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Hierarchy of Students' Reflective Thinking Levels in Mathematical Problem Solving

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ABSTRACT

Background: Reflective thinking plays an essential role in problem solving. However, the hierarchy of students' reflective thinking levels when solving problems is not yet known. **Objectives**: The study aims to determine the hierarchy levels of students' reflective thinking in solving mathematical problems. **Design**: This type of research was qualitative and employed an explorative approach. **Setting and Participants**: This research examined 104 reflective thinkers from Java, Sumatra, and Sulawesi. **Data collection and analysis**: We used analytical geometry instrument tests, observation sheets, and interview guidelines to refine the data. The instruments underwent a review process by validators experts in mathematics education research, qualitative research, and mathematical thinking skills research. We used the think-aloud

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method to capture the data, which were recorded through an audiovisual recording tool. The data were analysed through three stages: preliminary analysis, open coding, and axial coding. **Results**: The analysis reveals that reflective thinking has four levels: in-depth understanding, relating concepts, making errors and willingness to correct them, and being convinced of answers. to elevate it to a higher level. **Conclusions**: The research concluded four hierarchy levels of reflective thinking. Further research can focus on defragmenting students' reflective thinking during problem solving in an attempt to improve such thinking. Moreover, determining the steps to defragment students' reflective thinking at each level is necessary.

Keywords: Hierarchy of reflective thinking; Levels of reflective thinking; Mathematical problem solving; Reflective thinking.

Hierarquia dos níveis de pensamento reflexivo dos alunos na resolução de problemas matemáticos

RESUMO

Contexto: O pensamento reflexivo desempenha um papel essencial na resolução de problemas. No entanto, a hierarquia do nível de pensamento reflexivo dos alunos ao resolver problemas ainda não é conhecida. **Objetivos**: O estudo visa determinar a hierarquia do pensamento reflexivo dos alunos na resolução de problemas matemáticos. **Design**: Esta pesquisa foi qualitativa, com abordagem exploratória. **Ambiente e participantes**: Esta pesquisa analisou 104 pensadores reflexivos de Java, Sumatra e Sulawesi. **Coleta e análise de dados**: Usamos testes de instrumentos de geometria analítica, folhas de observação e diretrizes de entrevista para refinar os dados. Os instrumentos passaram por um processo de revisão por validadores especialistas em pesquisa em educação matemática, pesquisa qualitativa e pesquisa de habilidades de pensamento matemático. Para a captação dos dados, utilizamos o método think-aloud, que foi gravado por meio de uma ferramenta de gravação audiovisual. Os dados foram analisados em três etapas: análise preliminar, codificação aberta e codificação axial. **Resultados**: Este estudo concluiu que o pensamento reflexivo possui quatro níveis: compreensão em profundidade, relacionando conceitos, cometer erros e disposição para corrigi-los e convencer-se das respostas. Outras pesquisas podem se concentrar em desfragmentar o pensamento reflexivo dos alunos durante a resolução de problemas para elevá-lo a um nível superior. **Conclusões**: A pesquisa concluiu quatro níveis de hierarquia de pensamento reflexivo. Outras pesquisas podem se concentrar na desfragmentação do pensamento reflexivo dos alunos durante a resolução de problemas, na tentativa de melhorar esse pensamento. Além disso, é necessário determinar as etapas para desfragmentar o pensamento reflexivo dos alunos em cada nível.

Palavras-chave: hierarquia do pensamento reflexivo; nível de pensamento reflexivo; resolução de problemas matemáticos; Pensamento reflexivo.

INTRODUCTION

Thinking is an essential component in solving mathematical problems. It is a mental activity to relate to and sort concepts, insights, and information to succeed in increased consciousness (Kholid et al., 2020). Solso et al. (2014) argued that in acquiring thinking, one must apply attributes in thinking, such as abstraction, logic, imagination, and problem solving. Thus, thinking is a mental process regulated by the mind by relating some points to understand them further. Every problem solver has a diverse level of thinking. Rodgers (2002) represented reflective thinking as a reasoning activity that starts as a result of confusion and investigation to make a precise decision. In the study presented in this paper, problem solvers are challenged to analyse, assess, and deal with tough decision making (Gurol, 2011).

Reflective thinking is active and persistent, based on the knowledge one has to make decisions (Graham, 2017). It arises when problem-solvers experience doubts while solving problems. They analyse, evaluate, and motivate themselves to overcome these doubts (Gurol, 2011; Rodgers, 2002).

Research that focuses on reflective thinking has formed several conclusions. First, reflective thinking plays a role in minimising problem-solvers' weaknesses when they find difficulties and misunderstandings in problem solutions and conclusions (Agustan et al., 2017a; Kholid et al., 2021). Second, reflective thinking leads problem-solvers to rethink and reevaluate the strategies used to make informed decisions in solving problems (Gencel & Saracaloğlu, 2018; Ngololo & Kanandjebo, 2021). Reflective thinking helps problem-solvers in terms of self-control during the process of solving problems (Hong & Choi, 2011; Kholid et al., 2022).

Reflective thinking has not received much attention from students and lecturers. Both researchers and anecdotal evidence show that lecturers are more concerned with the final answers of their students, regardless of their processes to obtain these results (Sezer, 2008). Lecturers tend to reprimand students for incorrect answers without tracing how they obtained their answers (Biggs & Tang, 2011). On the other hand, students tend to utilise their knowledge without evaluating and developing it (Tutticci et al., 2017).

Reflective thinking plays an essential role in the performance of problem solvers. First, it compensates their deficiency in dealing with difficulties and misunderstandings (Agustan et al., 2017b; Gencel & Saracaloğlu, 2018; Muhammad Noor Kholid et al., 2019). Hong and Choi (2011) proved that reflective thinking assists problem solvers in terms of self-control to make precise decisions. In other words, the better the problem solver's reflective thinking ability, the better their achievements (Ghanizadeh, 2017; Hsieh & Chen, 2012; Kaune, 2006; Yang et al., 2016).

Reflective thinking can be measured quantitatively or described qualitatively. Hence, several studies have developed instruments for these purposes. Basol and Gencel (2013) created a reflective thinking scale that has been evaluated for validity and reliability. Ghanizadeh and Jahedizadeh (2017) and Kember et al. (2000) measure problem solvers' reflective thinking ability by employing a questionnaire with self-report measuring techniques. Hong and Choi (2011) used an observation sheet with indicators to check reflective thinking. Pennington (2011) developed a rubric for evaluating reflective thinking.

Rosmiati et al. (2020) conducted quantitative research on reflective thinking levels. Their results confirmed that problem solvers with worksheets based on hierarchy argumentation essays have better reflective thinking abilities. They suggested that further research could describe reflective thinking levels in feelings, mutual communication, and the relationship between facts and concepts. These aspects can be explored in depth with qualitative approaches.

Furthermore, this research bridges this gap in qualitative research on reflective thinking. The descriptions at each hierarchy will be depicted to understand the placement of problem solvers' reflective thinking abilities, which merits urgency. The following follow-up research entails defragmenting to improve the pupils' reflective thinking levels. In other words, the higher their level of reflective thinking, the better their achievements.

The study reported in this paper is targeted to answer the following research questions: (1) What are the hierarchies of students' reflective thinking levels in problem solving? and (2) What is the description of each reflective thinking level?

THEORETICAL BACKGROUND

Thinking: Notion, Hierarchy, and Cognitive Activity

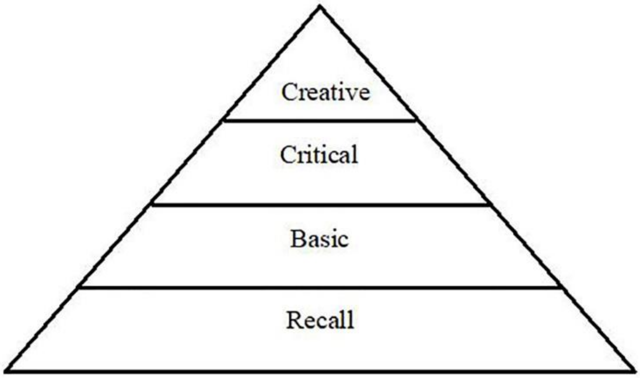
Thinking is the process of grasping, studying, storing, and recalling information (Slavin, 1997). It is also described as the process of matching, connecting, and sorting concepts, perceptions, and knowledge possessed to acquire new knowledge (Day & Goldstone, 2012; Rumelhart & Ortony, 2017).

Solso et al. (2014) added that problem-solvers need to involve attributes in thinking such as abstraction, logic, imagination, and problem solving to obtain such new knowledge.

Krulik et al.'s (2003) hierarchy of thinking involves four levels: recall thinking, basic thinking, critical thinking, and creative thinking (Figure 1).

Figure 1

Hierarchy of thinking levels by Krulik et al. (2003)



The lowest stage of thinking is recall. At this stage, problem solving takes place automatically without involving logical and analytical processes. For example, when the problem-solvers calculate 7 + 7, they automatically answer 14. The second stage of thinking is basic thinking, which is common. For example, when the problem-solvers determine the equation of a line that goes through two points, they not only substitute for both points in the equation formula of the line but also use reasoning with addition and multiplication operations. Critical thinking is the third stage of thinking. This stage is characterised by analysing problems, determining the adequacy of information in solving problems, and determining additional information in solving problems. For example, the problem-solvers check whether point $A\ (2, 1)$ is located on line $g: x + y - 3 = 0$. They substitute A into the line equation and sum up the terms and meanings of whether point A lies on line g. They also need to check their solution in other ways, such as by sketching. The fourth stage of thinking is creative thinking, characterised by the ability to solve problems in unique ways.

Mayer (1983) stated that thinking is divided into three cognitive activities. First, thinking is a cognitive activity in a person's mental, invisible but traceable behaviour. For example, the problem-solvers show their thought processes in sketching a parallelogram $ABCD$ using Cartesian coordinates. Second, thinking is a process that employs some manipulation of knowledge in cognition. Experiences and information stored in memories are interconnected to solve problems according to the situation. For instance, on parallelogram $ABCD$, AB is aligned with CD, so the AB gradient is similar to the CD gradient. Third, thinking activities are directed to solve problems appropriately. Although not all steps are successful, in general, problem-solvers make all efforts to solve the problem appropriately in their minds. Therefore, thinking is a process that involves some manipulation of knowledge in cognitive activities to lead to one proper conclusion in problem solving.

Reflective Thinking for Problem-solving

Many experts have studied reflective thinking. Dewey (1933) inspects reflective thinking as an intense, persistent, considerate, and conscious thinking activity that leads him to decisions that he believes to be accurate. Problem-solvers who employ reflective thinking are concerned with aspects of attitude and expertise to decide what to do when encountering confusion. Furthermore, Habermas (1971) defines reflective thinking as a thinking activity that requires insight and experience, both of which must be maintained to solve a problem adequately and efficiently. During reflective thinking, problem-solvers need to engage in communicative actions to control and comprehend situations. Another expert, Schön (1983), argues that during reflective thinking, problem-solvers employ knowledge and experience and evaluate the steps repeatedly to solve problems. Reflective thinking is a thinking activity that involves both the knowledge and experience one has to solve problems effectively and efficiently, even if the problem-solvers experience difficulties or confusion. In this study, reflective thinking is a mental activity that begins with confusion and concludes with repeated evaluations to solve problems. Confusion includes difficulties, doubts, or mistakes that the problem-solvers make when trying to find the solution to a problem.

Problem-solvers are said to engage in reflective thinking if they overcome their confusion when solving problems. Their reflective thinking skills are also different. The difference is in the way that problem-solvers overcome confusion. Suharna (2018) explains that there are three categories of reflective thinking used when solving mathematics. The three categories are productive reflective-thinkers, connective reflective-thinkers, and clarification reflective-thinkers. The difference between the three categories lies in *how* the problem-solvers overcome the confusion that arises in them. Productive reflective-thinkers overcome confusion by exploring alternative approaches to solving the problem. Connective reflective-thinkers, on the other hand, overcome confusion by conceptualising. The conceptualisation in question matches all mathematical concepts, principles, and processes related to a problem. Unlike productive reflective-thinkers, clarification reflective-thinkers overcome the confusion by clarifying the problem with related concepts.

Many experts have examined the reflective thought process. Reflective thinking is defined as a mental activity to overcome confusion in solving problems by employing knowledge, experience, repeated evaluation, and responsibility (Dewey, 1933; Habermas, 1971; Schön, 1983). Moreover, Rodgers (2002) defines reflective thinking as a mental activity that arises from confusion and repeated investigation to find a solution.

Hong and Choi (2011) identify three-dimensional models for understanding reflective thinking (i.e., timings, objects, and levels). Timing dimensions are conditions in which individuals engage in reflective thinking when solving problems. During problem solving, problem-solvers need to examine the relationship between timings and reflection patterns. An excellent mathematical problem can produce iteration cycles (Adams et al., 2003). Individual iteration habits are positively correlated with problem-solving outcomes (Atman et al., 2005). Iterations of personal reflections can affect the performance and quality of problem-solving works. The dimensions of objects are involved so that the reflective thinking process of the individual is more concrete, and following the context of problem-solving, each individual can view a problem with a different understanding (Visscher-voerman & Procee, 2006). The object’s components in question are self-object, artefact object, and circumstances object—the level dimensions for understanding the reflections performed by the individual. Hong and Choi (2011) distinguish between reflective thinking levels in single-loop, double-loop, and triple-loop learning. Each individual can achieve a different level of reflective thinking. Each level characterises an individual’s ability to solve problems. The higher the presence of an individual’s reflective thinking level in solving problems, the better the performance and results. The three-dimensional model is illustrated in Figure 2.

Figure 2

Three-Dimensional Model Illustration of Reflective Thinking

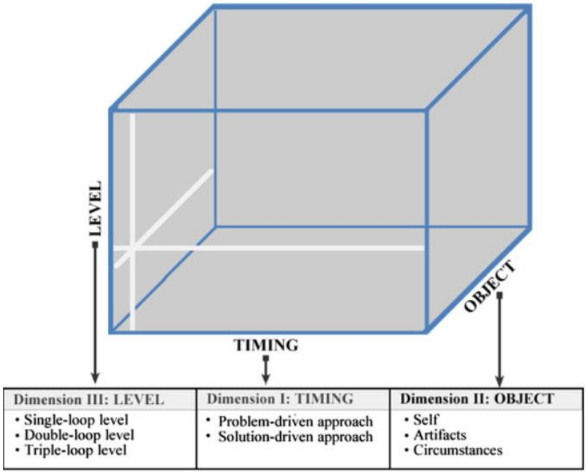


Figure 2 illustrates the three dimensions of reflective thinking: timing, objects, and levels. In summary, the timing dimension indicates the timing of the beginning sign of an individual's reflective thinking in problem solving. The dimension of the object indicates the differences in the individual's perception when faced with problems. Meanwhile, the level dimension measures to which extent individuals perform reflective thinking when solving problems that may affect their problem-solving outcomes. In other words, the three dimensions play their respective roles in describing problem-solvers' reflective thinking.

Reflective Thinking Aspects

Dewey (1933) states that the stages that problem-solvers go through when conducting reflective thinking include: an experience, a spontaneous interpretation of the experience, the naming of the problem, the generation of possible explanations for the problem, ramifying the descriptions into full-blown, and a hypothesis. Of course, problem-solvers encounter challenges when engaged in reflective thinking. The encounter involves not only the individual's knowledge; the individual's attitude when solving problems that are considered challenging. The spontaneous interpretation of the experience aspect is a state in which the individual can spontaneously decipher the experience. Analysing the experience in question is remembering your experience and thinking about how to use the experience to solve the problem. By recognising the challenge, the individual will begin to identify the difficulty and think of ways to start solving the problem. In this aspect, the individual's attitude is tested, whether the individual will continue to solve the given situation or not (Schoenfeld, 1985). When generating possible explanations for the problem, the individual sorts out various potential solutions for problem-solving and then chooses one of the most appropriate possibilities. The next aspect of ramifying the explanations into full-blown is a state where problem-solvers can explain the possibility of solutions chosen based on consideration of knowledge and experience. Furthermore, the conditions in which problem-solvers execute possible solutions into problem-solving solutions are aspects of experimentation.

Lee (1999) concludes that problem-solvers engage in the following activities when undergoing reflective thinking: understanding the problem context, defining and reframing the problem, seeking a possible solution, and experimenting. When trying to understand the problem context, they evaluate the problem. They engage in experiences they have had when solving similar problems. By defining and reframing the problem, the individual once again understands the problem in question but with clearer language. At the same time, the individuals measure their ability to solve the problem; indeed, the individuals will decide whether or not they will proceed to solve the problem.

Furthermore, when seeking possible solutions, problem-solvers think about potential solutions to a given problem. Moreover, in this aspect, problem-solvers also review all the possible answers with multiple considerations. Individuals will decide on the best way to solve the problem. In the last aspect, by experimenting, they will try and observe the solution. They can assess whether the solution is appropriate. When the solution is adequately arranged, the series of aspects in which they engage will be used as new schemes of knowledge and experiences for reflective thinking.

Rodgers (2002) adds that a problem-solver goes through three aspects during reflective thinking. These aspects, which are related to experience, are (1) description of the incident, (2) analysis of the experience, and (3) experimentation. By describing the incident, a problem-solver identifies a problem. Students can measure their ability and capacity to solve the problem. In this aspect, problem-solvers decide whether to keep problem-solving or not. By analysing the experience, problem-solvers consider all the possible solutions to a problem. They believe in the suitability of all possible solutions. Only through experimentation will they choose a solution that best suits the problem. Next, he or she begins executing the selected solution until the problem can be solved.

Table 1

Reflective thinking aspects

Dewey (1933)	Lee (1999)	Rodgers (2002)	Zehavi & Mann (2005)
An experience	Problem context	Presence to experience	Selection of techniques
Spontaneous interpretation of the experience	Problem definition/Reframing	Description of experience	Monitoring of the solution process
Naming the problem			
Generating possible explanations for the problem	Seeking possible solution	Analysis of experience	Conceptualisation
Ramifying the explanations into full-blown Hypothesis			Insight
Experimenting or testing the selected hypothesis	Experimentation	Intelligent action/experimentation	
	Evaluation Acceptance/rejection		

During reflective thinking in solving mathematics, problem-solvers employ four aspects: technique, monitoring, insight, and conceptualisation (Zehavi & Mann, 2005). The technique aspect consists of activities that they complete when choosing which techniques to employ to solve problems. This aspect also comprises how they devise strategies to solve mathematical problems with effective and efficient principles (Setyaningsih et al., 2019). Monitoring is the activity of re-assessing the solution to the mathematical problem. It indicates that problem-solvers are re-examining whether the resolution of the mathematical problem is correct or not. Insight is a state in which they use their ingenuity and emotions to solve problems. This aspect involves their motivation and persistence to keep trying to solve the problem, even when experiencing confusion. Thus, the insight aspect combines knowledge and attitudes. Conceptualisation involves their ability to connect with and understand several concepts and meanings. This aspect also consists of their power to process their knowledge and skills to decide how to solve problems appropriately.

Four experts have presented explanations related to aspects of reflective thinking. When observed, the aspects offered by the four experts are identical. Nevertheless, they give different names to each aspect of reflective thinking. The correspondences of the four aspects of reflective thinking in question are summarised in Table 1.

State of the Art

According to Schön (1983) and Thompson and Thompson (2018), reflective thinking is active, persistent, and careful thinking based on knowledge to reach conclusions. This means that reflective thinking involves attitude in addition to knowledge. In other words, reflective thinking is the integration process among thinking ability, experience, and insight (Funny et al., 2019; Pagano, 2009; Yoke et al., 2018).

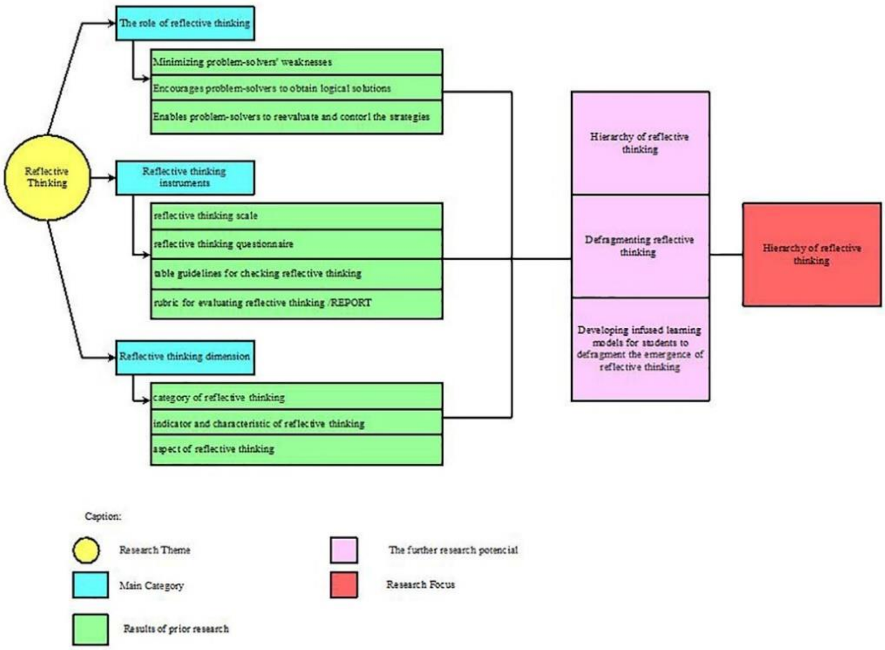
Studies on reflective thinking focus on three areas. The first area is research on the relationship of reflective thinking with the problem-solvers' performance. The results conclude that reflective thinking (1) reduces problem-solvers' inadequacies when they encounter difficulties (Agustan et al., 2017b; Choy, 2012), (2) facilitates problem-solvers to obtain logical solutions (Coulson & Harvey, 2013; Ghanizadeh, 2017), (3) enables problem-solvers to reevaluate their strategies (Hsieh & Chen, 2012; Rosmiati et al., 2020), and (4) helps problem-solvers control themselves during the problem-solving process (Maksimović & Osmanović, 2019; Yang et al., 2016). The second is research studies that produce instruments to measure reflective thinking, including the following: (1) reflective thinking scale (Basol & Gencel, 2013; Forman, 2020),

(2) reflective thinking questionnaire (Ghanizadeh & Jahedizadeh, 2017), (3) table guidelines for checking reflective thinking (Hong & Choi, 2011), and (4) rubric for evaluating reflective thinking (Pennington, 2011). The third area is research studies that identify and categorise reflective thinking: (1) categorising reflective thinking (Suharna, 2018), (2) producing indicators and identifying characteristics of reflective thinking (Kholid et al., 2022), and (3) presenting aspects of reflective thinking (Zehavi & Mann, 2005).

Based on the three aforementioned focuses of the study, some additional potential research on reflective thinking can be made, as illustrated in Figure 3.

Figure 3

Potential research on reflective thinking.

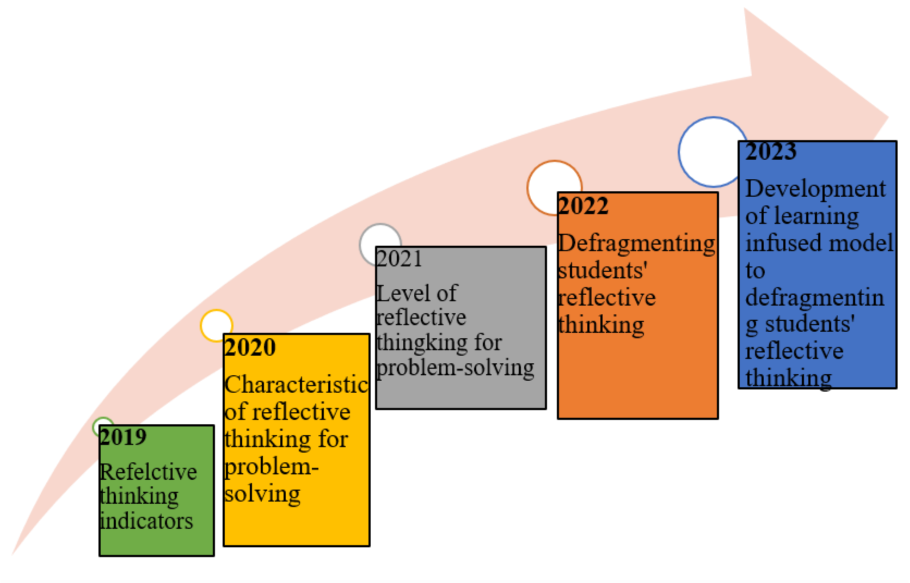


Research Position

In 2019, researchers posited reflective thinking indicators in solving problems (Kholid et al., 2020) in terms of techniques, monitoring, insight, and conceptualisation. In 2020, these researchers conducted a study on the characteristics of reflective thinking (Kholid et al., 2022). In 2021, the same researchers examined problem-solvers' level of reflective thinking in solving problems. The findings will be used as a guideline for the researchers in 2022 and 2023. The research plan in 2022 is to defragment reflective thinking in the problem-solving process. In 2023, the study will focus on developing infused learning models for students to defragment the emergence of reflective thinking in solving problems.

Figure 4

Research roadmap



Research Urgency

There is an urgent need for research that uncovers the hierarchy of reflective thinking. These findings can then be compiled into a theoretical guideline to describe reflective thinking in mathematical problem-solving. In addition, the conclusion can be used as a guideline for lecturers to develop learning models that employ reflective thinking when solving mathematical problems, especially in analytical geometry. Another underlying urgency is research related to the hierarchy reflective thinking, in line with the formation of competitive characters by the National Research Master Plan/NRMP (Rencana Induk Riset Nasional/RIRN), contained in the NRMP document, page 84. The document states that in the period 2017-2045, the focus of research in Indonesia is directed at the theme of Social Humanities-Arts Culture-Education, which also covers aspects of education and culture. More details of the theme are illustrated in Figure 5.

Figure 5

Themes and Topics for The Focus of Social Research Humanities-Arts Cultural-Educational

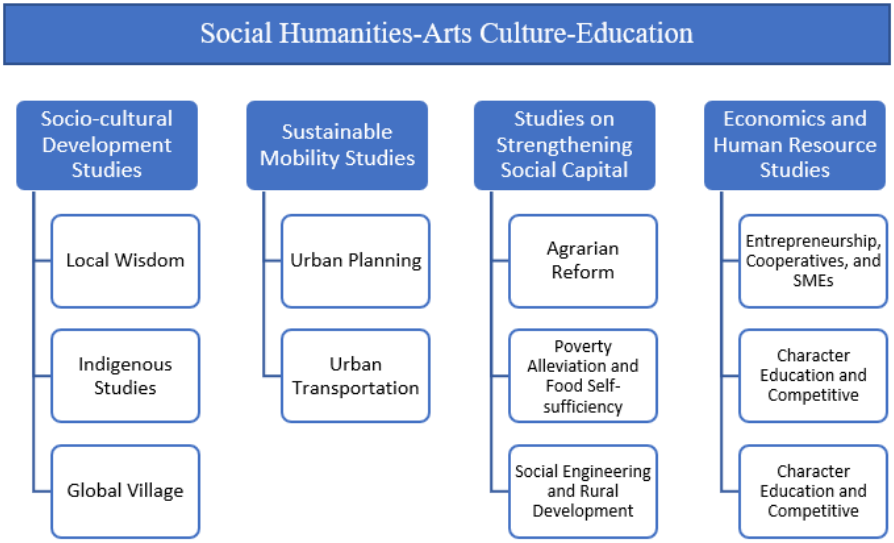


Figure 5 suggests that Indonesia is focusing on developing participatory technology research to build the identity of the nation through several research topics. The research topics are 1) socio-cultural development studies, 2) sustainable mobility studies, 3) studies on strengthening social capital, and (4) economic and human resource studies. The study of reflective thinking as a sub-focus of educational research is aligned with the objective of producing human resources of character and high competitiveness. This indicates that reflective thinking research needs more attention from the state.

METHODOLOGY

This research was qualitative with an explorative approach, where we investigated the reflective thinking levels of students in solving problems. Therefore, we employed analytical geometry tasks, task-based interviews, the think-aloud method, and observation sheets as data collection tools. In a task-based interview, one subject meets the interviewer, who introduces the assignment to the subject. We used audio and video to capture the verbal expressions of the participants we analysed later and recorded their mental activity while they solved problems. The task-based interviews were semi-structured because the interviewer asked the subject pre-planned questions during the interview. In addition, the research met the declaration of Helsinki 1975, Council for International Organizations of Medical Sciences (CIOMS), and World Health Organization (WHO) 2016 so that it was declared ethically approved by the Health Research Ethics Committee Faculty of Medicine Universitas Muhammadiyah Surakarta Indonesia.

Context of the Study

This study was part of Kholid et al.'s (2020) and Kholid's (2022) follow-up research on reflective thinking processes when students solve geometry problems. The study posits several opportunities to approach levels of reflective thinking. Therefore, it focuses on how students apply reflective thinking levels in solving problems. These students may exhibit varying levels of thinking when engaging in reflective thinking to obtain answers.

Participants

As many as 104 students from levels 2 and 4 from various universities in Indonesia participated in this study. Their ages ranged between 19 and 20 years old. The students took turns completing an assignment by applying the think-aloud method within 45 minutes each. We had contacted colleagues at the universities to ensure that their students worked on the questions given. All the participating students were enrolled in analytical geometry classes, and most of the students were taught to solve problems of a procedural nature (e.g., searching for derivatives of a function, sketching graphs with known function formulas). Therefore, they were rarely allowed to develop their ideas about analytical geometry concepts further. Consequently, many students had difficulty when analysing the given problems. A demographic picture of the participants is shown in Table 2.

Table 2

Demographic structure of participants

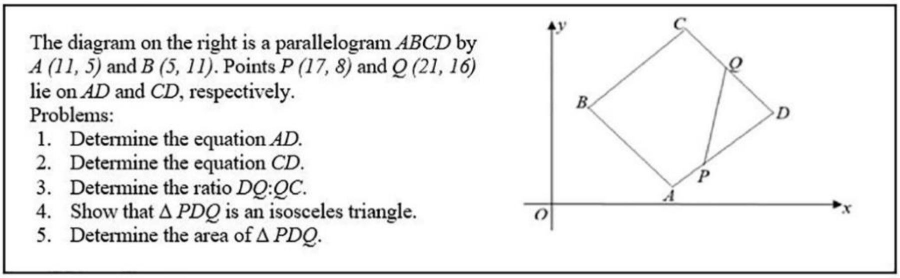
Island of origin	Number of participants
Sumatera	18
Java	62
Sulawesi	24
Sum	104

Instruments and Data Collection Method

In addition to the think-aloud method to solve the geometry tasks, the most important data collection tool was geometry tasks and included interview guidelines to clarify the subjects' answers. One task with references to geometry is shown in Figure 6.

Figure 6

Mathematical Problem


The diagram on the right is a parallelogram *ABCD* by *A (11, 5)* and *B (5, 11)*. Points *P (17, 8)* and *Q (21, 16)* lie on *AD* and *CD*, respectively.
Problems:
1. Determine the equation *AD*.
2. Determine the equation *CD*.
3. Determine the ratio *DQ*:*QC*.
4. Show that Δ *PDQ* is an isosceles triangle.
5. Determine the area of Δ *PDQ*.

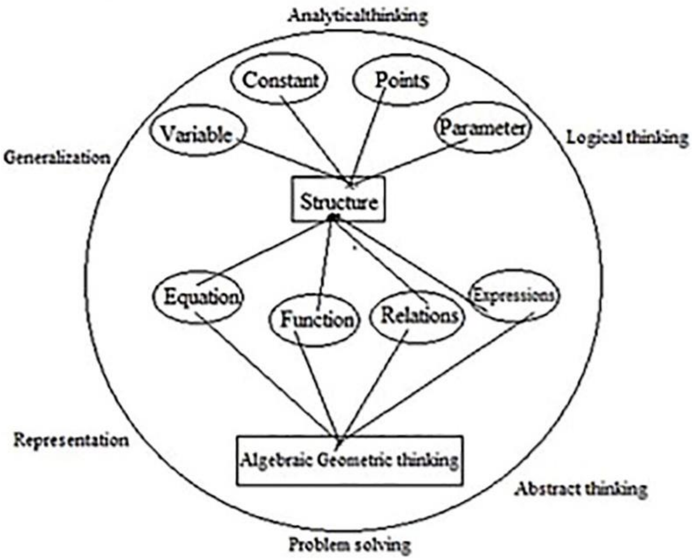
The tasks were formulated with many fundamental considerations. To increase the confidence level of the designed instrument, we hired three expert validators to ensure that the created task could measure the students' reflective thinking levels. Based on the solutions they produced, 12 participants were invited for follow-up interviews to clarify their answers. We interviewed students whom we thought showed compelling reasons for doing the work. During the interview, the researchers showed their answer sheets. Next, the students were asked to provide detailed descriptions of the ideas they wrote down. Each interview lasted 10–15 minutes and was audio-recorded and transcribed. After the students completed the task, the researchers asked some other questions that had not been asked during the study. Interviews aim to sharpen how and why each student completes an assignment in that way. Typical questions in our interview guidelines included the following: "What do you think?", "Why do you think so?", "Can you give us another reason?", and "Why were you quiet when you finished that?" We also asked the following: (1) "What does each point of the information of the task mean?"; (2) "What sort of knowledge do you need to solve the problem?"; and (3) "How can you use such knowledge?"

Why Analytical Geometry?

Reflective thinking begins with confusion on the part of the problem-solver. Mathematical problems that can invoke reflective thinking trigger such confusion, such as non-routine problems (Hong & Choi, 2011). Unfamiliar questions cause problem-solvers to perform reflective thinking (Hidajat et al., 2019). Non-routine problems are usual in analytical geometry, which was first discovered in the seventeenth century by René Descartes, a French mathematician (Khalil et al., 2019). Problem-solvers require reasoning to solve geometry problems with algebraic concepts (Evans, 2014).

Figure 7

Analytical geometry structures by Lew (2004)



Lew (2004) found that each analytical geometry object is a structure that produces various forms, such as expressions, equations, relationships, and functions. Geometrically, each structure is an algebraic form of variables, parameters, and constants.

Data Analysis

The data collection and analysis processes were based on a constructivist point of view. The constructivist theory states that student knowledge consists of a set of schemes based on previous experience (Dubinsky, 2002; Von Glasersfeld, 1995), which implies that we do not have direct access to students' learning. We can only model their interpretations based on the observed think-aloud and interview results (e.g., verbal expressions, behaviours/gestures, resulting graphs). Thus, our analysis reflects our best efforts in levelling their reflective thinking in solving problems. Our data analysis is aligned with Corbin and Strauss's (2010) description of grounded theory, where students' reflective thinking levels arise from data analysis. This analysis consists of three stages, as follows.

Preliminary Analysis

Initial data analysis began after a think tank and interview. In this case, we made initial guesses based on the students' verbal expressions, gestures, and graphic sketches. These initial conjectures were used as a guide to follow-up questions. Based on the students' responses to these follow-up questions, the initial conjectures were either corrected or changed. Additional questions were asked until all the data was collected sufficiently.

After conducting the interviews with the students, the research team met to talk about the results. More discussions were completed, resulting in the discovery that the students' reflective thinking levels can be polarised. Specifically, after several interviews, the research team found that four levels emerged from the students' reflective thinking while solving problems. We paid particular attention to situations that triggered the students to use reversible reasoning and clarified information about their verbal expressions during the interviews.

Open Coding

After conducting the interviews, we analysed students individually and the transcripts and video recordings in detail, creating open coding of the students' interpretations during each problem-solving process. We also analysed the transcript results by developing codes to decrypt the most essential and relevant parts of reflective thinking. Furthermore, these codes were enhanced by adding and expanding the initial encoding to determine the students' reflective thinking levels. This process continued until we analysed the data of 12 students. In the end, four themes stood out: (1) level 1: in-depth understanding; (2) level 2: relating among concepts; (3) level 3: willing to correct; and (4) level 4: convinced. All four themes were found to be general descriptions and characterised the reflective thinking of the students. Level 1 is the lowest level in reflective thinking, so level 4 is the highest level. This step is in accordance with Krulik et al. (2003).

Axial Coding

Once these four themes were established, we refined these two findings using axial coding. To perfect the definitions of these four themes, we compared triggering situations and verbal expressions that indicated that the students had used reflective thinking. Next, we compared the students according to the different themes to develop more detailed descriptions of our analysis. Finally, we re-encoded the transcript results using enhanced codes and used these four themes to frame the findings of the students' reflective thinking levels in solving problems. The analysis implies that we have conducted a reliability test of the found themes, where each research team helped analyse the student transcripts separately. Furthermore, these analyses were put together and compared for the researchers to discuss their differences.

RESULTS AND ANALYSES

After the data analysis, the findings show four hierarchies of reflective thinking levels: in-depth understanding, relating among concepts, "I made errors and am willing to correct them," and "I am convinced of my answer."

Table 3 shows that the higher the reflective thinking level, the lower the distribution of subjects. The hierarchy levels of reflective thinking can be illustrated as a pyramid in Figure 8.

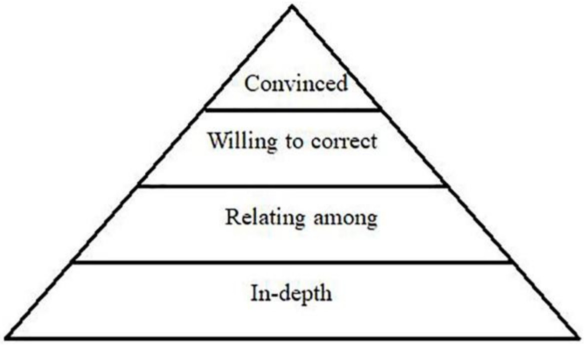
Table 3

Distribution of the subject

Hierarchy level	Island of origin	Numbers	Sum
First level: In-depth understanding	Sumatera	7	38
	Java	23	
	Sulawesi	8	
Second level: Relating among concepts	Sumatera	5	31
	Java	19	
	Sulawesi	7	
Third level: "I made errors, and I am willing to correct them"	Sumatera	4	23
	Java	14	
	Sulawesi	5	
Fourth level: "I am convinced of my answer"	Sumatera	2	12
	Java	6	
	Sulawesi	4	

Figure 8

Hierarchy Levels of Reflective Thinking



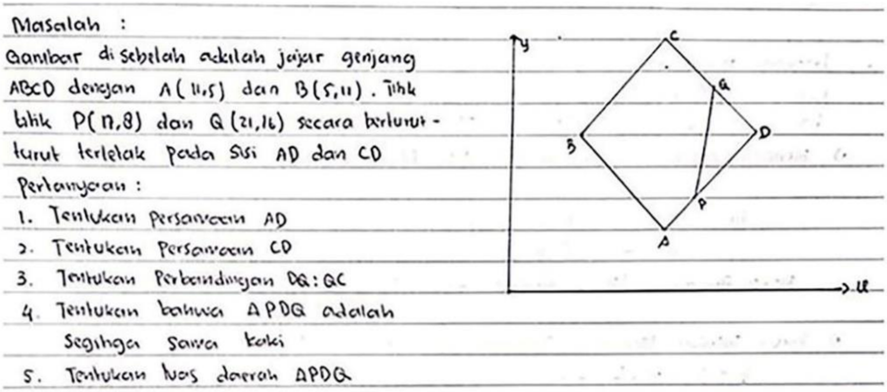
First Level: In-Depth Understanding

In the first level, the problem solvers understand in depth the meaning of the available information, the questions to be solved, and how to solve the problem. A total of 38 out of 108 problem-solvers were categorised to be at the first level. They review the available information and the questions to be answered. The transcripts of the students employing the think-aloud method for problem solving served as evidence of the first level. They said, "Known . . . parallelogram $ABCD$ with $A\ (1, 5)$, and $B\ (5, 11)$. There is a point P and Q. Then I have to determine the equation AD and CD, comparison DQ: QC. Prove that PDQ is a triangle isosceles and determine the area of triangle PDQ. I see . . .". The first level is also evident in the student answer sheet presented in Figure 9.

Figure 9

Problem Solver Employs In-depth Understanding

Original Version:


Masalah :
Gambar di sebelah adalah jajar genjang ABCD dengan A(11,5) dan B(5,11). Titik titik P(17,8) dan Q(21,16) secara berturut-turut terletak pada sisi AD dan CD
Pertanyaan :
1. Tentukan persamaan AD
2. Tentukan persamaan CD
3. Tentukan perbandingan DQ : QC
4. Tentukan bahwa ΔPDQ adalah segitiga sama kaki
5. Tentukan luas daerah ΔPDQ

Translated Version:

The diagram on the right is a parallelogram *ABCD* by *A (11, 5)* and *B (5, 11)*. Points *P (17, 8)* and *Q (21, 16)* lie on *AD* and *CD*, respectively.

Problems:

1. Determine the equation *AD*.
2. Determine the equation *CD*.
3. Determine the ratio *DQ*:*QC*.
4. Show that Δ *PDQ* is an isosceles triangle.
5. Determine the area of Δ *PDQ*.

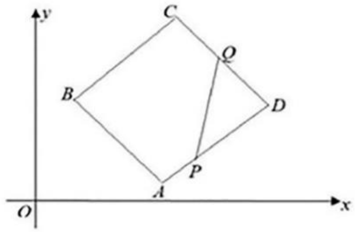


Figure 9 shows that the participants rewrote the known information, created sketches, and wrote down the solved problem. The purpose of the solver is to receive and review the information and questions to gain a deep understanding of them. This statement is derived from the following interview excerpts.

> *Researcher: Why did you rewrite the information and questions?*
>
> *Problem-solver: Before answering a question, I always [rewrite the] information and questions. I did it to gain an in-depth understanding of the complete steps that I will take.*
>
> *Researcher: Are these efforts effective at gaining a deep*

understanding?

Problem-solver: Of course. In addition to rereading the information and questions, I also read [them] over and over . . . A deep understanding led me to come to the correct conclusion.

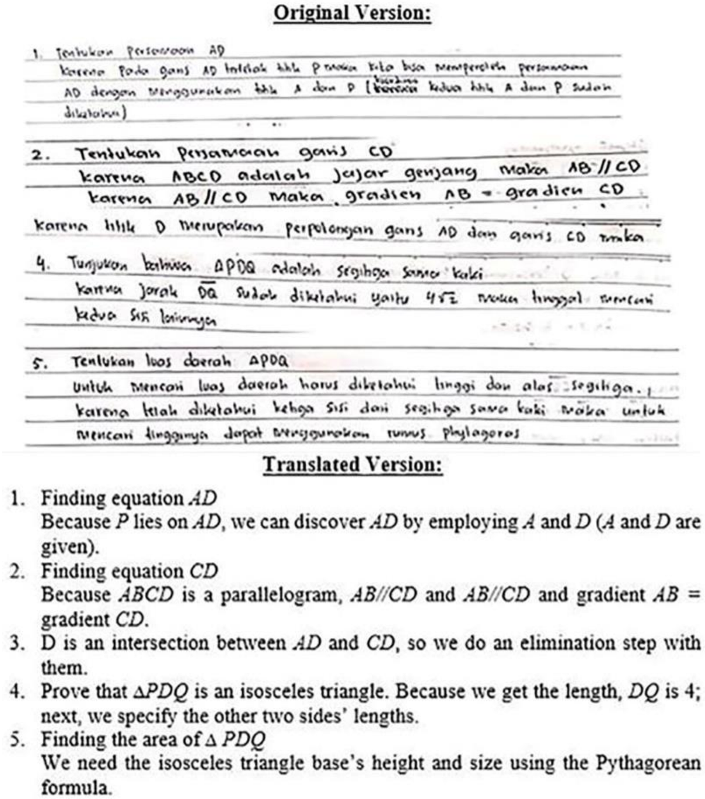
The interview excerpts show the students' efforts to understand the provided information, such as rewriting and rereading it. These efforts assisted them in drawing the correct conclusions. We conclude that students need time to understand the information provided and the intent of the question (Hidayati et al., 2020), and they do it by working on the provided information in different ways, such as by rereading and redrawing it. These efforts improve their performance significantly (Hidayati et al., 2020; Pradana et al., 2020). When associated with the problem-solving step by Polya (1973), the first level is identic with the first stage of problem solving, i.e., understanding the problem. At this stage, the problem-solvers identify what information is known and what information is needed. The visible activity at this stage is drawing a figure.

Second Level: Relating among Concepts

At this level, the problem solvers connect and use mathematical concepts to solve the problems. A total of 31 out of 108 problem-solvers reached the second level. In other words, they went through the first and second levels but could not get to the next. They recalled these concepts and wrote them down on the sheet before they conducted a calculation, in addition to writing down the reasons for every decision made. In solving Problem 1, the subject recalled how to determine the line's equation through two points, whereas in solving Problem 2, the subject understood that AB is aligned with CD, so the AB gradient is similar to the CD gradient. The subject knew that point D is the intersection between AD and CD, so they used the concept of elimination to solve the problem. In showing that PDQ is an isosceles triangle, the subject recalled that for this type of triangle, two of the three sides are equal in length. If the length of DQ is 42, then the subject determined PD and PQ. In solving Problem 5, the subject used the Pythagorean theorem to determine the height and area of the triangle.

Figure 10

Problem solver employs relation among the concepts



Original Version:

1. Tentukan persamaan AD
karena pada garis AD terdapat titik P maka kita bisa memperoleh persamaan AD dengan menggunakan titik A dan P (karena kedua titik A dan P sudah diketahui)

2. Tentukan persamaan garis CD
karena ABCD adalah jajar genjang maka AB // CD
karena AB // CD maka gradien AB = gradien CD

karena titik D merupakan perpotongan garis AD dan garis CD maka

4. Tunjukan bahwa ΔPDQ adalah segitiga sama kaki
karena jarak $\overline{DQ}$ sudah diketahui yaitu $4\sqrt{2}$ maka tinggal mencari kedua sisi lainnya

5. Tentukan luas daerah ΔPDQ
Untuk mencari luas daerah harus diketahui tinggi dan alas segitiga.
karena telah diketahui ketiga sisi dari segitiga sama kaki maka untuk mencari tingginya dapat menggunakan rumus phytagoras

Translated Version:

1. Finding equation *AD*
Because *P* lies on *AD*, we can discover *AD* by employing *A* and *D* (*A* and *D* are given).
2. Finding equation *CD*
Because *ABCD* is a parallelogram, *AB//CD* and *AB//CD* and gradient *AB* = gradient *CD*.
3. D is an intersection between *AD* and *CD*, so we do an elimination step with them.
4. Prove that Δ*PDQ* is an isosceles triangle. Because we get the length, *DQ* is 4; next, we specify the other two sides' lengths.
5. Finding the area of Δ *PDQ*
We need the isosceles triangle base's height and size using the Pythagorean formula.

Figure 10 shows that each subject linked several concepts in problem solving. The transcription of the students employing the think-aloud method for problem solving served as evidence of the second level. They said, "AD and AP are concurrent lines. It means to determine the equation AD just as I determine the equation AP… because $ABCD$ is a parallelogram. As a result, $AB//CD$, so their gradient is the same… I see that D is the intersection point of AD and CD. We can employ an elimination concept to solve this item problem… How to prove… PDQ is an isosceles triangle by investigating whether $PD = DQ$ or $PD = PQ$ or $DQ = PQ$? I have a long DQ. So, I have to find the [lengths of] PD and PQ first. Let's do it… There are indeed several strategies for determining the area of the PDQ triangle. Because PDQ is an isosceles triangle… the Pythagoras theorem applies…”. The results of the interviews that support the second-level findings are presented as follows:

> *Researcher: Once you understand the information and the problem, what do you do?*
>
> *Student: I plan a strategy on how to solve that problem.*
>
> *Researcher: What actual steps do you take in planning a strategy?*
>
> *Student: First, I thought of possible concepts that can be employed to solve the problem. Then I consider which concept I should use. For example, a concept that I understand better.*
>
> *Researcher: Is it effectively used in solving problems?*
>
> *Student: Yes, very effective. This method, I always use to minimise errors in solving problems.*

In the second level, students began to think about how to solve the problems by recalling the concepts that they have learned and deciding on the appropriate concepts to apply in problem solving (Suharna et al., 2020), also known as conceptualisation (Schoenfeld, 1985; Zehavi & Mann, 2005). This activity involves the problem solver’s ability to connect several concepts and understood meanings and to cultivate their knowledge and skills to make decisions to solve the problems appropriately. It is relevant to the research by Dündar and Yaman (2015). Some problem solvers cannot perform conceptualisation because they are more skilled at procedural solving and are not used to solving new problems. When associated with the problem-solving step by Polya (1973), the second level is identic with the second and third stages of problem solving, i.e., devising and carrying out a plan. At this stage, the problem-solvers identify what concepts can be employed, whether there are additional concepts to solve the problem, and how to solve the problem more efficiently.

Third Level: “I Made Errors, and I Am Willing to Correct Them”

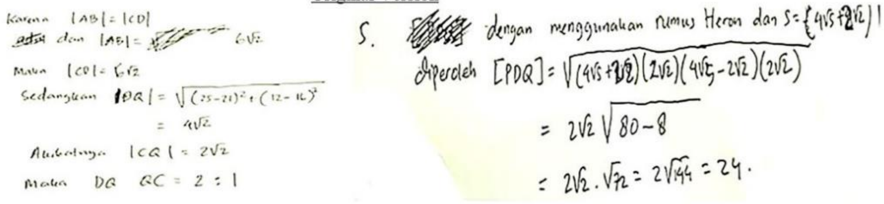
The third level is when problem solvers make mistakes or experience confusion. A reflective thinker is willing to overcome errors and confusion until the problem has been solved. A total of 23 out of 108 problem-solvers reached the third level. In other words, they went through the first to third levels but could not get to the next.

As the audiovisual recording evidenced, some problem solvers show signs of confusion when they stop for a few seconds. Other problem solvers seem to make errors based on the answer sheet's scribbles, but they must be able to detect and correct the errors.

Figure 11

Problem Solver Makes Errors and is Willing to Correct Them

Original Version



Translated Version

Because $|AB| = |CD|$
~~It will~~ and $|AB| = \cancel{\sqrt{7}} = 6\sqrt{2}$
So $|CD| = 6\sqrt{2}$
Whereas $|DQ| = \sqrt{(25-22)^2 + (12-16)^2}$
$= 4\sqrt{2}$
As a result, $|CQ| = 2\sqrt{2}$
So, DQ: QC = 2: 1

5. ~~[PQR]~~ By employing Heron and $S = 4\sqrt{5} + 2\sqrt{2}$
We get $[PDQ] = \sqrt{(4\sqrt{5}+2\sqrt{2})(2\sqrt{2})(4\sqrt{5}-2\sqrt{2})(2\sqrt{2})}$
$=2\sqrt{2}\sqrt{80-8}$
$=2\sqrt{2}\sqrt{72}$
$=2\sqrt{144}$
$=24$

The subject said, "Length $AB = \sqrt{36}$ ... Uh, wrong. Wait a minute. It should be 6… Value $S = 4\sqrt{5} + 4\sqrt{2}$ … so area PDQ should be [equal] to… I have made a mistake. I'll check first… Should be an S ...". Apart from audiovisual recordings, the subject's errors are indicated by scribbles on the answer sheet. Figure 11 shows that the subject made a mistake in the missing calculations, marked with scribbles on their answer sheet. The subject also openly admitted that he had made an error in solving the problem. The subject was willing to correct it. Excerpts of the interview transcriptions support these findings.

Researcher: Are you having trouble solving the problem?

Student: Yes. I made an error.

Researcher: What error?

Student: I miscalculated. But I realised it.

Researcher: Do you find it difficult to correct that error?

Student: No. When I make an eror, then I will look back at my answers step by step. I made sure of my error. Then [I will] fix it for sure.

Researcher: Do you always correct your errors in solving mathematical problems?

Student: Of course. I fixed it so that I could come to a very precise conclusion.

Excerpts of the interview transcriptions show that the student made an error and corrected it. The student realised his/her error and then looked back at their answers. After learning of the error, they reevaluated it to plan repair efforts and ensured that a concept could be employed to fix it. They had the motivation to correct it and to get the right answer.

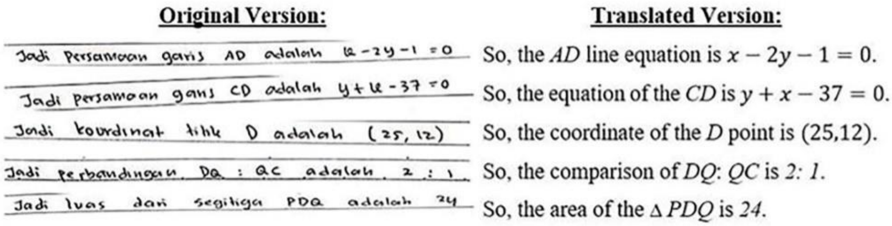
In the third level, students can identify errors and correct them. This level reflects the main description of a reflective thinker, namely, a problem solver who encounters problems but perseveres by considering various strategies for these problems to the end (Rodgers, 2002). Furthermore, the third level can be attributed to the monitoring aspect (Zehavi & Mann, 2005), i.e., the activity of monitoring the solution of the math problem. Ozsoy and Ataman (2009) and Schneider and Artelt (2010) stated that monitoring helps problem solvers build strong knowledge domains to get the correct answer. When associated with the problem-solving step by Polya (1973), the third level is identic with the fourth stage of problem solving, which is looking back. At this stage, the problem-solvers evaluate their arguments and answers at a second glance or become extra observant. Evaluation may implement a precise visual illustration (Kholid et al., 2022)

Fourth Level: "I Am Convinced of My Answer"

The fourth level entails the emergence of belief in one's answers. Here, problem solvers show that they are optimistic about the validity of their solutions, demonstrating optimism and confidence in their written conclusions. A total of 12 out of 108 problem-solvers reached the fourth level. In other words, they went through the first to fourth levels, reaching the highest level in reflective thinking. Figure 12 depicts the confidence of the subjects regarding their answers. They displayed such confidence by providing conclusions on the final answer to each item.

Figure 12

Problem Solver Writes Conclusions to Show Confidence in Answers



Original Version:	**Translated Version:**
Jadi Persamaan garis AD adalah $x-2y-1=0$	So, the *AD* line equation is $x-2y-1=0$.
Jadi persamaan garis CD adalah $y+x-37=0$	So, the equation of the *CD* is $y+x-37=0$.
Jadi kourdinat titik D adalah (25, 12)	So, the coordinate of the *D* point is (25,12).
Jadi perbandingan DQ : QC adalah 2 : 1	So, the comparison of *DQ*: *QC* is *2: 1*.
Jadi luas dari segitiga PDQ adalah 24	So, the area of the Δ *PDQ* is *24*.

The subject wrote down the conclusion of the answer to each item of the problem. The interview excerpts show that this method helps the problem solver gain confidence in the solutions achieved.

Researcher : *Did you provide a conclusion to the answer to each item of the problem?*
Student : *Yes, that's right.*
Researcher : *Why did you do it?*
Student : *I did it to give myself confidence that I had solved the problem according to the question.*

At the fourth level, students manage to solve the problems and highlight each point of their conclusion to show that they have answered the question appropriately. When associated with the study by Hong and Choi (2011), the problem solver reaches the triple-loop position, which is the highest position for reflective thinkers. They are considered capable of solving new problems, overcoming difficulties (Önder, 2016), and showing the correct conclusion performance through the implementation of strategies and abilities.

The study found four levels of reflective thinking in solving problems. A description of each level is presented in Table 4.

Table 4

The descriptors of each student's reflective thinking level of problem-solving

Hierarchy level	Descriptors
First level: In-depth understanding	The problem solver does not fully understand the problem. The problem solver explores the information and understands the meaning of the problems.
Second level: Relating among concepts	The problem solver recalls the math concepts that they have studied. The problem solver sorts out the concepts that can be applied to the problems. The problem solver decides on the concept/s used to solve the problem.
Third level: "I made errors, and I am willing to correct them"	The problem solver experiences confusion in the form of doubt or errors in answering the problems. The problem solver conducts monitoring to identify errors. The problem solver thinks about how to fix these errors. The problem solver fixes the errors.
Fourth level: "I am convinced of my answer"	The problem solver solves the problem. The problem solver gives a conclusion to every answer. The problem solver gains confidence in their answer.

CONCLUSIONS

We successfully determined four hierarchy levels of reflective thinking. The first level is the in-depth understanding of the information available and the intent of the question. The second level entails the relation of concepts through recalling concepts that have been studied, selecting the appropriate ones, and deciding which ones to use. The third level entails the subject making an error and being willing to correct it. The fourth level is when the subject is convinced of their answer. Further research can focus on defragmenting students' reflective thinking during problem solving to improve such thinking.

Moreover, determining the steps to defragment students' reflective thinking at each level is necessary.

AUTHORS' CONTRIBUTIONS STATEMENTS

M.N.K is a research coordinator and contributed to developing ideas, A.S, N.I, A.N, S.W, and E.P. were responsible for developing theory and designing instruments, M.I and T.T.L developed research methods and reviewed the final manuscripts, T.R.M contributed to collecting data on Java island and data analysis, M contributed to collecting data on Sulawesi island and data analysis, and A.P.W contributed to collecting data on Sumatra island and data analysis.

DATA AVAILABILITY STATEMENT

The data presented and supporting this research results are available at a reasonable request to the first author, M.N.K.

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