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Handwork and Geometry in the Early Years: Curricular Movements (1890-2020)

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ABSTRACT

Background: Research in the history of mathematics education makes it possible to identify the links between manual practices and the teaching of geometry in the early years since the insertion of the subject Handwork, at the end of the 19th century, to the present time. **Objectives:** Analyse the interconnections between manual practices and the teaching of geometry developed by the school culture of primary education in São Paulo, since the early Republic. **Design:** The curricular regulations of the state of São Paulo and the national guidelines – PCN and BNCC – and school manuals that circulated in each historical moment are mobilised together with the results of research already developed in the area. The documents were collated and analysed to produce an understandable and sustainable historical narrative, based on evidence and controls. **Data collection and analysis:** The study inventoried research results and analysed teaching programs in the State of São Paulo, national guidelines at the end of the 20th century, and school manuals from different periods. **Setting and participants:** The research sources are documentary, most of them available in the UFSC Digital Repository. **Results:** The documental examination indicated that manual practices actively participate in the school culture and cause changes, from time to time. Handwork change its status from a school subject to a geometry teaching methodology, intended for the early school years, which we observe to this day. **Conclusions:** This research concluded that the understanding of the historical perspective contributes to reflection on the current educational problems, on the debate about how to mobilise tools for geometry teaching, questioning the feasibility or not of handwork acting as an appropriate methodology for the first explorations of geometric properties.

Keywords: State of São Paulo. Curricular regulations. Textbooks. Handwork.

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Trabalhos manuais e geometria nos anos iniciais: movimentos curriculares (1890-2020)

RESUMO

Contexto: Pesquisas em história da educação matemática permitem identificar inúmeras articulações entre práticas manuais e o ensino de geometria nos anos iniciais desde a inserção da matéria Trabalhos Manuais, final do século XIX, até os tempos atuais. **Objetivos**: Analisar interconexões entre práticas manuais e o ensino de geometria elaboradas pela cultura escolar do ensino primário paulista, em contextos escolares a partir da República. **Design**: Mobilizam-se as normativas curriculares do estado de São Paulo e as orientações nacionais – PCN e BNCC – assim como manuais escolares que circularam em cada momento histórico, juntamente com os resultados de pesquisas já desenvolvidas na área. Os documentos foram cotejados e analisados para a produção de uma narrativa histórica compreensível e sustentável, a partir de provas e controles. **Coleta e análise de dados**: O estudo inventariou resultados de pesquisas, assim como analisou programas de ensino do Estado de São Paulo, orientações de âmbito nacional, manuais escolares de diferentes épocas do período republicano. **Ambiente e participantes**: As fontes de pesquisas são documentais, em grande maioria disponibilizadas no Repositório Digital da UFSC. **Resultados:** O exame documental indicou que as práticas manuais participam ativamente da cultura escolar e dinamizam alterações, a cada tempo. Os Trabalhos Manuais modificam seu *status* de matéria escolar para uma metodologia de ensino de geometria, destinada aos primeiros anos escolares, que observamos, até os dias de hoje. **Conclusões**: Conclui-se que a compreensão da perspectiva histórica contribui para refletir sobre os problemas educacionais da atualidade, no debate acerca de como mobilizar ferramentas para o ensino de geometria, problematizando a viabilidade ou não de os trabalhos manuais atuarem como uma metodologia apropriada para as primeiras explorações de propriedades geométricas.

Palavras-chave: Estado de São Paulo. Normativas curriculares. Manuais escolares. Trabalhos Manuais.

FIRST CONSIDERATIONS

> But the cultural vices that crushed our Sloyd have not gone away. Few understood that the hands offer a secret passageway to knowledge. One day, when we get out of this damned Covid, we will be able to think seriously about education. Would it be a chance to use our hands to boost our teaching? (Castro, 2021).

The previous extract was taken from the article “O curioso trajeto da faquinha sloyd” [The Curious Path of the Sloyd Knife], published in the newspaper *O Estado de S. Paulo*[1] on May 2, 2021, in which the researcher in education gives a brief report on the insertion of the sloyd tool[2] in the school curriculum in Sweden and comments on its appropriation for Brazilian teaching. In the end, Castro provokes us with the question: "Would it be a chance that we used our hands to boost our teaching?" Without intending to answer the question, this article presents research results in the area of the history of mathematics education that allow us to better reflect on that provocation. We assume that knowledge of the past, built on the basis of historiographical investigations, can bring arguments and subsidies to this discussion.

In this way, this article aims to walk through the various school programs in comparison with textbooks of the first years of schooling from the beginning of the Republic (1889) until reaching the present time, trying to capture evidence of emphasis or distancing of proposals that take handwork[3] as a determining element for geometry teaching in the state of São Paulo, Southeast Brazil.

A historical narrative that intends to build a representation of a long period of investigation, over a hundred years, necessarily needs to limit the sources examined. More than that, it demands the clarity of being a path that can be interconnected without any intention of representing the totality of Brazilian school realities. In addition, the proposed objective seems to be an ambitious task, given that there is a lack of further research to better understand the complexity of such relationships and curricular movements in Brazil. Even so, we take responsibility for the methodological procedures regarding the presentation and justification of the sources inventoried for the analysis, the process of organisation, systematisation, and comparison with the results of

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[1] Press newspaper with wide circulation in the state of São Paulo.

[2] The word slöjd, of Swedish origin, comes from the adjective "slog", which means dexterous, skillful, and aims to provide a method of teaching handwork that represents true "educational handwork". It consists of exercises that incite the triple intellectual, formal, and hands-on education without the educational goal of offering the child a trade. Several nomenclatures can be found for this word: sloyd, slöjd, and slöyd, among others (D'Ávila, 1967).

[3] Since the beginning of Brazilian primary schools (19th century) to the present day, we can observe different conceptions of the "use of hands" in primary education in São Paulo and Brazil. This article analyses the handicrafts that involve the making, the elaboration of objects and works that educationally provide handwork, not using the hands simply for observation.

research already completed[4], exposing arguments that support the challenge of producing a first historiographical narrative about the relationship between manual practices and proposals for teaching geometry in primary school.

As a starting point of our narrative, we take the Republic, as it was when the subject Handwork[5] was created, first in Rio de Janeiro, back then the Federal District of Brazil, in 1891. However, the proposal of a space in the school for hands-on tasks comes from long ago. Étienne Schmitt, an important name in the dissemination of handworks in the Scandinavian countries and Germany in 1888, exposed the teaching framework of this that would become a school subject in several countries of the world:

> All persons concerned with children's education recognise the usefulness of handwork in primary school; but when it comes to determining the objective and choosing the means to achieve the desired result, the agreement is not unanimous. Some see the manipulations of wood and iron only as direct preparation for a specific craft; its sole purpose is to teach even the elementary school child the profession of carpenter, locksmith, and fitter. Others, faced with an unknown future, seek to prepare children for all social positions that destiny may reserve for them; tend to make them acquire the various qualities they will need in the struggle of life: dexterity of the hand, the precision of the eye, the flexibility of the body, development of physical forces, the good direction of moral dispositions, and rational culture of intellectual faculties. (Schmitt, 1888, p. V, our translation)

The panorama of divergent positions on the insertion of handwork in schools was nothing new. D'Ávila (1967) points out that in 1524, Martin Luther was the first to discuss the movement for the schooling of handwork. He evidenced handwork as an element of human education, which should be

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[4] Particularly noteworthy are the research projects developed by the History of Mathematics Education Research Group (Grupo de Pesquisa de História da Educação Matemática - GHEMAT).

[5] The differentiation of the writing Handwork with initial capital letters will be used to designate the teaching subject for the primary school, which we will discuss below. The rubric handwork. with lowercase initial letters refers to all manual exercises without mentioning the subject. The same criteria will be adopted in the Geometry and geometry writings, as well as for Shapes and shapes.

related to the teaching of other subjects in the primary course, however, without the status of a school subject. Years later, other educators, philosophers, and education professionals[6] began to advocate different conceptions of the purpose of inserting handworks in primary school. However, the Swiss educator Johann Heinrich Pestalozzi's ideas changed the way of conceiving the schooling of handwork. Pestalozzi affirmed and applied his pedagogical guideline: "The heart develops by loving, the spirit by thinking, the hand by working" (D'Ávila, 1967, p. 192), to provide each child with the instruments of their autonomy, so that handwork would be at the heart of his teaching proposal.

Supported by Pestalozzi's ideas, in the mid-1800s, Froebel proposed creative games that sought to offer the maximum opportunities for child's education in the pre-primary course, of their gifts[7]. And based on the ideas of Pestalozzi and Froebel, other educators began to defend handwork at school. In 1866 in Finland, Ugo Cygneus established Handwork as an independent subject within the primary school programme, and in Sweden, in the 1870s, Otto Salomon proposed that the school subject begin to be considered as an educational instrument and no longer an income.

The true meaning of "educational handwork", under the name of sloyd, gained recognition in 1875, in studies done by Salomon at the Nääs Seminary, a handwork school in Sweden. From then on, in the mid-1880s, other countries such as Denmark, Germany, Switzerland, the United States of America, Italy, and France began to introduce Handwork as a mandatory subject in their primary courses (Revista Pedagógica, 1891, n. 13).

Brazil began to drink from Swedish innovations about 25 years later, in 1890, with the participation of Manoel José Pereira Frazão, a Brazilian primary teacher, in the summer course at the European centre for the education of teachers of Handwork (Nääs Seminary), based on a pedagogical trip to approach educational policies and get in touch with innovative and successful practices in Europe (Leme da Silva et al., 2021).

In the same period, with the first republican primary education programme, Handwork was introduced as a school subject in Brazil, opening the debate on manual practices and their relationship with the educational

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[6] Further detail can be obtained in Frizzarini (2018).

[7] Leme da Silva and Camara (2021) present a more detailed study on the appropriations of Froebel's method regarding mathematics teaching and its circulation in Rio de Janeiro, São Paulo, and Paraná.

process. In our study, we analysed and synthesised responses and reinterpretations elaborated by the school culture of primary education, current fundamental education – initial years – about manual practices and the teaching of geometry over more than 100 years, from the Republic to the present day.

This article is inserted in the history of mathematics education and, more specifically, in what is called the history of school subjects (Chervel, 1990), which has become an important area of study, whose potential lies in providing a renewed look at the school of the past and that goes beyond the history of ideas and pedagogical discourses. Chervel (1990) states that the school is a space for creation, rather than the reproduction of values and that the disciplines are produced within the school in its relationships with the school culture.

The history of school subjects presents a fundamental nucleus, school culture, understood as

> a set of norms that define knowledge to be taught and behaviours to be inculcated, and a set of practices that allow the transmission of this knowledge and the incorporation of these behaviours; norms and practices coordinated with purposes that may vary from time to time. (Julia, 2001, p.10)

In this study, we collate educational norms and indicative proposals for practices analysed in school textbooks aiming to understand better the relationship between the teaching of geometry and manual practices constructed and reconstructed by school culture.

To this end, we carried out a first selection of the various programmes that guided the pedagogical practices in São Paulo[8]. We analysed the main educational regulations of the state of São Paulo since 1894, and from the end of the 20th century, we examined the national-scope legislation - the National Curricular Parameters (PCN) and, more recently, the National Common Core Curriculum (BNCC). Our choice was due to the fact that many of the completed studies addressed that state, which was considered to be at the forefront at the beginning of the Republic, with the creation of school groups, a model that was later expanded to many other states (Souza, 2009). We selected once more in view of the long period of analysis. Thus, we examined how the content of

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[8] In Brazil, since the Republic (1889), the different Brazilian states were autonomous to elaborate their curriculum reforms.

plane and spatial geometric shapes was proposed in the different normative and didactic productions[9].

For our narrative, considering school curricula, we classified the period in three phases, namely: (1) the existence of Handwork in São Paulo curriculum (1894 to 1949/50); (2) the exclusion of Handwork from the São Paulo curriculum and the MMM (1969 to 1975) and (3) national curriculum documents (1998 to 2018). The textbooks selected in the first two phases certainly circulated in the state of São Paulo, with several editions, and in the third phase, books approved by the National Textbook Programme (PNLD), which also have wide circulation in the state of São Paulo.

HANDWORK IN SÃO PAULO PROGRAMMES (1894 TO 1949/50)

In this first phase, we analysed the seven programmes for primary education in the state of São Paulo[10], referring to the years 1894, 1905, 1918, 1921, 1925, 1934, and 1949/50, all of them destined to the model of "grupos escolares" (schools groups)[11].

The school subject Handwork was timidly introduced in the 1894 programme (Decree n. 248), with few delimitations but clear objectives: to contribute to the construction of the citizen and also to strengthen ties with

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[9] Many other selections could be made, for example, the interactions between the social context and the Handwork subject are equally pertinent. For example, social relations in the proposition of manual activity in schools, since Brazil, since the beginning of colonisation, understood handwork as "things of slaves" (CUNHA, 2000), culminating in the contempt and prejudice against the crafts in schools, which many believed to be a punishment. However, our option was to prioritise the school culture, especially educational norms and practices, expressed in the two sources of analysis.

[10] Primary education, in this period, constituted the first stage of Brazilian school education, in which children who attended schools were seven years old or older.

[11] The "grupos escolares" (schools groups) were created in the context of the reform of public education in 1893 from the gathering of isolated schools, grouped by their proximity. With the purpose of organising students by age and grade, homogenising them, and establishing a teacher for each grade, the "Grupos Escolares" reveal a new presentation of teaching programmes, organisation, and method, thus, increasingly detailed indications turn to the standardisation of school programmes (Souza, 2009).

geometry in constructions by hands. We identified interactions in the modelling, cartoning, folding, cutting, or weaving proposals in the "Shapes"[12] subject, with plane or spatial geometric figures, based on their observation to understand their shapes and properties. However, the regulations do not explain who should do such activities, teacher or student (Fizzarini, 2018).

In the following prescription, from 1905 (Decree n. 1281), the bond was maintained, however, without the "Shapes" subject. In Handwork, we identified the proposal for modelling geometric solids, the same ones that were being studied in geometry, which, in this matter, once again, had the sole purpose of observing properties and shapes. Building squares, rulers, and angle brackets also showed a close relationship with geometry, but also reinforced the real reason for the insertion of handwork in primary school, to encourage work, in this case, carpentry classes (Frizzarini, 2018).

The following São Paulo programme, from 1918 (Decree n. 2944), only in modelling and some carpentry constructions, Handwork approached Geometry. It seems that the relationship, more apparent in 1894, was now strained. Differentiated activities began to be added, such as notions of horticulture and gardening, and the modelling emphasised the usual objects compared to geometric solids.

Since its first edition, published in 1894, the book by Olavo Freire da Silva, entitled Primeiras Noções de Geometria Prática [First Notions of Practical Geometry], presented the outstanding characteristics of geometry teaching in Brazilian classrooms. As Leme da Silva (2021) points out, the textbook, with a wide circulation in São Paulo, highlighted its practical proposal, as stated in the title of the textbook, the construction of geometric figures with a ruler and compass. And just like in the programmes presented during this period, there was little articulation with handwork, only in specific exercises, but which, at the same time, showed their importance.

Figure 1 brings images from the chapter, which deals with the study of prisms and pyramids, illustrations of common objects that have the shape of those geometric solids and together with their flatness, highlighting the importance of this type of handwork. In the exercises, students were asked to make some solids, such as exercise 23 "Make a triangular, hexagonal,

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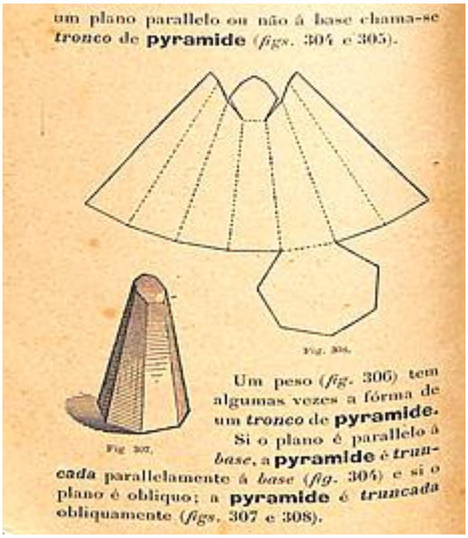
[12] The 1894 programme was structured in four years. The geometric contents were arranged in two distinct subjects: "Shapes", with discussions of plane and spatial geometric figures based on the study of geometric solids, and Geometry, which presented a more in-depth study of geometric properties.

quadrangular, pentagonal, octagonal pyramid out of cardboard" (Silva, 1907, p. 167), following the examples of planning, presented in the textbook, as shown in Figure 1.

Figure 1

Cartonnage by Olavo Freire. Silva (1907, p. 166)



And almost 100 years ago, in 1921, to eradicate illiteracy, primary courses in São Paulo, which previously had four years duration, were shortened to two years (Decree n. 3356). Although condensed, the proposal was very similar to the previous recommendations, with a close relationship between the study of geometric solids and their construction through the subject of Handwork. Even with such condensation, Handwork continued as a subject in the programme, denoting its relevance for teaching. The characteristic bias of promoting taste and love for work remained evident, mainly due to the differentiation of content in rural and urban schools. The former proposed teaching horticulture, arboriculture, and gardening, whereas the latter included weaving, cartoning, and modelling, characteristic of manufacturing functions.

And after a period of not-so-evident relationships, in the 1925 programme, we can notice explicit articulations of handwork and geometry.

The programme returns to the old four-year structure and expresses broad indications of how the contents should be taught in confluence with the intuitive teaching methodology[13] and, at the same time, expresses indications of the educational movement, of the new school pedagogy[14], by presenting at various times a relationship with the student's life, with the child's activity, with interaction outside of school.

In the 1925 programme, we observe the "return" of "Shape" subject, present only in the 1894 programme. In this subject (intended only for the first and second grades), the focus was on three-dimensional and two-dimensional figures, leading the student to build, observe, and manipulate objects that represented those figures. In geometry (intended for the final two grades), geometric properties had greater visibility and from them, students were invited to build figures with instruments, lines, and develop the properties learned, giving the subject a formal and conceptual characteristic.

But the biggest change comes with Handwork, which, despite being present in the four years of the programme, no longer had the constructions, models, and cartons that were included exclusively in this school subject. In "Shapes", the study of solids was proposed with modelling in clay or plastic mass, and then their faces were stamped on paper or drawn, to be later cut and folded, forming the geometric solid again in composition and decomposition of it. Thus, the first contacts with the shapes of plane and spatial figures were proposed through children's handworks, i.e., the "Shape" would fulfill the role of preparing students for the geometry of the final years.

In Handwork, such constructions were revisited and reinforced, since the objective was to "untangle the children's fingers and give them dexterity and manual skill" (Programa de Ensino para as Escolas Primárias de 1925, 1941a, p. 18). We observed the proposed modelling of solids, already studied

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[13] Based on the principles of Modern Pedagogy, the intuitive method of teaching has, in children's intuition, as the name itself says, the precept of teaching, in which the student would learn by observing things. Thus, the making of handworks itself expresses the intentionality of this teaching moment, in the construction of models, cuts, folds (Fizzarini, 2021).

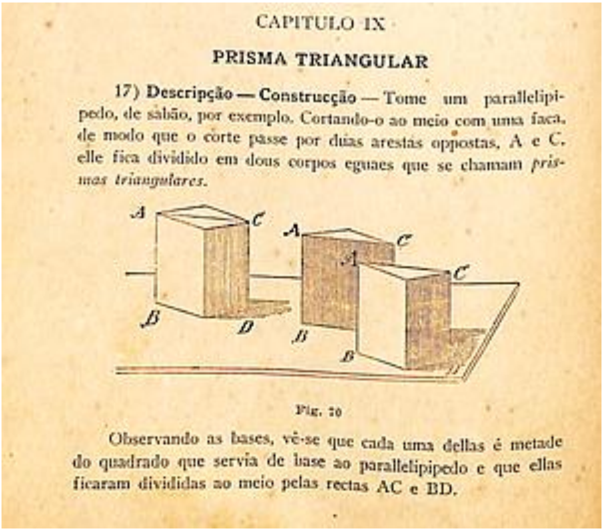
[14] Brazilian educational movement that gained strength in Brazil in the 1930s, bringing experimental psychology issues to teaching, involving tests to measure the difficulty of the content, and proposing an education closer to the child's reality. The student was placed at the centre of teaching, and different methodologies were put to the test by the teachers so that the child's interest was the protagonist of education (Monarcha, 2009, p. 69).

in the subject “Shapes”, and of usual objects that resembled their forms, in addition to folding squares, rectangles, and equilateral triangles, cutting out symmetrical figures, cartoning geometric solids, weaving, working with rope or twine and sloyd (which in this programme was understood as woodworking techniques).

Heitor Lyra da Silva's handbook[15], published in the 1920s, connected handcraft with geometry teaching in the constitution of the idea of a teaching methodology, in which the constructions (of modelling and cartoning, for example) would allow the student to recognise, observe, and identify the properties of the figures, as shown in Figure 2.

Figure 2

Modelling. (Silva, 1923, p. 40)


CAPITULO IX

PRISMA TRIANGULAR

17) **Descripção — Construcção** — Tome um parallelipipedo, de sabão, por exemplo. Cortando-o ao meio com uma faca, de modo que o corte passe por duas arestas oppostas, A e C, elle fica dividido em dous corpos eguaes que se chamam *prismas triangulares.*

Fig. 20

Observando as bases, vê-se que cada uma dellas é metade do quadrado que servia de base ao parallelipipedo e que ellas ficaram divididas ao meio pelas rectas AC e BD.

In Silva’s proposal (1923), making solids was accompanied by the critical study of their properties and elements, thus configuring handwork as a methodology for teaching geometry. For example, exercises that asked

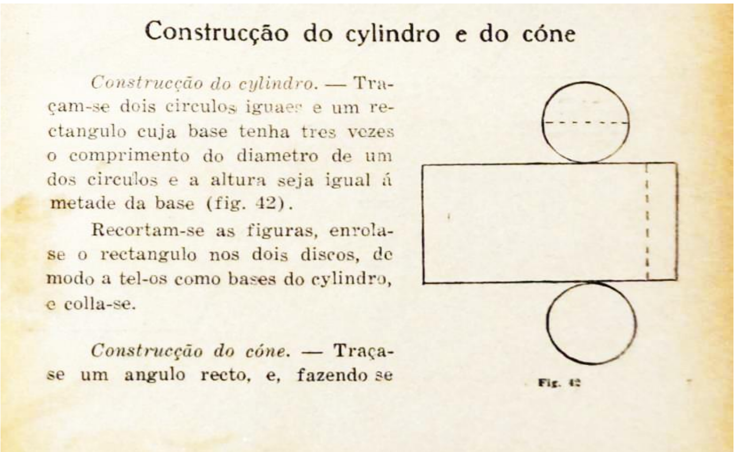
[15] The handbook indicated innovative methods of presenting content based on the idea of “concentric circles”, for more information see Leme da Silva (2021).

students: “How can you build two triangular prisms to form a cube? Are the side faces of a triangular prism parallel to each other? And the bases? And the edges?” (p. 42).

Those interactions of geometric contents with handworks could also be explicitly observed in the *Manual do Ensino Primário* [Primary School Manual], by Miguel Milano, published in 1938. The textbook did not present a chapter dedicated to the study of Handwork, but it is possible to identify constructions and manual practices in the subjects of “Shapes” and Geometry[16], which evidenced this relationship (Fizzarini, 2014), as shown in Figures 3:

Figure 3a

Proposal of tasks for the 1st grade. (Milano, 1938, p. 129)



Construcção do cylindro e do cóne

Construcção do cylindro. — Traçam-se dois circulos iguaes e um rectangulo cuja base tenha tres vezes o comprimento do diametro de um dos circulos e a altura seja igual á metade da base (fig. 42).

Recortam-se as figuras, enrola-se o rectangulo nos dois discos, de modo a tel-os como bases do cylindro, e colla-se.

Construcção do cóne. — Traça-se um angulo recto, e, fazendo se

Fig. 42

Mixtures of methodologies and proposals were still confused in the following programme, from 1934, known as the minimum programme and which was adopted with the forced reduction of school periods. Again, despite the reduction, Handwork kept its contents and delimitations almost in its entirety, because, as the text of the programme highlighted, “as the teacher should always give preference to works that students can perform with raw material easily found in school locality” (Programa de Ensino para as Escolas

[16] Which worked along the same lines as the 1925 programme.

Primárias, 1941b). We highlight the verb execute as a priority in the proposal; in other words, the making, the production of materials should be prioritised.

Figure 3b

Proposal of tasks for the 2nd grade. (Milano, 1937, p. 99).

— Se eu tivesse um serrote, serraria esta esphera ao meio. Não importa. Cortarei uma laranja. Façam de conta que ella seja a esphera.

Aqui está. Em quantas metades ficou dividida a esphera?

— A esphera ficou dividida em duas metades.

— Cada uma destas metades tem o nome de **hemispherio** (fig. 15-b). A palavra **hemispherio** quer dizer metade da esphera.

Como estão vendo, se eu assentar os **hemispherios** sobre a mesa, pelo côrte feito, elles não rolarão. Porque?

— Porque têm uma superficie **plana**, como a taboa da mesa.

— Quantas superficies tem, então, um hemispherio?

— Duas superficies: uma **curva** e outra **plana**.

— Muitissimo bem.

Both the 1925 and 1934 programmes consolidated the interactions between handwork and the teaching of geometry pointed out in previous programmes. The proposal of globalised education[17] gave rise to a change in the school's purpose of teaching handwork, which was very close to promoting a taste and love for work and, at this moment, showed that hands-on practices, previously restricted to Handwork (more technical), started to be developed mainly by the subject of "Shapes" to favour the teaching of plane and spatial geometric figures, as Fonseca (1929) pointed out:

> We see that, far from being a new subject, independent and alongside others, handwork is part of all of them, as a didactic means. Handwork is a methodology, the methodology *par excellence* of the active school (Escola Ativa), and represents, despite the materiality of working with one's hands, a mental task rather than a material task. (...) Mathematics is one of the

[17] Globalised teaching is a term popularised in the mid-1930s, and refers to attempts to develop teaching programmes based on centres of interest, advocating integration between different subjects and their contents (Souza, 2009).

> subjects that offer the greatest scope for handwork. In any handwork, there are always measurements and calculations to be made, so there is always geometry, arithmetic, and even algebra to be applied. Generally speaking, whatever the occasional didactic application they may have, handworks are always a course of applied mathematics, of executed mathematics. (pp. 26-36)

The didactic approach became more explicit with the programme of 1949/50 that followed, in which primary education received a new rubric: fundamental primary education, lasting five years. The Geometry subject (there is no subject “Shapes” in 1949/50) indicated “modelling, building, drawing, colouring, and cutting out geometric figures” (Act n. 17, 1949, p. 88), and Handwork reiterated the importance of these confections:

> Nothing is more suited to a child's impulsive and creative nature than handwork (...). Handwork was closely connected with the teaching of the other subjects in the programme, often taking place as an auxiliary means of that teaching. By cutting, collecting, modelling, and building on the sand tray, the child can make, in these concrete manifestations of ideas and feelings, more complete, clearer knowledge acquired in other classes, and fix it better. (Act n. 65, 1950, p. 22)

In 1949/50, Handwork was more “concerned” with promoting the globalisation of teaching, using the knowledge of hands-on work to implement the teaching of the different areas and subjects set out in the programme, as in the example of modelling, highlighted in the proposal to the 2nd grade, “Modelling: The teacher should awaken the child's attention to shapes and proportion. The works will be done on a clipboard, with wax, plastic putty, or clay and also on a sand board” (Act n. 65, 1950, p. 30).

Examining the set of programmes over almost the first 60 years of the Republic allows us to perceive the articulations between hands-on practices and proposals for teaching plane and spatial geometric figures. Starting with specific approximations in 1894, throughout the programmes, we identified continuities, deepening, and readjustments of purposes, so that in 1925 and 1934 we could verify effective signs of a marriage between Handwork and Geometry, which, in 1949/50, consolidated the insertion of handwork in the subject of Geometry, acting as a true teaching methodology.

The 1949/50 programme was the last in São Paulo to include Handwork, which represented a break from the curriculum point of view. On the other hand, it left marks and legacies[18]. Its proposals began to be incorporated for other purposes in other subjects, such as geometry or mathematics, as discussed below.

THE END OF HANDWORK AND THE MMM IN THE STATE OF SÃO PAULO (1969 TO 1981)

The decades from the 60s to the 80s of the 20th century comprise the movement of international scope known as the Modern Mathematics Movement (MMM). The movement was characterised as an articulation between advances in cognitive psychology and mathematics, organised through algebraic structures, having as an icon the inclusion of set theory in the school curriculum (Valente, 2013).

The Hungarian mathematician and psychologist Zoltan Dienes, one of those responsible for drawing up a proposal for teaching geometry in the early years, defended the beginning of the study of geometry through the notions of "inside" and "outside", openings", etc.". Children should be interested in the topological properties of space, borders, doors, spaces, and domains, without special attention to measurement (Valente, 2013).

We can infer that the significant change in the teaching of geometry in the first school years, to meet the MMM ideals, was the introduction of topological notions in the Brazilian curricula. However, the study by Valente (2013) concluded that "the MMM finds everyday life ready to incorporate new elements of geometry, without effectively abandoning Euclidean geometry. (...)

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[18] As explained earlier, many other approaches could have been directed to this research, for example, gender relations in which works exclusive to boys and others to girls would be evident, and as emphasised by Frizzarini (2018), those involving simple mathematical concepts could be done by girls, but modelling and complex work, which required abstract knowledge such as perspective and depth, were only aimed at boys. In addition, the change in perspective of the purpose of Handwork throughout its entire schooling makes explicit that this hands-on work could have its "death" in some spaces stopped at the expense of other teachings, such as the case of technology, which is presented and discussed in French researcher Lebeaume's (1994) study on handwork in France. However, our approach values establishing relationships with the teaching of geometry and handwork.

the rudiments of topology come to be seen, by all indications, as pre-geometry" (p.175).

The state of São Paulo played a leading role, in particular, through the creation of the Study Group on Mathematics Teaching (Grupo de Estudos do Ensino da Matemática - GEEM) in 1961[19]. The GEEM provided numerous contributions to consolidate the MMM in Brazil, such as the translation and production of didactic materials, conducting teacher education courses, and participating directly or indirectly in the state's curricular reforms. Four norms guided the curriculum of the period[20], in 1969, 1975, 1981, and 1986.

The objectives for teaching Geometry in the programme published in 1969, for the first two grades, were to create conditions for distinguishing figures in the plane from figures in space and identifying curves, polygons, quadrilaterals, and triangles. In the list of contents related to the study of geometric shapes, we identified: figures in space – recognise sphere, cylinder, and cube; figures on the plane – recognise square, rectangle, triangle, and circle. Curves – open, simple closed – recognise the inside and outside of a simple closed curve. For the final two series, the objectives were: to recognise plane figures and spatial figures, polygons; identify flat regions; recognise and name, among the figures of space – cube and parallelepiped, pyramid, cylinder, cone, and sphere. And for that, it was necessary to study: simple and not simple closed curves as a set of points – draw and identify. Polygons as simple closed curves, formed by line segments. Space figures as sets of points with plane faces (prisms) and no plane faces (cylinder, cone, and sphere).

Clearly, in all series, we can evidence the topological relationships proposed within the scope of the MMM in the 1969 regulation, as a differential compared to the 1949/50 programme. We highlight the verbs that appear in the regulations: identify, recognise and name plane and spatial geometric figures. The study process does not mention making plans, modelling, or representing figures through other manual activities.

The next regulation, the Curricular Guidelines, published in 1975, inaugurated a new phase in the organisation of teaching in São Paulo and Brazil. Law 5692/71 ended the segmentation of primary and secondary education,

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[19] A more in-depth study on the GEEM can be read in Lima and Passos (2008).

[20] França's (2007) thesis aimed to analyse the curriculum changes and teaching legislation directed to mathematics teaching in primary schools in São Paulo from 1960 to 1980. The selection of regulations considered this study.

creating the elementary education course (Ensino de 1.º Grau), which lasted eight years. Mathematics in the 1975 Guidelines was organised into four themes: I – Relationships and Functions, to acquire a language and concepts that constitute unifying elements of mathematics; II – Numerical fields; III – Equations and Inequalities, IV – Geometry. Figure 4 indicates the proposal for teaching geometric figures over the eight years of elementary school.

Figure 4

Tema IV Geometria – Guias Curriculares, 1975 [Theme IV Geometry – Curriculum Guidelines]. (França, 2007, p. 159)

	NÍVEL I				Nível II			
Conteúdo	1ª	2ª	3ª	4ª	5ª	6ª	7ª	8ª
1- Figuras geométricas								
A. Noções topológicas: interior, exterior e fronteira; regiões, conexidade.	X	X	X	X	X	X		
B. Noções projetivas: retas, intersecções, convexidade.			X	X	X	X		
C. Noções afins: paralelismo, semelhança.			X	X	X	X	X	
D. Noções euclidianas: distâncias, ângulos.			X	X	X	X	X	X

The study of geometric figures, as shown in Figure 4, reiterated Dienes' proposal to begin studies with topological notions. Topology became the guiding principle for teaching geometry in the early years. According to França's (2007) analysis, the authors of the Curriculum Guidelines recommended:

> the development of the contents proposed in a totally intuitive way, from the first grades to the last grade the construction of geometric knowledge proposed by the observation and exploration of the physical space, with the manipulation of convenient didactic materials. (p. 163)

The 1975 regulation also contained in the bibliography a significant list of materials[21] previously built for the manipulation and exploration of

[21] In the Space topic, the following materials are listed: Dienes Logical Blocks, Multibase Blocks, Colour Factors, Montessori Gold Material, Kosmos Geometric

mathematical and geometric concepts (França, 2007). The manipulation of objects should be carried out using ready-made materials, without the invitation to make them. The focus was on manipulating objects with pre-established didactic purposes, different even from the manipulation of geometric solids observed in 1894, which aimed at observation to understand their properties.

The *Subsídios para implementação do guia curricular de matemática* [Subsidies for the Implementation of the Mathematics Curriculum Guide] were published in 1981, with minor changes. The topological relations (open and closed curves, simple and not-simple, interior, exterior, and border, among others) remained more markedly in the first two series. From the third series onwards, previous proposals resumed how to recognise and classify solids, based on assembling and disassembling models, i.e., with indications of a proposal for handwork in the making of solids through flattening.

The regulations, briefly commented, indicated the reception of the insertion of topological notions, since 1969, as an alignment of the curricula of the state of São Paulo to the MMM. New contents were inserted, and topological notions entered primary education.

To better understand the spirit of the proposal that should subsidise the new concepts, we analysed extracts of activities proposed by the collection *Curso Moderno de matemática para o 1.º grau* [Modern Mathematics Course for elementary school], signed as GRUEMA[22]. The textbooks were organised by a sequence of tasks proposed to the students. For a summary, we elaborated Table 1, to indicate how the Geometry contents were distributed in the collection.

We highlight the non-presence of the study of plane geometric figures in the 1st grade, and space figures are only in the last three tasks of the textbook. In the other three grades, geometry was included in a set of consecutive tasks, always in the middle of the work. In the 2nd grade, we visualised an example of a task on the notions of open and closed, simple and non-simple curves; in the 3rd grade, tasks on parallel and concurrent lines and mathematical languages and a last one, as shown in Figure 5:

Material, Kugeli Material, Magneto – Super-Magneto, Cuisenaire Color Numbers, Perforated Plates and Pegs, Malmbero Polygons and Polyhedrons, Quadrimath, Geometric Solids, and Trimath.

[22] A detailed study on GRUEMA and the Collection can be read in Villela's thesis (2009).

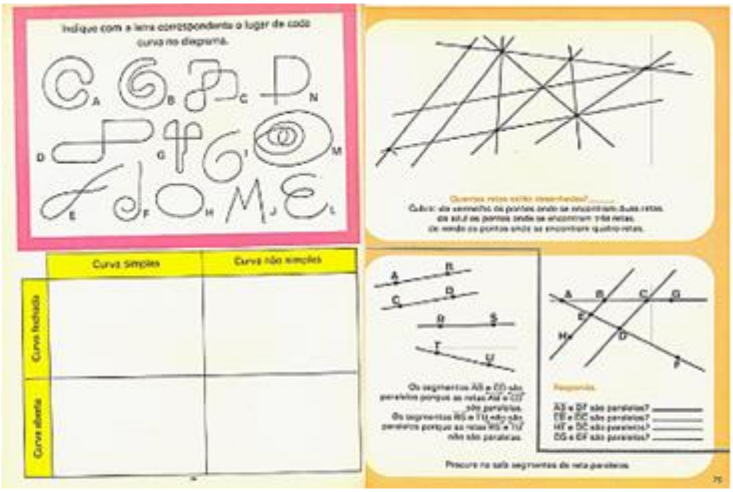
Table 1

Distribution of Geometry in the four volumes of the GRUEMA collection. (Adapted from Sanchez and Liberman, 1974)

Books	Presence of Geometry
1.ª Série [1st grade]	Spatial geometric figures in correspondence with children's everyday objects.
2.ª Série [2nd Grade]	Curves and regions.
3.ª Série [3rd Grade]	Point, Line segment, Polygons, Parallel line and competitors, Classification of quadrilaterals.
4.ª Série [4th Grade]	Congruent figures and angles, Parallelogram and trapezoid, Perpendicular lines, Rectangle, rhombus, and square, Isosceles, equilateral, and right triangle.

Figure 5

Examples of tasks in the GRUEMA Collection. (Liberman et al., 1974, p. 74 and Sanchez and Liberman, 1974, p. 75)



In 1985, the Secretary of State published a review document with two criticisms of the teaching of geometry in the first grades:

> - the topological notions of interior, exterior, border of a geometric figure suggested for the two series of level I, the open, closed, simple, not simple curves, did not give rise, in any way, to an interesting work; on the contrary, insisting on asking a child whether a dot, or even a mouse or a cat, is inside or outside a "curve" is so obvious as to perplex them;
>
> -in turn, the work based on three-dimensional objects and geometric solids, although not proposed by the Guidelines, appears in the Subsidies of 1st. the 4th. grades in a very interesting way and has been used by teachers; the same can be said of the work of classifying polygons and especially quadrilaterals, exploring the notions of parallelism and perpendicularity basic to the study of geometry. (Em debate uma proposta curricular, 1985, p. 9)

We can infer, at the close of the MMM period, after analysing São Paulo normative documents and the collection of textbooks, that the insertion of topological notions, particularly in the first two years, may have diverted the exploration of the properties of geometric figures, whether plane or spatial, manually. The focus of the studies shifted to topological relationships, involving more advanced and abstract mathematical concepts and languages, such as the difference between a straight line and a ray, the definition of a polygon before its exploration in material models. In addition, the dismissal of the Handwork subject may also have contributed to the almost absence of proposals for making and producing materials, despite the emphasis on the use of many ready-made teaching materials. Another aspect to be noted was the prioritisation of plane figures to the detriment of the articulation between plane and spatial figures.

The debate and criticisms pointed out in the 1985 document certainly had an effect on the *Proposta Curricular* [Curriculum Proposal], published in 1986[23]. Structured in three major themes: Numbers, Geometry, and Measurements, that proposal announced and brought changes to the theme of Geometry, especially for the first two grades, designated as the basic cycle:

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[23] The 1st edition of the proposal is from 1986; however, the analysed text refers to the 2nd edition, published in 1992.

- Identify the similarities and differences between objects and develop the perception of “shape” as an attribute of physical objects familiar to children.

- Represent and build objects in different ways.

- Classify figures according to different criteria. (Proposta curricular..., 1992, p.23)

Topological notions (simple and non-simple closed curves, interior and exterior, etc.) completely disappeared from the contents, compared to the 1975 program, which suggests that this approach was not well accepted by the school culture. On the other hand, the recognition and identification of properties of geometric figures resumed, focusing on aspects of physical objects familiar to the children's universe. We also emphasise the presence of the verb to build as an indication of handwork, but in a new context.

Some of the suggestions indicated by the 1986 proposal refer to carrying out experiments that promote the understanding of the shape of figures as an attribute. For this, it suggests the reproduction of objects using different materials (modelling clay, soap, clay, cardboard, sticks, geoplane, mirrors, games, Tangram, etc.) in situations of cutting, gluing, folding, and playing games, among others. Such proposals announce changes that are consolidated in the 1997 national guidelines.

NATIONAL REGULATIONS – PCN E BNCC (1997 A 2018)

In the 1990s, new educational guidelines were drawn up, in compliance with the 1988 Constitution, which determined that it was the duty of the State to establish a common national basis for basic education. In this context, in 1997 the National Curricular Parameters for Basic Education (PCN) were published, aimed at the first and second cycles of elementary education - currently from the 1st to the 5th grades[24].

[24] In 1998, the PCNs for the third and fourth cycles of elementary education were presented – currently from the 6th to the 9th grades. The PCNs for high school were presented in the year 2000.

The issue of curriculum organisation was placed at the centre of the debate as fundamental to qualifying school education in the country, and the role of mathematics was understood as essential

> in the formation of intellectual capacities, the structuring of thought, streamlining students' deductive reasoning, its application to problems, situations of everyday life and activities in the world of work, and supporting the construction of knowledge in other curricular areas. (PCN, 1997, p. 29)

Pires (2008) states that considering broader principles, the proposals presented in the PCN did not mean a total break with the proposals of the 1980s but brought some new elements by introducing research contributions to mathematics education[25].

Thus, the PCNs for Mathematics present new trends and conceptions in relation to the teaching and learning process. Problem solving came to be seen as a fundamental didactic strategy used by the teacher to trigger the understanding of mathematical concepts, ideas, and methods. In addition to problem solving, the history of Mathematics, the use of games and technology were also present in the document.

The PCNs for Mathematics for the first years of elementary school were structured in two cycles: the first referred to the 1st, 2nd, and 3rd grades; and the second to the 4th and 5th grades. The contents were organised in four blocks: “Numbers and operations”, “Space and Shape”, “Quantities and Measures”, and “Information Processing”. The teaching of geometry started to be contemplated in the “Space and Shape”[26] block under the justification that through geometric knowledge, the student develops a special type of thinking that allows them to understand, describe and represent their world in an organised way (PCN, 1997).

For the study of geometry, in the 1st cycle, they indicated the observation of geometric shapes present in natural elements and objects created by man, including their characteristics, besides establishing comparisons between objects of physical space and geometric objects. Furthermore, it was important for them to observe similarities and differences between geometric

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[25] Numerous references are mentioned in the PCNs, among which we cite authors such as Freudenthal, Piaget, Polya, Van Hiele, Vergnaud, and Durand.

[26] A historical study of the various designations for the study of geometry in the first years of schooling can be read in Frizzarini and Leme da Silva (2016).

shapes (two-dimensional and three-dimensional) and to construct and represent objects in different ways.

For the 2nd cycle, the idea was to observe the characteristics of three-dimensional and two-dimensional figures, identifying similarities and differences between them and exploring characteristics such as triangular rigidity, parallelism, and perpendicular sides, etc. The composition and decomposition of figures, the enlargement and reduction of figures through the use of meshes, the exploration of the flatness of three-dimensional figures, the identification of symmetries, the representation of geometric figures, and the perception of geometric elements in the forms of nature and artistic creations were also included in the indications of contents proposed by the PCNs.

The *Orientações didáticas* [Didactic Guidelines] point out that geometry activities could contribute to the development of visual estimation procedures without the use of drawing or measuring instruments, which could be done through activities with clippings, folding, mirrors, stacking, and modelling in clay or dough. They also recommended the construction of models and the description of what was represented in them and the use of *software*, saying that it was a way to help students with geometric reasoning (PCN, 1997).

The indication of hands-on practices can be observed in a task proposed in the textbook *Matemática - Pensar e Viver* [Mathematics - Think and Live], by Ana Maria Bueno and Antonieta Moreira Leite (2006) (Figure 6).

In the task, children were asked to model the shapes of the objects in the image, using play dough or clay, and then these shapes were identified with geometric shapes (sphere, cube, parallelepiped, and cylinder). According to the PCNs (1997), the space was presented to the child, initially, in a practical way. They would build spatial notions through the senses and movements. This should be the perceptive space, in which the knowledge of objects depends on contact with them, and which would allow a representative space when the child manages to evoke a certain object even in its absence. However, the point, the straight line, and the plane did not contemplate the perceptive and sensitive space, but the geometric space.

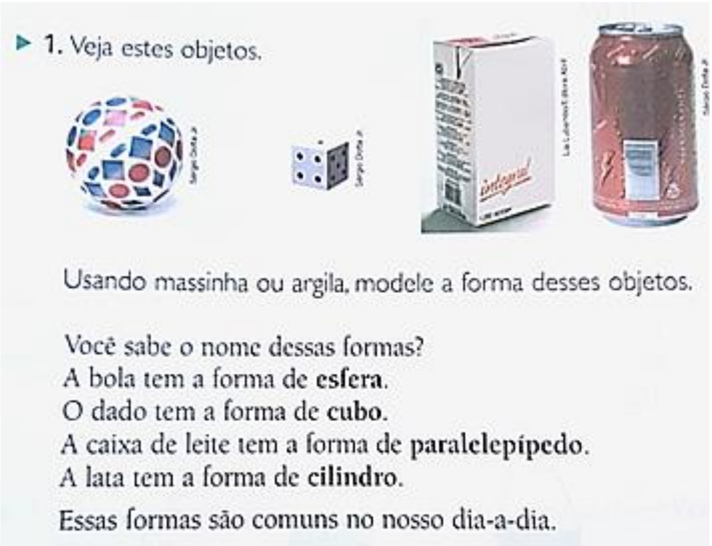
The discussion raised by the PCNs was that through experimentation, the child would be able to distance themselves from the sensorial or physical space and approach the geometric space, i.e., the world of abstraction.

> On the one hand, experimentation makes it possible to act, anticipate, see, and explain what is happening in the sensible space, and, on the other hand, it makes it possible to work on

> the representations of objects in geometric space and, thus, to detach oneself from the manipulation of real objects to reason about mental representations. (PCN, 1997, p.126)

Figure 6

Object modelling. Bueno and Leite (2006, p. 61)



For this, it was necessary to develop situations in which children, through experimentation, could move from concrete to abstract objects. In this context, Piaget (1975) helps us when he clarifies the movement of concrete and abstract:

> Mathematics, however, consists first and foremost in actions exerted on things, and the operations themselves are always actions too, but well coordinated with one another and simply imagined, rather than being performed materially. Without a doubt, it is essential to reach abstraction, and this is even absolutely natural in all fields during the mental development of adolescence; but abstraction will be reduced to a kind of deceit and deviation of the spirit if it does not constitute the culmination of an uninterrupted series of previous concrete

> actions. The true cause of the failures of formal education stems essentially from the fact that it starts with language (accompanied by drawings, fictional, or narrated actions, etc.) instead of doing it with real and material action. (p. 67)

For Piaget (1975), the concrete was essential in the development of knowledge during childhood, and the abstractions developed through the manipulation of objects occur due to the actions performed on them, therefore, being the basis for the following levels of knowledge.

In this sense, the objects that appear in space are the main elements of the work in the "Space and Shape" block, and the observation and construction of shapes through various experiences would lead the student to approach abstract geometric knowledge. Hands-on practices such as cutting, folding, and modelling, among others, were methodological resources suggested in the PCNs to assist in the construction of more abstract geometric knowledge.

In a more recent context, in 2018, the National Common Core Curriculum (BNCC)[27] was approved. In it, mathematical literacy was understood as a process that involves competencies and skills related to representation, communication, and argumentation that favour "the establishment of conjectures, the formulation and resolution of problems in a variety of contexts, using concepts, procedures, facts, and mathematical tools" (BNCC, 2018, p. 264).

For the Mathematics area, the document was organised into five thematic units: Numbers, Algebra, Geometry, Quantities and Measures, and Probability and Statistics. As privileged processes to develop mathematical literacy, for the teaching of Mathematics, the BNCC proposed Problem Solving, Investigation, Projects, and Modelling.[28]

When analysing the skills related to Geometry for the 1st and 2nd grades related to geometric shapes, we verified the need for children to identify, recognise, name, and compare geometric figures in drawings presented to them, and relate spatial geometric shapes to objects of everyday life in the physical world. In the 3rd grade, they must relate spatial geometric figures to their plans

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[27] The BNCC was organised in stages: Early Childhood Education, Elementary School, subdivided into initial years (1st to 5th grades) and final years (6th to 9th grades), and High School.

[28] Here, Modelling refers to a methodology for teaching Mathematics that is part of the research field in Mathematics Education.

and recognise congruent figures using overlapping and drawings in checkered or triangular meshes, also including the use of digital technologies.

For the 4th grade, the document indicated the use of folding, squares, or geometry software to recognise straight and non-straight angles in polygonal figures and the construction of congruent figures with the use of checkered meshes and geometry software. Finally, for the 5th grade, students must draw polygons using drawing materials or digital technologies and enlarge and reduce polygonal figures in checkered grids and, likewise, using digital technologies.

The use of geometry software, strongly indicated by the BNCC, are didactic resources that can contribute to creating, visualising, and manipulating geometric objects in a virtual environment (computer screens, tablets, or smartphones), favouring the exploration of geometric properties. Research has pointed out that by allowing the dynamic manipulation of geometric representations, the software can facilitate the learning of geometric concepts (Gravina, 2001). However, its use is just a means to assist learning. The teacher is the mediator of the process, being responsible for proposing activities that help construct geometric knowledge and meaningful learning.

As the BNCC document (2018) does not include specific didactic guidelines for the development of skills and competencies, it becomes difficult for us to understand the resources, strategies, and methodologies that should be used. Therefore, regarding our object of study, we found few ideas about handwork, except for folding and models, which are very evident.

The textbook collection *Matemática: Joamir* (2017), which has on the cover “Nos caminhos da BNCC” [On the paths of the BNCC], brings some situations that indicate the first interpretations of the BNCC. An example of a task presented by the author is the construction of a dice (Figure 7), indicated for the 3rd grade.

To carry out the proposed task, the student must build shapes from the outline drawing of the faces of a cubic shape. Then, he/she must cut out the drawn figures, compose the flat pattern and mount the die, indicating the die points on each face of the die.

Besides the intense manual activity carried out, there is a concern with identifying the plane shape – square – that forms the faces of the cube. And, in the teachers’ guidelines, the author indicates more configurations of cube unfoldings than the one shown in the task image and suggests that they draw them on the board. The author also suggested to expose, in the classrooms or in a specific place in the schoolyard, the cube representations obtained, sharing them with the other students.

Figure 7

Dice construction by cutting and folding – 3rd grade. (Souza, 2017, p.43)

This type of task refers to some existing elements in the Maker movement, originated in the United States in 2005. It concerns the various ways people gather in physical spaces and/or online to use digital and/or analog technologies to produce some artefact. The conceptions of Maker vary, but there is a consensus on two characteristics: building some kind of digital or physical artefact and sharing the manufacturing process of the created product with a community (Cohen et al., 2017).

Research by Paula et al. (2021) showed the growing influence of the *Maker* culture in Brazilian education. The "do it yourself" or "hands-on" culture encourages students to build, modify, and repair their own objects, which fosters creative, interactive, and proactive approach to learning. This type of approach seeks to make students take the leading role in their teaching and learning process, as it promotes creativity and explores varied knowledge from different areas of the curriculum. This trend may be influencing new approaches in teaching and learning mathematics, and, in our view, this deserves further research in mathematics education.

The analysis of the PCNs (1997) emphatically shows the importance of a child's experimentation, indicating that hands-on practices can help in the development of geometric knowledge. The BNCC indications (2018) restrict references to handwork, focusing their suggestions on digital technologies and, specifically, geometry software[29]. On the other hand, when analysing the textbook focusing on the BNCC, we could identify handwork practice for geometry teaching with a probable approximation of building with one's own hands based on the Maker culture, bringing evidence of handwork in classroom practices. However, this is a recent story in which more sources are being elaborated, which will help us understand in future research how handwork will be mobilised in school culture after the publication of the BNCC.

FINAL CONSIDERATIONS

We reached the end of this narrative that allowed us to look into the interrelationships and connections of handwork, understood as manual work that involved the making of plane and spatial geometric figures and which enabled the appropriation of these activities as a methodology for geometry teaching. Despite being particularised for the state of São Paulo, in specific periods, without intending to cover the reality of Brazil as a whole, the synthesis presented here brought clear indications and evidence that the school is not a place of stagnation.

Quite the contrary, the panorama allows us to reiterate the legacy of many historians, who, like Bloch (2001), express historical construction as a *continuum* and a perpetual change. In the same way that the understanding of school culture changes and is constituted in its own way, it cannot be acknowledged without the analysis of the relationships they maintain at each period of its history and the place of speech (Julia, 2001).

[29] In France, the movement towards the entry of technologies into schools in the mid-1980s is seen by Houssaye (2000) as the "death" of Hand Craft as a school subject, since knowledge of astronomy, physics, chemistry, geology, biology, and information technology gain the space that previously belonged to Hand Craft, under the name of Science and Technology.

At every historical moment, we could identify in the educational proposals both legacies from the past and innovations from the present. Understanding elements of the construction of school culture prepares us to better deal with today's educational problems. Because history is not linear, several questions from the past are revisited, reconfigured, to dialogue with the present.

The example of the insertion of Handwork as a social demand that met the context of the end of the 19th century, through a movement of international scope, indicated the relationships between school culture and society at that time. However, society has changed, and the school has developed its way of incorporating the proposal of hands-on teaching practices, altering over time and the needs inside and outside of the school, its *status* from a school subject to a teaching methodology at the service of other subjects in the early school years. We found, as Chervel (1990) points out, that the school is not limited to reproducing what is outside of it, but adapts, transforms, and creates knowledge and cultures specific to a given historical moment.

We have reached the 21st century, and other demands are currently challenging us. Returning to the opening quote, educator Castro highlights "cultural addictions" as responsible for the difficulty in understanding that "hands offer a secret passage to knowledge". The current moment invites us to reflect on past practices left aside in the face of so many discoveries in the 20th century, particularly new technologies. Perhaps it is time to resume practices and proposals from the past, such as "using your hands to boost teaching" (of geometry?), not to be reproduced but re-dimensioned based on the known experience of the past and immersed in new knowledge and technologies of the present.

Finally, we can also witness how the results of studies on the history of mathematics education can contribute to current discussions, in the debate on how to mobilise tools for teaching geometry. Then, wouldhandworking be an appropriate methodology for the first explorations of geometric properties?

The history of mathematics education does not present ready-made answers for the present, but it leverages reflections, and critical understandings of the complexity of educational movements and curricular reforms. It also reveals the exchanges and interactions between professionals from different disciplinary fields, movements that bring together a collective in favour of educational proposals, always aiming to bring answers to the complex task of developing curricula and programmes that dialogue with the school culture and that can be contemplated in the educational context.

The history of mathematics education also makes it possible to observe how the school culture responded to the different proposals, which were those that most closely touched the school culture, and those that were most quickly discarded by the school. It strengthens us with the knowledge that can be mobilised as arguments for the current debate.

AUTHORSHIP CONTRIBUTION STATEMENT

The first stage of the article was conducted by CRBF's study, the second phase, referring to the modern mathematics movement, by research by MCLS, and the third, more current phase, referred to the PCNs and BNCC, by AC's study. The three authors systematised the analysis, narratives, and final considerations from the conception of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study are openly available in the Digital Content Repository of the Research Group on History of Mathematics Education (Grupo de Pesquisa em História da Educação Matemática - Ghemat – Brazil) through the link https://repositorio.ufsc.br/xmlui/handle/123456789/1769 in a format that can be read and processed automatically by a computer.

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