identification

We analysed 12 textbooks, and the information and the number of proposed tasks can be seen in Table 1.

Table 1

Books information and tasks analysed

Code	Authors	Title	Publishing House	Number of tasks
T1	Isoda, M.	Suma Primero 1° básico. Texto del estudiante. Tomo 1	Gakko Tosho Co	28
T2	Isoda, M.	Suma Primero 1° básico. Cuaderno de actividades. Tomo 1	Gakko Tosho Co	6
T3	Isoda, M.	Suma Primero 2° básico. Texto del estudiante. Tomo 1	Gakko Tosho Co	21
T4	Isoda, M.	Suma Primero 2° básico. Cuaderno de actividades. Tomo 1	Gakko Tosho Co	10
T5	Urra, A., Córdova, C. y Quezada, C.	Matemática 3° básico. Texto del estudiante.	Santillana	25
T6	Silva, M. y Pastén, A	Matemática 3° básico. Cuaderno de ejercicios.	Santillana	3
T7	Rodríguez, R. García, D., Romante, P. y Verdejo, A.	Matemática 4° básico. Texto del estudiante.	SM	11
T8	Castillo, P., Huaracán, E. y Zambrano, R.	Matemática 4° básico. Cuaderno de ejercicios	SM	5
T9	Ho, F., Kee, G. y Ramakrishnan, C.	Matemática 5° básico. Texto del estudiante.	Santillana	4
T10	Ho, F., Kee, G. y Ramakrishnan, C.	Matemática 5° básico. Cuaderno de ejercicios.	Santillana	1
T11	Castro, C.	Matemática 6° básico. Texto del estudiante	Santillana	0
T12	Castro, C.	Matemática 6° básico. Cuaderno de ejercicios	Santillana	0
		Total		114

Task analysis

According to Godino, Batanero, and Font (2020), the institutional genesis of mathematical knowledge is investigated in the OSA through *1) the identification and categorisation of problem situations that require a response*;

2) the description of the sequences of practices that are put into play in the resolution. With these notions, elements for the analysis of textbooks have been proposed, for example, the levels of analysis of the mathematical activity (Godino, Beltrán-Pellicer, Burgos, & Giacomone, 2017) or the analysis focused on didactic configurations (Burgos, Castillo, Beltrán-Pellicer, Giacomone, & Godino, 2020).

In this research, we used the levels of analysis of mathematical activity. At level 1, *phenomenological-anthropological analysis*, we considered the context, i.e., we identified whether the task corresponds to the intra-mathematical or extra-mathematical context (Font, Breda, & Seckel, 2017; Font & Godino, 2006). We also recognise the use of the material in the task, and a distinction was made between 1) representing the cardinality of a set; 2) explaining a procedure; 3) explaining a concept; 4) representing a number; 5) comparing numbers.

At Level 2, *onto-semiotic analysis of practices*, we focused on identifying the plot of mathematical objects and relationships that are put into play in each of the mathematical practices, which constitute the system of practices expected in solving the problem situation. To identify that plot of objects, the onto-semiotic configuration tool was used (Godino et al., 2017), allowing us to identify emerging primary objects in the resolution of tasks, in which the following elements are highlighted. Also, an element was added to identify the function of the multibase blocks in the task.

- *Statement and sequence of elementary practices to solve the task.* The statement and sequence of operational and discursive practices that are important in solving the task are organised.
- *Use and intentionality of the practices*. Its purpose is to recognise the role of each mathematical practice in solving the task posed.
- *Objects mentioned in the practices*. The plot of primary objects (concepts, language, procedures, propositions, and arguments) identified in each of the mathematical practices is shown.
- *Function of the multibase blocks*. The function that the multibase material has in the practices is identified.

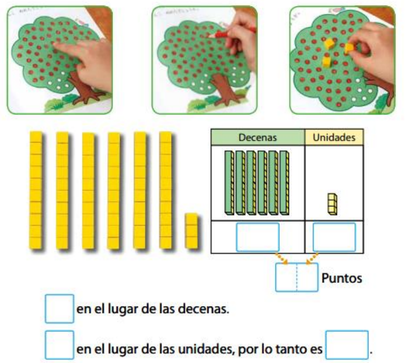
Next, we describe the use of multibase blocks according to the levels of analysis of the mathematical activity.

Represent the cardinality of a set

In these tasks, multibase blocks are used to organise the recounting of object collections (Figure 5). For example, in the task presented in Figure 5, the number of points in the tree is asked to be quantified using bars and cubes. Below, the expected response, the sequence of practices and the onto-semiotic analysis are presented (Table 2).

Figure 5

Use of multibase blocks to represent the cardinality of a set



Expected answer: There are 63 points on the tree.

Sequence of practices

1. Observe the image and count the points on the tree.
2. Arrange the recounting in groups of 10 points, using the yellow strips.
3. Identify that 6 strips and one strip with 3 cubes were used in the recounting.
4. Arrange the strips in the table of units and tens.
5. In the white space at the bottom of the tens, write the number of bars, in this case, 6, and the number of units, i.e., 3.
6. If we put together the number of tens and ones, we get 63.
7. Given that, 6 tens and 3 units is 63.
8. Therefore, there are 63 points on the tree.

Table 2

Onto-semiotic analysis of the practices

Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
Observe the image and recount the points on the tree.	Understand what the task requires and perform a first recount of points.	Concept: numerical sequence, recounting. Language: verbal representation of numbers. Procedure: Count one at a time.	
Arrange the recounting in groups of 10 points, using the yellow strips.	Recount the points on the tree using multibase material.	Language: verbal representation Procedure: 10 by 10 counting. Proposition: 10 points on the tree are organised into a strip of 10 cubes.	Arrange the recount

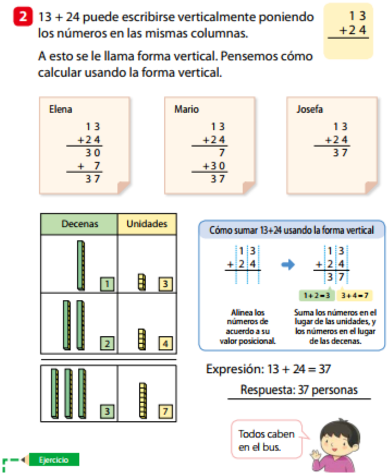
Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
Identify that 6 strips and one strip with 3 cubes were used in the recounting.	Indicate how many strips with 10 cubes were used and how many cubes were left loose.	Procedure: 10 by 10 counting.	Arrange the recount
Arrange the strips in the table of units and tens.	Recognise how many units and tens were obtained in the recount.	Language: verbal and symbolic representation, place value table. Concept: unit and ten. Proposition: 1) a ten is represented with a bar with 10 cubes. 2) the units are the cubes that were left loose.	Represent tens and units
In the white space at the bottom of the tens, write the number of bars, in this case, 6, and the number of units, i.e., 3.	Relate the place value with the conformation of the figures of a number.	Concept: unit, ten, place value, cardinality. Procedure: count the number of bars (tens) and the number of cubes (units). Proposition: there are 6 tens and 3 units.	
If we put together the number of tens and units we get 63.	Indicate how the number indicating the cardinality of the set of points is structured.	Concept: place value. Proposition: 6 represents the number of tens and 3 the number of units.	
Given that 6 tens and 3 units is **63**.	Argument to justify the conformation of the number.	Proposition: the place value determines the conformation of the digits in a number.	
Therefore, there are 63 points on the tree	Task response.		

Explain a procedure

In this case, multibase blocks are used as a manipulative model to explain procedures to perform any of the operations with natural numbers (addition, subtraction, multiplication, or division). Figure 6 presents the sequence of practices for the addition of two-digit numbers and the onto-semiotic analysis of the practices (Table 3).

Figure 6

Use of multibase blocks to explain the procedure for adding two natural numbers



Sequence of practices

Display *the vertical form* as a procedure to perform the addition of natural two-digit numbers.

1. Align the numbers according to their place value.
2. Add the figures in the place of the units and the figures in the place of the tens.

In the table of units and tens, the procedure performed is represented.

3. Identify how many tens (bars) and units (cubes) make up each number.
4. The 13 is represented with a bar, and 3 cubes (row 2) and the 24 is represented with 2 bars and 4 cubes (row 3).
5. The number of cubes, in this case, 3+4, and the number of bars 1+2, are added.
6. The result is 3 bars and 7 cubes (row 4), i.e., 37.

Expected answer: 13+24=37

Table 3

Onto-semiotic analysis of the practices

Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
Align the numbers according to their place value.	Arrange the numbers for addition.	Language: symbolic representation of numbers. Concept: place value Procedure: Vertical form to add numbers. Propositions: a) number 13 is made up of 1 ten and 3 units; b) number 24 is made up of 2 tens and 4 units; c) number 1 is aligned with 2 and 3 is aligned with 4.	
Add the numbers in the place of the units, and the numbers in the place of the tens.	Calculate the sum	Proposition: a) 1+2= 3 and 3 + 4= 7; b) the result of the sum is 37.	

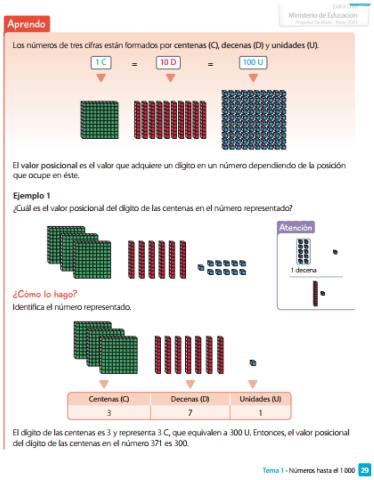
Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
Identify how many tens (bars) and units (cubes) make up each number.	Represent the numbers with multibase material.	Concept: unit, ten and place value.	Represent the units and tens.
The 13 is represented with a bar and three cubes (row 2) and the 24 is represented with two bars and four cubes (row 3).	Represent the number 13 and 24 with multibase material.	Concept: units, ten, place value. Procedure: Additive decomposition. Propositions: a) the 13 is represented with 1 ten (bar) and 3 units (cubes); b) the 24 is represented with 2 bars (tens) and 4 cubes.	Represent a number.
The number of cubes, in this case, 3+4, and the number of bars 1+2, are added.	Calculate the result of the addition.	Propositions: a) units with units and tens with tens are added; b) 1+2 = 3 bars; c) 3+ 4 = 7 cubes. Arguments: place value of the figures of a number.	Justify the procedure to add.
The result is 3 bars and 7 cubes (row 3). That is, 37.	Result of the sum.		

Explain a concept

In this case, multibase blocks are used as a resource to explain the concepts of *place value*, *commutativity in addition, equivalence between units and tens*, for example, 1 ten = 10 units or 1 hundred = 10 tens. In this case, the analysis of the multibase blocks is presented to explain the concept of place value (Figure 7) and the onto-semiotic analysis of the practices (Table 4).

Figure 7

Use of the multibase blocks to explain a concept



Expected answer in the example: the place value of the hundreds digit is 300.

Sequence of practices

1. Three-digit numbers consist of hundreds (C), tens (D), and units (U).
2. A hundred is made up of 10 tens or 100 units, i.e., 1C= 10D= 100U.
3. The place value is the value that a digit acquires in a number depending on the position that it occupies in it.

In the example,

4. Identify the number represented with multibase material,
5. There are 3 blocks, i.e., 3 hundreds.
6. There are 6 bars, i.e., 6 tens.
7. There are 11 cubes, i.e., 11 units, but 10U = 1D, then there are 7 tens and one unit.
8. The number represented is 371.
9. So, the digit of the hundreds is 3 and represents 3C, which is equivalent to 300 U.
10. Therefore, the place value of the digit of the hundreds in the number 371 is 300.

Table 4

Onto-semiotic analysis of the practices

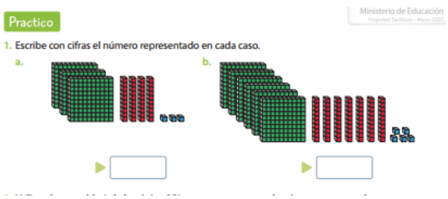
Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
Three-digit numbers consist of hundreds (C), tens (D), and units (U).	Explain the structure of three-digit numbers.	Language: digit, hundred = C, ten= D, unit= U. Concept: unit, ten and hundred. Proposition: a) the hundreds is represented with the letter C; b) the tens is represented with the letter D; c) The units with the letter U.	
A hundred is made up of 10 tens or 100 units, i.e., 1C= 10D= 100U.	Show equivalence between hundreds, tens and units.	Proposition: a) 1C = 10D = 100U	Represent equivalences between units, tens and hundreds
The place value is the value that a digit acquires in a number depending on the position that it occupies in it.	Define the concept of place value.	Concept: place value	

Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
Identify the number represented with multibase material,	Objective of the example	Language: representation of a three-digit number with multibase material.	Represent a three-digit number
There are 3 blocks, i.e., 3 hundreds.	Relate the number of blocks with their numerical meaning.	Proposition: 3 blocks represent 3 hundreds.	Represent the hundreds of a number.
There are 6 bars, i.e., 6 tens.	Relate the number of blocks with their numerical meaning.	Proposition: 6 bars represent 6 tens.	Represent the tens of a number.
There are 11 cubes, i.e., 11 units, but 10U = 1D, then there are 7 tens and one unit.	Relate the number of blocks with their numerical meaning.	Proposition: a) 1D=10U; b) 11U= 1D+1; c) then there are 7D and 1U. Concept: unit, ten and hundred. Argument: place value	Represent the equivalence between tens and units
The number represented is 371.	Identify the number represented with multibase material.	Language: symbolic representation.	
So, the digit of the hundreds is 3 and represents 3C, which is equivalent to 300 U.	Activity response.		

Represent a number

Figure 8

Use of multibase blocks to represent a number



Expected answer: a) 343, b) 675

Sequence of practices

For the response to subparagraph a)

1. The green blocks represent the number of hundreds.
2. There are 3 green blocks, then the number has 3 hundreds or 300 units.
3. The red bars represent the tens.
4. There are 4 red bars, then the number has 4 tens or 40 units.
5. The blue cubes represent the units.
6. There are 3 blue cubes so the number has 3 units.
7. The number represented by the multibase material is 3C +4D +3U, i.e., 300+40+3 or 343.
8. Then the number represented in the multibase material is 343.

Multibase blocks are also used in textbooks to represent a number, for example, we start from the representation of the number with the multibase material and ask to write the indicated number with figures. Below, an example of the analysis of a task is given (Figure 8), as well as the onto-semiotic analysis of the practices (Table 5).

Table 5

Onto-semiotic analysis of the practices

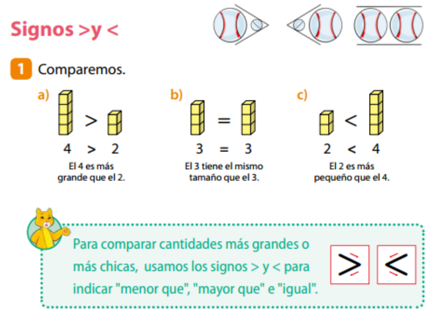
Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
The green blocks represent the number of hundreds.	Identify the value of the blocks.	Proposition: 1 green block = 1 hundred. Concept: hundred	Represent the digit of the hundreds.
There are 3 green blocks, then the number has 3 hundreds or 300 units.	Identify the value of 3 green blocks.	Proposition: 3 green blocks = 3 hundreds= 300 units. Concept: hundred, units, equivalence.	Represent 3 hundreds
The red bars represent the tens.	Identify the value of the bars.	Proposition: 1 red bar = 1 ten Concept: ten.	Represent tens
There are 4 red bars, then the number has 4 tens or 40 units.	Identify the value of 4 red bars.	Proposition: 4 red bars = 4 tens= 40 units.	Represent 4 tens.
The blue cubes represent the units.	Identify the value of the blue cubes.	Proposition: 1 blue cube = 1 unit. Concept: units	Represent the units.
There are 3 blue cubes so the number has 3 units.	Identify the value of 3 blue cubes.	Proposition: 3 blue cubes = 3 units.	Represent 3 units.
The number represented by the multibase material is 3C +4D +3U, i.e., 300+40+3 or 343.	Calculate the number represented by the multibase material.	Language: symbolic representation of numbers. Concept: unit, ten and hundred. Procedure: sum of the place value of the figures of a number. Proposition: 3C+4D+3U= 3(100) +4(10) +3(1) = 300 + 40 + 3= 343.	Represent a three-digit number
Then, the number represented in the multibase material is 343.	Task response.		

Compare numbers

Finally, another use that the material has is to compare two numbers, this use is perhaps the least usual, and then, we present the analysis of the practices (Figure 9) and their onto-semiotic analysis (Table 6).

Figure 9

Use of multibase blocks to compare two numbers



Sequence of practices

1. The 4 is larger than the 2.
2. The 3 is the same size as the 3.
3. The 2 is smaller than the 4.
4. To compare larger or smaller quantities, we use the signs to indicate "less than" (<), "greater than" (>) and "equal" (=).

Table 6

Onto-semiotic analysis of the practices

Sequence of practices	Use and intentionality of the practices	Objects mentioned in the practices	Function of the multibase blocks
The 4 is larger than the 2.	Compare two numbers.	Language: symbolic representation of numbers. Concept: greater than, Procedure: comparison with multibase blocks. Proposition: 4 is greater than 2. Argument: order in the numerical sequence.	Compare two numbers.
The 3 is the same size as the 3.	Indicate when two numbers are equal.	Language: symbolic representation Concept: equal to Procedure: comparison with multibase blocks. Proposition: 3 is equal to 3. Argument: equality of natural numbers.	Indicate when two numbers are equal

The 2 is smaller than the 4.	Compare two numbers.	Language: symbolic representation of numbers. Concept: less than, Procedure: comparison with multibase blocks. Proposition: 2 is less than 4. Argument: order in the numerical sequence.	Compare two numbers
To compare larger or smaller quantities, we use the signs to indicate “less than” (<), “greater than” (>) and “equal” (=).	Presentation of the symbols >, < o = to indicate when a number is greater, lesser or equal.	Proposition: a) the symbol > represents "greater than"; b) the symbol < represents "less than"; c) the symbol = represents "equal to".	

RESULTS

In the analysis, we found that multibase blocks are used to 1) represent the cardinality of a set; 2) explain a procedure; 3) explain a concept; 4) represent a number; and 5) compare numbers. The information on the number of tasks included in each category is shown in Table 7, from which we observe that multibase blocks are used in the first five grades of primary education, concentrating on the first three and is done, mostly, to explain a procedure and represent a number.

Table 7

Use of the multibase blocks in each school grade

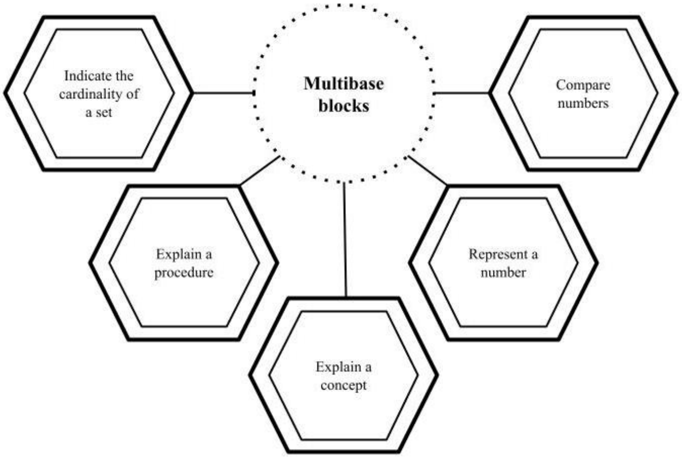
Grade/ code	Indicate the cardinality of a set	Explain a procedure	Explain a concept	Represent a number	Compare numbers	Tasks total
1st (T1, T2)	6	14	0	14	0	34
2nd (T3, T4)	6	22	0	1	2	31
3rd (T5, T6)	0	11	5	11	1	28
4th (T7, T8)	0	7	1	8	0	16
5th (T9, T10)	0	5	0	0	0	5
6th (T11, T12)	0	0	0	0	0	0
Total	12	59	6	34	3	114

Epistemic configurations of the multibase blocks in primary education

According to the analysis of tasks, we found that different primary objects are activated, according to the use that the multibase material has in the different situations proposed in the texts. Their organisation is presented in epistemic configurations (Figure 10).

Figure 10

Epistemic configurations of the use of multibase blocks



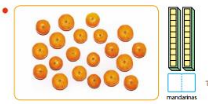
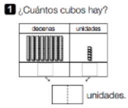
Below, we specifically present the type of problem-situations (SP), language (L), procedures (P), concepts (C), propositions (PS), and arguments (A) associated with each epistemic configuration.

Epistemic configuration 1. Indicate the cardinality of a set

The primary objects involved in this epistemic configuration (Table 8) report the use of *cubes and bars* as a means to organise the recounting of object collections. This use is one of the first interactions that the texts propose for multibase material.

Table 8

Epistemic configuration 1.

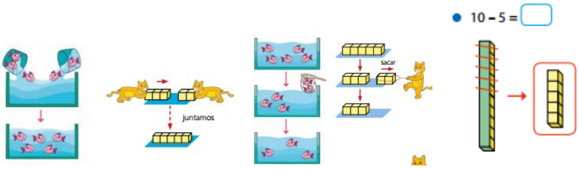
Primary objects	**Meaning**
SP	Relate up to three representations of the natural number (multibase material, symbolic and iconic). Indicate the cardinality of a collection of objects (dragonflies, eggs, mandarins, backpacks, among others) using multibase material (cubes and bars). Determine the number of cubes in the multibase material.
L	Symbolic representation: 6, 7, 8, 9 and 10; positional value tables, iconic representation of sets of objects.    
P	-Evoke the numerical sequence 1, 2, 3,... and assign each object a number in the conventional order, the last number mentioned corresponds to the cardinality of the set. This number is represented with multibase material (cubes and bars). -Make groupings of 10 objects, leaving some loose objects ungrouped. The symbolic representation of the number is obtained by putting the loose elements as the first-order digit (units) and the number of groups formed as the second-order digit (tens).
C	Cardinality, set, unit, and ten.
PS	Each grouping of 10 elements of a set of objects is represented with a bar, if there are single elements these are represented using cubes; 1 bar= 10 cubes=1 ten; 1 cube= 1 unit
A	Knowledge of the conventional numerical sequence, place value.

Epistemic configuration 2. Explain a procedure

Textbooks emphasise tasks that involve multibase material (cubes, bars, plates, and blocks) as a manipulative model to explain and justify procedures for the addition, subtraction, multiplication, and division of natural numbers. Table 9 shows the characteristics of the primary objects in this use.

Table 9

Epistemic configuration 2

Primary objects	Meaning
SP	Additive problems of type change-increase, change-decrease, and combination with an unknown in the whole, where the multibase material (cubes and bars) is used as a model to solve addition and subtraction operations.  Situations to address multiplication as a repeated addition. Situations to address the partitive meaning of division.
L	+, =, join, add; iconic representation, symbolic representation. Structure of the addition: *adding 1 + adding 2 = sum or total (horizontal form);* place value tables. Point symbol to indicate multiplication (example: 231. 3), conventional division algorithm (for example 264: 2).
P	*Procedure for horizontal addition (situations of change-increase, change-decrease)* -Represent the initial number of objects with cubes and bars. Taking into account that 10 objects are represented with a bar and the remaining objects with cubes. -This initial number of cubes add or remove as many as the problem indicates, the total or remaining number of cubes will be the result of the addition or subtraction. *Vertical addition procedure* -Represents the summands with multibase material (cubes, bars, plates, or blocks) in a place value table. -Taking into account that the first digit from right to left represents the number of units or loose cubes: the second digit represents the tens or bars; the third digit represents the number of plates, or hundreds and the fourth digit represents the number of blocks or units of thousands. -Let us add the number of cubes vertically, taking into account that 10 cubes become a bar and vice versa. -Let us add the number of bars, taking into account that 10 bars become a plate and vice versa. -Let us add the number of blocks, taking into account that a block is made up of 10 plates.

	- The necessary decompositions and/or groupings are carried out in the case of the addition or subtraction carried out. - The numerical representation of the sum is obtained by placing the number of cubes as the number of units, the number of bars as the number of tens, the number of plates as the number of hundreds, and the number of blocks as the number of thousands. *Procedure for multiplying* -Represent with multibase material the multiplicand as many times as indicated by the multiplier, then perform the procedure for the addition, exposed above. *Procedure for the division* Represent the dividend with multibase blocks (cubes, bars, and plates). Make as many groups as indicated by the divider, taking care that each one contains the same number of cubes, bars, and plates. The quotient or result of the division will be the number represented in each group formed.
C	Cardinality, unitary and binary conception of addition, binary, and unitary conception of subtraction, unit, ten, hundred, units of thousand; multiplication as repeated sum; multiplicand, multiplier; dividend, divisor, quotient.
PS	1 block= 10 plates; 1 plate= 10 bars; 1 bar= 10 units.
A	Place value, knowledge of the conventional numerical sequence.

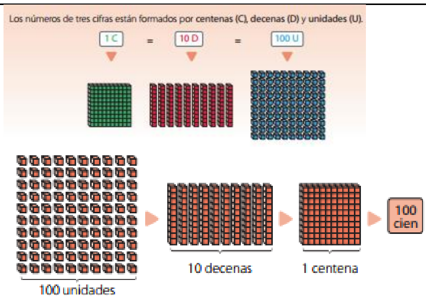
Epistemic configuration 3. Explain a concept

Multibase blocks are also used to visualise features of some concepts or properties. Table 10 shows the primary objects involved.

Table 10

Epistemic configuration 3

Primary objects	Meaning
SP	Situations to address concepts of place value, equivalence between hundreds, tens and units; commutativity in the sum of two numbers.



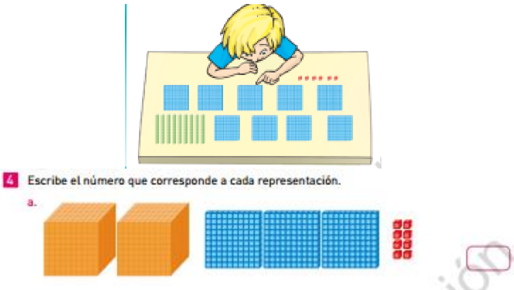
L	Symbolic representation of numbers up to four digits; place value tables; 1C= a hundred, 1D= a ten, U= units, 7C= 700U.
P	-In the case of the *place value*, a four-digit number is represented with multibase material. Subsequently, in a table of positional value, in the section of units, place the number of loose cubes; in the section of tens, place the number of bars; in the place of hundreds, the number of plates, and in the place of thousand units, place the number of cubes. The number formed is obtained by placing the number of units, tens, hundreds, and thousands from right to left. -For the *equivalence between hundreds, tens, and units*, represent a number with multibase material, starting from numerical expressions such as 6D, which represents 6 tens. Subsequently that number is represented with 6 bars, and those bars are decomposed into units and the cubes are recounted. The result of the recounting indicates the number of units, in this case, 6D= 60 units. -For the *commutativity in the sum of two numbers*, represent the summands in both members of the equality with multibase material. Calculate the result of the addition in the first member of the equality and in the second member and verify that the order of the summands does not affect the result of the addition.
C	Commutativity of the sum, place value, unit, ten, hundred, unit of thousand, cardinality, numerical sequence.
PS	a+b= b+a; 10D= 100 units= 1C;
A	Commutative property of addition; place value; cardinality; numerical sequence.

Epistemic configuration 4. Represent a number

In the tasks included in this epistemic configuration, multibase blocks are used as a means to represent numbers of up to four digits. Table 11 shows specifically the primary objects involved.

Table 11

Epistemic configuration 4

Primary objects	Meaning
SP	Identify the number represented with multibase material and transit to its symbolic, verbal, or additive decomposition representation. Represent a number with multibase material given its symbolic representation; identify the value of a digit in a number of up to three digits. 
L	Symbolic, verbal representation and additive decomposition; place value table. For example, 4C+3D+2U= 400+30+2= 432
P	The number of blocks represents the number of thousands; the number of plates, the number of hundreds; the number of bars, the number of tens; and the number of loose cubes, the number of units. The number represented is written from left to right, starting with the highest order number.
C	Place value, unit, ten, hundred, and unit of thousand.
PS	1C=100, 5D= 50, 3U=3,...
A	Natural numbers can be decomposed according to the place value of their figures.

Epistemic configuration 5. Compare numbers

Multibase blocks are also used as a means to compare numbers and allows you to visualise when a number is greater, lesser, or equal to another. Table 12 shows the primary objects.

Table 12

Epistemic configuration 5

Primary objects	Meaning
SP	Situations to compare numbers and show the meaning of the symbols > (greater than), < (less than), or = (equal to) using as a means the representation of numbers with multibase material (cubes, bars, and plates). 
L	Symbols >, < o = to indicate the order relationship between the numbers.
P	Represent the numbers to be compared with multibase material. Use the size of the number represented to indicate which is larger (greater than), smaller (less than), or the same size (equal to) and represent this with a symbol.
C	Greater than, less than, equal to.
PS	The symbol < indicates when a number is lesser than another. The symbol > indicates when a number is greater than another. The symbol = indicates when two numbers are equal.
A	Relation of order in natural numbers.

CONCLUSIONS

This research aimed to analyse the use of multibase blocks in primary education textbooks in Chile. Based on that, we see that they are used to 1) indicate the cardinality of a set; 2) explain a procedure; 3) explain a concept; 4) represent a number; and 5) compare numbers. In this study, the levels of analysis of mathematical activity and epistemic configuration were essential tools that allowed the breakdown and organisation of mathematical practices associated with the resolution of problems related to the use of manipulative material.

The literature had reported that multibase blocks were used to carry out addition and subtraction operations, to work on double and half concepts, and to help solve daily problems that involve operations with natural numbers (Alcalde et al., 2014; Ndlovu & Chiromo, 2019). However, the results allow complementing it, since we found that the textbooks are also used to carry out *multiplication and division operations, work on place value concepts, represent numbers of up to four digits, and compare numbers.* With the above, the importance that the curricular guidelines in Chile give to the use of this type of material in mathematics teaching is remarkable. Consequently, this research contributes to the knowledge of the mathematics teacher, since it allows to show an overview on how to approach the teaching of the SND by using the multibase material presented in its *graphic* version in the texts.

On the other hand, we agree that multibase material is a motivational tool that can affect classroom performance (Leguizamón et al., 2015; Murillo et al., 2016) and that it can improve the teaching of the *numerical sense* (Lommatsch et al., 2017). It is also important to guide teachers on the use of this type of material (Brandao, 2015; West, 2018), since, to achieve effectiveness in its use, it must be accompanied by adequate teaching to ensure good learning experiences. In this sense, including the *tangible* and *virtual* version of the multibase material can bring positive effects on teaching-learning.

Finally, the theoretical tools of the OSA showed to be effective in the analysis of the use of multibase material in textbooks. The levels of analysis of the mathematical activity (Godino et al., 2017) and the epistemic configuration of primary objects (Font & Godino, 2006) are constructs that together provide a microscopic vision of the primary objects involved in each use of the multibase material, and can serve as the basis for guiding the practice of the mathematics teacher through formative interventions (e.g., Burgos, Godino, & Rivas, 2019) focused on the development of knowledge and competencies on the different uses of the material identified in this research.

AUTHORSHIP CONTRIBUTION STATEMENT

L.M.G. and D.D.L. conceived the idea of the research presented. D.D.L. collected the data. The two authors (L.M.G. and D.D.L.) actively participated in the development of the theory, methodology, organisation and analysis of the data, discussion of results, and approval of the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, L.M.G., upon reasonably previous request.

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