ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7123

Challenges faced by Preservice Teachers in Planning and Exploring Tasks that Promote Mathematical Reasoning

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Received for publication 26 Apr. 2022. Accepted after review 18 May 2022
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Mathematical reasoning is fundamental for mathematics learning from the first years of schooling. It is a challenge for students and teachers, so it is relevant to deepen ways to develop this ability with the prospective teachers. **Objectives**: Identify the challenges in supervised practice with a view to developing students' mathematical reasoning, seeking to answer the following question: What challenges do prospective teachers face in planning and exploring tasks that promote mathematical reasoning? **Design**: It is based on a formative experiment and follows an interpretive methodology. **Setting and participants**: This experiment was conducted during 13 sessions of the curricular unit (CU) Didactics of Mathematics of the 2nd year of the master's course in Pre-School Education and Teaching of the 1st Cycle of Basic Education, in a class of 25 students. The participants were four students - two pairs in teaching practice- whose selection followed the following criteria: not having any of the researchers as teaching practice supervisors and regularly intervening in class. **Data collection and analysis:** the data were collected through participant observation of the CU classes, interviews, and document collection. **Results**: Students face more challenges associated with mathematical reasoning during monitoring phases of the exploration of the tasks and their final discussion. **Conclusions**: These results point to the need for initial training programs to prioritise activities that support prospective teachers in the understanding of mathematical reasoning processes and that involve them in the planning of tasks and analysing practical exploration that will enhance their development.

Keywords: Mathematical reasoning; Tasks; Challenges; Initial formation.

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Desafios dos Futuros Professores na Planificação e Exploração de Tarefas que Promovem o Raciocínio Matemático

RESUMO

Contexto: O raciocínio matemático é fundamental para a aprendizagem da matemática desde os primeiros anos de escolaridade. Constitui um desafio para alunos e professores, pelo que é relevante aprofundar formas de desenvolver esta capacidade com os futuros professores. **Objetivos**: Identificar os desafios na prática supervisionada com vista ao desenvolvimento do raciocínio matemático dos alunos, procurando responder à seguinte questão: Que desafios enfrentam os futuros professores para planificar e explorar tarefas promotoras do raciocínio matemático? **Design**: Baseia-se numa experiência de formação e segue uma metodologia interpretativa. **Ambiente e participantes**: Esta experiência decorreu durante 13 sessões da unidade curricular (UC) Didática da Matemática, do 2.º ano do curso de mestrado em Educação Pré-Escolar e Ensino do 1.º Ciclo do Ensino Básico, numa turma com 25 estudantes. Os participantes quatro estudantes pertencentes a dois pares de estágio, cuja seleção seguiu os seguintes critérios: não terem como supervisora de estágio alguma das investigadoras; habitualmente intervirem nas aulas. **Coleta e análise de dados**: Os dados foram recolhidos através da observação participante das aulas da UC, entrevistas e recolha documental. **Resultados**: Os estudantes deparam-se com mais desafios associados ao raciocínio matemático nas fases de monitorização da exploração das tarefas e da sua discussão final. **Conclusões**: Estes resultados apontam para a necessidade de os programas de formação inicial priorizarem atividades que apoiem os futuros professores na compreensão dos processos de raciocínio matemático e que os envolvam na planificação de tarefas e análise da sua exploração na prática que potenciem o seu desenvolvimento.

Palavras-chave: raciocínio matemático; tarefas; desafios; formação inicial.

INTRODUCTION

General curricular perspectives that focus on the individual's overall education (Martins et al., 2017; UNESCO, 2017) agree in seeing reasoning and problem-solving skills as essential for the education of 21st- century citizens. In terms of specific curriculum guidelines for mathematics, the importance of those skills is reaffirmed, specified, and developed, emphasising the importance of mathematical reasoning (MR) for mathematics learning with understanding and effectively experiencing what mathematics is, as mentioned in *Novas Aprendizagens Essenciais de Matemática para o Ensino Básico* [New Essential Mathematics Learning for Basic Education] (ME-DGE, 2021).

The recommendation of the NCTM (2007), stressing that it is essential that mathematics teaching centres on solving and discussing tasks that promote

MR and allow the use of different ways of exploring and solving them, is shared by many authors and a priority for learning mathematics expressed in various curricula (Jeannotte & Kieran, 2017).

This article is part of the *Raciocínio Matemático e Formação de Professores* [Mathematical Reasoning and Teacher Training] project (REASON)[1], in which reasoning mathematically is understood as making justified inferences (Mata-Pereira & Ponte, 2017) and the processes of generalising, justifying, classifying, conjecturing, and exemplifying are highlighted. Generalising and justifying are central processes of MR. The former consists in asserting that an idea, property, or procedure is valid for a specific set of objects or stating that a property is common to a group of objects. The latter consists in presenting a logical argument based on mathematical ideas to support a specific claim or to refute it (Jeannotte & Kieran, 2017). Conjecturing involves formulating statements (conjectures) that are expected to be true, although their veracity must be validated (Lannin et al., 2011). These conjectures stem from the search for regularities, similarities, or differences, with the aim of establishing relationships (Jeannotte & Kieran, 2017). Classifying is a process that can be triggered by looking for similarities or differences between mathematical objects and involves making inferences about classes of objects based on their properties and definitions (Jeannotte & Kieran, 2017). Finally, exemplifying is an auxiliary mathematical reasoning process (MRP) meant to support others and is especially important when working with younger students. It consists of presenting examples that support the search for similar and different aspects or the validation of a statement (Jeannotte & Kieran, 2017).

Research shows that many students cannot adequately explore tasks that involve MRP and that teachers also face several challenges in implementing teaching aimed at developing them (Stylianides et al., 2013). Given this context, there is broad consensus on the relevance of working with MR in initial teacher education. Consequently, it is essential to deepen knowledge about ways to work the MR with prospective teachers (Desfitri, 2018). In particular, understanding the challenges they face in practice aimed at developing MR may contribute to better understanding the aspects that deserve special attention in initial training programmes to favour this development.

[1] http://www.ie.ulisboa.pt/projetos/reason

This paper is based on a study that focuses on identifying the challenges that prospective teachers face during supervised practice to develop students' MR. Specifically, starting from the analytical model used by Stylianides et al. (2013) in two of their groups of categories, we seek to answer the following question: What challenges do prospective teachers face in planning and exploring tasks that promote MR?

We see the same meaning of challenge as that attributed by Stylianides et al. (2013), i.e., problematic situations that prospective teachers face in supervised practice and what they consider to be problematic. Those situations can translate into fears, doubts, difficulties, and ambivalences (Delgado, 2013).

THEORETICAL BACKGROUND

Prospective teacher's knowledge and the development of the MR

Recognition of the practical nature of teacher knowledge (Ball et al., 2008) and the complexity of the knowledge required from them (Livy & Downton, 2018) have triggered numerous studies on initial training (Ponte & Chapman, 2006). Those studies seek, mainly, to understand what knowledge prospective teachers should develop to teach – which is an aspect that should not be considered on the sidelines of practice (Ponte & Chapman, 2006). Several works confirm prospective teachers' weak mathematical knowledge (Ponte & Chapman, 2006), particularly those who teach in the first years of schooling (Stylianides et al., 2013).

Some authors have also considered the implications of the poor mathematical knowledge of prospective teachers in the various areas of their teaching knowledge (Herbert et al., 2015). For example, prospective teachers' mathematical knowledge of teaching topics influences their knowledge of the type of answers students can give when solving a given task (Tirosh, 2000). Moreover, making specific key concepts and ideas visible to students when they solve tasks or identifying the origin of their difficulties is strongly related to this knowledge dimension (Morris et al., 2009).

Given the MR importance as an essential process to support students' mathematics learning, some of those studies focus on prospective teachers' knowledge to identify students' MR and promote their development (e.g., Livy & Downton, 2018; Maher et al., 2014; Stylianides et al., 2013). In addition, MR is a somewhat complex topic, both in terms of its meaning and the variety of associated processes (Herbert et al., 2015). Teachers' understanding of MR is diverse and sometimes limited – it ranges from merely corresponding to ‘thinking’ the idea of making, justifying, and validating conjectures and establishing connections between different mathematical ideas (Herbert et al., 2015). However, knowledge and understanding of MR are fundamental to promoting its development in students (Herbert et al., 2015; Livy & Downton, 2018), having implications, for example, on what teachers can accept as a valid justification and how they can support students’ reasoning (Livy & Downton, 2018).

Associated with limited knowledge about MR and its processes, teachers, particularly those who teach in the first years of schooling, present what Stylianides et al. (2013) call their teaching ‘counterproductive beliefs’. These translate into a reluctance to teach students to reason mathematically because they see some of the MRPs, namely justification and proof, as not accessible to their students (Stylianides et al., 2013).

Challenges prospective teachers face in the development of MR

To develop students’ MR, it is important to opt for an exploratory approach to teaching, in which two elements stand out as fundamental - the tasks proposed to students and the teacher’s specific actions that can make the MR emerge (Brocardo et al., in press).

The importance of tasks lies not in the task itself but in the type of activity in which students engage in solving it (Ponte et al., 2014). In particular, to promote MR, tasks must allow the use of different strategies, encourage the use of a variety of representations, and encourage reflection on the MRP used (REASON, 2020). The complexity of choosing tasks for this purpose is even higher as they must satisfy a set of characteristics related to specific MRPs. For example, it is important that the tasks include questions that lead to the formulation of generalisations, that encourage justification of answers, strategies, or mathematical statements, and ask for the justified identification of the truth or falsity of mathematical statements (REASON, 2020).

In an exploratory approach, the class is usually structured in the following stages: presentation of the task, students’ individual resolution of the task, and discussion of resolutions and systematisation of the learning that resulted from the exploration of the task (Stein et al., 2008). This approach requires an understanding of how the class is organised and conducted and, consequently, class preparation that encompasses the anticipation of student work and collective discussion (Ponte et al., 2014). Planning and conducting a class with these characteristics are indeed complex and pose a challenge for many teachers (Ponte et al., 2014), but given their lack of teaching experience, it becomes even more challenging for prospective teachers (Adeeb, 2020; Santos et al., 2019). Besides the anxiety inherent to their lack of experience, the challenges are related to the difficulties the prospective teachers have in applying in the classroom the teaching theories they have studied during the initial training course and in managing classroom situations due to their lack of authority compared to experienced teachers. Moreover, they may have a weak teaching knowledge of the content, evidenced in how they plan and conduct their classes (Adeeb, 2020).

In this approach, it is also fundamental to create a specific classroom culture in which students share and justify mathematical ideas, and the teacher manages those interactions to encourage discussion (McNeal & Simon, 2000). This classroom culture takes time to build and is not something imposed by the teacher (McNeal & Simon, 2000). In this sense, contexts that have not developed this culture can be more challenging for prospective teachers when adopting an exploratory approach to teaching and, consequently, when developing students' MR (McNeal & Simon, 2000; Stylianides et al., 2013).

Next, we focus on the teachers' specific challenges in two stages of work around the tasks – lesson preparation and planning and exploration of the task in class.

Challenges in class preparation and planning

The tasks proposed to students must be carefully selected according to the learning objective established for the class (Morris et al., 2009; Ponte et al., 2014). One of the challenges that prospective teachers face when selecting tasks is precisely to ensure that they are aligned with what they intend to teach (Desfitri, 2018; Santos et al., 2019). Being able to make this connection between what is proposed to students and the objectives intended requires some experience in "decomposing" the mathematical concepts inherent to the learning objectives in mathematical subconcepts and using this information to plan, teach, and assess (Morris et al., 2009). For Desfitri (2018), this difficulty in selecting tasks according to learning objectives is related not only to the prospective teachers' inexperience, but also to their fragile mathematical knowledge.

When selecting problems (with a non-mathematical context), prospective teachers want these contexts to be real. However, the challenge is that the problems must be associated with the students' daily life so they can attribute relevance/meaning to them. Also, prospective teachers find it hard to regulate the complexity of the problems to match the students' knowledge and, simultaneously, corresponding to the level of demand compatible with their level of education. Furthermore, they must face the challenge of identifying tasks that can be solved through different strategies (Mallart et al., 2018).

Another fundamental aspect that must underlie the preparation of a lesson is adapting planning elements to the specific needs of the class. Prospective teachers have trouble planning lessons to align their decisions with the students' learning dispositions (König et al., 2020), tending to use a poorly adaptive teaching style, programmed step by step, as if it were a recipe (Chizhik & Chizhik, 2018). Even when they value exploratory teaching in their discourse, referring to interactions between students and discussion as crucial moments in a math class, they tend to develop teacher-centred plans, which do not include space for these moments (Martins et al., 2021).

For teachers to adopt an exploratory teaching perspective, they must engage in a lesson planning process that includes anticipating students' difficulties when solving tasks, so they can feel more prepared to help students overcome such difficulties and they feel more confident to conduct collective discussion (Morris et al., 2009; Ponte et al., 2014). However, anticipating students' difficulties is challenging for prospective teachers because of their little teaching experience and knowledge about students (Santos et al., 2019).

Challenges in exploring tasks in class

During the first phase, exploring the task, i.e., presenting the task, beyond the teachers' organisation of the work students should carry out (determining the time they should dedicate to the different phases, managing the resources to be used, and defining the modalities of the students' work, etc.), teachers must make sure that students understand the task and feel motivated to solve it (Anghileri, 2006). Many teachers find it demanding to ensure that students understand the task context and the mathematical terms of the statement while involving and stimulating them to solve it without reducing the degree of challenge (NCTM, 2017; Stein et al., 2008).

In the second phase, students' autonomous task resolution, the teacher's role is to monitor their work and support their progress in solving the task (NCTM, 2017). Once again, the challenge that both teachers and prospective teachers usually face is to provide this support, trying to give suggestions or ask questions that do not reduce the level of demand of the task (NCTM, 2017; Santos et al., 2019).

Finally, the third phase, collective discussion of task resolutions and systematisation of learning is the most complex, according to research on the exploratory approach (Delgado, 2013; Ponte et al., 2017; Santos et al., 2019), as it involves the decision-making associated with diverse - intellectual, temporal, and social - issues (Lampert, 2001). Indeed, the teacher must create an order in which the students' solutions are presented, ask them questions, encourage them to justify their statements, help them establish connections between the presented solutions, synthesise the relevant mathematical ideas associated with solving the task, and promote students' reflection on what they have learned (Ponte et al., 2003; Stein et al., 2008). The connections established between the students' different solving strategies stand out as particularly challenging, and the teacher needs to create connections between the different representations used and the systematisation of the mathematical ideas associated with the presented solutions (Delgado, 2013; Delgado et al., 2017). As mentioned by Lampert (2001), we need to be able to 'extract' all the mathematics used by students. In addition, students must have balanced participation, so teachers must help some of them while managing the interventions of those who spontaneously present their contributions (Lampert, 2001). This dual concern of managing students' participation and involvement in the discussion of the task and making this discussion relevant from the point of view of mathematical ideas is one of the challenges of teaching MR highlighted by Brodie (2010), who calls it "linking learners with the subject" (p. 168).

The collective discussion stage is particularly challenging for the prospective teachers due to its inherent demands and unforeseen circumstances (Ponte et al., 2017; Santos et al., 2019). In fact, selecting and sequencing students' strategies, involving all students in the discussions, and exploring what they say, especially what is mathematically important, does not seem to be easy for prospective teachers (Santos et al., 2019).

During this stage there is intensified idea sharing, search for arguments that either validate, or invalidate, a specific conclusion or proposal of new explanations and generalisations and that is why this stage is essential for the development of MR and students' mathematical understanding (NCTM, 2017). It is natural that the teacher cannot anticipate all kinds of questions the students may ask, nor has he thought of some of the students' conjectures, especially when dealing with open tasks for which the solution may lead to a diversity of resolution paths. Ponte et al. (2003) mention that one of the problems the teacher may face is how students justify their ‘unexpected’ conjectures, which are sometimes unclearly formulated and that, besides requiring reformulation, need to be tested. In these situations, the teacher will also have to reason mathematically, being able to make the decision to discuss these conjectures immediately or in a later class (Ponte et al., 2003). Dealing with these challenges requires a solid mathematical knowledge of reasoning and proving, which we often do not see in novice teachers or sometimes even in more experienced teachers (Stylianides et al., 2013).

Finally, another exploratory teaching challenge that prospective teachers face both in moments of autonomous performance of tasks and in the collective discussion is related to time management (Santos et al., 2019). We need to decide how much time to allow students to explore the task autonomously and at what point in the lesson we should initiate the discussion. This decision depends on whether students are tired and on the assessment of the progress of the task (Ponte et al., 2003). Also, in the collective discussion moment, we need to continue to manage the time for the presentation of the students’ work and to monitor the discussion taking into account the end of the class period (Lampert, 2001).

METHODOLOGY

In this article, we analyse data collected in a training experiment with four prospective teachers (6-10 years), from a class of 25 students of Didactics of Mathematics, an annual CU of the 2nd year of the master’s degree in Pre-School Education and Teaching of the 1st Cycle of Basic Education. The training experiment took place over 13 sessions of 90 minutes each and had three different components.

In the first phase, which lasted seven classes, the prospective teachers explored tasks and texts that could potentially promote the MR of 1st-cycle students and were designed or adapted by the REASON project team. The materials used had been analysed and improved after the first phase of a training experience carried out in the previous school year (2019-2020) with another 2nd -year class of that same master’s degree. We used training tasks that included a task statement intended for 1st-cycle students and the formulation of questions related to the analysis of students’ concrete answers and the teacher’s possible actions, as exemplified in the task “Compare perimeters” (Appendix 1),[2] which constitutes the last and sixth training tasks (TF6). In the second phase, which lasted four classes, the focus was on planning an intervention in a teaching practice class from the 3rd or 4th year of education, in which the prospective teachers would use a task with the potential to promote children’s MR. In this context, they could select or adjust one of the tasks explored in the CU or choose another one that they felt was more appropriate. In the third phase, which lasted two classes, prospective teachers shared and reflected on how the class with the children went.

This investigation follows an interpretive methodology (Erickson, 1986) as it focuses on the meanings attributed by the study participants to the lived situations, explained either verbally or in writing. The participants are four prospective teachers in training who worked in pairs: Júlia and Rute, from a 4th year and Carla and Maria, from another class of the same year. The group’s choice resulted from combining two criteria: 1) that both pairs did not have the researchers[3] as their practice supervisors; and 2) that they included students who usually actively participate in classes. The justification for Criterion 1 is that we wanted to ensure that the data reflected the conditions of the training experience and were not contaminated by other training interventions within the scope of lesson planning. Criterion 2 is related to the intention of ensuring the collection of data in various contexts, so counting on prospective teachers’ spontaneous contributions in class was important.

The data collection techniques were the participant observation of the 1st Cycle Mathematics Didactics CU classes, the interviews, and document collection of different student productions (task resolution, task planning, and reflection on its accomplishment in the classroom).

In this article, we analyse[4] the data from the teaching practice experiment on planning and carrying out the task “Compare Perimeters” in two 4th-grade classes, focused on:

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[2] Adapted from Battista, M. (2017).

[3] The teacher practice supervisor supports the prospective teachers in carrying out their plans.

[4] The Free and Informed Consent Term was signed by the study participants. The investigation followed the guidelines of the Letter of Ethics for Research in Education and Training of the Institute of Education of the University of Lisbon (http://www.ie.ulisboa.pt/download/carta-etica-e-regulamento-da-comissao-de-etica), institution proposing the research project in which this study is part. The Ethics

- transcription of the four classes of the second phase of the training experience;
- transcription of two interviews, one for each practice pair, carried out at the end of the training experience (script – Appendix 2);
- written reflection with a paired component and an individual component of practice, including a discussion on the suitability of the task given the defined learning objectives and the class in question (in pairs); discussion of the options taken that may have contributed to the results achieved (in pairs); reflection on the contribution of the class to the students' learning in the teaching topic(s) covered in this class (individual); reflection on experience in teaching or observing this class (individual).

We start from the analytical model used by Stylianides et al. (2013), considering the four groups of challenges identified: i) the cooperating teacher's classroom context; ii) task planning; iii) task exploration; iv) the prospective teachers' knowledge; and v) others, such as learning assessment or time management.

In the first data analysis phase, the challenges related to each of the previous categories were identified and characterised by a data collection instrument. This first analysis was discussed by the three authors of the article, aiming at its validation and refinement. On the one hand, we found that the "challenges related to the context" and "other challenges" had a very limited significance, since the entire focus of the study was on class planning and management. Thus, we decided to consider a single global category called "others". On the other hand, we clarified that an affirmation was considered a challenge for the students whenever a fear, difficulty, doubt, ambivalence, or hesitation towards the experience they had lived was explicitly indicated.

Finally, the numerical accounting of each challenge was done, taking into account the way they were characterised in each episode. For example, during the final interview, Maria and Carla identified the challenge related to understanding the objectives of the task, which was counted as two occurrences. In the first part, the first occurrence, their dialogue focused on the challenge of having identified too many objectives for the task:

Committee of the institution mentioned above considered that ethical principles and the ethical guidelines for research are fully taken into account.

> *M: [...] The problem is that setting goals for this task was a little difficult.*
>
> *C: Because we were writing down a lot of objectives. The problem is that we set too many objectives.* (Maria and Carla, Interview)

In the second part, counted as the second occurrence, Maria goes further, being more specific about the challenge, showing that they had general and specific objectives and that it was necessary to select, a task which she finds was not easy.

> *M: Yes, we ended up with few, because we had that one... we knew what we were going to work on, and we put those right away, but then we got to have too many... we had some that were very general and others that were very, very specific (...) we then started to select what makes the most sense, but it was complicated. (Maria, Interview)*

Accounting for the frequency of each challenge was considered a sensitive aspect of the data analysis, which was, therefore, the subject of several later adjustments and refinements. In any case, we emphasise that, in this article, this quantification is, above all, considered to assess the relative weight of the challenges associated with MR.

In the second data analysis phase, the challenges were labelled with a brief description of what characterised them, such as: challenge - managing to lead students to justify a conjecture; challenge – knowing to what extent to explain, or challenge - get the child to exemplify.

Finally, the description of the data analysis focused on the categories related to the MR identified in this labelling and its empirical illustration.

CHALLENGES IN TASK PLANNING AND EXPLORATION

Challenges related to planning

The planning of the lesson "Compare perimeters" involved moments of autonomous work by each pair and moments of collective work. The task the prospective teachers chose had already been explored in the Didactics of Mathematics class, i.e., solved and analysed from the MR point of view.

The students had three 90-minute classes to plan an intervention that consisted of exploring the task with 4th-grade students. In these three classes, the four students worked in a parallel room where two authors of this article were present, and the third author occasionally visited. They carried out autonomous work between each class, trying to progress with their planning. In a fourth class, held after exploring the task in the 4th-grade classes, the prospective teachers, together with all the other Didactics of Mathematics colleagues, participated in a global reflection class on the intervention experience in each pair's practice.

The data relating to each pair's written plans and individual reflections did not constitute a data source for this aspect. In fact, we found that due to the nature of the written plans, they do not inform about the challenges as understood in this text. As for the written reflection, since its focus was the exploration phase of the task with the students, there is no information about task planning.

Challenges related to solving the task

Data analysis revealed that during class, when they explore the group task and organise the planning, the students explicitly identify the most challenges. However, in the interviews, the students also mentioned several challenges, especially those related to their difficulties in solving the task.

Table 1

Challenges associated with solving the task and predicting students' solutions

Moment	Challenges associated with solving the task		Challenges associated with predicting students' solutions	
	Reasoning	Total	Reasoning	Total
N (classes)	8	19	1	13
N (interviews)	1	13	2	4

As seen in table 1 there are more challenges 'in action' (during class) - 19+13 - than the ones 'when reflecting' (during the interview). There is only one exception related to reasoning.

The students had already solved the task before, but during the planning, they quickly recalled their difficulties. One of them concerns the doubts regarding the MRP used in a specific part of the task resolution and which they wanted to explain in their planning. When referring to an aspect that had already been discussed in previous classes, Carla wondered whether saying “isoperimetric figures” would not be associated with the MRP of “classifying”. This doubt was promptly clarified by Júlia, who sought to distinguish the common understanding of classifying from that of classifying as an MRP:

> *C: Because we are confusing the “classifying” that we commonly use in everyday language and the “classifying” of mathematical reasoning, because “classifying” means organising in orders, isn’t that right, teacher?*

Understanding what a conjecture is, is also not clear to some students. As illustrated in the following episode, initially, Júlia does not realise that it is in question 3 of the task statement that a conjecture is formulated (“Maria says that there are many figures with six sides that have the same perimeter as B. Do you agree? Explain why”) and that what is being asked of the children is their justification and not the formulation of a conjecture.

> *J: I think the only thing missing is “conjecture”. Because Maria says that there are many 6-sided figures with the same perimeter. She’s making a conjecture, right? In the question, and then, we are asked if we agree, if the children agree or not.*
>
> *M: We’re not asking them to guess.*
>
> *J: Yes, but they can perhaps start from a conjecture to explain, right?*
>
> *M: But the conjecture was made by Maria.*
>
> *C: Well, that’s it. They have to give examples...*
>
> *M: To agree or not with Maria.*
>
> *J: So, they can’t do like the one I did a while ago of “are there several figures with the same number of sides and the same perimeter”?*

Maria explained that students were not asked to formulate a conjecture. Carla seemed to understand as she considered that she could generate justifications (“starting from a conjecture to explain”). However, in her last intervention (“So they can’t do like the one I did just now, “are there several figures with the same number of sides and the same perimeter"?") she seemed to go back to her initial confusion.

The identification of MRPs that children could use also reveals students' doubts:

> *C: Justification, exemplification, and I think I put generalising. I don't remember anymore.*
>
> *PC: So, justification, exemplification...*
>
> *C: Generalising, I don't know if it's so right.*
>
> *PC: And generalising.*
>
> *C: I don't know if it turns out to be a thing... I think so.*

Another challenge was explaining what a generalisation could be, which arose from the question that one of us asked, "What the generalisation would you like children to reach?", to which Júlia answered, "isoperimetric figures". This answer led Carla to suggest as a generalisation, "maybe the part where the sides must stay... for there to be many different figures, two sides must stay... they can't move. And then only the others vary", revealing that she was also unable to formulate the generalisation that she would like the students to reach. Then, Júlia, when proposing as a generalisation, "several, infinite. I mean, I don't know whether they are infinite, but there are several", showed that she was beginning to understand what might constitute a generalisation, but that she was not sure which one could be reached by exploring the task. Let us note that Júlia's doubt was at the level of her knowledge ("I don't know if they are infinite") and not at the level of what students were required to indicate as a generalisation.

Finally, we identified several challenges associated with understanding the MRP to justify, explicitly considered the most difficult MRP. The students mentioned difficulties associated with justification – "but we couldn't give a reason, and in that situation none of us was able to get to the why"; and doubts about the distinction between explaining and justifying – "Explaining can be: I thought this way, and then, the reason that comes next is the justification".

In the interview with the pair formed by Maria and Carla, Maria explained the only challenge she found regarding the MR ("I think that was our only difficulty, that we were there thinking and thinking in different ways, but we had no justification") and which could be considered as expressing, once again, the difficulty they mentioned with the MRP of justifying.

During planning, when trying to predict students' difficulties in solving the task, the prospective teachers returned to their most significant difficulty - justifying - and they hesitated when specifying students' difficulties, asking themselves whether it would be explaining or justifying:

C: "Difficulty with justifying...", isn't it...

PC: What does the question say?

J: "Do you agree? Explain why."

PC: It's in explaining, isn't it?

C: ... explain. "In explaining..." But justifying was also fine, wasn't it? Justify your opinion, no...

PC: Explaining may not be the same as justifying.

C: Well...

J: Explaining is OK: I thought this way and then, why what comes next is the justification.

C: So, it can be both! Explain, no.

J: (...) But then it says, "Explain why".

PC: Okay, so if you say, "Explain why", it's explain and justify.

C: Explaining and justifying, exactly, can be both. "Explain and justify (...) their reasoning".

In the interviews, it is Júlia and Rute's group that explained two challenges focused on MR, related to predicting student resolutions. Júlia considered that there was a general lack of focus on MRP when they prepared the class. She stated, "we also knew from the start that children might not be very comfortable with mobilising mathematical reasoning processes... in generalising or making conjectures...". Although with low expectations about students' ease in reasoning mathematically, they recognised that they were not able, in their planning, to focus on MR as would be necessary for students, who are not used to working on this aspect.

Challenges related to the justification for choosing the task and its objectives

Although we explicitly asked for the planning and exploration (with the students of the class in which they were doing their internship) of a task that potentially impacted the development of the MR, this aspect is not highlighted in the justification for their choice or the definition of the learning objectives.

Table 2

Challenges associated with the justification for choosing the task and defining learning objectives

Moment	Challenges associated with the justification for choosing the task		Challenges associated with defining learning objectives	
	Reasoning	Total	Reasoning	Total
N (planning)	2	7	3	16

Of the seven challenges the prospective teachers explained, the two related to the MR were formulated, above all, in terms of doubt, through the terms "maybe" or "I believe".

C: Maybe the part where this type of task provides the reasoning and mathematical argumentation and...

J: Communication...

C: Mathematical communication and maybe that can also be one of the topics to work on, right?

C: We can't forget that this has a lot to do with reasoning, can we? That the main objective is to develop mathematical reasoning. Therefore, I believe that this is also the case.

In the interviews, the reference to the reasons for choosing the task is not formulated in terms of challenge. The students described what led to this choice without mentioning the doubts or hesitations they had.

Júlia's and Rute's references to the MR corresponded to the intervention, "And also the processes of mathematical reasoning, of course". Although the expression "of course" could be interpreted as indicating that the choice of the task was obvious due to its potential in terms of MR, it seems that intentionality did not 'stand out', since it was only mentioned after a list that included the rationales for selecting the task according to i) the subject and topic, ii) familiarity with the task, iii) students' difficulties, iv) the nature of the task and the competences it develops, v) the type of work it provides (group work and discussion) and vi) adapt to the exploratory teaching methodology.

Carla and Maria did not mention potential in terms of the MR, and we identified some hesitation in their consideration:

> PC: How do you see that your knowledge of what mathematical reasoning is and the reasoning processes influenced the choice of this task?
>
> C: Maybe... I don't know. Maybe, the exemplification part, exemplifying. It is knowing how to exemplify.

The formulation of learning objectives during the class raised many dilemmas for students. Those that included references to the MR all fall into one of the categories considered, which concerns the level of generality of the learning objectives and is associated with their understanding. Students began by asking themselves if they could formulate objectives associated with general ideas such as "deepening the perimeter concept" or "anything like problem-solving". Realising that it was important to carry out the learning in which that task intentionally focused, they hesitated, moving forward with more specific objectives. However, they doubted whether they could be included in the task. The following two episodes illustrate this dilemma of generality versus specificity of learning objectives, the only one in which aspects related to MR were identified:

> *C: So, in this one, of the reasoning processes, do we just keep it that way or do we also refer to the ability to resort to mathematical reasoning? Or is it all implied?*
>
> *C: And we were also missing... processes.*
>
> *PJ: And what do you think they are?*
>
> *C: So, justifying and exemplifying, yes.*
>
> *A: Justifying, perhaps...*

(...)

C: Generalising, maybe...

J: Because they can generalise, like, for example, there can be more figures with six sides and the same perimeter.

PJ: And is that a generalisation?

C: Yeah, I think it's conjecture. No?

In the interviews, the students were well aware of the challenges they faced in formulating the learning objectives. Maria's and Júlia's interventions illustrate this awareness of the generality versus specificity dilemma.

M: Yes, we ended up with few because we had that one… we knew what we were going to work on and we put those right away, but then we got to have many… we had some that were very general and others that were very, very specific and teacher Joana said "no" on the spot and that's when we started to select what makes the most sense, but it was complicated.

J: The difficulty was also defining specific goals and sometimes not so general, exactly, sometimes not so general, because we went too far in the general objective and didn't focus on specific objectives.

At this stage, none of the references specified the MR, which can be interpreted as indicating that the challenges related to the definition of learning objectives were common to all topics, not having any particular characteristics associated with the MR.

Challenges related to the anticipation of the teacher's actions

Both during lesson planning and in interviews, anticipating the teacher's actions seemed to be an aspect that the prospective teachers tended not to feel as a challenge. Overall, the anticipation of the teacher's actions was only made explicit from questions that the prospective teachers posed without any challenges being evidenced, as illustrated in the following episode, in which one of the prospective teachers asked the others to explain whether what they had previously mentioned was related to teacher's actions.

PC: And who does that? Is it the teacher?

> *C: The teacher can challenge a student to do it.*
>
> *J: Exactly. Confront different perspectives about what area and perimeter are.*
>
> *C: These aspects of overcoming it can already be done in the collective discussion. I remember that in the last class, I created a task like this and called a student to the board to explain their reasoning, and then they confronted the various hypotheses, and then the teacher's role is to validate and organise.*

Júlia, agreeing with Carla that the teacher could challenge the student, described what her action could be, which seemed to have occurred to her because she had already used it.

As Rute says: "Well, we didn't do that, we did it in general, and not taking into account each one's difficulties", there is a clear tendency to anticipate very general actions, not specifying particular or material issues that could help clarify possible doubts of the students.

Challenges related to exploring the task in the classroom

The data sources analysed in this section are: (i) the fourth Didactics of Mathematics class, held after all the students had explored a task potentially focused on the MR in their practice classroom; (ii) the final interviews with each student pair and (iii) the written reflections.

In the analysis of the class, we verified that the contributions of the four students focus mainly on the description of what happened in the practice class, identifying only one contribution formulated in terms of challenge, not associated with the MR, in which Júlia showed she was not sure she had given clear instructions to the students to register in writing all their attempts:

> *J: Teacher, can I just say one thing? It has already been mentioned here, but I think my children also didn't register all their attempts, for example, the drawings of the figures... some left the figures and others didn't, but I think it was my fault, because I told them to register all their thoughts and calculations, but perhaps it was not clear [for them] what to do.*

Data analysis revealed that it is during the interviews that the students mention most of the challenges, seeking to explain their outlines. However, they also explain some challenges in their written reflections (Table 3).

Table 3

Challenges associated with the presentation of the task, the monitoring of the individual task completion, and the guidance of the collective discussion

Moment	Challenges associated with the presentation of the task		Challenges associated with the monitoring of the individual task completion		Challenges associated with the guidance of the collective discussion	
	Reasoning	Total	Reasoning	Total	Reasoning	Total
N Interviews)	0	5	12	33	5	11
N (Written reflections	0	0	0	4	1	6

In the Didactics of Mathematics class focused on the presentation and analysis of the experiment of exploring tasks potentially promoting MR, no specific formulation of challenges was identified, so in the previous table, we did not consider that moment of work with the students.

Challenges associated with the task presentation

As with the written reflections, no challenges focused on the MR were identified during the interview. The challenges described in the interviews focus on the difficulty in understanding how far the prospective teachers should explain to their students what to do during task exploration and in finding different ways to support them.

> *R: So, Júlia made a systematisation. Basically, Júlia said, "Ok, here in question one you should do 'so and so', in question two you should do 'so and so', so now you can start and try to do*

it on your own", but they had a lot of doubts and Júlia was going crazy because she couldn't do it… she wanted to reach everyone and then she would go to a group and they were having a hard time, then she would move on to another group and there were already other groups calling her, then the group she had already helped was already in need of help again. Everyone wanted to call the teacher, everyone was struggling.

The previous transcript illustrates the ambivalence Júlia experienced when launching the task. Initially, Júlia presents the task, as foreseen in her planning (without explaining what they are supposed to do). However, as many doubts came up, she chose to explain question by question what they should do, hoping the children could move forward. However, the introduction phase of the task did not seem to include significant challenges, since none of the written reflections refers to this phase of the class.

Challenges associated with monitoring the individual completion of the task

Four out of the twelve challenges associated with MR highlighted the difficulty in being able to support students in justifying their reasoning. They feel that they had thought about general support "we should have thought about more specific issues", which made it difficult for them to act in specific support. One of the aspects mentioned concerns the support for the justification of the conjecture in the task statement.

The following transcript illustrates how Julia recognised that the general questions they posed did not support students in coming up with justifications that could either reject or accept Maria's conjecture:

J: I think we started by asking the children at the time it was "so do you agree or disagree with Maria?", I'm not sure what the girl's name was, but… and then, they said yes or no. Most said "yes", of course. And then, why: "so why do you agree?" and I think some answers… because there were also discrepancies. There were also children who said, "because I made and built figures and therefore" (...) we could ask "So, is it just because of that?", "What else is missing? Read the question again".

Rute mentioned the challenge of being able to help students give examples to support generalisation. She felt that the short time to explore the task further and not having foreseen how they could do it justified the difficulty they felt.

Challenges related to supporting MR, in general, are referred to by identifying various points of view. Rute feels that “We also didn’t know how to identify well, at least, we didn’t reveal that”, so they necessarily had difficulties supporting the students. Júlia agrees with Rute: “The processes were explored, but I think there was a lack of work in this regard” and adds that her expectations were “that children could better justify their reasoning and better mobilise the mathematical reasoning processes” which made it more difficult to support students. Once again, the too general preparation is advanced as also justifying the difficulties experienced in supporting the students’ MRPs.

Maria wonders about the help that identifying the MRP can give to support students in exploring a task that focuses on the MR: “I don’t know how much it helped us identify the processes. Maybe it did help, we don’t know, but when we were identifying, I don’t know to what extent it helped us after we made the task more dynamic with the class”. This intervention of hers can be interpreted as a note in which Maria intends to emphasise that the difficulties she felt supporting the students were complex, even doubting that a simple prior identification of the MRP could effectively help overcome the difficulties felt in the class.

Finally, we highlight Maria’s note about the challenge inherent to the tasks that affect the MR and which led her to realise that this type of task required a careful prior solution: “I think that from then on, we always solved everything, all the problems of reasoning”.

Challenges associated with the guidance of the collective discussion

In the interviews, the proportion of challenges that affect the MR is 5 out of 11 (45%), the highest of all were the ones included in Tables 2 and 3. This data can be interpreted as indicating that in the final discussion and synthesis, the students felt it was necessary to focus on the MR. This raised doubts, hesitations, or ambivalences that they promptly mentioned.

One of the challenges they identified was related to the introduction of aspects of the MR in the class systematisation, which is illustrated by one of Rute’s contributions: “In the final synthesis, it was necessary to integrate the mathematical reasoning processes (...) it is not that I think we didn't mention them, I think we didn't explain them according to… (being) integrated, so to speak".

In line with this point, several of the students' contributions identified the challenge of better exploring the MR, either because they had difficulties exploring them themselves (Rute) or because they could not move forward based on what the children had said (Júlia).

> *R: I think that we… I think that we didn't explain it well, I think that when it was time for the students to explore, we didn't either… how can I say it? We didn't know how to identify it well either, at least we didn't show it, that's what I think. I don't know. Jessica, what do you think?*
>
> *J: Yes, I agree to some extent with what you are saying because I think that later on, in reflection (...) aspects that we could have explored better, but for example, there was also an attempt here to… I asked the children, like, "so do you think...how many figures do you think we can make with the requirements" and a boy told me, "Teacher, there are infinite ones" and I said "oh, are there? So, explain your opinion better", but the child was like that... because it was almost... it seemed to me that it was very intuitive.*

Finally, justification was also mentioned in this phase of the class by Júlia, who identified the difficulty of justifying assuming all the examples, ending up making an incomplete justification, based on just one example: "Ah, and then, there was a doubt, because, in the justification, we justified with the examples from the previous figures, but I think we ended up just choosing one of them".

It was in this final phase of the class that the only challenge that could be associated with MR was identified and which was mentioned in the written reflections:

> *In this sense, I believe that the greatest challenge in this process was to create a rich collective discussion, since not everyone put into words what they had written in their statements, and not even in group conversations.*
>
> *(...) in many cases, the students had found the answers, but they could not justify how they did it. A fact that is somewhat*

difficult to manage, since the answers in some cases were "I don't agree, because we couldn't do it" or "I agree, because we did many". (Júlia)

Júlia mentioned the difficulty of organising a proper collective discussion, not based only on the reproduction of written records that the groups had made during the exploration phase of the task. In this context, she identified her difficulty getting students to propose justifications for their reasoning, going beyond a justification based on the examples they were able to propose or not.

Challenges related to the teaching practice context

The indication of challenges associated with aspects related to the teaching practice, while it exists, is not very significant. However, all the students mentioned that the students with whom they worked during their practice found adequate thinking and justification of their statements hard. The prospective teachers' observations were based on the appreciation the cooperating teacher conveyed and their observation during the practice.

In the description and analysis of the experience of exploring the task, the students also mention how independent they were from the cooperating teacher. They believe the teacher gave them the freedom to plan and manage the class as they thought to be most appropriate, an aspect that the prospective teachers appreciated.

The challenges related to the classroom context of the cooperating teacher are doubly complex. On the one hand, children had difficulties in terms of habits of thinking, and, on the other hand, they do not have the effective support of the cooperating teacher, who is more experienced, in order to propose actions to support them. However, the prospective teachers did not seem to feel this double complexity, who conveyed the idea that they appreciated the freedom they were given in the practice.

CONCLUSIONS

Table 4 shows the frequency of challenges per moment (Planning classes, Interviews, or Written Reflections) and per phase related to planning and exploring the task with the children.

Table 4

Frequency of challenges associated with planning and exploring the task

Challenges associated with …	Classes RM/Total	Interviews RM/Total	Reflection RM/Total	Classes, Interviews and Reflection RM/Total
solving the task	8/19	1/13		9/32 (28%)
predicting students' solutions	1/13	2/14		3/27 (11%)
the justification for choosing the task	2/7	0/0		2/7 (29%)
defining learning objectives	3/16	0/2		3/18 (17%)
anticipation of the teacher's actions	0/0	0/0		0/0 (0%)
presentation of the task	0/0*	0/5	0/0	0/5 (0%)
the monitoring of the individual task completion	0/0*	12/33	0/4	12/37 (32%)
the guidance of the collective discussion	0/0*	5/11	1/6	6/17 (35%)
students' way of working and thinking habits	0/0	0/2	0/0	0/2 (0%)

* These data refer to a class in which all future teachers shared the experience of exploring the task in the context of an internship and in which the intervention time of the four participants in the study was necessarily reduced.

The previous table allows us to conclude that:

- the challenges associated with the MR were not present in the task presentation phase, which was naturally more focused on promoting understanding of the task;

- during the monitoring phase of the exploration of the task, there were more challenges associated with MR in absolute terms (12);
- in the final discussion phase, the proportion of challenges associated with the MR was higher (35%), although very close to the proportion related to the monitoring phase of the autonomous exploration of the task (32%).

Authors such as Ponte and Chapman (2006) and Ball et al. (2008) have mentioned limitations in the prospective teachers' mathematical knowledge, which tend to be more noticeable in prospective teachers who teach the first years (Stylianides et al., 2013). The data from this study confirm the authors' conclusions and suggest that they should persevere. Even as in this study, when the prospective teachers solved a task they had previously explored, they indicated 32 challenges, nine related to the MR. During planning, the aspect in which the prospective teachers identified the most challenges was precisely in solving the task they selected to explore with the students, explicitly mentioning doubts regarding the MRP used. We highlight the difficulties in understanding the formulation of a conjecture, which was confused with a constant affirmation of the statement that should be justified and in formulating a generalisation that they considered adequate. However, it is interesting to note that justification is the MRP that they explicitly considered more difficult since they hesitated to realise whether they were able to propose adequate justifications. They understand that they found it hard to justify but not conjecture, generalise, or classify since, when solving the task, they did not feel it was "missing", as was the case with the justification.

The potential of the MR to justify the choice of the task seems to be an aspect that was still unclear to the students, who mentioned it mainly because they were asked to do academic work focused on it. Also, the identification of learning objectives of the task linked to the MR was considered a challenge with identical contours to the definition of all the learning objectives, marked by the initial tendency of very general formulations. Thus, the difficulty with the connection cited by Morris et al. (2009) between what was proposed to students and the actual objectives was also present in the aspects associated with MR.

Regarding the exploration of the task, students reported more challenges associated with the MR in the phase of support for their individual performance. They highlighted the challenge of supporting students in the justification, explaining that the answers they obtained did not allow them to advance in formulating a justification.

In the discussion phase of the task, the prospective teachers explained the challenge of integrating aspects related to the MR and, in particular, to the justification, identifying the difficulty of organising a rich discussion in which the students' participation went beyond the simple enunciation of its resolution.

Overall, the way the students explained the challenges (associated or not with MR) seems to be related to their difficulty organising a teaching that Chizhik and Chizhik (2018) identify as poorly adaptive, which tends to be used by the prospective teachers. The frequency of 33 in the challenges associated with monitoring the autonomous performance of the task stands out, suggesting precisely what those authors indicate and what the students participating in this study verbalised, recognising their difficulties in implementing paths that effectively support the students.

The conclusions of this study suggest several recommendations for the initial training of teachers that should be included in their programs: i) the analysis of mathematical tasks, paying particular attention to the clear identification of their learning objectives (Morris et al., 2009); ii) the planning of tasks that enhance students' MR development (Livy & Downton, 2018); iii) the analysis of the teacher's actions in the classroom that contribute to the emergence of MR (Mendes et al., 2021). In this sense, it is equally important that initial training contributes to prospective teachers being able to deal with classroom cultures that are typically unfavourable to student involvement in mathematical activities that promote MR (Stylianides et al., 2013).

ACKNOWLEDGEMENTS

The research was supported by the FCT – Fundação para a Ciência e Tecnologia, Portugal, through the REASON Project – Mathematical Reasoning and Teacher Training (PTDC/CED-EDG/28022/2017)

AUTHORSHIP CONTRIBUTION STATEMENT

The three authors actively participated in the development of the study's theory and methodology, as well as in the discussion of results and conclusions.

DATA SHARING DECLARATION

The data supporting the results of this study will be made available by the corresponding author (FM) upon reasonable request.

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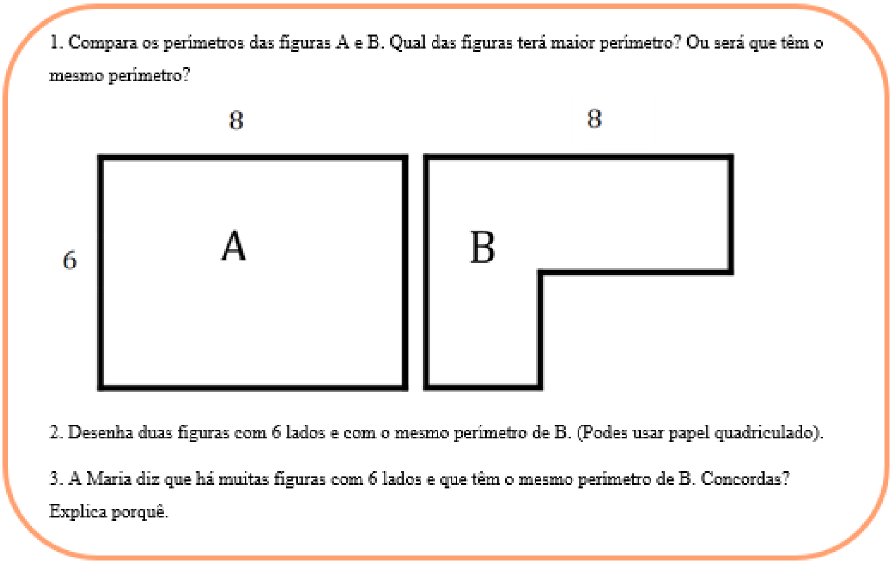
APPENDIX A



1. Compare the perimeters of Figures A and B. Which one has the biggest perimeter? Or are both the same?
2. Draw two figures of 6 sides and the same perimeter as B. You can use squared paper.
3. Maria says that many figures have 6 sides and the same perimeter as B. Do you agree with her? Explain.

APPENDIX B

Guião de entrevista

1. Seleção da tarefa

   Recordar os critérios que as levaram a escolher a tarefa; Depois deste trabalho alteraria os critérios? E a sua escolha? Porquê?

2. Resolução da tarefa

   Quando exploraram autonomamente a tarefa: O que conseguiram fazer/perceber sem qualquer dificuldade? O que foi complicado perceber? Como ultrapassaram as (eventuais) dificuldades que tiveram? Como pensaram para propor resoluções que os alunos podiam apresentar? Tiveram em conta as vossas próprias dificuldades? Tiveram em conta o que já observaram nos alunos? Tiveram em conta textos teóricos sobre a aprendizagem do tema focado na tarefa? O que aprenderam?

3. Definição de objetivos

   Dificuldades na definição dos objetivos; O que aprenderam?

4. Antecipar dificuldades dos alunos

   Como conseguiram antecipar as eventuais dificuldades dos alunos? Tiveram em conta as vossas próprias dificuldades? Tiveram em conta o que já observaram nos alunos? Tiveram em conta textos teóricos sobre a aprendizagem do tema focado na tarefa? Que "tipo" de ações do professor previram para ultrapassar as dificuldades dos alunos? Envolviam guiar o aluno a partir de perguntas sucessivas? Envolviam recorrer à explicação de outro aluno? Envolviam propor a análise de uma resolução parcial da tarefa? Envolviam prever o uso de materiais concretos? O que aprenderam?

5. Apresentação da tarefa

   O modo como pensou organizar a apresentação da tarefa teve em conta que aspetos (o que é habito, o que pensa ser mais produtivo, o que conhece acerca dos alunos, ...)? Após esta experiência alteraria o modo de o fazer? Em que aspetos? O que aprenderam?

6. Trabalho autónomo

   Durante o trabalho autónomo surgiram algumas das dificuldades dos alunos que tinha previsto? Surgiu alguma nova? Como avalia a sua previsão de ações para as ultrapassar? Operacionalizou-as? Sentiu que faltavam muitas alternativas? De que tipo? O que aprenderam?

7. Síntese final

   Como a pensou organizar (apresentações de todos os grupos, seleção de alguns feita com que critérios, sequenciação das apresentações, ...). O que alteraria depois da experiência? Considera que articulou as diferentes apresentações entre si? Explicar como pensa tê-lo feito. Na síntese final conseguiu focar o raciocínio matemático, especificando os processos usados pelos alunos que apresentaram as suas resoluções? Organizou uma síntese final? Se sim, teve em conta as apresentações dos alunos e os objetivos relativos aos conteúdos matemáticos que tinha previsto na planificação? Preparou previamente algum suporte para apoiar a síntese final? Se não preparou, pensa que ele poderia ser útil? O que aprenderam?

8. Raciocínio matemático

   Ao resolveram a tarefa associaram as vossas resoluções a processos de raciocínio? Quais? (e esse aspeto influenciou a antecipação dos processos de raciocínio usados pelos alunos?) De que modo é que o vosso conhecimento sobre o que é o raciocínio matemático e os processos de raciocínio influenciou a escolha da tarefa? Houve algum (alguns) processos usados/não usados pelos alunos que as tenham surpreendido? Quais? Porquê? Vários alunos conseguiram dar dois exemplos e generalizaram imediatamente sem justificar. O que fizeram e o que pensam que poderiam ter feito para apoiar os alunos a elaborar uma justificação matemática? Alguns alunos exemplificaram, mas não generalizaram. O que fizeram e o que pensam que poderiam ter feito para apoiar os alunos a generalizar? Globalmente, que ações consideram que foram (ou que poderão ser) fundamentais para apoiar o uso de processos de raciocínio dos alunos?

Script of the interview

1. Task selection

   Remember the criteria that led you to choose the task. After that, would you change the criteria? And your choice? Why?

2. Task resolution

   When you explore the task by yourselves (autonomously): What could you do/perceive without difficulty? What was difficult to understand? How did you overcome the (occasional) difficulties you had? How did you think to propose resolutions the students could present? Did you consider your own difficulties? Did you consider what you had already observed in the students? Did you consider theoretical texts about the learning of the theme addressed in the task? What did you learn?

3. Definition of objectives

   Difficulties in defining the objectives: What did you learn?

4. Anticipating the students' difficulties

   How did you anticipate the students' occasional difficulties? Did you take into account your own difficulties? Did you consider what you had already observed in the students? Did you consider theoretical texts about learning of the theme addressed in the task? What kind of teachers' actions did you foresee to overcome the students' difficulties? Did they include guiding the students from successive questions? Did they include referring to other students' explanations? Did they include proposing an analysis of a partial resolution of the task? Did they include foreseeing the use of concrete materials? What did you learn?

5. Presentation of the task

   Did the way you thought about organising the task presentation consider the following aspects: what habit is, what you think to be more productive, what you know of the students etc.? What did you learn?

6. Autonomous work

   Did the students present some difficulties you had foreseen during the autonomous work? Did any new one emerges? How do you evaluate your prediction of actions to overcome them? Did you operationalise it? Did you feel that you missed many alternatives? Of what kind? What did you learn?

7. Summing up

   How did you think in organising it (presentations of all groups, selection of some done with which criteria, sequencing of the presentations)? What would you change after the experience? Do you consider that you articulated the different presentations? Explain how you think you did it. In the final synthesis, did you focus on the mathematical thinking, specifying the processes used by the students that presented their resolutions? Did you organise a final summing up? If so, did you consider the students' presentations and the goals related to the mathematical content you had planned? Did you prepare in advance some support to help the summing up? If not, do you think it could be useful? What did you learn?

8. Mathematical reasoning

   When solving the task, did you associate your resolutions with reasoning processes? Which ones? (and did this aspect impact the anticipation of the students' reasoning processes?) How did your knowledge of mathematical reasoning and reasoning processes impact on the task choice? Was any process used or non-used by the students that caught your attention? Which one(s)? Why? Several students could give two examples and immediately generalise them without justification. What did they do, and what did you think you could have done to support students and elaborate on a mathematical justification? Some students justified but did not generalise. What did you do, and what did you think you could have done to support the students in generalising? Overall, what actions were (or may be) essential to support students' use of reasoning processes.