ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7179

The Fabrication of Scientists through School Geometry

Melissa Andrade-Molina

Pontificia Universidad Católica de Valparaíso, Instituto de Matemáticas, Valparaíso, Chile.

Received for publication 1 May 2022. Accepted after revision 13 Dec. 2022
Designated editor: Maria Celia Leme da Silva

ABSTRACT

Context: Visuospatial abilities that derive from perception, for example, are not usually considered one of the mathematical abilities developed in the national school geometry curricula. **Objective**: This work aims to explore school practices that seek to guide the ways of being and acting of students in Chile. **Design**: This is done by mapping the geometry school curriculum to trace the system of reason that makes possible the fabrication of particular kinds of people. The study uses a Foucaultian toolbox to address school geometry as a technology of government that has power effects in shaping students' subjectivities. **Data**: School textbooks distributed by the Chilean Ministry of Education, MINEDUC, from the final six levels of school education (13 to 18 years) were analyzed. **Data analysis**: The school textbooks were explored, through discourse analysis, to map the connections between the activities proposed in the geometry units for each level in order to trace the paths that allow for guiding students' ways of being and acting. **Results**: Evidence compiled from an analysis of current Chilean mathematics textbooks shows that geometry school practices are constructed to train the eye so that students conceive and experience space not through their senses but through reason and logic, which inserts students in the modern process of training scientific minds of the future.

Keywords: school geometry; curriculum; technology of government; conduction of conduct; eyes of reason.

Fabricación de personas científicas mediante la geometría escolar

RESUMEN

Contexto: Las habilidades visuo-espaciales que derivan, por ejemplo, de la percepción usualmente no son consideradas como parte de las habilidades matemáticas a desarrollar en los currículos nacionales de geometría escolar**. Objetivo**: Este trabajo tiene como objetivo explorar las prácticas escolares que buscan conducir las formas de pensar y actuar de estudiantes en el sistema educativo chileno. **Diseño:** Esta exploración se realiza al mapear el currículo escolar de geometría para evidenciar el

Corresponding author: Melissa Andrade-Molina. Email: melissa. andrade@pucv.cl

sistema de razón que hace posible fabricar tipos particulares de personas: personas de ciencia. El estudio utiliza una caja de herramientas Foucaultiana para abordar la geometría escolar como una tecnología de gobierno que tiene efectos de poder en la formación de las subjetividades de estudiantes en etapa escolar. **Datos**: Se analizaron textos escolares distribuidos por el Ministerio de Educación de Chile, MINEDUC, de los últimos 6 niveles de enseñanza escolar (13 a 18 años). **Análisis de datos:** los textos escolares fueron explorados, mediante un análisis de discurso, para mapear las conexiones entre las actividades propuestas en las unidades de geometría sugeridas en cada nivel, de modo de trazar los caminos que permiten conducir la conducta de tales estudiantes. **Resultados**: La evidencia recopilada a partir de un análisis de los libros de texto de matemáticas chilenos actuales muestra que las prácticas escolares de geometría contribuyen al entrenamiento del ojo. Tal entrenamiento supone fabricar a personas de ciencia que conciban y experimenten el espacio no a través de sus sentidos, sino que a través de la lógica y la razón. Lo que inserta a estudiantes en etapa escolar en el proceso moderno de formación de las mentes científicas del futuro.

Palabras clave: geometría escolar; currículo; tecnología de gobierno; conducción de conducta; ojos de la razón.

A fabricação de cientistas através da geometria escolar

RESUMO

Contexto: As habilidades visuo-espaciais que derivam, por exemplo, da percepção usualmente não são consideradas como parte das habilidades matemáticas a serem desenvolvidas nos currículos escolares nacionais de geometria. **Objetivo**: Este trabalho tem como objetivo explorar as práticas escolares que buscam conduzir os modos de pensar e agir dos alunos no Chile. **Design**: Isso é feito mapeando o currículo de geometria da escola para traçar o sistema de razão que torna possível fabricar tipos particulares de pessoas. O estudo usa uma caixa de ferramentas foucaultiana para abordar a geometria escolar como uma tecnologia de governança que tem efeitos de poder na formação das subjetividades dos alunos. **Ambiente e participantes**: Foram analisados livros didáticos distribuídos pelo Ministério da Educação do Chile, MINEDUC, dos 6 últimos níveis de educação escolar (13 a 18 anos). **Recolha e Análise dos dados:** os livros didáticos escolares foram explorados, por meio da análise de discurso, para mapear as conexões entre as atividades propostas nas unidades de geometria para cada nível, a fim de traçar os caminhos que permitem conduzir os modos de pensar e agir dos alunos. **Resultados**: Evidências compiladas a partir de uma análise de livros didáticos de matemática chilenos atuais mostram que as práticas escolares de geometria são construídas para treinar o olho para que os alunos concebam e experimentem o espaço não por meio de seus sentidos, mas por meio da lógica e da razão. O que insere os alunos no moderno processo de formação das mentes científicas do futuro. **Conclusões:** Se a intenção da escola é potencializar as possibilidades de estudantes no estágio escolar optarem por carreiras STEM, é necessário replantar e compensar os efeitos de poder da geometria escolar.

Palavras-chave: geometria escolar; currículo; tecnologia de governo; condução da conduta; olhos da razão.

INTRODUCTION

The development of mathematics skills in school has been considered essential for solving day-to-day problems, an ability acknowledged as vital for relating daily practices with mathematics (OCDE, 2014). Particularly in geometry, spatial thinking has been proposed as a fundamental competency for solving problems situated in the mathematics classroom (Mevarech & Kramarski, 2014). From this, visuospatial reasoning (Taversky, 2005) or spatial visualization becomes indispensable to reading, interpreting, and discerning information within two-dimensional or three-dimensional images (OCDE, 2012). As an example, according to Arcavi (2003), visualization becomes

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (p. 217)

Spatial visualization has been linked to important scientific contributions such as those by Galileo, Faraday, and Einstein, as well as the discovery of the structure of DNA (Miller, 1986; Kozhevnikov, Motes, & Hegarty, 2007; Newcombe, 2010). Furthermore, spatial thinking has been considered as necessary as mathematical thinking (Wai, Lubinski & Benbow, 2009; Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2014). Research in the field of mathematics education has focused on exploring and understanding the complexity of visualization processes in learning school mathematics (for example, Swoboda & Vighi, 2016). Thus, visualization has become a form of significant reasoning both in the learning and teaching processes of school geometry (Sinclair et al., 2016). Recognizing this, some research suggests developing spatial visualization as part of the school curriculum (Sinclair & Bruce, 2015). Other investigations discuss whether these skills are accepted, encouraged, and valued in the mathematics classroom (Presmeg, 2014).

While a wide range of activities have been proposed to improve and develop spatial skills—from introducing concrete object manipulation activities such as ones using building blocks to proposing to restructure the entire school geometry curriculum (Clements, 2008; Hauptman, 2010; Prieto

& Velasco, 2010)—, school geometry is framed in Euclid's axioms and the Cartesian system. Thus, school geometry is conceived in a reduction of space that leads to possible misconceptions (Skordoulis et al., 2009; Andrade-Molina, 2017). However, the kinds of skills that should be developed in school not only respond to the structure of the geometry curriculum by positioning themselves as areas of interest (for example, by justifying the importance of specific skills for STEM careers) but also outline who the desired subject is (Andrade-Molina & Valero, 2015).

A few years ago (Andrade-Molina, 2017a), I discussed how school geometry becomes a technology of government (Foucault, 1988) that drives the ways of acting and thinking of students. This analysis focused on the effects of power in producing the desired student by inserting them into school practices aimed at training the eye through school mathematics (Andrade-Molina & Valero, 2017). In this fashion, students are positioned in contexts that mimic life experiences but ignore the information collected through the senses to accommodate mathematical practices based on logic, deduction, and reason. For this reason, this article explores how the mathematics curriculum in Chile has changed regarding the training of the eye through school geometry after the introduction of transnational concerns such as attention to diversity and inclusion to assure quality of and access to education (see UN, 2015). The study collects data from the latest update of school textbooks for students carried out by the Chilean Ministry of Education (MINEDUC) to explore how school geometry becomes a technology of government that conducts the ways of thinking and acting of students. In the conduct of their ways of being and acting, students in the Chilean educational system are inserted into practices that seek to perceive space through trained eyes via Euclidean school geometry.

SCHOOL CURRICULUM AND TECHNOLOGIES OF GOVERNMENT

School textbooks are explored to understand how school geometry becomes a technology of government. This is achieved by mapping the curricular structure that allows the conduct of students' ways of thinking and acting. According to Foucault (1993), governing people

> is not a way to force people to do what the governor wants; it is always a versatile equilibrium, with complementarity and conflicts between techniques which assure coercion and

> processes through which the self is constructed or modified by himself. (p. 204)

In this regard, school geometry should not be understood as a pedagogical device that seeks an absolute imposition of students' ways of thinking to which they must submit without offering resistance. For Foucault (1984), to govern is not to force, dominate, or impose, but rather depends on the freedom that people feel to be and act for themselves, to decide for themselves. Thus, the conduct of conduct operates through systematized and regulated technologies of power that include forms of self-regulation (Foucault, 1997). Then, technologies of power allow people to change and develop their thoughts and direct their ways of being within a process of fabricating their subjectivities (Foucault, 2008). For example, Kollosche (2014) explores how school mathematics is a technology of government through logic and calculation practices that drive the behaviors of students to be “able and willing to think and speak logically and act bureaucratically” (p. 1070).

In other words, people accept certain models "that [the subject] finds in his culture and are proposed, suggested, imposed upon him by his culture, his society, and his social group" (Foucault, 1984, p. 291) in a negotiation process –a constant tension between subjection and subjectivation– and act with them in a productive way. Here, modern education has been pointed out as a fundamental space in the governing of the subject. In particular, school mathematics practices insert norms of reasoning in both restrictive and productive ways (Valero & García, 2014). That is, in schools, students learn the mathematics determined by transnationals and governments as essential for the fabrication of active citizens so that: (1) each student can decide their future according to their own interests and possibilities, but (2) are restricted to a system of reason that allows us to conceive the future only according to parameters in which school mathematics takes a gatekeeping role. In this light, school geometry governs spatial and visual perception under certain norms that make it possible to fabricate the desired subject (Andrade-Molina & Valero, 2017).

SCHOOL GEOMETRY AND THE CONDUCT OF CONDUCT

This exploration analyzes the school textbooks distributed by MINEDUC. Only the texts comprising the final six levels of school education – aimed at students between 13 and 18 years old – have been reviewed. These texts are complemented by the study programs released by MINEDUC for each level on the website https://www.curriculumnacional.cl/docentes/. It should be noted that school texts are free educational materials distributed by MINEDUC to public and subsidized educational establishments in order to promote quality, equitable access to education and guarantee equal opportunities for all students (Government of Chile, 2018). Other establishments, for example, private schools, can use school textbooks from other publishers, which implies an elevated cost for parents and guardians of students who attend such schools (see Gonzáles & Parra, 2016).

The exploration was carried out by analyzing the problems in school textbooks that involved a confrontation between school geometry, perception, and spatial visualization. For this, analysis focused on identifying the unit's objective, the content to be addressed, and how the proposed problems for each lesson allow the application of geometric knowledge of each lesson. More specifically, the exploration of school textbooks answers the following questions: what elements should students put into play in school to solve problems contextualized in everyday life? In what way does school geometry allow for conducting students' conduct towards training the eye?

The results are presented below, discussed by level. This exploration accounts for the elements that are emphasized when solving contextualized geometry problems in everyday life situations. Such problems make the development of spatial visualization impossible.

First level: Séptimo año básico

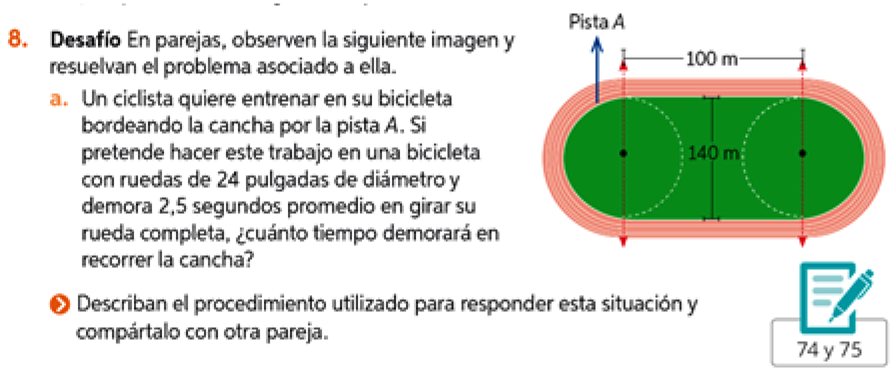
According to the study program for this level (MINEDUC, 2016a), this unit must focus on working with two-dimensional figures, angles, lines, and circles – the calculation of areas and perimeters. In addition, it is expected to incorporate the notion of vectors through placement games.

The first activity of the geometry unit of this school text (Iturra, Manosalva, Ramírez & Romero, 2019) seeks the recognition of plane figures, lines, and angles in three-dimensional objects. Students are expected to "activate" their prior knowledge through activities such as constructing triangles using triplets of given segments, classifying and adding angles, and identifying transformations. For example, in Lesson 10, students must classify polygons (as regular and irregular) in relation to the number of sides and type of angles, as well as recognize polygons in the environment. The unit continues with the calculation of areas and perimeters of plane figures. The connection with phenomena of everyday life occurs in terms of performing these calculations on known objects.

The following figure (see figure 1) shows a problem in which students must calculate the perimeter of a figure, particularly *the perimeter of the circle* (Lesson 12). Students should estimate the time it would take a cyclist to complete a lap knowing the diameter of the bicycle's wheels and the time it takes for the wheel to turn. The context of the problem is confusing in practical terms: if a cyclist wants to determine how long it takes to ride the track, the cyclist will most likely use a stopwatch.

Figure 1

Perimeter calculation of an everyday object. (Iturra et al., 2019, p. 137)

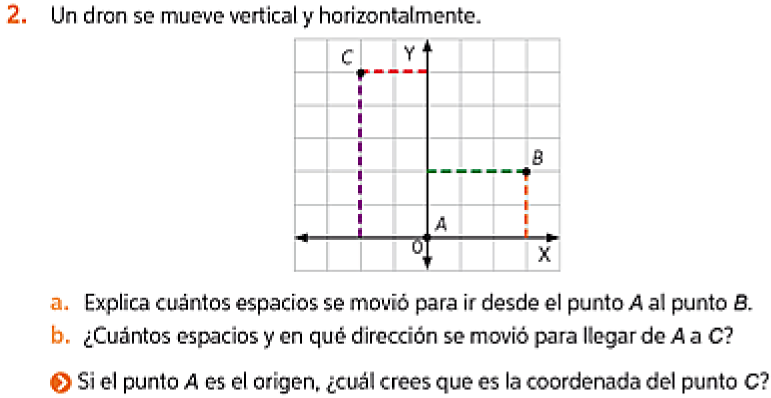


However, the method that students at this level are expected to use requires that, in addition to calculating the perimeter of the track (using the information provided by the green surface), they calculate the speed of the cyclist (using the information provided in the problem). Thus, the techniques applied in daily life begin to be incompatible with the techniques applied in daily life inserted in a school context.

Later, in Lessons 13 and 14, the Cartesian plane and vectors are introduced, respectively. In these lessons students must identify and locate points and vectors on a plane. The problems involve an abstraction of three-dimensional space to conceive it two-dimensionally. As an example, problem 2 in Lesson 13 leads students to assume that the displacement of a body occurs two-dimensionally in a space conceived by the X and Y axes. That is, when an everyday object, such as a drone, is part of a school problem, its trajectory is restricted to vertical and horizontal movements within the same plane (see figure 2).

Figure 2

Bidimensional displacement of an object (Iturra et al., 2019, p. 146)


2. Un dron se mueve vertical y horizontalmente.

a. Explica cuántos espacios se movió para ir desde el punto *A* al punto *B*.
b. ¿Cuántos espacios y en qué dirección se movió para llegar de *A* a *C*?

Si el punto *A* es el origen, ¿cuál crees que es la coordenada del punto *C*?

Figure 3

Translation vectors in an everyday context. (Iturra et al., 2019, p. 152).


4. Observa la situación y realiza las actividades solicitadas.

En la Zona Sur de Chile habita el pingüino magallánico, un ave que nada a gran velocidad y se zambulle desde elevadas alturas.

a. Recrea en un plano cartesiano, con puntos y vectores, la siguiente situación: Usa solo el cuadrante IV y toma como posición inicial el punto (0, 0).
   - Un pingüino se sumerge en el mar. En este movimiento se desplaza cinco lugares hacia abajo y dos lugares a la derecha.

b. ¿Cuáles son las coordenadas del vector desplazamiento que representa el movimiento?

This technique is used in Lesson 14: *vectors*. In this lesson one must establish the translation vectors of a figure. The activities involve objects such as helicopters as well as flat figures (in this case triangles). Here, everyday life situations are presented in such a way as to frame them in a school context that justifies being approached through vectors. For example, the following problem (see figure 3) is presented from a local context: the great speed and trajectory of a Magellanic penguin from southern Chile are reduced to being imagined by means of a movement of quadrants.

Second level: Octavo año básico

According to the syllabus for this level (MINEDUC, 2016b), the geometry unit should focus on discovering and applying surface and volume formulas by working with networks of three-dimensional geometric shapes and the Pythagorean theorem, in addition to approaching them through the Cartesian plane to integrate translation, rotation, and reflection movements.

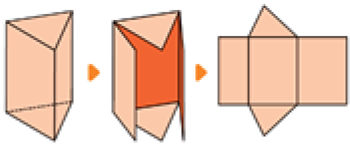
In the school textbook (Torres & Caroca, 2019), the unit begins by linking mathematics and art through a brief reflection followed by calculating areas and perimeters of plane figures and identifying the number of faces of two given prisms. The unit continues calculating areas and perimeters of prisms and cylinders. As in the previous level, *séptimo año básico*, an attempt is made to approximate reality through a restricted view of daily practices so they make sense in the classroom setting. Example 1 from Lesson 1 shows how, inside the classroom, people act entirely differently than they would outside a school context. This problem (figure 4) assumes that to determine the amount of paper needed to wrap a box, the total area of the figure must be calculated, rather than superimposing the unassembled box on a piece of paper or wrapping the box with the paper and visually estimating the amount required. In this problem, the total area of the box will not necessarily coincide with its net shown in the image, and the technique to solve the problem is reduced to calculating areas using a given algorithm: calculate the area of each figure that makes up the geometric net and add them to find the total surface area of the box. How helpful is it to know the total surface area of the box if what students need is a rectangle of paper with the same width and height of the deconstructed box? Here it is necessary to discard the techniques acquired outside school and accept the techniques acquired through the practices of school geometry.

Figure 4

Prism surface calculation. (Torres & Caroca, 2019, p. 119).

Ejemplo 1

Tamara tiene una caja en la que guarda sus útiles escolares. Ella quiere forrarla con papel. Para ello, desarma la caja, como se muestra en la siguiente imagen.



¿Cómo podría saber Tamara cuánto papel requiere para forrar la caja?

Para saber cuánto papel utilizará, Tamara desarma la caja obteniendo una red formada por 2 triángulos y tres rectángulos. Entonces, si calcula el área de cada una de estas figuras y las suma, tendrá el área total, y podrá dar respuesta a su interrogante.

Like the previous example, the following problems regarding the volume of prisms are posed to be solved by applying formulas to everyday life contexts that are typically addressed with other types of tools. For example, instead of calculating the volume of a container (in this case, a box), a problem is presented in which the volume is fixed, but the parameters of one side of the base of the container change.

> A designer makes a sketch of a box 18cm high whose base is a right triangle, and its legs measure 4cm and 3cm. If he is unsatisfied with this sketch and doubles the length of the shorter leg, how must the height of the box change to maintain the same capacity? (Torres & Caroca, 2019, p. 135)

The unit continues with the Pythagorean theorem and its applications in everyday life. Some problems align with the traditional way of approaching mathematical objects in a school context, for example, by calculating the maximum possible distance in a rectangle (the diagonal). This is done by posing problems involving movement, such as calculating the maximum possible path in a swimming pool if one can only swim in a straight line and knows the dimensions of the swimming pool (Example 2 from Lesson 2). This level includes problems that lead to a non-traditional application of the Pythagorean theorem. This approximation includes practices from other civilizations (see figure 5) regarding the construction of angles based on the Pythagorean theorem. However, instead of promoting the use of the rope to understand how right angles can be drawn using the artifact, the figures are presented and students must justify the mathematical argument behind the method. This leads to mechanizing the problem and limiting the possibilities of developing spatial visualization strategies. In this way, students are expected to superimpose their geometric knowledge to justify the construction of angles with "valid knowledge" in a school setting.

Figure 5

Construction of right angles. (Torres & Caroca, 2019, p. 143).

In the next lesson, *isometric transformations*, the problems are technical. Translations, rotations, and reflections must be applied to two-dimensional figures in the Cartesian plane and to three-dimensional shapes – the three-dimensional shapes are not situated in the XYZ coordinate system. The connection with the phenomena of daily life occurs from the coordination with known objects (i.e., animals, videogames such as Tetris, paintings) although the treatment of these scenarios continues to restrict the practices that occur in an everyday context. Question 5 of the evaluation section of Lesson 3 shows multiple ships sailing. Students must find the isometric transformations applied to these ships. The situation is framed, perhaps without such intention, in a Cartesian plane comprising the equator and the prime meridian. In this way, increasingly, the techniques introduced require thinking about reality in terms of coordinates on a Cartesian plane and not in the trajectories that ships would possibly follow when navigating from one point to another.

Figure 6

Isometric transformations. (Torres & Caroca, 2019, p. 169)

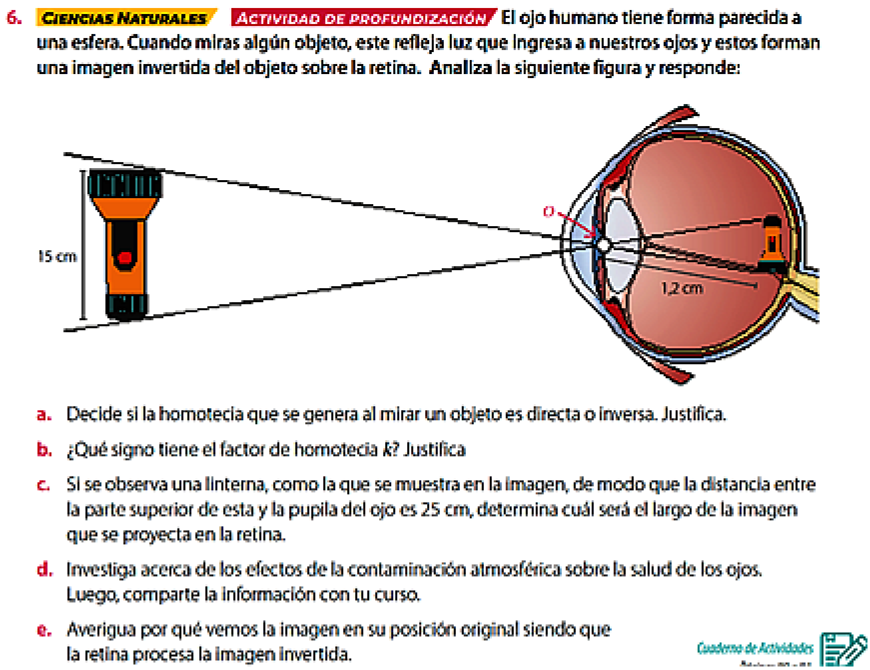
Third level: Primer año medio

According to the curricular guidelines for this level (MINEDUC, 2016c), this geometry unit should focus on developing the formulas for areas and perimeters of circular sectors and the formulas for the area and volume of a cone. In addition, students must relate dilation —homothecy— with perspective, understand their properties, develop Thales's theorem through the properties of homothecy, similarity, and proportionality, and represent dilation in vector form.

In the school textbook (Fresno, Torres & Ávila, 2020), the unit begins by presenting the abandoned rails of a railway in San Pedro de Atacama in northern Chile to problematize Euclid's fifth postulate and the vanishing point at which parallel rails seem to converge on the horizon because of perspective. Subsequently, various ratio and proportion problems are presented that are unrelated to the previous discussion on Euclid's postulates. The unit continues with technical problems regarding homothety and Thales's theorem, such as problems that require calculating the dilation factor. The exercises proposed in this unit, as in previous levels, restrict daily life practices according to the parameters necessary for a context to function based on the mathematical concept to be taught. Activity 6 of Lesson 8 presents a problem about the human eye and the way it processes images. The first three questions are limited to calculating the dilation factor, the length of the resulting image, and determining whether the dilation is inverse or not (see figure 7). The reflection questions do not delve into why this phenomenon occurs, nor do they refer to other more familiar situations in which it occurs (i.e., the reflection on a spoon). The dilation occurs only in terms of recognizing the coordinates of the figures (original and resulting) in vector form.

Figure 7

Homothecy application. (Fresno et al., 2020, p. 113).


6. CIENCIAS NATURALES ACTIVIDAD DE PROFUNDIZACIÓN El ojo humano tiene forma parecida a una esfera. Cuando miras algún objeto, este refleja luz que ingresa a nuestros ojos y estos forman una imagen invertida del objeto sobre la retina. Analiza la siguiente figura y responde:

a. Decide si la homotecia que se genera al mirar un objeto es directa o inversa. Justifica.

b. ¿Qué signo tiene el factor de homotecia *k*? Justifica

c. Si se observa una linterna, como la que se muestra en la imagen, de modo que la distancia entre la parte superior de esta y la pupila del ojo es 25 cm, determina cuál será el largo de la imagen que se proyecta en la retina.

d. Investiga acerca de los efectos de la contaminación atmosférica sobre la salud de los ojos. Luego, comparte la información con tu curso.

e. Averigua por qué vemos la imagen en su posición original siendo que la retina procesa la imagen invertida.

Cuaderno de Actividades

Thales's theorem is presented in a way that makes it possible to calculate the heights of distant objects, as is typically used in school. The everyday context is given by the objects involved in solving a problem, such as araucaria trees in southern Chile (see figure 8). This problem requires an understanding that everyday life situations are accessed by drawing coordinates, lines, and segments over the given object to calculate the dimensions of said object.

Figure 8

Thales's theorem. (Fresno et al., 2020, p. 125).

Fourth level: Segundo año medio

According to the curricular guidelines for this level (MINEDUC, 2016d), the geometry unit should focus on developing formulas for the area and volume of the sphere, trigonometric ratios, and their application to various contexts. In the student textbook (Díaz et al., 2020), the unit begins by presenting a globe and asking about the differences between continents. It continues with questions about calculating the area and perimeter of plane figures, cones, and cylinders and the similarity of triangles and vectors. By introducing the sphere as a solid of revolution that results from rotating a semicircle around its diameter allows for the posing of problems focused on calculating the diameter and radius of circles and on using the rotation of plane figures to generate three-dimensional shapes. Subsequently, the relationship between the volume of a sphere and the volume of a cone and a cylinder is established.

In general, this lesson's questions focus on calculating the volume of the sphere and hemisphere even when the problems are contextualized in everyday life. For example, elements of daily life are taken as part of the context of the problem and are reduced to the application of an algorithm: in this case, the calculation of the volume knowing the diameter of the sphere, the Géode cinema in Paris (see figure 9).

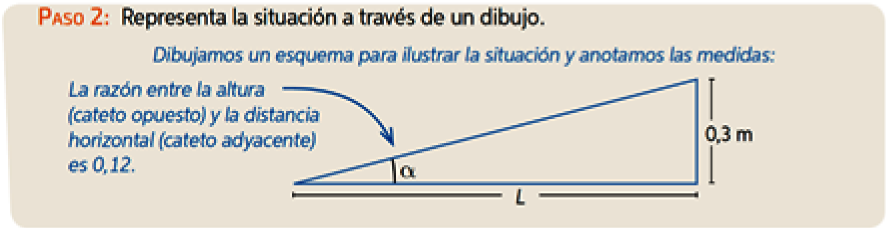
Figure 9

Calculation of the volume of a sphere. (Díaz et al., 2020, p. 100)

The text then moves on to Lesson 9, *trigonometry*, which focuses on trigonometric ratios in right triangles; thus, the problems involve right triangles. Example 1 of the section on applications of trigonometric ratios sets the problem of constructing an access ramp to a building knowing that the Chilean law requires that the ratio between the ramp height and the horizontal distance be a maximum of 0.12 (figure 10). This example's solution suggests representing the problem's context with a right triangle.

Figure 10

Trigonometry. (Díaz et al., 2020, p. 110)



Finally, the relationship between vectors and trigonometry is presented; vector decomposition occurs by using trigonometric ratios. This technique is applied to establish the movement of three-dimensional shapes according to the trajectory angle with the horizontal axis. This problem (see figure 11) requires the proposal of a scheme. During previous levels, students have established relationships between objects of daily life and flat figures or geometric three-dimensional shapes that can be adjusted to these objects. On the other hand, the curricular structure and proposed problems contribute to diagramming (using a scheme) the situations of daily life in order to accomodate the content being addressed in each lesson.

Figure 11

Vectors and trigonometric ratios, (Díaz et al., 2020, p. 110)

9. Un objeto se desplaza a una rapidez de 17 $\frac{m}{s}$ de modo que forma un ángulo de 60° con respecto a la horizontal.
   a. Realiza un esquema que ilustre la situación.
   b. ¿Cuáles son las componentes horizontal y vertical de su velocidad?

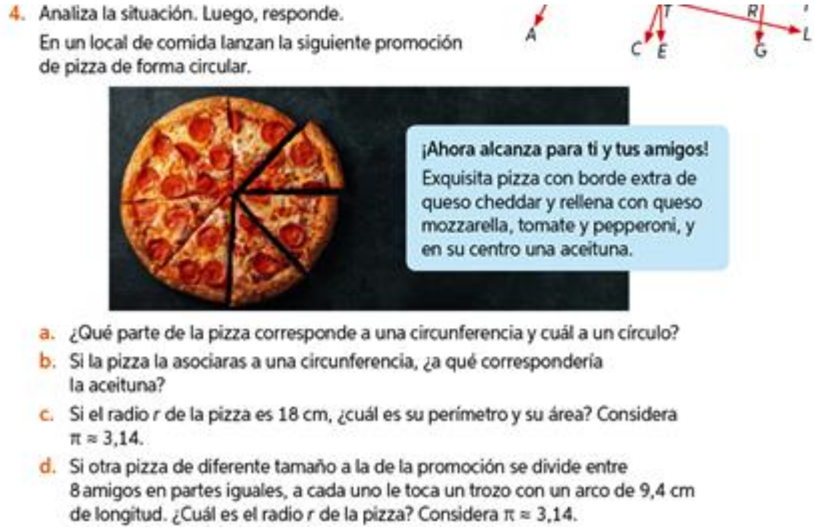
Fifth level: Tercer y cuarto año medio

According to the curricular guidelines for the third and fourth year of secondary education (MINEDUC, 2019), 3D geometry must be presented through Euclidean, Cartesian, and vector formulations. In this way, the goal is that students develop their spatial thinking by solving problems and modeling situations that affect the size, shape, and position of objects.

In the school textbook (Osorio et al., 2019), the unit begins by reflecting on metric relationships in a circle. The unit aims for students to solve problems with angles and segments in the circle. As in previous levels, the problems in this unit are contextualized in everyday life situations that result in restricting people's experiences in those situations in order to adjust them to the context of the problem. For example, an activity is presented to activate prior knowledge that involves calculating a pizza's perimeter, area, and radius. This activity expects students at this level to remember previous units by posing a problem with familiar objects that becomes unusual: students must calculate the radius of a pizza using the information that each person receives a piece of pizza with a 9.4-centimeter arc (see figure 12).

Figure 12

Area, perimeter, and radius of a circle. (Osorio et al. 2019, p. 58).

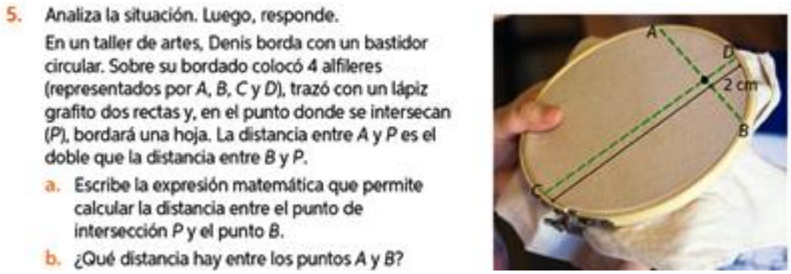


4. Analiza la situación. Luego, responde.
En un local de comida lanzan la siguiente promoción de pizza de forma circular.

a. ¿Qué parte de la pizza corresponde a una circunferencia y cuál a un círculo?
b. Si la pizza la asociaras a una circunferencia, ¿a qué correspondería la aceituna?
c. Si el radio r de la pizza es 18 cm, ¿cuál es su perímetro y su área? Considera $\pi \approx 3{,}14$.
d. Si otra pizza de diferente tamaño a la de la promoción se divide entre 8 amigos en partes iguales, a cada uno le toca un trozo con un arco de 9,4 cm de longitud. ¿Cuál es el radio r de la pizza? Considera $\pi \approx 3{,}14$.

Figure 13

Segments calculation. (Osorio et al. 2019, p. 70)



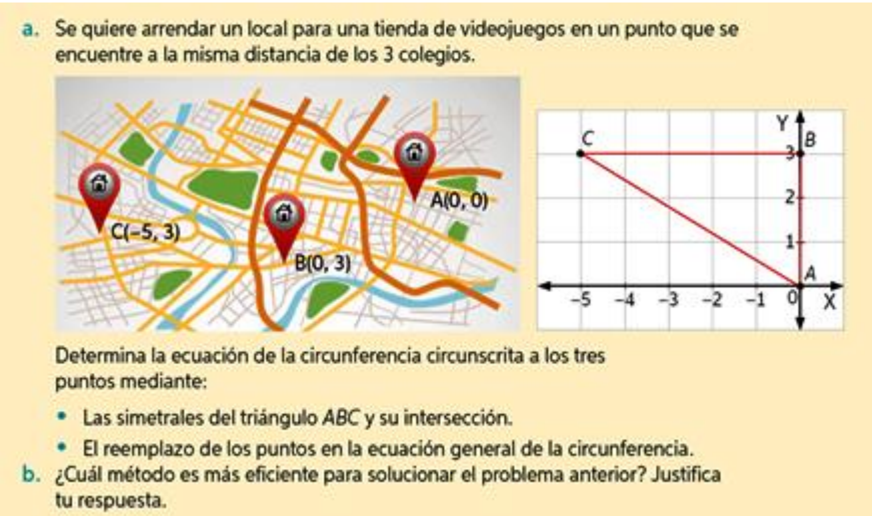
5. Analiza la situación. Luego, responde.
En un taller de artes, Denis borda con un bastidor circular. Sobre su bordado colocó 4 alfileres (representados por A, B, C y D), trazó con un lápiz grafito dos rectas y, en el punto donde se intersecan (P), bordará una hoja. La distancia entre A y P es el doble que la distancia entre B y P.
a. Escribe la expresión matemática que permite calcular la distancia entre el punto de intersección P y el punto B.
b. ¿Qué distancia hay entre los puntos A y B?

The unit focuses on the calculation of angles –interior, exterior, inscribed, semi-inscribed– and segments in the circle. Most of the problems raised in this unit are related to the application of formulas. The activities that seek to make a connection between the content and daily life reduce daily practice to recognizing certain elements that allow working with mathematical abstractions, leaving context aside. For example, one problem asks to identify specific points in the context of an embroidery frame to facilitate the calculation of distances (see figure 13).

The following unit, *geometry with coordinates*, seeks to solve problems regarding lines and circles in the Cartesian plane. It begins with a reflection on the ALMA observatory, the position of the antennas, and the stars. Prior knowledge is activated through exercises on calculating triangle segments, locating points and graphing lines on the Cartesian plane, and factoring algebraic expressions. The first lesson in the unit aims to calculate the distance between points and locate lines in the Cartesian plane. The lesson finishes with the equation of the circumference of a circle.

Figure 14

Equation of the circumference and equidistant points (Osorio et al. 2019, p. 208).



As in previous levels, the relationship between everyday life and mathematics occurs as a reduction of daily life practices. A proposed problem requires finding an equidistant point from three different places, in this case, a video game store that is equidistant from 3 schools. Students at this level are expected to calculate the equation of the circle circumscribed on the three points and determine the symmetrical lines and their intersection. The problem restricts the trajectories of possible clients to symmetrical ones that do not coincide with travel routes (considering the configuration of the streets and traffic regulations) that determine the time it would take to reach the video game store from each of the three points of departure.

The way in which school geometry is structured and problems are proposed conducts the behavior of students. According to the Chilean curricular structure, students must develop spatial abilities, understand space and forms, and use spatial visualization skills during their initial 12 years of schooling. The curricular guidelines suggest that these are achieved through certain school practices that require comparing, measuring, estimating magnitudes, analyzing properties and characteristics of three-dimensional shapes and figures, and representing coordinates in the Cartesian plane. The conduct of conduct occurs when activities and problems are progressively intertwined so that students can operate with the techniques acquired in previous levels: linking flat figures and everyday objects, discriminating variabilities and conservations, establishing a scheme based on a theorem or formula, and representing a situation in a Cartesian plane, among others. In this way, space is restricted to a Cartesian understanding, where perception is reduced to XYZ coordinates. In this sense, there is a mismatch between the skills to be developed and the proposed techniques to describe positions and movements based on coordinates and vectors.

THE TRAINING OF SCIENTISTS AND THE TRAINING OF THE EYE

The exploration of school textbooks as pedagogical devices reveals that the school geometry curriculum contributes to conceiving space as an abstract product as a result of school mathematical practices. In this conception of space, there is no place for students' perception or experiences of daily life. This leads to problematizing the development of spatial abilities and spatial visualization that MINEDUC declares fundamental for developing geometric thinking. Apparently, the curricular structure does not allow the connection between spatial visualization and school geometry. In this way, routine practices framed in a school context require navigating space in terms of XY and XYZ and observing the environment to identify two-dimensional figures and formulas.

Additionally, though the emphasis on a school curriculum based on Euclidean geometry has been recognized as detrimental to the understanding of other notions (NRC, 2006), school geometry is still mainly based on Euclidean geometry. For example, a study by Mevarech and Kramarski (2014) reveals that, particularly in geometry,

> students are presented with the properties of shapes and theorems for proof […] all the information needed is given in the problem, and the students are asked to apply the theorems in what has to be proven. […] The skills needed to solve these types of problems are limited, and teaching these skills usually consists of demonstrating the appropriate technique followed by a series of similar problems for practice". (Mevarech & Kramarski, 2014 , p. 24)

Therefore, a mismatch or gap structures a geometry curriculum that lacks a connection between what students see and what they should see according to school geometry – the reality perceived through Cartesian planes. Such disconnection occurs even from an early age. For example, Clements and Sarama (2011) show that the development of spatial skills has been ignored in formal school settings. A possible justification, from a historicization of the present, could be that a large part of geometry content, including skills such as visualization, was eliminated from the school curriculum as one of the consequences of the need to train "workers with particular skills" as part of the industrialization, urbanization and capitalism agenda (Whiteley, Sinclair & Davis, 2015). Therefore, an active and productive citizen is not conceived as a person who has developed their spatial visualization or who has managed to establish a link between perception and school geometry.

This allows for rethinking whether this conception of productive citizenry and the restricted vision of space (and the environment) affect the low levels of student achievement in national and international measurements. For example, the TIMSS 2019 results from Chile show that performance in geometry is significantly lower than students' overall performance (ACE, 2020). Moreover, Sinclair et al. (2016) have discussed the need for more clarity about how school curricula address visualization assessment in standardized tests and teacher guides. That spatial visualization practices have been limited in school experiences to which students have access suggests that school geometry, as a technology of governance, shapes the subjectivities of such students in order to negotiate practices in which they must see using another type of eyes in the classroom: the "trained eyes" (Andrade-Molina & Valero, 2015). In other words, such students must recognize the information gathered through their senses as subjective in order to accept the objectivity of the information obtained by the eyes of reason and logic. In this way, the spatial experiences of such students outside the classroom contrast sharply with the experiences that emerge from school practices. It becomes necessary to question how schooling processes continue to promote an approach to space through reason and logic that is far from the senses and close to a Platonic conception of school mathematics.

In this sense, Chilean school geometry has power effects on the subjectivity of such students, not in terms of oppressing and eradicating the meanings of school practices, but through negotiation processes in which students agree to conceive space that way: i.e., accepting that the movements of a body occur two-dimensionally to be traced through vectors in a Cartesian plane. These processes occur as the conduct of conduct, using techniques that regulate habits and desires: "structuring things so that people, following their own interest, do what they should" (Scott, 1995, p. 202). Government technologies allow students to change and develop their own thoughts and direct their ways of seeing but inserted in the logic of who is the desired subject and who is not. Following Foucault (1988), these technologies

> Permit individuals to effect by their own means or with the help of others a certain number of operations on their own bodies and souls, thoughts, conduct, and way of being, so as to transform themselves in order to attain a certain state of happiness, purity, wisdom, perfection, or immortality. (p. 18)

Thus, such students must be able to recognize the advantages of conceiving the restricted space of school geometry; they must learn to play the game of training their eyes in order to be successful in terms of national and international assessments, but if and only if it is derived from their own interest. In other words, by accepting these practices, such students will not only acquire the skills required to become the desired citizen but will also obtain techniques to successfully navigate the space defined by the school. In addition, such students must not only accept to model daily life situations using geometric deductions and see space through reason and logic that emerges from Euclidean geometry but also must recognize the importance of school mathematics to ensure the future, as stated by transnationals such as the OECD (2014)

> All adults, not just those with technical or scientific careers, now require adequate mathematics proficiency for personal fulfilment, employment and full participation in society. […]

> Students about to leave compulsory education should thus have a solid understanding of these concepts and be able to apply them to solve problems that they encounter in their daily lives. (p. 32)

Accordingly, school mathematics and geometry, in this particular case, are key tools for fabricating a productive citizenry. Therefore, high performance in geometry is desirable for modern society in order to make mathematically supported decisions, a desirable quality for a reflective citizenry (OECD, 2014). On the other hand, this training of the eye is desirable for the formation of the scientific minds of the future (i.e., the relationship between certain scientific discoveries and spatial visualization, as previously mentioned). At school, these processes occur in different ways – as evidenced in the analysis of school textbooks. Spatial visualization, which makes some scientific discoveries possible, remains restricted under parameters that can be measured and guided by teachers. Then, not only do students have to accept this negotiation on the space conceived by the school, but also in-service teachers must agree to teach these conditions to their students. The processes of subjectivation, or conduct of conduct, occur for both.

Under this understanding, subjectivity does not imply the repetition of an implicit agreement naturalized through the school curriculum but rather that human beings become subjects through the objectifying effects of scientific knowledge (Foucault, 1982), that is, the practice of knowing generates power effects in the subject that is knowing (Daston & Galison, 2007). In school, such subjectivation occurs to fabricate scientific thought. In this sense, Euclidean geometry allows us to approach a geometry that occurs in the mind, in the Platonic world of ideas, where objects are not achievable in everyday life except through formulas. In school geometry, for example, the notion of a point is characterized as a massless object that does not exist outside of its definition. By contrast, in everyday life, a point can be physically manipulated.

In this way, Euclidean geometry is not just a particular way of looking at space; it is much more than learning a series of mathematical concepts and rules (see Andrade-Molina & Valero, 2015; Andrade-Molina, Valero & Ravn, 2018). The school curriculum is structured in a similar way to that in which The Elements are presented. Euclid's Elements display a deductive system based on postulates, proofs, and demonstrations; they become a concrete model of the scientific method. In other words, The Elements becomes a template for the path to becoming a scientist. It is not because schools seek to train scientists, but rather that students are expected to follow these paths (which the school perceives as the approximation of a scientific being) to think based on logical and deductive parameters and solve problems based on reason. This is evident when analyzing the type of geometric problems that are posed to students.

The questions that such students face allow the deployment of a pattern in which key prior knowledge is activated to answer questions with progressive cognitive difficulty in which they must apply the mathematical conditions (definitions, theorems, formulas, among others) that were previously presented to them. However, the actions and strategies they can employ are limited to the conception of space in the classroom, a space where everything can be approximated to the closest flat figure or geometrical three-dimensional shape and calculating the area and perimeter of objects, ratios, isometric transformations, and vectors is enough for such students to solve more complex daily life problems. Here I refer to problems like the following (see figure 15).

Figure 15

Surface of a semisphere. (Díaz et al., 2020, p. 104)

ACTIVIDADES DE PROFUNDIZACIÓN

ARQUITECTURA

9. En los domos geodésicos se utilizan triángulos isósceles para construir con una forma similar a una semiesfera. Se quiere determinar la cantidad de material necesario para cubrir el piso y las paredes de un domo. Este se asemeja a una semiesfera compuesta por 60 triángulos isósceles y su base es aproximadamente una circunferencia de radio 1 m.
   a. Si el domo fuera una semiesfera, ¿cuál sería su área?
   b. ¿Cuál es el área de cada triángulo?
   c. ◆ ¿Cuánto material estimarías necesario para construir el domo?

In this example, it is assumed necessary to conceive the dome as a hemisphere to calculate the needed material to build the dome; this is because students are working with said geometric shape in this lesson–the hemisphere. This allows questions such as "if the dome were a hemisphere, what would its surface area be?", which have meaning and relevance only in school. However, solving this problem implies that students must think logically and deductively; they must make hypotheses and formulations and validate their assumptions and calculations. It is here where school geometry becomes a government technology that allows conducting the students' ways of thinking and acting who agree to participate in these practices while thinking about obtaining a better future. That is, the scientific training of such students occurs to fabricate the desired citizenry for society moved by their own interest and determination, not by a curricular imposition of domination (it is in this sense that the effects of power occur, according to Foucault). Such students learn to train their eyes to see the environment framed in a Cartesian plane (Andrade-Molina & Valero, 2017) in which they calculate trajectories through vectors and navigate space through the eyes of reason and logic, fundamental features of the scientificisation of students (Andrade-Molina, 2017).

The discussion is not about whether or not spatial visualization should be included as part of the range of tools that students in STEM careers should possess. The discussion focuses on how school geometry becomes a government technology to insert students into the practices of the scientific method. These practices are deployed through school geometry to activate logical and deductive thinking to the detriment of spatial visualization and its connection with perception. If school intends to enhance the possibilities that students choose STEM careers, it is necessary to rethink the power effects of school geometry.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, MAM, upon reasonable request.

REFERENCES

Andrade–Molina, M. (2017). *(D)effecting the child: The scientifization of the self through school mathematics*. Published doctoral dissertation. Aalborg University Press.

Andrade–Molina, M. & Valero, P. (2017). The effects of school geometry in the shaping of a desired child. In H. Straehler–Pohl, N. Bohlmann, & A. Pais (Eds.). *The disorder of mathematics education–Challenging the socio-political dimensions of research* (pp. 251- 270). Springer.

Andrade–Molina, M. & Valero, P. (2015). The sightless eyes of reason: Scientific objectivism and school geometry. In K. Krainer, & N. Vondrová (Eds.). *Proceedings of the ninth Congress of European Research in Mathematics Education* (pp. 1551–1557). Charles University in Prague, Faculty of Education and ERME.

Andrade–Molina, M., Valero, P., & Ravn, O. (2018). The amalgam of faith and reason: Euclid's Elements and the scientific thinker. In P. Ernest & L. Kvasz (Eds.). *The philosophy of Mathematics Education Today*. Springer.

Agencia de la Calidad de la Educación [ACE]. (2020). *TIMSS 2019. Estudio Internacional de tendencias en Matemática y Ciencias. Presentación nacional de resultados.* Gobierno de Chile.

Clements, M. A. (2008). Spatial abilities, mathematics, culture, and the Papua New Guinea experience. In P. Clarkson & N. Presmeg (Eds.). *Critical issues in mathematics education: Major contributions of Alan Bishop* (pp. 97–106). Springer.

Clements, D. & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education, 14*(2), 133–148.

Daston, L. & Galison, P. (2007). *Objectivity*. Zone Books.

Díaz, E., Ortíz, N., Morales, K., Rebolledo, M., Barrera, R., & Norambuena, P. (2020). *Texto del estudiante. Matemática 2° medio.* (Edición especial para el Ministerio de Educación). SM.

Foucault, M. (1997). Technologies of the self. En M. Foucault & P. Rabinow (Eds.), *Ethics: Subjectivity and truth* (pp. 223–251). The New Press.

Foucault, M. (1993). About the beginning of the hermeneutics of the self: Two lectures at Dartmouth. *Political Theory, 21*(2), 198–227.

Foucault, M. (1988). Technologies of the self. En L. H. Martin, H. Gutman, & P. H. Hutton (Eds.), *Technologies of the self* (pp. 16–49). University of Massachusetts Press.

Foucault, M. (1984). The ethics of the concern of the self as a practice of freedom. In P. Rabinow (Ed.). *Ethics, subjectivity and truth, essential works of Michel Foucault, 1954-1984* (pp. 281–302). Penguin.

Foucault, M. (1982). The subject and power. *Critical inquiry, 8*(4), 777-795.

Fresno, C., Torres, C., & Ávila, J. (2020). *Matemática. Texto del estudiante. Primero medio.* (Edición especial para el Ministerio de Educación). Santillana.

Gobierno de Chile. (2018). *Aprueba bases administrativas, bases técnicas, anexos y contrato tipo de licitación pública ID N°592-12-LS18, sobre* *servicio personal especializado de elaboración de contenido de textos escolares destinados a estudiantes y docentes de educación parvularia, básica y media de establecimientos subvencionados del país y derecho de uso no exclusivo, año 2020.* Gobierno de Chile.

González, J. & Parra, D. (2016). Mercantilización de la Educación. Comentarios sobre la Reforma Educativa en Chile 2015. *Revista Enfoques Educacionales, 13*(1), 71–89.

Hauptman, H. (2010). Enhancement of spatial thinking with Virtual Spaces 1.0. *Computers and Education, 54*(1), 123–135.

Iturra, F., Manosalva, C., Romero, D., & Ramírez, M. (2019). *Texto del Estudiante. Matemática 7° básico.* (Edición especial para el Ministerio de Educación). SM

Kollosche, D. (2014). Mathematics and power: An alliance in the foundations of mathematics and its teaching. *ZDM, 46*(7), 1061–1072.

Mevarech, Z. & Kramarski, B. (2014). *Critical maths for innovative societies. The role of metacognitive pedagogies*. OECD.

Miller, A. I. (1986). *Imagery in Scientific Thought.* MIT Press.

MINEDUC. (2019). *Bases Curriculares, 3° y 4° medio*. Ministerio de Educación.

MINEDUC. (2016a). *Matemática. Programa de Estudio Séptimo Básico*. Ministerio de Educación.

MINEDUC. (2016b). *Matemática. Programa de Estudio Octavo Básico*. Ministerio de Educación.

MINEDUC. (2016c). *Matemática. Programa de Estudio Primero Medio*. Ministerio de Educación.

MINEDUC. (2016d). *Matemática. Programa de Estudio Segundo Medio*. Ministerio de Educación.

Naciones Unidas. (2015). *Transformar a nuestro mundo: la Agenda 2030 para el Desarrollo Sostenible.A/RES/70/1*. Naciones Unidas.

National Research Council. (2006*). Learning to think spatially: GIS as a support system in the K-12 curriculum*. The National Academies Press.

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator*, *34*, 2–29.

Osorio, G., Norambuena, O., Romante, M., Gaete, D., Díaz, J., Celedón, J., Morales, K., Ortíz, N., Ramírez, P., Barrera, R., & Hurtado, Y. (2019). *Texto del estudiante. Matemática 3° y 4° medio* (Edición especial para el Ministerio de Educación). SM.

Organisation for Economic Co-operation and Development [OCDE]. (2014). *PISA 2012 results: What students know and can do—student performance in mathematics, reading and science* (revised ed., Vol. I, February 2014). OECD Publishing. http://dx.doi.org/10.1787/9789264201118-en .

Organisation for Economic Co-operation and Development [OCDE]. (2012). *Connected minds: Technology and today's learners, educational research and innovation*. OECD Publishing. http://dx.doi.org/10.1787/9789264111011-en .

Presmeg, N. (2014). Contemplating visualization as an epistemological learning tool in mathematics. *ZDM*, *46*(1), 151–157.

Prieto, G. & Velasco, A. D. (2010). Does spatial visualization ability improve after studying technical drawing? *Quality and Quantity*, *44*(5), 1015–1024.

Sinclair, N., Bartolini Bussi, M., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., et al. (2016). Recent research on geometry education: An ICME–13 survey team report. *ZDM*, *48*, 691–719.

Sinclair, N. & Bruce, C. (2015). New opportunities in geometry education at the primary school. ZDM, *51*(3), 319–329.

Scott, D. (1995). Colonial governmentality. *Social Text*, *43*, 191–220.

Skordoulis, C., Vitsas, T., Dafermos, V., & Koleza, E. (2009). The system of coordinates as an obstacle in understanding the concept of dimension. *International Journal of Science and Mathematics Education*, *7*(2), 253–272.

Swoboda, E. & Vighi, P. (2016). *Early geometrical thinking in the environment of patterns, mosaics and isometries*. Springer.

Torres, C. & Caroca, M. (2019). *Texto del estudiante. Matemática 8° básico*. Edición especial para el Ministerio de Educación. Santillana.

Tversky, B. (2005). Visuospatial reasoning. In K.Holyoak & R.Morrison (Eds.). *The Cambridge handbook of thinking and reasoning* (pp. 209-240. Cambridge University Press.

Valero, P. & Garcia, G. (2014). Matemáticas escolares y el gobierno del sujeto moderno. *Bolema*, *28*(49), 491–515.

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2014). Finding the missing piece: Blocks, puzzles, and shapes fuel school readiness. *Trends in Neuroscience and Education*, *3*(1), 7–13.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, *101*(4), 817–835.

Whiteley, W., Sinclair, N., & Davis, B. (2015). What is spatial reasoning? In B. Davis & Spatial Reasoning Study Group (Eds.), *Spatial reasoning in the early years. Principles, assertions and speculations* (pp. 3–14). Routledge.