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The City Has Abandoned Geometry: The Countryside Has Not! Reflections on Geometry and Its Teaching from the Perspective of Countryside Education

Filipe Santos Fernandes [a] ORCID iD (0000-0002-2964-3582)
Flávia Cristina Duarte Pôssas Grossi [b] ORCID iD (0000-0001-5340-1308)
Maria de Fátima Almeida Martins [a] ORCID iD (0000-0001-9244-3404)

[a] Universidade Federal de Minas Gerais, Faculdade de Educação, Departamento de Métodos e Técnicas de Ensino, Belo Horizonte, MG, Brasil
[b] Instituto Federal de Educação Ciência e Tecnologia de Minas Gerais, Departamento de Matemática, Ouro Preto, MG, Brasil

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ABSTRACT

Background: On the one hand, the history of mathematics teaching in Brazil is marked by the gradual abandonment of Geometry teaching; on the other hand, the countryside has always assumed it as an essential ally in the political and identity affirmation of its populations. **Objectives:** This paper discusses how Geometry and its teaching can emerge, in the paradigm of Countryside Education, as a possibility of political-epistemic disobedience to a Mathematics Education referenced in the knowledge, procedures, attitudes, and values of urban, industrial, and market forms of life. **Design:** The research uses qualitative methodology with a focus on Countryside Education. **Setting and participants:** The investigation was developed in the context of an Undergraduate Teaching Degree in Countryside Education, based on records of students' activities. **Data collection and analysis:** The text mobilizes formative experiences of a degree in Countryside Education with emphasis on Mathematics to reflect on how Geometry can be articulated to rural peoples' territories and territorialities, aiming to contribute to pedagogical guidelines for teaching K-12 Mathematics. **Results:** The paper contributes to evaluating epistemological and educational positions regarding Geometry and geometric knowledge commonly established by school cultures, not necessarily limited to schooling in rural regions. **Conclusions:** Geometry in school cultures can be used to understand the social reality in which subjects, communities and collectives are inserted, mapping inequality relations, and proposing ways to overcome them.

Keywords: Geometry Teaching; Territory; territoriality; Sociocultural Practices; Rural.

Corresponding author: Flávia Cristina Duarte Pôssas Grossi. Email: fcdpossas@gmail.com

A cidade "abandonou" a geometria; o campo, não! Reflexões sobre a geometria e o seu ensino na perspectiva da Educação do Campo

RESUMO

Contexto: Se, por um lado, a história (urbana) do ensino de matemática no Brasil é marcada pelo gradual "abandono" do ensino da Geometria; por outro, o campo sempre a assumiu como importante aliada na afirmação política e identitária de suas populações. **Objetivos:** Este texto discute como a Geometria e o seu ensino podem surgir, no paradigma da Educação Campo, como possibilidade de desobediência político-epistêmica a uma Educação Matemática referenciada em conhecimentos, procedimentos, atitudes e valores das formas de vida urbana, industrial e mercadológica. **Design:** Utilizou-se uma abordagem qualitativa, centrada nos processos educativos e formativos junto à Educação do Campo. **Cenário e participantes:** A investigação se desenvolve no contexto de uma Licenciatura em Educação do Campo e a partir de trabalhos desenvolvidos pelos estudantes. **Coleta de dados e análise:** Mobilizam-se experiências formativas de estudantes do curso de Licenciatura em Educação do Campo com habilitação em Matemática para refletir sobre como a Geometria pode se articular aos territórios e às territorialidades dos povos do campo visando contribuir com orientações curriculares e didático-pedagógicas para o ensino de Matemática na Educação Básica. **Resultados:** O texto contribui com a avaliação de posições epistemológicas e educacionais a respeito da Geometria e dos conhecimentos geométricos estabelecidos pelas culturas escolas de um modo geral, não se limitando, necessariamente, à escolarização em regiões rurais. **Conclusões:** A Geometria presente nas culturas escolares pode ser mobilizada para a compreensão das realidades em que os sujeitos, as comunidades e os coletivos estão inseridos, mapeando relações de desigualdade e propondo caminhos para superá-las.

Palavras-chave: Ensino de Geometria; Território; Territorialidade; Práticas Socioculturais; Rural.

INTRODUCTION

One of Brazil's most disseminated narratives about the history of Mathematics teaching is the gradual "abandonment" of geometry teaching, mainly in the first half of the 20th century. Regina Pavanello (1993) pointed out that this "abandonment" took place through the incorporation of educational ideas and discourses that granted schools the task of qualifying urban workers. Therefore, even in non-vocational schools, education prioritized the logic of emerging industrialization and, until now, was connected to the financial market and neoliberal perspectives.

As some recent studies show, the discussion on a possible "abandonment" of Geometry teaching portrays a reductionist view of geometry

but not necessarily its absence in school culture. In this sense, we can say that the proposal of teaching Geometry – or, at least, teaching Geometry with no concern with the concepts, procedures, and the specific languages of Algebra or Set Theory – was not central or highly relevant in the educational project to urban settings, industry, and the international market. Thus, any evaluation of Geometry teaching should be connected to the understanding of political, social, and economic contexts that, on several levels, are undertaken by education through governmental measures, revealing the conditions and opportunities of different segments of Brazilian people.

This text is situated in these contexts that move Geometry teaching in Brazilian education. We start from the understanding that a political, social, and economic project that privileges the urban-industrial-market sector ignores or denies territories as spaces that produce school knowledge. In this scenario, Mathematics teaching – and, in particular, Geometry teaching – is one exclusion strategy. Hence, the need to invest in reflections and practices that revise the presence of this curricular component aiming to promote identity policies not restricted to urban, industrial, and market life forms but that consider rural peoples part of Brazilian society.

Thinking about Geometry and its teaching in the context of Countryside Education is a way to promote this investment. The qualification of the countryside that composes the name Countryside Education is related to policies of knowledge and identity and the historical processes lived by sociocultural groups in rural spaces. The countryside is not characterized as a non-urban perimeter. It is understood as a "space of possibilities that boosts the connection of human beings with the production of conditions for social existence and the endeavors of human society" (Brasil, 2001, p. 1). In this sense,

> Countryside Education refers to a multidimensional life space requiring broader educational policies and proposals. The diversity of what we call Countryside Education makes explicit the differences related to political, economic, ethical, and moral aspects. These specificities should be analyzed and considered when we organize our pedagogical activities. (Monteiro, 2014, p. 15)

Among the specificities of Mathematics teaching, the objects of knowledge connected to Geometry were broadly taken as favorers of the organization of pedagogical activities in the rural context. The geometric representations connected to the art and work of rural peoples, as well as the types of organization and experience of the space by rural workers, riverside

population, extractivism groups, and other countryside groups are gradually incorporated into educational guideless to Mathematics teaching in Countryside education. They are present in general curriculum regulations and teaching proposals targeting countryside schools. Thus, Geometry and its teaching emerge as a possibility of political-epistemic disobedience toward a Mathematics Education grounded on the knowledge, procedures, attitudes, and values of urban, industrial, and market life. If, on the one hand, the city has abandoned geometry, on the other, the countryside has always considered it an essential ally in its populations' political and identity assertion.

However, the studies that grant this task to Mathematics Education are recent. Therefore, we use formative experiences in an Undergraduate Teaching Degree in Countryside Education with Mathematics Emphasis to reflect on how Geometry can be articulated in the countryside territories to contribute to the pedagogical guidelines of K-12 Mathematics, not necessarily limited to countryside schooling.

This text is divided into three sections. First, we discuss how the concepts of territory and territoriality allow us to think of Geometry and its teaching from the perspective of Countryside Education. In the second section, we present the Undergraduate Teaching Degree in Countryside Education at *Universidade Federal de Minas Gerais* (UFMG), where the formative experiences discussed in section three emerged. Finally, we present three reflections about Geometry and its teaching from the perspective of Countryside Education.

SPACE, TERRITORY, AND GEOMETRY

In contexts of Mathematics knowledge production and circulation, at school or otherwise, Geometry is recurrently defined as a branch of Mathematics that deals with the study of space and its properties. The displacement of its functional role marks its history - as seen in the reports about ancient Egypt when the solution of problems related to measures stood out - to a unique axiomatic approach in Greek culture and Western Europe – according to the same reports. Thus, we can consider that, in traditional reports, “the Greek culture was marked by a division between theoretical and practical knowledge, and Plato’s thought is frequently invoked as proof that the Greek man perceived Mathematics as a superior knowledge to common sense” (Roque, 2012, p. 174).

Space is a theme of Western philosophy, mobilized to discuss knowledge, particularly Geometry. As Tatiana Roque (2020) highlights, the notion of abstract space in Plato's thought decisively builds a way of thinking about Geometry, but the hierarch that arises from the ways of understanding and practicing knowledge. According to the author, Geometry is the primary example used by Plato to approach the so-called hypothetical sciences, and "one of their distinctive traces is the use of visible forms with the sole end of investigating the absolute they enclose" (Roque, 2012, p. 117).

Nowadays, though not denying the possibility of a concrete dimension, we consider that geometric knowledge deals with abstract shapes, prevailing the idea that "the proof of a geometric truth can use sensitive shapes, such as drawings, only as auxiliaries" (Roque, 2012, p. 117). Thus, geometry is described by its approximation to ideals, abstractions, and, in algebraic geometry, generalizations.

> A square is not the square we draw on paper: it is an abstract shape, the "squared" shape. The basic geometric objects- such as the point, the line, and the plane- are also not concrete. The point is something with no dimension, which does not exist in reality. So, these objects can only be conceived through abstraction. […] When a geometry scholar researches the properties of a square by drawing in on the blackboard- a copy of the ideal square -it is the true square they intend to simulate and not a mere copy. The truths of the idea can only be seen through the eyes of thought, and by seeking the soul, we have to use the first principles, resulting in their consequences. (Roque, 2012, p. 117)

Adlai Detoni (2012, p. 188) highlights that "this ideal space has forged the tradition of considering it as an attribute of the human reason, therefore, as preexisting the physical word", grounded on Western science and educational environments. According to this author, besides "making us abdicate from the perception of space we have by being in the word" (p. 189), there is a clear option to study Geometry privileging a base that understands ideals, which reflects an understanding of space that follows the Greek tradition valued by West Europe.

In this sense, a group of studies revisits the notion of space used in school, rethinking the concepts, procedures, attitudes, values, and other dimensions that compose Geometry in school cultures.

In Countryside Education, beyond the discussion on space, the concepts of *territory* and *territoriality* can be discussed in Geometry and its teaching. This approximation aims to understand education concerning the construction of policies, knowledge, and identity that, in the educational ideals and discourses, establish the rural field as territory:

> The construction of the concept of field as territory, where several types of rural workers' organizations take place, and the organizational shape of capitalist agriculture, called agribusiness. The territorial meaning is broader than the sectorial meaning that understands the field merely as a space to produce goods. Considering the field as territory means understanding it as a living space or a geographic space in which all dimensions of human existence occur. The concept of 'field' as a living space is multidimensional and allows broader understandings and policies than the concept of countryside or rural, only as a space of goods production. The economy is not a totality; it is a dimension of the territory. (Fernandes, 2006, p. 2)

Therefore, the field as a territory emerges from the collective actions of rural groups and aims to construct inclusion strategies. These actions are ‘territorialized’ and ‘territorializing’, as they are grounded on "symbolic, cognitive mediations and practices between the materiality of places and social actions in the processes of territorial transformation and local development" (Dematteis, 2009, p. 35).

This notion of territory affects the ways of thinking about Geometry in school cultures, mainly in school education in the countryside. The relationship between materiality and social action reflects on an education that recognizes the histories and types of knowledge of specific groups, such as rural workers, and that brings to Geometry teaching the concepts, procedures, and languages that show the struggles for political and cultural identifications. Hence, this is related to the creation of strategies to revisit the logic of production and identity assimilation demanded and promoted by the hegemonic knowledge, which considers the field only as a sector of the economy and shapes the educational processes in school and society.

To align education through Geometry in the territory means to grant “the people who live there the awareness of their participation, provoking the feeling of territoriality that, subjectively, creates a confraternization awareness among them” (Andrade, 1995, p. 20).

In this sense, territoriality expresses the subjects' actions when constructing and working in the space. It marks the subjective process of raising the population's awareness as part of the territory regarding the "set of practices and knowledge of men concerning the material reality, the sum of the relations established by a subject with the territory (the exteriority) and the other subjects (the alterity)." (Dematteis, 2009, p. 34). To Milton Santos (1996, p. 168), territoriality is "in each moment, [and] there is always a mosaic of subspaces, completely covering the Earth's surface and whose drawing is provided by history: the space is no longer a geometric notion but conditioned by the time".

A challenge placed by Geometry and its teaching in their articulation with the notions of territory and territoriality is the selection of knowledge objects considered geometric. In many cases, the identity policies of rural groups are marked by their work practices – rural workers, *geraizeiros*, *vazanteiros*[1], extractivists, etc. – and, consequently, their understanding and appropriation of space pass by situations not limited to the curricular Geometry. The typical ways of knowing, doing, and being in these work environments offer knowledge, procedures, attitudes, and values that point out how Geometry and its teaching can articulate the experiences of specific social groups, as in the case of rural peoples.

An example is a separation, in the BNCC (National Curriculum Framework) and other documents, of the theme unities about Geometry and Magnitudes and Measurements. As we know, the approach to Geometry in school is commonly made by quantifications and measurements. A numeric obsession with little dialogue with the sensorial properties of these elements establishes the properties of these elements and their relations with life. Thus, from the didactic-pedagogical point of view, the set of knowledge objects referring to such unities favor and suggest, when separated, the need to discuss Geometry in school without restricting it to metrics.

Mainly in Countryside Education, there is an escalation of this scenario when geometric elements are mobilized to create rural mathematic problems "in which 'candies' are exchanged for ' corn sacks', bringing an agricultural context to the standard of already-established school standards" (Oliveira, 2010, p. 306). These problems in Geometry teaching express not only an approach to its concepts and procedures exclusively through the quantification and

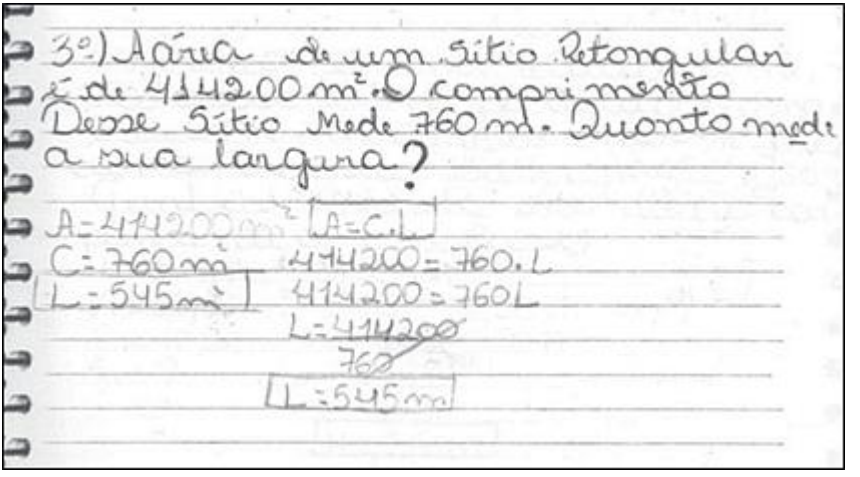
[1] Translation note (T.N.): *gerazeiro* refers to the people living in the grassland regions in the North of Minas Gerais. *Vazanteiros* are the people who plant and harvest by following the cycle of the rivers, in this case, the São Francisco River in Minas Gerais.

measurements but also neglect the forms of specialization and measurement that rural groups have developed and still develop.

The work of Aldinete Lima (2014), which followed teachers in the Dry and Hinder lands of Pernambuco (Brazil), brings an interesting example in this direction. In a student notebook analyzed by the researcher, there is the following activity (Figure 1)

Figure 1

Activity in a student's notebook. (Lima, 2014)[2]



In the activity, we can notice the reference to the rural area by the word 'field'. Lima (2014, p. 106) argues that "the fact that the statement brings a term

[2] Translation from Portuguese: The surface of a triangular field is 414,200 m². The length of the field measures 760 m. What is its width?

S = 414200 m²
L = 760 m²
W = 545 m²

S = L . W
414200 = 760 . W
414200 = 760W
$W = \frac{414200}{760}$
L = 545 m

related to the countryside might seem that the exercise integrates teaching to the students' reality". However, the procedures used to solve the problem reinforce metric Geometry, in which the measure of the surface (indicated by S) is equal to the product of the length (L) by the width (W) of the rectangular surface. The rural field spatial organization and the different forms of calculating the surface area in this context disappear for the sake of a general rule that favors little (or not at all) the discussion on the rural way of life or the context of the geometric element in question, connected to the shape of rural properties and the possibilities of measuring their surface.

Hence, approaching Geometry from the perspective of the territory together with Countryside Education demands thinking about the frontiers established within Mathematic itself as a knowledge area. This means assuming that limiting particular knowledge objects to specific theme units might limit the possibilities of a pedagogical work that considers rural peoples' ways of living as a principle and as a means. Thus, we defend that revisiting the policies of knowledge that establish Geometry in school cultures is a critical path to institute identity policies immersed in different territorialities. This perspective contributes to the struggle for a political and cultural identity that resists the logic of assimilation produced by the hegemonic knowledge and promoted in educational processes in the social and school environments.

THE UNDERGRADUATE TEACHING DEGREE IN COUNTRYSIDE EDUCATION (LECAMPO) AT UFMG: BRIEF HISTORY AND THE PRESENCE OF GEOMETRY IN MATHEMATICS TEACHERS' EDUCATION

The commitment of UFMG to the education of countryside groups is not recent. In 2005, UFMG held the first experience of a specific teaching degree in the context of Countryside Education through an undergraduate degree entitled 'Basic Education in Countryside Education: Pedagogy of Land'. The university partnered with the Ministry of Agriculture Development, the Landless Movement, the *Via Campesina* movement, and the ICRA (National Institute of Colonization and Agrarian Reform). The degree aimed to train teachers to work in Settlement Projects created by Proenera (National Program of Education in Agrarian Reform).

In 2007, MEC (Ministry of Education) created the *Programa de Apoio à Formação Superior em Licenciatura em Educação do Campo* (Procampo), which aimed to support the implementation of regular undergraduate teaching

degrees in Countryside Education. By MEC's invitation, UFMG, *Universidade de Brasília* (UnB), *Universidade Federal de Sergipe* (UFS), and the *Universidade Federal da Bahia* (UFBA) first offered these degrees, standing out in the national scenario of building a different teacher training for the rural context through Procampo. In 2007, the Teaching Degree in Countryside Education became a regular degree in UFMG, approved in the context of REUNI (Support Program to Restructure and Expand Federal Universities).

With a regular class offer since 2009, the degree aims to train pre-service teachers in Countryside Education to act in different areas of knowledge in Middle and High School. Four emphases currently structure these areas: Sciences of Life and Nature; Social Sciences and Humanities; language, Arts, and Literature; and Mathematics. We highlight that the training of Mathematics teachers has a specific emphasis, articulating the knowledge area to the curricular component, creating specific possibilities and challenges, as discussed by Fernandes (2019).

LECAMPO, sharing aspects of the political-educational project of other institutions in the country, is structured around three dimensions: 1. the protagonism of the subjects and their contexts; 2. the formation by knowledge area; and 3. the alternate organization of times and spaces.

According to Antunes-Rocha (2010), the protagonism of the subjects and their contexts of life production has two directions. First, the participation in the elaboration, execution and evaluation of the formative process. Second, the effective presence of the types of knowledge and practices created in the production and reproduction of life in the countryside as the structuring axis of their education and training. To guarantee such protagonism, the degree proposes a set of academic activities based on the dialogue between different agents, such as students, teachers, inhabitants of rural communities, members of social movements, governmental, non-governmental, unions, and religious groups.

The training by knowledge areas aims to break away or stretch out the disciplinary frontiers that guide knowledge production, organization, and circulation and problematize the spaces occupied by curricular components in countryside school education (Fernandes, 2019). According to Molina (2015, p. 153):

> the training by knowledge areas should develop with the broader intention of promoting strategies that can contribute to overcoming knowledge fragmentation, creating, proposing,

> and promoting teachers' interdisciplinarily-articulated actions, intrinsically associated with school transformations, and articulated to the demands of the rural community in the school. The proposal and the challenge are to materialize formative practices during the Teaching Degree in Countryside Education that can develop and promote in future educators the necessary abilities to contribute to the consolidation of an ideal school built by this educational movement headed by rural workers […].

From this consideration, focusing on the training of Mathematics teachers and questioning the prevalence of education centered on the disciplinary perspective, Fernandes (2019, p. 31) defends that:

> in Mathematics, considering it as a knowledge area supposes breaking away from the universalizing dimensions that delimitate its disciplinary contour, turning it into a human production, social-culturally situated and that, especially in rural situations, dissipates the characteristics of infallibility, rigor, and precision, and a powerful instrument of the modern world that were historically attributed to them. At the limit, these ruptures propose new dimensions that draw curriculum elements that involve typical ways of knowing, making, and fundamentally being.

The organization of the times and spaces of the degree are in alternance, articulating the School Time (ST) when the students are gathered at UFMG, and the study load is concentrated, and the Community Time (CT), when the students meet in their communities and the study load matches the working times of these places.

The alternance education follows the principle that the students' territories are also teacher-training agents, actively participating in their educational processes, overcoming the idea of the University as the privileged *locus* of training. An achievement was the inclusion of the Alternance education format in the General Undergraduate Rule of UFMG, besides the in-person and online education formats, granting institutional recognition and guarantees for the organization of this course.

In the Mathematics emphasis, the curriculum construction is a challenge given by the need to conjugate the conditions and expectations of training connected to the fight in favor of rural work life and the demands of

professionally teaching Mathematics in K-12 schools, marked by the disciplinary character prescribed in K-12 curricula. For this reason, the knowledge of the countryside and the scientific-academic ones are not opposed in this training process. They participate together to understand and value the knowledge from their sociocultural practices. However, it also recognizes the access to historically built knowledge disseminated by the capitalist, urban, industrial, scientific, and technological society in which our school inevitably participates.

In curriculum terms, different subjects include discussion on Geometry and Geometry teaching, as can be seen in the table below, which indicates the name of the subject, the syllabus, the study load (SL) measured in hours, distributed into School Time (ST) and Community Time (CT).

Table 1

Subjects that include discussions on Geometry and its teaching in LECAMPO/UFMG.

Subject	Syllabus	SL(ST)	SL (CT)
Space organization I: the study of shapes	Geometry: history and relations with sociocultural practices. The teaching of geometry and the educational project of social groups. Observation and organization of shapes. Functional and aesthetical approach. Study of shapes: classification, occurrence, and appreciation. Congruence, symmetries, and similarities.	40	20
Space organization II: shapes and measurements in a plane	Plane geometry: history and relations with sociocultural practices. The geometric shapes in the plane, their classification and properties. Congruence, symmetries, and similarities. Metric relations in the triangle and circle. Length and area: concepts and relations with field practices. Didactic strategies and resources in plane geometry teaching.	40	20

Space organization III: shapes and measurements in solid geometry	Solid Geometry: history and relations with sociocultural practices. The geometric shapes in space, their classification and properties. The problem with the plane. Metric relations in parallelepipeds, pyramids, cylinders, cones, and spheres. Volume and capacity: concepts and relations with field practices. Didactic strategies and resources in solid geometry teaching.	40	20
Space organization IV: analytical geometry	Analytical geometry as a possibility of modeling phenomena. Cartesian graphics: elements and possibilities. Analytical geometry in a surface: straight lines, circumferences, and distances. Didactic strategies and resources in analytical geometry teaching.	40	20
Study of the proportion and metric relations	The problem of measure. Magnitudes, unities, and measurement devices. Metric systems. Proportional magnitudes. Reason and proportion. Situations and procedures to measure in the field. Didactic strategies and resources in teaching proportional relations and measurements.	40	20

We should observe two crucial points when analyzing the syllabus of the subjects in the table above. First, the curriculum organization of the Teaching Undergraduate Degree in Countryside Education of UFMG emphasizes teachers' professional development focusing on school Mathematics, approaching processes, resources, representations, and criteria related to what we call Mathematics in school cultures. In this sense, we can observe the disposition of concepts, procedures, and languages characterized as

geometric from the criteria present in the K-12 curricula and not according to the delineations of Geometry developed in scientific-academic spaces.

The second point, more relevant to this text, refers to the guidance for a work involving Geometry's presence in history, sociocultural practices, and educational projects in different social groups, mainly rural workers. Such guidance corroborates Kátia Gonçalves (2014) that, in the scope of countryside schools, advocates for the need to deconstruct the disciplinary and compartmentalized way of thinking Mathematics and consider in the pedagogical practice the "actions and relationships that people and groups keep with each other that are reflected in rules of living, maintaining, and transforming society" (p. 30). Here, we indicate proposals of Geometry teaching guided by and developed with the traditions of rural peoples in sociocultural practices and experiences in and by communities where the educational space is located.

Besides the subjects, some Undergraduate Dissertations contribute to reflecting on Geometry and its teaching from the perspective of Countryside Education. These are academic products aiming to express dimensions of the students' formative process and the development of their production, whose contents generally describe, discuss, and propose questions and actions, dialoguing with processual, historical, and conceptual references in Countryside Education.

In these cases, as discussed in the following section, the problems emerging in rural territories allow the reevaluation of epistemological and educational positions regarding Geometry and the geometric knowledge established by school cultures.

We point out that the works used in this text, given their nature and internal regulations of the places they were produced, do not need the previous ethical evaluation by the Councils connected to the university. However, we considered them while developing this work. Thus, the authors exempt *Acta Scientiae* from any resulting consequences and possible compensation for any damage to research participants, according to Resolution nº. 510, from April 07, 2016, of *Conselho Nacional de Saúde do Brasil*.

SUBJECTS AS ARTICULATORS BETWEEN GEOMETRY AND THE COUNTRYSIDE TERRITORY

We can perceive in the syllabus of the subjects presented in the previous section the concern to articulate the discussions about Geometry and the sociocultural practices experienced and headed by rural workers in their territories. In this sense, the syllabuses propose a dialogue between the contents involving ideas, rules, procedures, definitions, representations and/or languages associated with what we recognize as school geometry and rural territory. Hence, they aim to politicize the teaching of mathematical concepts and contribute to constructing a social and emancipatory project of Mathematical education in, from, and for the Countryside.

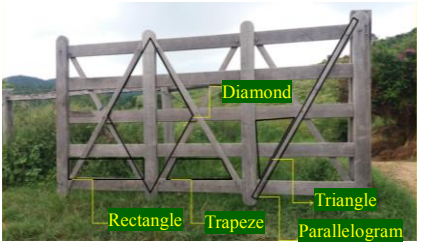
This dialogue is provoked and problematized in the activities held during School Time and/or through work proposals, investigations, and activities in the field during Community Time. In this section, we illustrate this dialogue been (de)constructed and established by the undergraduate students of Countryside Education, UFMG, in an experience of the subject 'Space organization I: the study of shapes'. To do so, we present some results of an activity created and proposed by the subject's professor to be developed during the first semester of 2018 during Community Time.

In this activity, the students photographed five objects or landscapes that were part of their daily routine in the countryside. Then they had to identify in these records some plane geometric shapes studied during January 2018, during School Time (triangle, square, parallelogram, rectangle, trapeze, heptagon, hexagon, and circles). Finally, they wrote a small comment (maximum ten lines) about the recognition and use of these plane geometric shapes in their territories, their functional advantages and/or their aesthetic appeal.

We highlight below some excerpts written by the undergraduate students answering an activity proposed by the professor:

Figure 2

Answers to the activities of the subject Space Organization I: study of shapes.



Gate in the corral entrance.

"When looking at the gate we can perceive several geometric shapes developed to close a certain space. People normally say that vertical and horizontal wood planks are needed to guarantee their firmness, which does not justify the geometry shapes as a utility or aesthetic form… I believe that the people who created them did not perceive the several geometric shapes that appear in the objects but were thought with a use value, guaranteeing their safety, stability, and convenience."

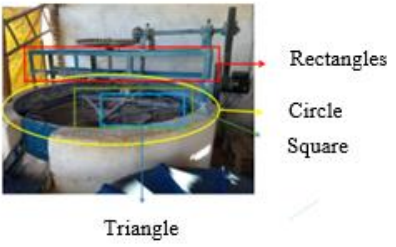
Plow to scratch the ground

"The plow is used to scratch the ground. It is widely used and helps the workers and the animals that pull it because its triangular shape easies its use, making perfect scratches on the ground."

Disintegrator engine feeding tube

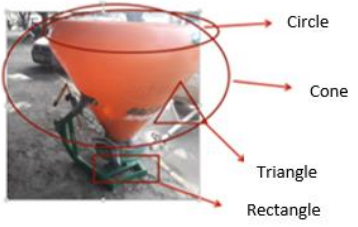
"A disintegrator engine is used to grind grains, grass, and sugar cane. The feeding tube of the disintegrator, in a trapezoid shape, is used to ease the grinding of grass or cane, because it makes it easier to distribute the force to easily introduce the product in the tube, avoiding the producer to hold a large amount of product to be ground. The square fits another squared piece that eases the grinding of grains. It avoids waste and workers' injuries, as when one turns the engine on and

starts grinding, the grains are ejected if the piece is not in place."



Oven to roast flour

"Another machinery we also use, which does the work that is still done manually in other places, is the oven to roast the flour. There are many aesthetic and use differences between manual and motorized ovens. The shape is different to ease the work done in each one. The motorized one has a circumference shape, while the manual is rectangular, because the engine allows the circular movement of the paddles to spread the flour while being roasted. In the manual oven, people do this movement with rakes."



Single row planter

"Its functional advantage is that the planter itself and its cone shape is tapered, thus one has the exact control of how to spread the seeds and the fertilizer at the same time. It can plant from 15 to 20 seeds per linear meter."

By bringing here the undergraduates' answers, we do not aim to point out their correct or wrong answers, though this was one of the pedagogical aims of the activity. We aim to highlight how the students, immersed in these different territorialities, relate the study of shapes with their ways of producing life in the countryside.

In the selected excerpts, the identification of plane geometric shapes emerges not only as the answer for a school activity. The disciplinary content is approached as a way through which the identity policies of rural peoples are produced, marked by their construction, needs, and work practices. Thus, we understand that the rural ways of knowing, doing, and being, typical of the

culture of each rural community, mobilize the identification of geometrical shapes. This identification reflects their knowledge, procedures, values, and decision-making, portraying how Geometry and its teaching can articulate several countryside life experiences.

UNDERGRADUATE DISSERTATIONS AND THE DESCRIPTION OF RURAL PRACTICES

In this section, we use the expression 'geometrically describe the practices in the countryside' articulated with the notion of mathematically describing a practice developed by Miarka (2013). The author questions the research proposals that intend to describe the mathematics existing in practice. In other words, proposals aiming to identify the forms of hegemonic knowledge in specific cultural practices, such as those exclusively based on Mathematics as a disciplinary field.

For the author, even if these references are situated within a set of concepts, procedures, and languages delimited by Mathematics as a disciple, this theme (the description of human activity according to mathematical criteria) emerges from the descriptor's cultural background which can coincide or not with the understanding of activity created by the users. In this sense, "Mathematics in its Western structure can be considered a theme. However, we must be aware that such mathematics is not necessarily a knowledge category present in other cultures" (Miarka, 2013, p. 42).

Mathematically describing a practice would be recognizing that the thematization of practice as mathematics is not static but dynamic and immersed in the experiences, choices, and intentions of the descriptor and its cultural background. Assuming the non-universality of the description built is a political task considering that the "participants of the practice might not recognize their knowledge in the description or even consider that production meaningless for them" (Miarka, 2013, p. 42). The author also states that:

> "mathematically describing a practice" means […] reporting the expression of an action perceived by a subject – a practice – guided by a thematization, in this case, mathematics. It is important to highlight that this practice description can be done differently depending on the thematization, such as religious, pragmatic, mythological, and so on. To be meaningful, the theme choice is related to the experiences of those describing the practice. If we think socially, we can say these themes

> emerge in culture. Summing up: a thematization emerges in the cultural background of the descriptor. (Miarka, 2013, p. 42)

However, this does not mean that the descriptions centered around disciplinary Mathematics cannot occur. However, we highlight the importance of establishing the cultural background from where emerges the thematization that will guide the viewpoints over the investigated practice. In the cases presented below, the descriptors are graduates from an Undergraduate Teaching degree in Countryside Education, with emphasis on Mathematics, schooled with Mathematics as a school subject. When developing their undergraduate dissertations, they worked or intended to work as K-12 Mathematics teachers. Hence, the geometric descriptions of rural practices articulate ways of knowing, doing, and being of the rural subjects and peoples with recognizable knowledge objects, such as Geometry in school cultures, using in the description concepts, procedures, languages, and identities involved in these practices.

We bring here two works that dialogue with the proposal of geometrically describing countryside practices. Therefore, these practices are connected to the authors' communities and are understood as situated in their territories, articulating space, identity belonging, and possible geometry knowledge, inextricably present in school cultures.

Circles, movements, and shapes in extractivism and the processing of pequi in the community Geraizeira

Neusita Agostinho's (2017) work investigated the extractivism and processing of *pequi*[3] to characterize and describe the types of knowledge involved in these processes, mainly those recognizable as mathematical ones. The study took place in the traditional community of Geraizeira Água Boa II, located approximately 18 km from the city of Rio Pardo de Minas in the state of Minas Gerais.

> According to Seu Antônio [research informer], people always arrive in Areião in silence so no one will notice them, after they start to shout to establish the territory, alerting others that this area is taken. When someone arrives first in someone else's

[3] T.N.: Pequi (scientific name: Caryocar brasiliense) is an edible fruit typically consumed in the North of Minas Gerais and the Brazilian Center west region. Usually, the trees are not commercially planted, and the fruits are collected (extractivism) by family members around their houses and the woods.

> space, the family has to find another place. They are often disappointed until they can find a place that was not gathered that day. Starting the gathering, the families enter [the area], stopping and gathering the *pequi* in sacks or buckets. The animal, with or without the wagon, stays tied. The members of each family divide themselves, and each goes to a side of the area. They return and pour everything into the wagon. If they notice some *pequi* around, they collect around the animal. The animal is left in an open space, so they can keep it in sight and find it quickly. When they notice no more *pequis* around, they walk a bit more, always taking the animal and keeping it as a spatial reference. When they can collect a good amount of *pequi*, they stop to rest. (Agostinho, 2017, p. 37-38)

The author's description of the gathering of *pequi* indicates the possibilities of dialogue with the concept of a circle. However, in the description, the concept appears differently than usually seen in the curricula and school material. Here, the circle is not openly defined as a plane surface limited by a circumference but as an attempt to cover a specific area (Areião) from a fixed point (the tied animal). This way of organizing the practice of *pequi* gathering allows everyone to have the animal "as a spatial reference", coming back and dropping the gathered fruit on the wagon. Furthermore, collecting "around the anima" keeps all family members close to the same spatial reference until "they notice there are no more *pequis* around, they walk a bit more".

We understand that the geometry present in Agostinho's (2017) description is not necessarily committed to establishing relationships – such as comparisons, ordering, operations, etc. – in a space that would lead us to a generic conceptualization of curriculum. The identity *'Geraizeira*' establishes the author's way of living and seems to be defined in the experiences with space during the extractivism practice. The circle is not a generic geometric being but a particular experience of a space that establishes itself as a territorial identity. In this direction, we understand that the knowledge, attitudes, and values described by Agostinho (2017) in the procedure of *pequi* gathering express the particular symbolic value of this territory (Areião) in whose belongings the *geraizeiros* establish their territoriality.

Though Agostinho's (2017) work invites us to think there is a curriculum concept of Mathematics in that practice, we have opted to understand that this concept is a thematization aligned with the author's

territorial identity. It has similarities with the relationships established by the Geometry predicted in the school but, fundamentally, establishes itself in procedures and languages unique to the Geraizeira community and their ways of living in their territory. However, the author also offers the reader what she indicates as "geometric representations" present in the gathering and processing of *pequi,* describing the movements and shapes in this activity.

> We notice circular movements in peeling *pequi* and when removing the fat to separate the fat from the excess water. Still, on geometry, we highlight that some tools represent geometric shapes, such as • wood container in the shape of a cube; • strainer in the shape of a cone; • rake in a crescent shape with a wooden cable, similar to a spade; • a liter container, of kerosene or soy oil, in the shape of a cylinder; • 18-liter can in the shape of a rectangle […] We can also highlight the trough, the plastic box, the wagon, the wheelbarrow, the water tank, the bucket, among others. (Agostinho, 2017, p. 61-62)

Agostinho (2017) argues that the systematization of fruit processing has valuable educational possibilities when directly relating to people's living conditions and environment. The author emphasizes the importance of inserting elements of the *Geraizeira* culture into school education – and, as indicated, in the teaching of concepts, procedures, and languages typical of Geometry – as a way of "giving back to the community an understanding and strengthening of their cultures and traditions". This would also highlight "the richness of the interconnection of several life areas, such as mathematics, history, culture, nature, commerce, present in the life of *Geraizeiros* and the tasks of rural peoples" (Agostinho, 2017, p. 64).

"Millimeter the milk" as a change in the way of life of subjects from a settlement

Raquel Mendanha (2020) is a rural worker living in the Agrarian reform settlement 'Hebert de Souza' in Paracatu (MG), a graduate with a Mathematics emphasis on the Countryside Education degree of UFMG. She faced the challenge of connecting the ways of living in her settlement and the practices recognized as Mathematics. The researchers used Oral History to give more visibility to camped and settled groups' fight processes, reaffirming these people's rural identity.

At a moment of the research (Mendanha, 2020, pp. 46-47), one of the participants in the formation process of the settlement declares:

> *In the beginning, first, we take the rope, the cable, the rope is about 50 meters. We opened the roads of 50 meters because we knew that if we didn't open them, when INCRA arrived, the road measurements wouldn't match. We knew there needed to pass the water and light, so we measured. Our technique was the rope. Later INCRA came and measured with devices. What we measured was milk. For example, if you had two children and earned a certain amount of milk, if there were three, then it was that much milk, we measure by eye the same amount per child, it was not much, we used the disposable litter […] When my mother lived here, where now is Amarildo's land, there was no scale. There was nothing. She used a plate, a regular plate, that enameled one weight approximately one kilo. There are other ways, for example, you will make a fence there and put up one kilometer, right? Things change a lot, and I'm old, if you go to Silvio's to buy a fish, for example, he has an electronic scale, it is no longer that one in which you have to put the weight here and there if you send the milk, the measure is on the ruler, the ruler is divided in millimeters. For instance, if there are 100 liters and nine millimeters, it appears in the ruler. Things really changed in this issue of measure of weight. Before you would sell cheese, you would sell the piece. Today you don't sell the piece of cheese anymore, you sell the kilo, the grams. Changes are necessary. The change is not bad nor good, I believe that is our process of life. Think about it, the man who studied until Year 5 was the school principal.*

In his lines, the rural worker in the settlement considered that weighting and measuring are different actions. These actions are referenced in enunciation places marked by territorial transformations during the time and their influences in measuring. In the past, in what he called measure, typical rural instruments were used, such as different-sized heads, the relationship between people, and the act of measuring was based on trust and camaraderie. Today, in weight, there is an approximation of a measurement form based on formal acts of commercialization. This change substitutes the feelings of sharing and mutual help for financial measures, determined by mechanisms and commercial processes immersed in the capitalist system, dissociating the act of

measure from their rural relations that he, as a rural settled worker, believes to be ideal.

We understand that to *'millimeter the milk',* as indicated by the worker, relates to the territory, connected with materiality and social action. This procedure reinforces the incongruence between the acts of measuring during more collective times and those experienced in the approximation to the State and capitalism as forms of collective authority, guiding behaviors and values through measuring practices.

This understanding brings some educational unfoldings, particularly regarding the curriculum. Sachs and Fernandes (2018) highlight that in official documents, such as the BNCC (National Curriculum Framework, there are expressions such as "objects of daily use" and "non-standard measurement units", valuing and respecting the local culture or more standard units in sociocultural contexts. According to the authors, these expressions point out that a pedagogical work makes no sense if it approaches "only units, instruments, and measurements connected to urban contexts (as some urban-centric didactic books suggest), with typical city artifacts and unities and tools of standard and conventional measures". They highlight the need to recognize the rural spaces "as a field of possible knowledge so that they can identify themselves in these activities and share their ways of life in the school context" (Sachs & Fernandes, 2018, p. 258). In this direction, Gonçalves (2014, p. 40) stresses that:

> The practice of measuring is one of the most important in the countryside. Often these communities use non-standard units of measure arising from their historical isolation. Far from representing a problem, these are creative practices that sought and seek to solve, in a simple way, the problem of selling and buying goods or lands.

However, often, there is a tendency to think that rural peoples exclusively use non-standard measurements. Contrariwise, Mendanha's (2020) work indicates that studying the incorporation of conventional units and tools, when aligned to the notions of territory and territoriality, can favor the awareness of these subjects on their participation in production, the use and dispute of intentions, arguments, attitudes, and values. We understand that this participation is not only related to using concepts and procedures connected to measuring the space, as in the case of milk. It also refers to the incorporation of identity policies demanded or assimilated in the production movement of rural territory that starts to use these measurement units and tools.

REFLECTIONS ON GEOMETRY AND ITS TEACHING FROM THE PERSPECTIVE OF COUNTRYSIDE EDUCATION

Based on the discussion so far, we present as final remarks three reflections on Geometry and its teaching from the perspective of Countryside Education. These reflections aim to contribute to the evaluation of epistemological and educational positions regarding the geometry knowledge established by school cultures in general, not necessarily limited to schooling in rural areas.

The first reflection refers to the epistemological issues related to Geometry. The movement between the notions of space and spatiality – for example, the resumption of the human dimension – might indicate ways to deconstruct historical and educational processes that establish the so-called geometry knowledge. From the perspective of Countryside Education, we believe that the association of these notions to territory and territoriality is an alternative that does not alienate Geometry from social, cultural, and political conditioners of the space and the territory of struggles from the subjects and collective groups. In this direction, the policies of knowledge that establish Geometry may contribute to affirming the identity policies of those groups who have claimed the land as a way of living within a long history of exclusion, exploitation, and extermination.

The second reflection is related to the different ways Geometry is associated with curriculum guidelines for Mathematics teaching. Every curriculum is a part that determines which life forms will be discussed at school. A part selects, separates, organizes, and distributes spaces and times, designating which types of knowledge will be central and which will be subordinated. Thus, every curriculum construction is a political action determined by power, knowledge, and being relations. The institutions that act in the curriculum creation are never naïve or uninterested but guide specific identity policies for the sake of others.

When dealing with the experiences in space, the possibilities of a work articulated with Countryside Education are often represented in curriculum guidelines by expressions such as "non-conventional", particularly themes involving magnitudes and measures. In this sense, it is vital to think that "non-conventional" might indicate broader discussions that question rural peoples' social, economic, and territorial inequalities beyond the concepts, procedures, and languages typical of these groups. Another critical point is understanding

how the study of "conventional" might express such inequality and how the imposition of hegemonic sociocultural traditions significantly affects different groups.

The third and last reflection, didactic-pedagogical, refers to the relationship between Geometry and sociocultural practices in rural areas and the intentions of mathematically describing them. The mobilization of experiences of rural peoples in Mathematics classes places investigation as a central element of the pedagogical work, seeking to grant autonomy to the student and allowing a collective construction of school knowledge. In this process, the educator has the mediating role of organizing and developing a conducive learning environment. In other words, the educator should not only transmit established knowledge but create an atmosphere that provokes the student to relate the concepts, procedures, and languages from the studied sociocultural practices with those already established by tradition, affirming the value of one life among other forms of living.

Finally, we defend that the Geography present in school cultures should be committed to the affirmation of rural peoples, those who have never abandoned it. The geometric knowledge mobilized should move toward understanding social reality in which the subjects, the communities, and the collective groups that compose the school map unequal relations and propose ways to overcome them.

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The authors F. S. F., F. C. D. P. G. e M. F. A. M. were responsible for conceiving, debating, and writing this article.

DECLARATION OF DATA AVAILABILITY

The data supporting the results of this study will be made available by the author F. C. D. P. G., upon reasonable request.

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