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A Quantitative Study of French 19th Century Geometry Textbooks

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ABSTRACT

Background: School textbooks constitute a valuable material of investigation in the history of education, which has developed in many disciplines, in connection with their history. In history of mathematics education, some studies are based on a qualitative analysis of a small number of textbooks, whereas others use quantitative arguments in addition. In this framework, my approach is to analyse the use of areas in French geometry textbooks of the 19th century, by articulating qualitative and quantitative methods, from a corpus that I have built. **Objectives:** In my study paper, I analyse information from statistical data collected on this corpus, and I put these results in perspective with other studies conducted in the history of school publishing and mathematics education. **Design:** My methodology consists in starting from hypotheses accepted in related studies, confronting them with my quantitative data, and then putting results into a qualitative perspective. **Setting and participants:** This work was carried out in conjunction with training on quantitative tools in human and social sciences provided by PROGEDO Loire in Nantes. **Data collection and analysis:** I first built up a database of geometry textbooks using a general source and exploited it statistically. Then, I studied specifically a more restricted corpus, focused on the notion of area, using multivariate analysis tools. **Results:** My results confirm some of the assumptions assumed in other studies (evolution of school publishing, porosity of primary and secondary education orders), challenge others (opposition between theoretical and practical geometry) and sometimes deepen them with additional information. **Conclusion:** This work answered questions but raised new ones, notably on links between sustainability, target audience and content on areas of a textbook.

Keywords: history of mathematics education; geometry; textbook; area; quantitative methods; 19th century; France.

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Um estudo quantitativo e qualitativo dos livros didáticos franceses de geometria do século XIX

RESUMO

Contexto: Os manuais escolares constituem um valioso material de investigação na história da educação, os quais se desenvolveram em muitas disciplinas, em conexão com a sua história. Na história da educação matemática, alguns estudos são baseados em uma análise qualitativa de um número restrito de livros didáticos, enquanto outros usam argumentos quantitativos. Nesse estudo, a pesquisa analisou o uso de áreas nos livros didáticos de geometria franceses do século XIX, articulando métodos qualitativos e quantitativos, a partir de um corpus construído. **Objetivo**: O objetivo foi analisar informações de dados estatísticos coletados nesse corpus, e colocar esses resultados em perspectiva com outros estudos realizados na história da publicação escolar e da educação matemática. **Design**: A metodologia consistiu em partir de hipóteses aceitas em estudos, confrontando-os com os dados quantitativos e, em seguida, colocando os resultados em uma perspectiva qualitativa. **Ambiente e participantes**: O trabalho foi realizado articulando ferramentas quantitativas em ciências humanas e sociais oferecidas pelo PROGEDO Loire em Nantes. **Coleta e análise de dados**: O primeiro passo foi a construção de um banco de dados de livros didáticos de geometria usando uma fonte geral e a exploração estatisticamente. Em seguida, foi examinado especificamente um corpus mais restrito, focado no tema das áreas, utilizando ferramentas de análise multivariada. **Resultados**: Os resultados confirmaram alguns dos pressupostos assumidos noutros estudos (evolução da edição escolar, porosidade das ordens do ensino primário e secundário), e, desafiaram outros (oposição entre geometria teórica e prática) e por vezes permitiram aprofundar informações adicionais. **Conclusão**: O trabalho respondeu às questões, todavia, levantou novas possibilidades, nomeadamente sobre as ligações entre sustentabilidade, público-alvo e conteúdo em áreas de um livro didático.

Palavras-chave: História da educação matemática; Geometria; Livro didático; Área; Métodos Quantitativos; Século XIX; França.

INTRODUCTION

In the 19th century, French educational structures gradually developed from Guizot law in 1833 establishing free public primary education, through the Ferry laws of 1880-82, which made primary school compulsory, until the 1902 reform, proposing substantially modified curricula for a more varied secondary education (Chapoulie, 2019; Mayeur, 1981). This evolution necessitated the development of appropriate teaching contents at all levels of primary (schools, upper primary schools and primary teacher training schools), secondary (middle and high schools) and even vocational education, particularly in mathematics (Belhoste, 1995a; D'Enfert, 2003b).

These successive reforms have required a more efficient organization of the educational publishing world, to be able to provide a varied offer of textbooks, and in large quantities, to an increasing number of students. This editorial evolution over time is documented by Jean-Yves Mollier in his essay on the history of French publishing enterprises, *Une autre histoire de l'édition française* (Mollier, 2015), and in Alain Choppin's history of school textbooks (Choppin, 1992). They both describe waves of textbook publication in the 19th century, which will be compared to our own data.

The history of mathematics education has been particularly renewed in the last decades and, in this context, textbooks constitute relevant sources of study[1]. This is particularly obvious in recent work in the history of mathematics education (Bernard & Proust, 2014; Karp & Schubring, 2014), although according to Alexander Karp "it would not be wrong to say that historical studies in these topics [organization of the instructional process, teaching practices, and the function of textbooks] in many countries are only beginning" (Karp & Schubring, 2014, p. 13). In this framework, geometry textbooks are interesting sources insofar as they are related to the current school curricula and to the mathematical production of their time (Choppin, 1992, p. 19). That is why I chose geometry textbooks to analyse how the notion of area and its uses were supposed to be taught in the 19th century in France.

The following issues examined here are first, how has the number of published geometry textbooks changed over time and for what reasons? Second, who are the authors of these textbooks, who are their target audiences, and who is writing for whom? Last, can we characterise geometry textbooks dealing with areas, according to common features, and possibly group them into clusters?

My goal is to extend recent work questioning certain presuppositions about the history of education and investigating some links of causality or opposition that are generally accepted. For example, I would like to assess the link between the production of geometry textbooks and institutional prescriptions, as stated by Alain Choppin (Choppin, 1986). I also question the

[1] There are, however, older studies of textbooks, such as Hugo Grosse's research on calculation books (*Rechenbücher*) in 1901 (Grosse, 1901) or Jean Dhombres' 1985 work on French mathematical textbooks (Dhombres, 1985).

supposed imperviousness of the two orders[2] of education, primary and secondary, through the prism of the teaching of area geometry, as Renaud D'Enfert has done on the subject of local teaching offers and the circulation of teachers (D'Enfert, 2013). In particular, in the continuity of Valérie Legros' study of arithmetic textbooks (Legros, 2019), I propose to consider the types of authors of geometry teaching books, their audiences, and to cross-reference the data obtained. Finally, I would like to confront my corpus with the current hypothesis of a possible opposition between a so-called practical geometry, linked to measurement actions, as exposed in the work of Renaud D'Enfert (D'Enfert, 2003a), and a geometry commonly designated as theoretical[3].

An exhaustive survey of geometry textbooks between 1811 and 1902, based on the records of the *Bibliographie de la France* is presented in this paper, with more than 2000 titles identified. After having exposed the statistical analyses of this corpus, giving information on geometry textbooks in general, I will explain the constitution of a more reduced sub-corpus of 39 books, allowing to study the place of areas in the geometry taught in the 19th century.

The quantitative methodology that I applied to the whole corpus and to the more restricted one was intended to give me some elements of answers to my questions. In her 1999 article, Catherine Goldstein analysed the potential and limitations of quantitative methods in the case of the history of number theory (Goldstein, 1999)[4]. She showed their legitimacy, effectiveness, and heuristic character, provided that they are used in a reasoned manner, by explaining the construction of the sources to which they are applied and by permanently linking the quantitative to the qualitative. More recently, Karp argued that in the history of education, quantitative methods are "feasible and useful" but that they should not be pitted against other types of methods and

[2] "Order" is the most literal translation of the word "ordre" in French, which refers to the two distinct and autonomous school systems of primary and secondary education. Indeed, the terms "cycle" or "level" do not correspond to this reality.

[3] The history of the distinction between theoretical and practical geometry is traced by Dominique Raynaud in the book he recently edited (Raynaud, 2020, p. 9‑20). Similarly, Marc Moyon reminds us of the medieval origin of the distinction between scholarly geometry on the one hand and the *Practica Geometricae* (practical geometry) linked to measurement on the other, despite the obvious interactions between these two traditions (Moyon, 2017, p. 15‑16).

[4] Catherine Goldstein herself refers to the work of Jroslav Folta and Luboš Nový of 1965 (Folta & Nový, 1965), initiating the quantitative approach in the history of mathematics.

assumed to be more reliable (Karp & Schubring, 2014, p. 19). It is in this spirit that I conduct my study based on statistical inquiry, relying on quantitative methods developed for several decades in the humanities, and particularly in history (Lemercier & Zalc, 2019). Furthermore, this methodological work aims to show the fruitfulness of a quantitative approach to the history of education, but also to discuss its limitations.

GEOMETRY TEXTBOOKS PUBLISHED BETWEEN 1811 AND 1902: QUANTITATIVE EVOLUTION, AUTHORS, AND AUDIENCES

In this section, the evolution over time of the number of geometry textbooks published will be analysed, to measure the impact of school laws and reforms on the editorial production. The sociological aspects of geometry textbooks will also be portrayed, by studying the professional categories in which their authors find themselves, as well as their school and non-school audiences, according to the order and level of education. Finally, the cross-referencing of these data will make it possible to know who writes for whom. To this end, I will apply univariate and bivariate statistical analysis tools to a global corpus of geometry books for teaching purposes.

Nature of the source and collection of data

The *Bibliographie de la France* is the first French national bibliography, created in 1811, which lists all the publications entered by legal deposit until 1971 at the Bibliothèque Nationale de France under a form of an annual volume (Seckel, 2011). Even if it appears irregularly over time, the *Bibliographie de la France* is a periodical which offers an annual summary issue containing all books published officially during the year.

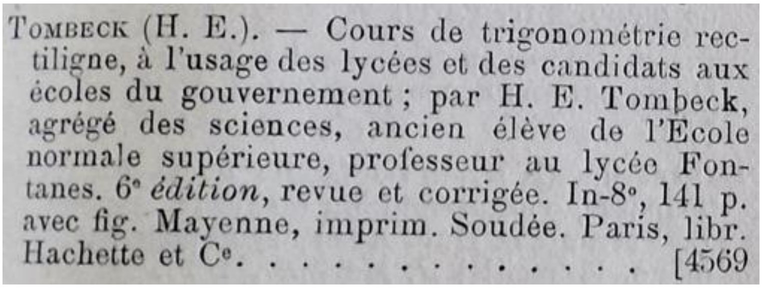
I systematically scanned these annual volumes, relying on their tables of contents when they existed, and created a database, including author's name, title (sometimes including target audience), professional and/or academic status (profession, academic credentials, or membership of an institution) and edition number.

Until 1856, there was a section "Mathematics", then from 1868, two categories "Mathematical Sciences" and "Scientific Education". For the other years, I searched by keywords in the titles, such as "geometry", "surveying", "levelling" or "trigonometry" (Cercle de la librairie & Bibliothèque nationale, 1811). Then, among this list, the teaching books were located either by the table of contents (in particular, from 1868 onwards, under the heading "science teaching") and/or by key words in the title or in the references of use, such as "courses", "lessons", "for the use of pupils of", "school", "high school", "college", etc. The resulting database included 2328 geometry textbooks published between 1811 and 1902 (including reprints) and referenced in the *Bibliographie de la France*.

A typical entry in the Bibliographie de la France (Figure 1) do not always include information on the author's status: among the 2328 titles of geometry textbooks, 1831 have such a designation of their authors[5]. Similarly, target audiences are often, if not always explicitly given, at least increasingly so over time. The information actually available was supplemented by consulting official sources, such as curricula (Belhoste, 1995; D'Enfert, 2015) or official lists of textbooks authorized for use in classrooms with a specified level (Giraud, 1851; LO, 1851). Finally, target audiences' data were found for 1694 of the 2328 textbooks surveyed.

Figure 1

Entry for Henri-Étienne Tombeck's (1827-1878) Cours de trigonométrie rectiligne*. Source: Bibliographie de la France, 1898, 85th year, 2nd series, volume XLII, p. 283.* (Cercle de la librairie & Bibliothèque nationale, 1811)



TOMBECK (H. E.). — Cours de trigonométrie rectiligne, à l'usage des lycées et des candidats aux écoles du gouvernement ; par H. E. Tombeck, agrégé des sciences, ancien élève de l'Ecole normale supérieure, professeur au lycée Fontanes. 6e *édition*, revue et corrigée. In-8o, 141 p. avec fig. Mayenne, imprim. Soudée. Paris, libr. Hachette et Ce. . . . . [4569

[5]In some cases, the name itself indicates the status of the author, such as a military rank, or an acronym for religious members of a congregation (such as the "Frères des écoles chrétiennes", *i. e*. Friars of the Christian Schools).

Number of geometry textbooks published and evolution over the period

In his book, Mollier recounts the rise to power of certain publishers, such as Hachette, brought about by the Guizot law of 1833 and the orders for textbooks then received from the ministry[6]. For example, Vernier's *Petite arithmétique raisonnée* (Vernier, 1834) was ordered from Hachette in 55,000 copies in 1832-33. It was later included in the list of recommended books for teaching arithmetic in elementary schools. Moreover, the number of so-called “classical” titles edited or reedited by this publishing house per year averaged 14 titles between 1827 and 1832, then 58 until 1839 (Mollier, 2015, p. 133–160). Alain Choppin confirms this first great wave of textbook publishing in the 1830s and also describes "a second boom" at the beginning of the Third Republic (Choppin, 1992, p. 54-70), which Mollier refers to as "the second take-off of school publishing" (Mollier, 2015, p. 152). During the 1880s, school book publishers multiplied and the peak of school publishing was reached with about 800 titles per year by the end of the decade (Mollier, 2015, p. 154). It was during this decade, more precisely in 1882, that Paul Leyssenne saw his arithmetic textbook for schools *L'année préparatoire* printed in 140,000 copies (D’Enfert et al., 2017, p. 65-66).

The production of geometry textbooks is fully in line with the overall educational context described in the first section. The 2328 referenced titles (including reprints) correspond to an average of 25 publications per year. As shown in Figure 2, the number of textbooks published in geometry increases globally over the period, with a maximum in the decade 1880 with 250 books published in 5 years. The graph even shows three peaks, the first after 1833 (year of Guizot’s law), the second after 1852 (Fortoul’s reform) and the third, the highest, after the Ferry’s laws of 1880-82.

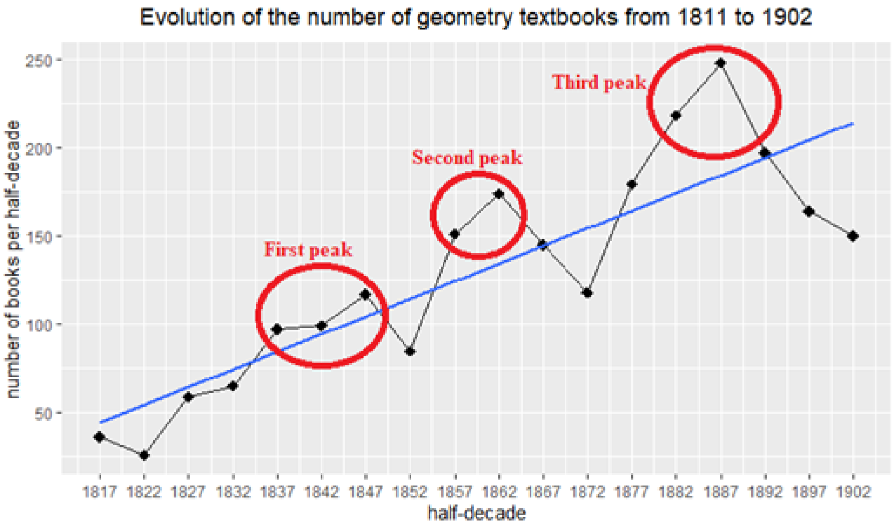
The orders of magnitude obtained, only for geometry textbooks, correspond to the general data found by Mollier and Choppin. Indeed, our data also include re-editions, unlike theirs. Above all, the two periods of growth in school publishing that they describe, in the 1830s and 1880s, are clearly represented in my case. My data also show another increase in publishing in the 1850s, following the Fortoul’s reform. This obviously confirms the

[6] For Hachette, these orders are estimated at 5 million copies by its detractors, who blame it for its quasi-monopoly in connection with its collusion with the “Commission des livres classiques” (French Classical Textbooks Commission).

prescriptive nature of educational laws and reforms, which automatically result in a more intensive production of textbooks (Choppin, 1986).

Figure 2

Evolution of the number of French geometry textbooks published in the 19th century.



Professional environment of the authors of the geometry textbooks

In her study of arithmetic textbooks for elementary school in the 19th century, Valérie Legros notes that most of their authors work in the world of teaching, whether primary or secondary (Legros, 2019, p. 128-130). More generally, Alain Choppin observes that the writing of school textbooks is not a minor activity, since many teachers and, also, senior public education officers and academics have invested themselves in this activity (Choppin, 1992, p. 58□59). Is the same true for geometry textbooks, not restricted to primary education? From which professional field do their authors come and how can we analyse it?

To answer this, I decided to classify authors into six main fields: army, church, civil society (including engineers, architects, surveyors, artisans, farmers, etc.), administration of public education (school headmasters, rectors, inspectors, etc.), teachers of all levels and scientists who do not teach (Ph.D in science, members of scientific institutions, alumni of "Grandes écoles", etc.). These fields have been prioritized, so that an author only belongs to one of these categories, the army and the church prevailing over civil society, which prevails over the administration of public education, which prevails over teachers, who prevail over non-teaching scientists[7]. The goal of this hierarchy is to highlight authors from fields other than education and scientific institutions, even if they also teach.

It is to be noted that this information, taken solely from the *Bibliographie* records, is not always explicit and that the same term may cover different situations. For example, the source does not allow knowing if the mention "engineer" refers only to a degree, or to the real exercise of such a function. The choice of the proposed partition as well as its underlying hierarchy was therefore made with an awareness of these limitations.

A first sorting according to these criteria shows that a little more than three quarters of the textbooks are authored by members of the public education system (about 60% by teachers and 18% by administrators). One book in ten is written by a member of the clergy, about 5,4% by members of civil society, roughly 4% by scientists, and the rest by the military (Figure 3).

The commitment of education professionals and, to a lesser extent, of the scientific community, noted in previous studies, is confirmed in the writing of geometry textbooks for which they represent almost 82% of the authors. My statistical study also shows three categories of authors who are in the minority, but nevertheless present. Clerics were identified as a significant community of authors by Legros for books on arithmetic, despite the difficulty of quantification due to the anonymity of many of them (Legros, 2019, p. 126-127).

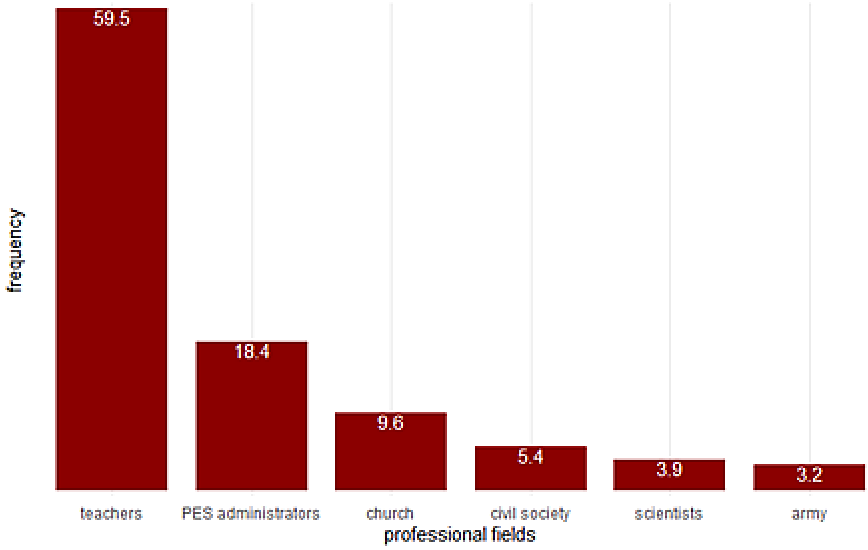
Authors from civil society and military who emerge in my study seem to be more specific to a potentially higher level of education and to a subject, geometry, more related to concrete applications in professional fields. For example, military authors can be found writing books on geodesy, topography, or trigonometry (in relation to navigation), or preparing for entry into military schools. Similarly, authors from civil society are often interested in applied

[7] Thus, an author who is both a teacher and a school headmaster will be referenced in the public education administration, an engineer who teaches will be classified in civil society.

mathematics such as surveying, levelling, descriptive or analytical geometry, taught at higher levels.

Figure 3

Professional fields of geometry textbooks authors.



Target audiences for geometry textbooks

In the 19th century, French primary and secondary orders are not meant to the same audiences. The primary order was attended by children from age 6 to over 18 and from lower social backgrounds: it included village schools, upper primary schools, and primary teacher training schools (*écoles normales primaires*). The secondary order was attended by children of higher social categories: it corresponded to middle and high schools of cities, including elementary classes for the youngest pupils. These two systems are generally considered impermeable in the history of education, but recent research challenges this and tends to show their porosity.

Briand and Chapoulie describe the official resurrection of upper primary schools in 1880, breaking with the teaching of high schools. The pitfall to be avoided was that they "became a counterfeit" of the special secondary education developed in the high schools to offer a more scientific and applied alternative to classical studies. In reality, these two systems were brought closer together in the field, as shown for example by the convergence of the two curricula at the Lycée Chaptal (Briand & Chapoulie, 2012, p. 369-382).

This was also noted by D'Enfert when he studied the local provision of mathematics education in Troyes during the first half of the 19th century (D'Enfert, 2015). Teachers practising at the college of Troyes were also solicited to give lessons in the upper primary and primary teachers training schools of the area, and imported methods from secondary education, in particular in geometry, whereas primary education curricula of 1833 had been created in a context of differentiation with those of the secondary order, both in methods (concrete pedagogy) and in aims (mainly practical) (D'Enfert, 2003a). About arithmetic textbooks, Legros notes that authors from secondary education wrote for a primary audience, and some for both, like Tarnier and Vernier. Nevertheless, the proportion of primary teachers writing books for primary school increased during the century (Legros, 2019, p. 128-130).

Table 1

Audience's categories, description and repartition by order and level.

Audience (abbreviation)	Description	Freq. (in %)
pro	professional education, in or out of school	14.3
all	when it says 'for the use of all educational institutions'	6.7
elem	elementary education, that means elementary classes of colleges and high schools	2.0
class	classical secondary education in colleges and high schools, with Latin and Greek	43.0
spe	special secondary education in colleges and high schools, without Latin and Greek but with more science	11.0
prim	primary education in schools	19.5
eps	upper primary school, 'école primaire supérieure' which correspond to the upper classes of primary schools	11.8
enp	'école normale primaire', i.e. training school for primary teachers	9.4
high	higher education	26.0
girls	female education, whatever the level or order	0.7

Distribution by order	Included categories	Freq. (in %)
Primary Order	prim, eps, enp, all	36.4
Secondary Order	elem, class, spe, all	53.1
Other	high, pro, girls	23.5

Distribution by level	Included Categories	Freq. (in %)
Level 0	pro	17.0
Level 1	all, prim, elem	24.0
Level 2	eps, class, spe	46.0
Level 3	high, enp	13.0

I analysed and cross-referenced data in my corpus to question this contrasting relationship between the two orders. Contrary to professions, and as shown in the previous example, the audience of a book can be multiple, and I have chosen to keep this information as rich as possible. Each textbook has therefore been classified according to one or more of the following categories: "pro", "all", "elem", "class", "spe", "prim", "eps", "enp", "high" and "girls", as described in Table 1. Therefore, the total number of frequencies exceeds 100, as a book can be aimed at several audiences.

This multiple designation makes it possible to classify textbooks by teaching order and by level. I was first interested in the distribution of books according to whether they were aimed at primary or secondary school, or possibly another audience (see “Distribution by order” in Table1). The results of this study are shown in Figure 4.

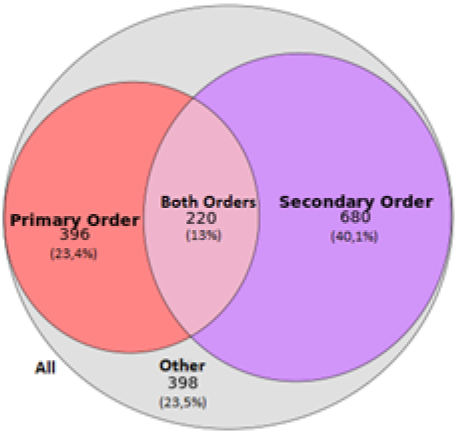
This diagram shows that primary and secondary orders have a large common part in terms of geometry textbooks, with 220 titles for students of both orders, *i.-e.* 13% of the production over the studied period. The large number of books for other audience is explained by the presence of many books for higher education as well as books for professional education. This defeats the preconceived idea of a total separation of the two orders in French educational systems, since authors of geometry textbooks do not hesitate to write books that can be used in both primary and secondary orders.

Figure 4

Distribution of geometry textbooks by order of instruction.

Numbers and frequencies of geometry textbooks by targeted order of instruction

Orders	Numbers	Frequency
Total primary order	616.0	36.4
Total secondary order	900.0	53.1
Total primary or secondary order	1,296.0	76.5
Total primary and secondary orders	220.0	13.0
Total other	398.0	23.5



As for the distribution by levels, it can be done according to the following distinction: level 0 (minimum required non-school level), level 1 (minimum required primary or elementary level), level 2 (minimum required secondary or upper primary level), and level 3 (minimum required higher level), as described in Table 1. This distribution shows that the most represented level in geometry textbooks is level 2 with nearly 800 books, *i.-e.* just under half of the titles, followed by level 1 (400 books, or one in four), then level 0 (280 books, or 17% of the titles), and finally level 3 (220 books, 13%). Here again, the large number of level 0 books is explained by a bibliography intended for a professional and potentially non-academic audience.

It is intermediate education, for students aged 11 to 18, that generates the largest volume of geometry textbooks. This concentration of Level 2 geometry books is probably due to the stronger presence of this study object in the middle school curriculum (Belhoste, 1995; D'Enfert, 2003b). To my knowledge, no quantitative study on mathematics textbooks has been carried out in this sense, most of them targeting a specific level, like the one by Valérie Legros (Legros, 2019).

Who writes for whom?

In 2017, in her chapter "History of teaching, history of mathematics: a recurrent fruitfulness", Hélène Gispert showed the richness of studies proposing to "think collectives of actors" (D'Enfert et al., 2017, p. 13–37). In this regard, she mentions various prosopographical studies mainly concerning teacher collectives. Indeed, these studies have become more and more numerous over time. Already in the 1990s, such an approach was the basis of the study "British mathematics a database", in which various data on English educational structures and their actors (teachers and students) from 1860 to 1940 were collected and analysed. This work gathered biographical data on scientists and on the courses taught but not specifically on textbooks (Barrow-Green, 1996). Legros' article also discussed the "Cirmath" project, a multidimensional study of mathematical circulation, focusing on journals and their audiences (Nabonnand et al., 2019).

In my study of geometry textbooks, authors are considered and analysed as a collective according to their professional backgrounds, as well as their potential audiences according to their education levels and orders. By cross-referencing these data, it is then possible to establish where these different educational actors meet, or in other words, who writes for whom.

The following table was produced by cross-referencing the authors' fields of practice and audience profiles for the 1473 books where both sets of information were available. Totals include all books with at least this category of audience. For example, the "total classical" includes all textbooks aimed at least in part at pupils in classical secondary education. Therefore, totals per line exceed 100%, as a book is potentially present in several categories.

Table2

Cross-tabulation of authors and audiences of geometry textbooks.

Authors' professional fields and audience										
	What is the target audience of the textbooks according to the professional fields of the authors?									
Professional fields	Total pro	Total all	Total elementary	Total primary	Total eps	Total classical	Total special	Total enp	Total high	Total girls
Army	**71.79**[1]	**0.00**	**0.00**	**0.00**	**0.00**	**2.56**	**0.00**	2.56	30.77	**0.00**
Church	**1.21**	0.61	**0.00**	**47.88**	**15.15**	37.58	0.61	**0.00**	**0.61**	**0.00**
Civil society	69.57	**13.04**	**0.00**	4.35	4.35	6.52	8.70	2.17	19.57	**0.00**
Education administration	4.79	10.86	1.60	25.88	14.06	**47.60**	**16.29**	12.46	17.25	**1.60**
Teachers	15.16	6.37	**2.78**	13.89	11.34	46.41	12.15	9.72	32.18	0.58
Scientists	4.35	4.35	**0.00**	10.87	10.87	39.13	2.17	**30.43**	**63.04**	**0.00**
Total	**14.26**	**6.65**	**1.97**	**19.48**	**11.81**	**43.04**	**11.00**	**9.44**	**26.00**	**0.68**

[1] Reading: About 71.79% of the books written by military authors are partly dedicated to a professional audience.

This table shows which categories of audience are over-represented among books written by a certain type of author. For example, in the first column, the row "Total" shows that books for a professional audience account for 14.26% of published textbooks, whereas these same books represent 71.79% of the titles published by military authors. It can therefore be said that military authors write preferentially for professionals, compared to other authors of geometry textbooks.

Similarly, books written for a primary audience are over-represented among religious authors (about 48% of books written by church members while they represent less than 20% of books published in total). Moreover, public education administrators seem to focus on secondary school audiences (classical or special) and scientists on books for higher education students from both orders (universities, engineering schools or primary teacher training schools).

The colour coding adopted in this table accentuates the under-represented categories, i.e. those whose partial percentage is lower than the total percentage (in blue) and those who are over-represented (in red). In addition, the maximum and minimum values for each category of audience have been highlighted in bold. This representation shows in particular that the classical secondary audience is under-represented among military and also among civil society authors, as is the professional one among clerics, educational administrators and scientists. The table confirms the interest of the military, but also of civil society in professional audiences. Religious people tend to focus on students in secondary schools, primary and upper primary schools, which are often the types of schools in which they teach. Education professionals (teachers or administrators) write more for secondary education, classical or special, but little for higher education or for professional education.

This study would tend to show that authors write for audiences they know, because they teach them, because they are part of them or because they themselves come from the same background. It also shows that certain audiences, undoubtedly more specific, are of interest to a smaller number of authors, such as the elementary classes of secondary schools (which probably use primary school textbooks), special secondary education or, even more markedly, girl's education, which at the time was very underdeveloped in geometry, according to the curricula (Belhoste, 1995; Decayeux-Cuvillier, 2019; D'Enfert, 2003b).

This analysis clarifies the function of the more specific authors of geometric textbooks, namely the military and civil society. It confirms the hypothesis that these authors are interested in parts of applied geometry since they are largely addressed to professional audiences, and at a rather high level since books for higher education are over-represented among military authors. It also specifies the role of religious authors, already highlighted by Legros in relation to arithmetic textbooks, by indicating more specifically the audiences they address, from the levels represented in the schools where they practice.

Textbooks in the corpus: specificities and similarities

I will now focus more specifically on a panel of 39 textbooks selected from the existing offer at the time, explaining the criteria by which I chose them, among the 2328 books listed in the "*Bibliographie de la France*".

Table 3

List of textbooks selected for the corpus.

Author	Title	Year	Prolific author	Bestseller	Curricula	Official Lists	Dep. Lists
Mauduit A. R.	Leçons de géométrie théorique et pratique	1790				X	
Lacroix S. F.	Eléments de géométrie	1808	X	X	X	X	
Legendre A. M.	Elémens de géométrie	1823		X		X	X
Dupin C.	Géométrie et mécanique des arts et métiers	1828	X		X	X	
Bergery C. L.	Géométrie appliquée à l'industrie	1828			X	X	
Terquem O.	Manuel de géométrie	1829				X	
Lancelot	Nouveau traité d'arpentage	1833		X			
Lefébure de Fourcy L.	Leçons de géométrie analytique	1834	X				
Desnanot P.	Pratique du toisé géométrique	1835				X	
Cirodde P. L.	Leçons de géométrie théorique et pratique	1836				X	
Lamotte M. L.	Traité élémentaire d'arpentage et du lever des plans	1839	X	X		X	
Sonnet H.	Nouvelle géométrie théorique et pratique	1839	X		X	X	
L. C. et F. P. B.	Abrégé de géométrie pratique, à l'usage des écoles chrétiennes	1840	X	X		X	X
Percin J.	Géométrie simplifiée	1842				X	
Idoux A. J.	Nouveaux éléments de géométrie	1842				X	
Vincent A. J. H.	Cours de géométrie élémentaire	1844				X	
Saigey J. F.	Eléments de géométrie d'après Bezout	1848			X	X	
Eysséric et Pascal	Traité de géométrie théorique et pratique	1850	X			X	X
Puille D.	Leçons normales de géométrie élémentaire théorique et appliquée	1855	X	X			X
Dupuis J.	Traité de géométrie appliquée, d'arpentage et de dessin linéaire	1857				X	X
Cirodde P. L.	Leçons de géométrie. . .	1858				X	
Briot C.	Arpentage, levé des plans et nivellement	1859	X			X	X
Amiot A.	Eléments de géométrie (6ème édition)	1860	X	X		X	X
Briot C. et Vacquant C.	Eléments de géométrie. Application	1862	X	X		X	
Harant H.	Géométrie appliquée aux arts et aux métiers	1863					
Briot C.	Eléments de géométrie. Théorie	1863	X				X
Rouché E., Comberousse C.	Traité de géométrie élémentaire	1866	X				
Bezodis A.	Notions élémentaires sur les courbes usuelles	1867	X			X	
Tarnier E. A.	Éléments de géométrie pratique	1872					
Méray C.	Nouveaux éléments de géométrie	1874					
Bos H.	Géométrie élémentaire	1875	X	X			X
André P.	Eléments de géométrie	1879	X	X			X
Bovier-Lapierre G.	Géométrie élémentaire, exposée dans ses applications	1879					X
Tombeck H. E.	Traité de géométrie élémentaire	1879	X	X			
Dalsème J.	Eléments de takymétrie	1880	X				X
Guilmin A.	Cours de géométrie élémentaire	1882	X	X			X
Bourget J.	Cours de géométrie	1885					X
Bert P.	Premiers éléments de géométrie	1886					X
Girod F.	Cours de géométrie théorique et pratique	1893	X	X			X

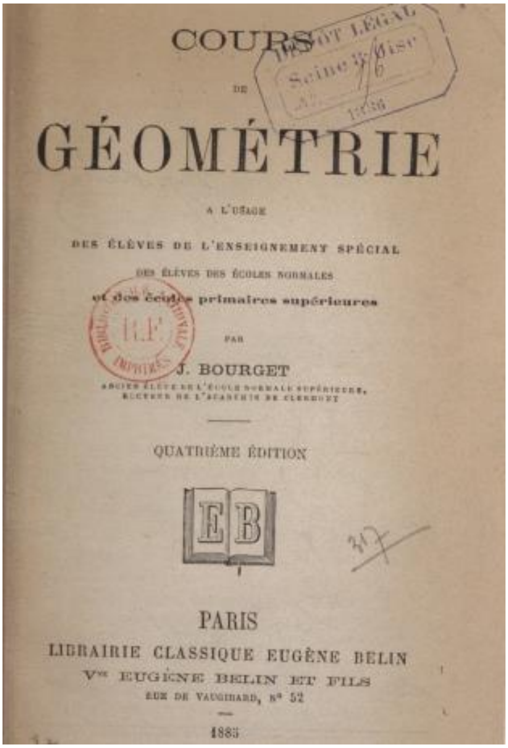
The selection criteria used are of an institutional and editorial nature, with an additional focus on content about areas. For example, textbooks cited in curricula were favoured, as were those on official lists of books to be used in the classroom and those on the French departmental lists drawn up by the teachers as being in use in their classes (LD1, 1883; LD2, 1883; LD3, 1883). The selection also includes Best Sellers, *i.-e.*, books that were very often reissued, and titles by very prolific authors over the period. All these textbooks have also been screened for their content on areas, which must be substantial enough to interest us.

Authors and audience of the restricted corpus

At this point, the idea is not to match the profile of the authors and the types of audience of the 39 books in the corpus to the distributions observed on all 2328 geometry textbooks, because the orders of magnitude of the numbers make the samples difficult to compare. However, I will look at these profiles on the restricted corpus to identify their specificities and to try to explain them.

Figure 5

Cover page of Bourget's Cours de géométrie (Bourget, 1885) *with indication of target audience.*



COURS
DE
GÉOMÉTRIE
A L'USAGE
DES ÉLÈVES DE L'ENSEIGNEMENT SPÉCIAL
DES ÉLÈVES DES ÉCOLES NORMALES
et des écoles primaires supérieures
PAR
J. BOURGET

QUATRIÈME ÉDITION

EB

PARIS
LIBRAIRIE CLASSIQUE EUGÈNE BELIN
V^ve EUGÈNE BELIN ET FILS
RUE DE VAUGIRARD, N° 52
1885

The distribution of audiences by level, using the same nomenclature as in the previous section, is as follows. Level 0: 10 books (26%). Level 1: 9 books (23%). Level 2: 19 books (49%). Level 3: 1 book (3%). Frequencies are comparable with the global panel on levels 1 and 2. Contrarily, level 3 is much less represented, in favour of level 0. This is because our study does not focus on higher education, but on intermediate education (secondary and upper primary).

This also affects the distribution by education orders. Indeed, in our corpus, only one book does not belong to any order, due to the exclusion of books dedicated to higher education. Books for both primary and secondary education are also more numerous in relative value (19 books, *i.e.* almost half of the titles), that even if books for higher and professional education were excluded from the overall panel. This notable difference can be explained by the fact that audiences are often better specified in books of the restricted corpus, for which I have the body of the text, and, in particular, cover pages, introductions, and prefaces.

In more detail, textbooks for mixed primary/secondary audiences are often those designated as "for use in all educational institutions" (Briot & Vacquant, 1859), or those included in official lists of books in use in both primary and secondary classes (Bergery, 1828; Legendre, 1794; Percin, 1842; Terquem, 1829). Others are fairly common combinations such as secondary education/primary teacher training (Cirodde, 1836) or special secondary education/upper primary education (Bourget, 1885; Bovier-Lapierre, 1879). These designations of use then correspond more to a logic of common level of the students or purpose of the teaching (more practical or more theoretical).

Importance of the theme of areas in corpus' textbooks

In her study of arithmetic textbooks, Valérie Legros mobilises the notion of "statement of knowledge" (Legros, 2019, p. 138). These statements include definitions, properties, explanations of techniques or methods, but also generic examples and exclude exercise and problem statements. She also discusses some of the formal characteristics of textbooks, such as their size. She points out, for example, that some long treatises are sometimes summarised in abridged form, and that increasingly normalised editorial rules make books more and more standardised (Legros, 2019, p. 133-136; 148). She also estimates the part of certain themes or concepts (numbering, operations, share of exercises) by counting the number of pages associated with them (Legros, 2019, p. 184-185; 212-214).

In my study on the teaching of geometry, I particularly focus on plane areas. This notion is situated at the crossroads of several key fields of mathematics such as arithmetic, geometry, algebra, and analysis. It thus constitutes a pertinent prism of study. Moreover, this object centralizes fundamental questions about numbers and quantities, and about the status of construction and demonstration in mathematics. From a pedagogical point of view, the learning of areas is part of the questions of the time between the more utilitarian or more educational aims of geometry teaching (Assude & Gispert, 2015; Buisson, 1882, p. 1162-1167).

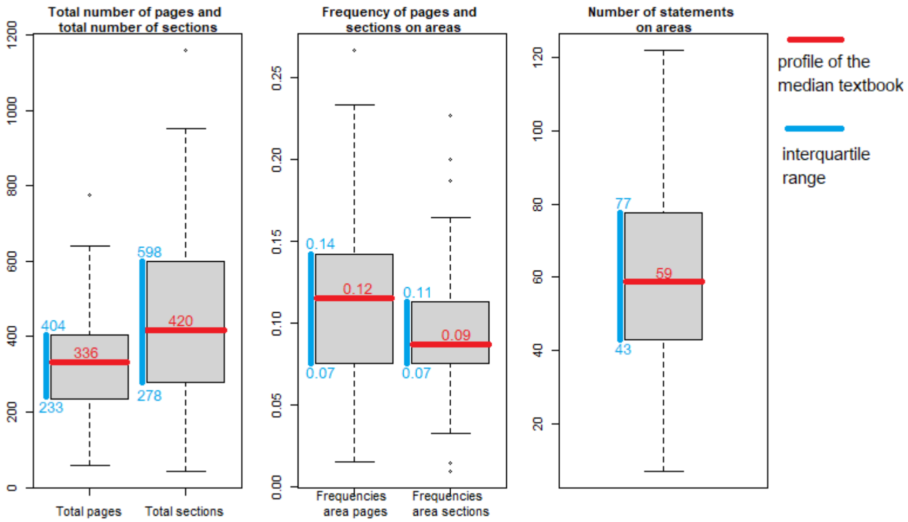
In my corpus of geometry textbooks, I systematized Legros' methods by quantitatively studying data such as the number of pages and sections (including the number of those concerning areas) and the number of statements on areas. After collecting them, I applied the main statistical indicators of position and dispersion (mean, median, quartiles, etc.) and then represented them in a box plot. This provided a typical profile of the form of geometry textbooks and permitted to measure the part of the theme of areas in their contents.

The median profile of a geometry textbook is a book with 336 pages and 420 sections, containing 12% of the pages and 9% of the sections on areas, and with nearly 60 statements dealing with this topic. A typical book of this median profile is Bos' one in 1875 (Bos, 1875) with 268 pages, 9% of which are on areas, 402 sections (11% on areas) and 61 statements. This representation also shows moderate heterogeneity, because while the ranges are large (from 44 to 1158 sections, for example), as are the inter-deciles gaps, the inter-quartile gaps (in blue on Figure 6) remain small. Indeed, half of the textbooks in the corpus have a number of pages between 233 and 400, a number of sections between 280 and 600, and contain between 7 and 14% of pages on areas (from 7 to 11% of sections) and from 43 to 77 statements on this topic.

On average, areas account for 10% of the volume of a geometry textbook, which is a significant but relatively small proportion. The study also highlights a common form for a large majority of the geometry books studied, despite more notable deviations at the margins. At the extremes, one find less typical books, such as Rouché and Comberousse (776 pages, 1158 sections, 5% on areas, more than 100 statements) (Rouché & Comberousse, 1866), or Dalsème (60 pages, 65 sections, a quarter of them on areas, 23 statements) (Dalsème, 1880).

Figure 6

Statistical indicators for textbooks in a box plot.



Multi-criteria cross-analysis of area theme weight and editorial longevity

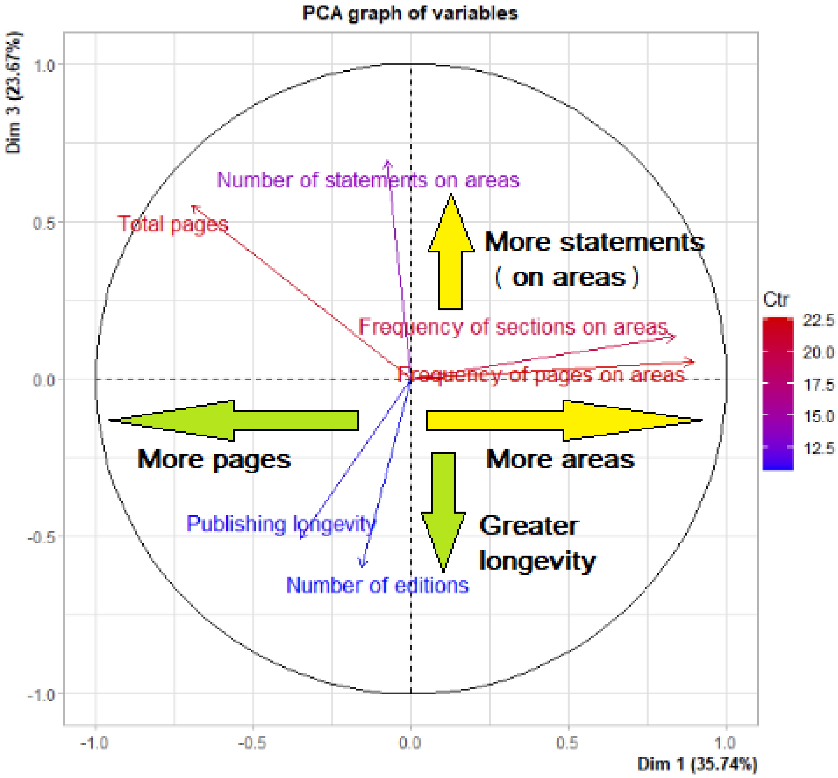
The analysis of quantitative characteristics studied in the previous section has drawn up a standard morphology of geometry textbooks, concerning their volume and the part they devote to the topic of areas. To go further, I had the idea of crossing this data with other quantitative characteristics, such as editorial longevity (difference between the last and the first year of publication of the textbook) and the number of reprints. To do this, I used one of the GDA (Geometric Data Analysis) techniques, the PCA one (Principal Component Analysis), which is adapted to quantitative variables[8].

[8] This method consists in representing variables as well as individuals in a plane and to summarise the information in two dimensions, whereas the real situation has many more. For example, in the following PCA, each of individuals is characterised by 6 variables, and must therefore be represented in 6 dimensions. PCA allows the best possible projection of the scatterplot of individuals along two axes by maximising

Figure 7 shows a double opposition between variables on horizontal and vertical axes. Indeed, books with the most content on areas in proportion are further to the right, while those with the largest number of pages are further to the left. Moreover, those containing the most statements are towards the top, while those with a greater longevity are towards the bottom.

Figure 7

PCA of the quantitative data of the corpus, graph of variables.



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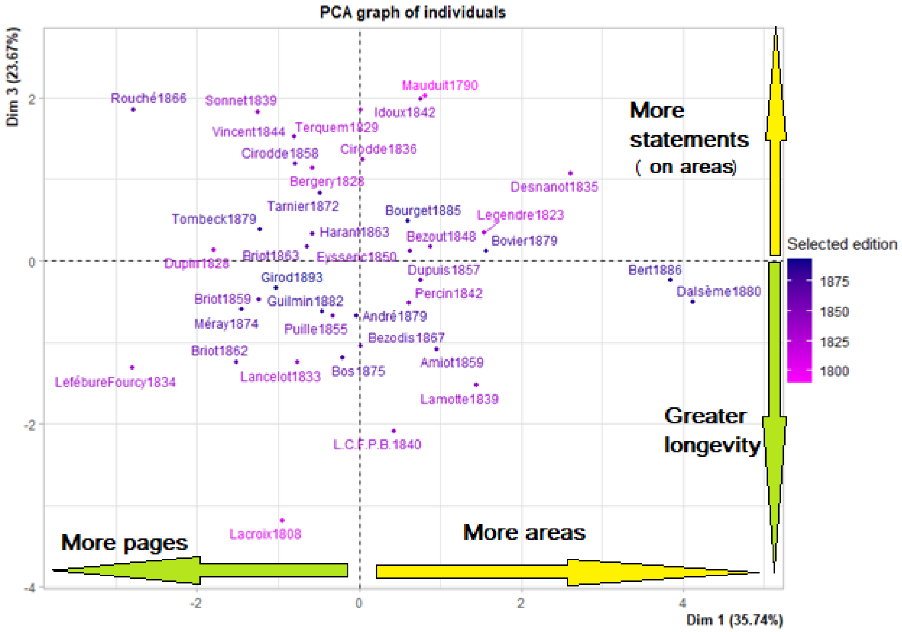
the inertia, *i.-e.* by allowing the best appreciation of the distances between individuals. Percentages on axes in Figure 7 indicate the inertia retained by the axes, in this case 35.7 and 23.7, *i.-e.* a total of almost 60%, which represents the share of the information summarised by the axes. This value is rather satisfactory in our framework.

In Figure 8, textbooks are distributed according to this double logic, such as Rouché's (Rouché & Comberousse, 1866), which is a very large book with many statements on areas (109), and a low number of editions (8) as well as its percentage of pages on areas, that is located at the top left. Similarly, Mauduit's (Mauduit, 1790) has only been reprinted three times, and has many statements on areas (104), but these account for 18.8% of the pages of the book. This explains its position at the top right. Lamotte's (Lamotte, 1839) is a small book (216 pages) with many pages on areas (22%), but few statements (40). On the other hand, it has been reprinted 20 times, and is therefore in the lower right-hand quarter. Finally, Lacroix's (Lacroix, 1808) is at the very bottom of the graph with 26 reprints over 115 years and a very modest number of statements on areas (41).

Figure 8

PCA of the quantitative data of the corpus, graph of individuals



Beyond this mapping, which can reveal proximities or, on the contrary, distances between our individuals on these criteria, this statistical method has raised new questions: why are the biggest books those containing the fewest parts on areas in proportion? why would books with the most statements about areas be the ones with the lowest overall longevity?

While I do not have definitive answers to these questions, I can nevertheless sketch out some explanations. Very large geometry textbooks often cover broad fields of geometry, such as spatial, projective, or spherical geometry. They are very comprehensive and therefore their parts on plane areas are not very high in proportion. These books do not offer any particular depth on the subject of areas, but rather develop other aspects of geometry.

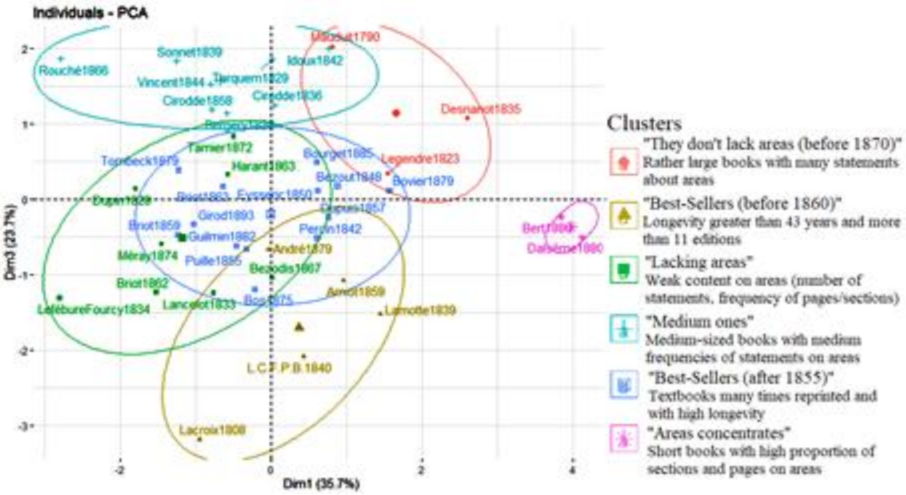
The opposition between longevity and a large number of statements on areas is more difficult to interpret. The presence of many statements undoubtedly indicates developments of points more on the margin, or even outside, of curricula. This is the case of Idoux' textbook, published only once, which includes 102 statements on areas (Idoux, 1842). It contains developments on minimum or maximum of isoperimetric figures (about twenty statements), notions which go beyond the official curricula. This lesser conformity to teaching contents standards could explain why this sort of textbooks do not persist over time. Less marked than the previous duality (contributions of these variables to axes are lower, according to Figure 7), it is contradicted by a few individual cases, such as André's (André, 1879), republished 38 times in 55 years, and presenting 108 statements on areas.

Basing on the weight of areas, the overall volume, and the longevity, I carried out a HAC (Hierarchical Ascending Classification) from the PCA presented and I identified an initial classification of our textbooks into six groups (Figure 9).

By statistical methods of analysis of variance, I could estimate the explanatory character of variables considered by a percentage. In our case, proportions of pages and sections on areas as well as the number of statements on this subject are the most characteristic of our partition. In Figure 8, the generational distribution of textbooks, highlighted by the purple shading, is not obvious. However, by calculating averages of each variable per cluster, I observed their chronological character, with the average years of publication practically increasing from cluster 1 to cluster 6. These two observations therefore legitimise the description of the clusters, by relative amount of area topic and time period, given above.

Figure 9

Classification of geometry textbooks on quantitative criteria.



Cross-analysis of audience, institutional scope, and "success" of textbooks

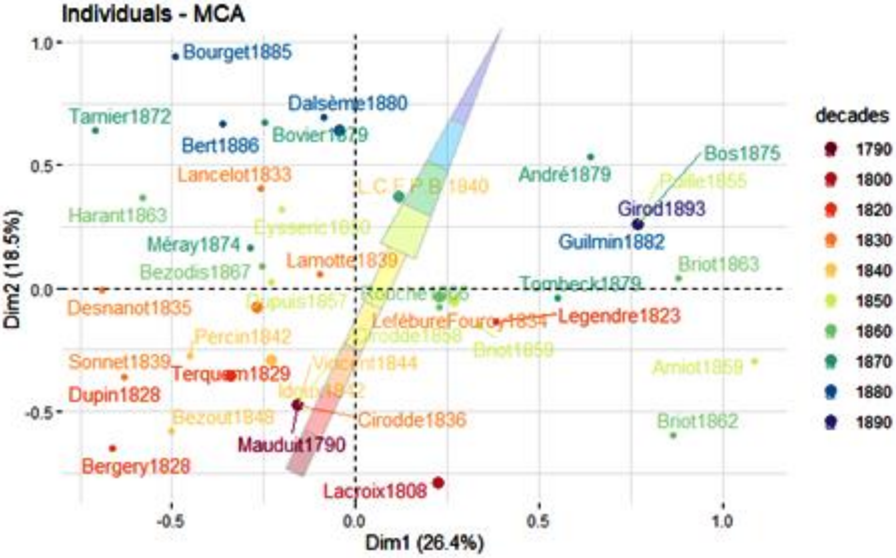
In her study, Valérie Legros highlights several successful arithmetic textbooks for primary audiences, such as those by Leyssenne, Father Bransiet (of the Friars of the Christian Schools) and Mutel (Legros, 2019, p. 164□173). In the same way, Jean Dhombres highlights the quasi monopoly of three authors of mathematics textbooks under the Empire: Lacroix, Bezout and Legendre. To do this, he relies on a quantitative study of the books published during this period, and in particular on their number of reissues (Dhombres, 1985). Moreover, we have seen that this success can be accentuated by an official recommendation from the public education authorities, as in the case of the Vernier's *Petite arithmétique*. For the geometry textbooks in my corpus, then, I observed the three characteristics of target audience, institutional reach, and editorial success together to see how they interact.

For this, I mobilised multivariate statistical methods of the GAD type, and more specifically an MCA (Multiple Component Analysis), adapted to variables of a qualitative nature. Indeed, my data here is Boolean in nature, depending on whether the textbooks belong to certain categories: citation in the official curricula, inclusion in the official lists for use in the classroom, inclusion in the departmental lists of works used by teachers, best-sellers, most prolific authors, classical secondary education, special secondary education, primary education (or upper primary education).

In this case, the map of individuals shows quite clearly an organisation of points by decades, in chronological order from the bottom left to the top right of the reference frame (Figure 10). The representativeness of axes is close to 45%. This is lower than in the previous PCA, which can be explained by the greater number of initial variables, generating a greater loss of information when passing through the map. The graph of variables is then of great help to interpret axes.

Figure 10

MCA of the qualitative data of the corpus, graph of individuals.

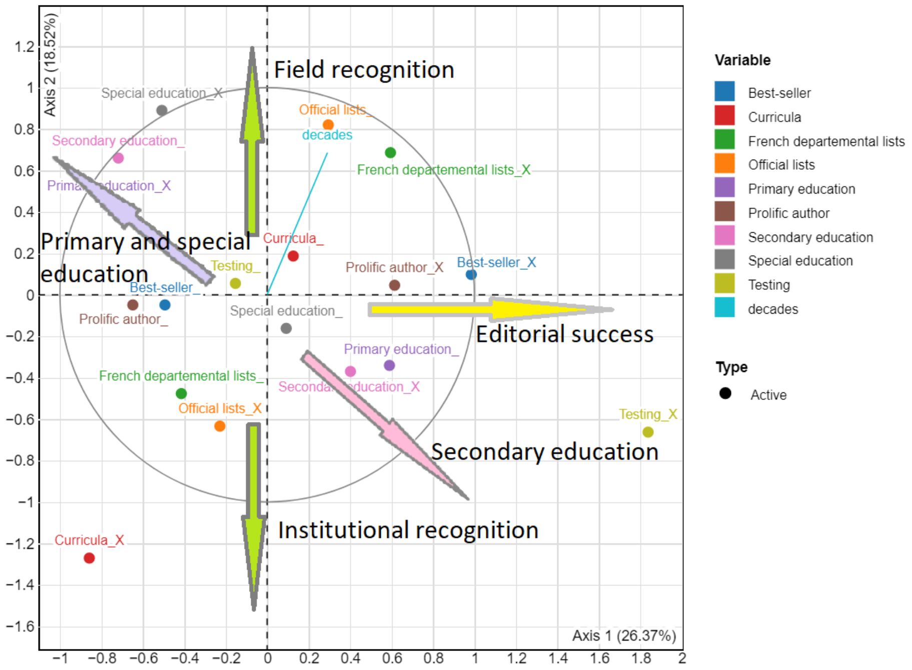


Indeed, the graph shows the positive character of editorial criteria on the horizontal axis, towards the right. Books on official lists tend to be at the lower end of the diagram, while those on French departmental lists compiled by teachers at pedagogical conferences tend to be at the higher end of the diagram. Another diagonal axis plays on the audience: primary and special at the top left, classical secondary at the bottom right (Figure 10).

The direction of the arrow indicating greater publishing success is closer to that of books for secondary audiences than that of textbooks for primary ones. However, the study of arithmetic books highlighted best-sellers for primary school (Legros, 2019, p. 106; 124). This seems to be different for geometry, which is more widely taught in the secondary education order. Similarly, the field recognition arrow is more related to books for primary (or special secondary) audiences. This is related to the nature of the source used, since these are the French departmental lists established by the teachers themselves during educational gatherings. Finally, the very distinct directions (horizontal and vertical) of arrows related to editorial success and institutional recognition show that this success does not seem to depend directly on the recommendations of authorities. This is the case, for example, of the Lancelot (Lancelot, 1833) or Rouché and Comberousse (Rouché & Comberousse, 1866) textbooks, not cited in the official documents consulted but which nevertheless enjoy good editorial representation.

Figure 11

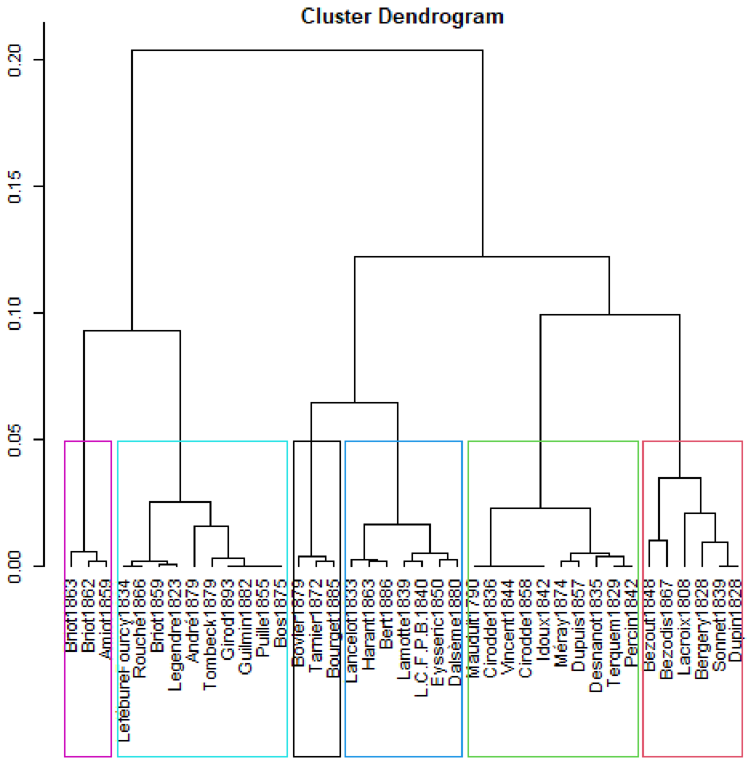
MCA of the qualitative data of the corpus, graph of variables.



This new mapping of textbooks in the corpus has been followed by a second classification, using CAH, which can be seen in the cluster dendrogram (Figure 12). The grouping observed at the end of this study is consistent with affinities that can be identified between books in a qualitative manner. For example, this classification groups together in the blue cluster more practical geometry titles aimed at a primary school audience like Lancelot's (Lancelot, 1833), Harant's (Harant, 1863) and Bert's (Bert, 1886). In the red cluster are books cited in official curricula, such as Sonnet's and Bezout's (Bezout, 1848; Sonnet, 1839) and Dupin's (Dupin, 1826). The pink cluster includes books that have participated in a classroom testing campaign. This is the case of Amiot's (Amiot, 1859) and Briot's (Briot, 1863; Briot & Vacquant, 1862).

Figure12

Classification obtained from the MCA.



These clusters can be compared with others based on qualitative criteria representing their content about areas, and I will analyse whether there is a match between these two classifications. For example, I will examine whether there is a typical profile of books (in terms of content) for a given audience, or whether books recommended by the institution have similar content.

Analysing textbooks by their content on areas: a work in progress

To continue and deepen the work of classification presented in the previous sections, I tried to compare the books of my corpus through their contents on areas, to measure the common features and the notable differences. This process is part of a strategy called "comparative approach" by Alain Bernard and Christine Proust in their introduction to their book about teaching contexts (Bernard & Proust, 2014). Here I present the first results of this study, which is a work in progress.

I focused first on similarities and dissimilarities between books in the corpus in terms of their statements on areas. To evaluate them, I used the statistical method of distance matrices, then visualised by a heatmap diagram. In concrete terms, I identified all knowledge statements about areas in each textbook in the corpus, gradually developing a common nomenclature. I then calculated an indicator of similarity between the books, by counting the number of statements they had in common[9]. This gave me a similarity rate, which, subtracted from 1, generated a dissimilarity rate, and therefore a distance, between books two by two. This method produced results that were difficult to use, probably because the study grid proved to be too precise, with a total number of statements identified of 445. It was therefore necessary at this point to group the statements by type to reduce the number of variables. I then referred to the classification of statements by Valérie Legros.

Indeed, she observed the methods used in arithmetic textbooks qualitatively, distinguishing theoretical from procedural knowledge. In the former, she included definitions, principles, theorems, propositions, demonstrations, and rules. In the others were practical calculation procedures and problems to be solved (Legros, 2019, p. 164–173). This distinction between theory and practice is common in geometry as well, as Teresa Assude

[9] The formula used was: 2*number of common statements/(number of statements in book 1 + number of statements in book 2).

and Hélène Gispert show in their 2015 article analysing the entry "Geometry" in Ferdinand Buisson's *Dictionnaire de pédagogie* (Assude & Gispert, 2015; Buisson, 1882). Practice, in both methods and objectives, seems to distinguish primary education from the more abstract and speculative secondary education.

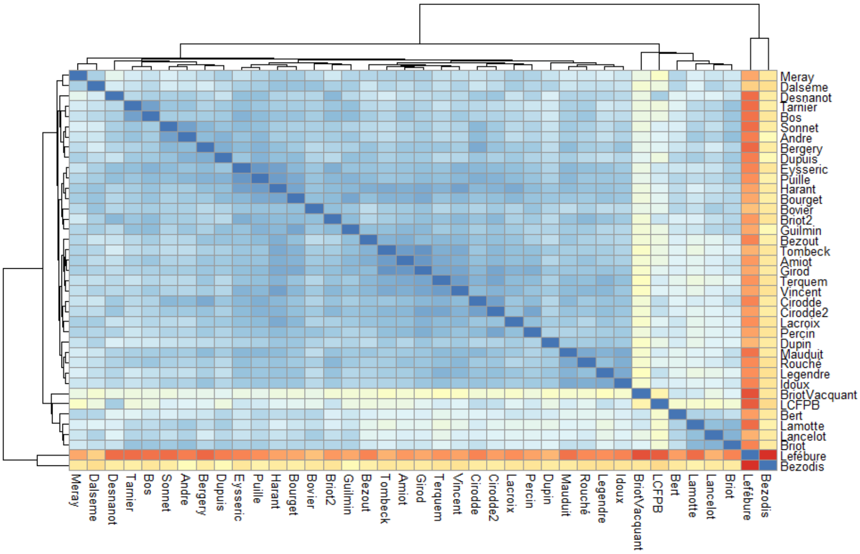
However, many titles of books on both arithmetic and geometry include the two terms “theoretical” and “practical”, showing that the frontier between the two is more porous than it seems. In my corpus, this is the case for 6 out of 39 geometry books, and in Valérie Legros' one, for 10 out of 63 arithmetic titles, which represents approximately the same proportion of 1 to 6. And, indeed, reading textbooks shows that theoretical statements coexist with more practical ones in their pages. In arithmetic books for primary education, Valérie Legros clearly indicates the scarcity of theoretical statements, without quantifying it (Legros, 2019, p. 164-173).

Therefore, I grouped statements by categories. On the one hand, I grouped the one of a theoretical nature such as definitions, theorems, principles, and proofs. On the other hand, I put together those of a more "practical" nature, distinguishing between those relating to calculations (formulae, numerical applications, calculation methods, for example) and those focusing on constructions (quadratures, partitioning, etc.). This typology highlighted 21 categories of statements, 9 of a theoretical nature, and 12 of a practical nature (5 concerning constructions and 7 concerning calculations). The percentage of statements of each type was calculated for each book and gave rise to a new distance calculation and a second heatmap diagram (Figure 13).

In Figure 13, the dark blue diagonal represents zero distances at the intersection of pairs of identical books. The warmer coloured cells show very different pairs, while the very colder ones show books with very similar contents. For example, Terquem’s book (Terquem, 1829) is very close to Tombeck’s (Tombeck, 1879) such as are the textbooks of contemporaries Esseyric and Puille (Eysseric& Pascal, 1850; Puille, 1855). Contrarily, Lefébure’s (Lefebure de Fourcy, 1834) is very different from all other books, as is Bezodis (Bezodis, 1867), to a lesser extent. This can be explained by the fact that they deal with subjects that are more marginal to the school curricula of the time, analytical geometry for the former and usual curves for the latter.

Figure 13

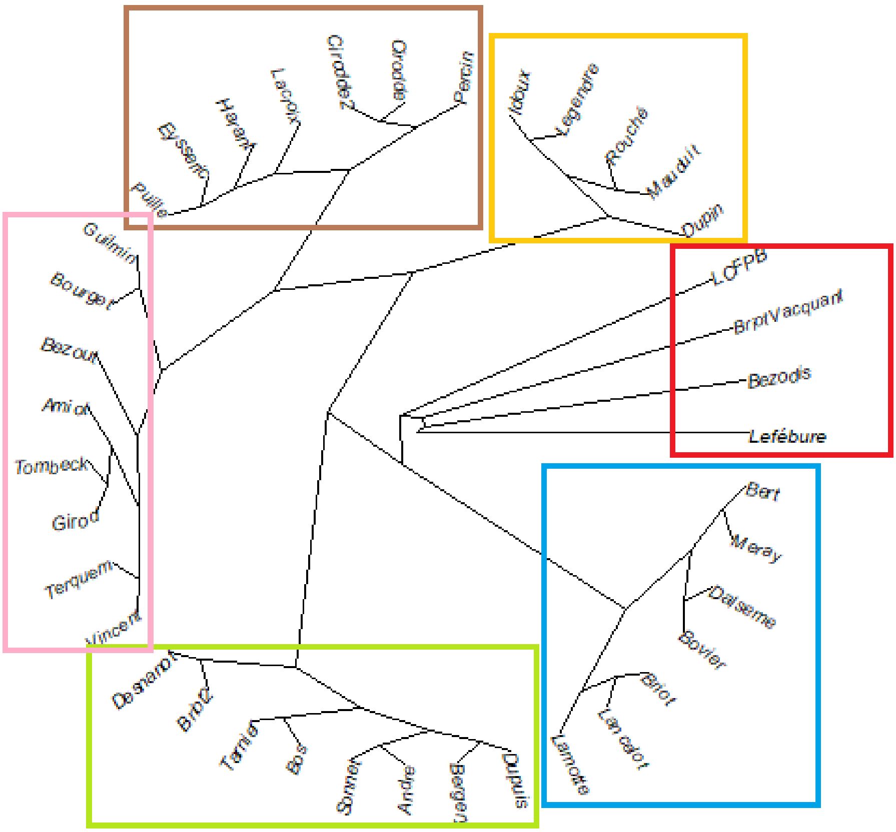
Heatmap of distances between books in the corpus.



I generated a new classification from this study, which can be seen in Figure 14. It should be noted that the cluster in red is a sort of false group, made up of individuals who cannot be matched with the other books, like that of the Friars of the Christian Schools (L. C. & F. P. B., 1840). Some of the groups generated by this method are clearly identifiable from the qualitative knowledge I have of these textbooks. This is the case, for example, of the yellow group, which contains books presenting an advanced theory of area optimisation of figures of given perimeter, for example Idoux's textbook (Idoux, 1842). Similarly, the blue cluster includes books on surveying (Lancelot, 1833) or geometry through experience (Méray, 1874) more for primary audience. As for the green cluster, it brings together books on practical or applied geometry, as their titles indicate (Desnanot, 1835; Dupuis, 1878; Tarnier, 1872). Finally, the group in brown gathers several of the authors who explicitly situate themselves in the double denomination of "theoretical and practical geometry" (Cirodde, 1836; Eysseric, 1850; Puille, 1855), a position that is not contradicted by the contents of the other textbooks of the cluster (Percin, 1842).

Figure14

Phylogenetic tree and clusters generated using the heatmap.



This last classification, which puts together books with very similar content on areas, is not at all the same as the two previously obtained from other criteria. In this sense, the hope of identifying identical or similar families of textbooks, regardless of the variable used, has not been realized. However, certain books remain systematically linked, whatever the classification, such as the author's trio Guilmin, Tombeck and Girod (Girod, 1893; Guilmin, 1882; Tombeck, 1879), the pairs Terquem and Vincent (Terquem, 1829; Vincent, 1844), Sonnet and Bergery (Bergery, 1828; Sonnet, 1839) or Dalsème and Bert (Bert, 1886; Dalsème, 1880). These small groups could be used as a basis for further analysis of similarities between books, perhaps using network modelling.

CONCLUSION

The use of quantitative methods in my research helped to answer my questions and test some commonly held assumptions. First, there is a real correlation between the editorial production of geometry textbooks and the official prescriptions, particularly following the reforms, as Mollier and Choppin pointed out for all French textbooks.

The imperviousness of the French primary and secondary education systems has also been questioned by different quantitative and qualitative studies. My results show that the line is far from being so clear-cut and that authors from one order do not hesitate to write for an audience from the other, or even for mixed readers, confirming the conclusions of Renaud D'Enfert's study (D'Enfert, 2003a). In fact, the target audience designated for the textbooks in our limited corpus is determined more by its level than by its order of education.

However, the analysis of my data beats the common distinction between theoretical and practical geometry, put forward in particular by Assude and Gispert (Assude & Gispert, 2015). There are certainly textbooks that are more theoretical and others that are more practical, but a significant number of them deal with both aspects. The classification by similarity of contents reveals a group of books specifically of this double nature.

In addition to confirming or not some of the initial hypotheses, this quantitative study brought to light patterns or new assumptions that I had not suspected, thus confirming the heuristic character of a quantitative analysis in the history of education[10]. In particular, this approach has made it possible to question the notion of publishing success by comparing it with other data, such as the size of the textbooks or the proportion of their content on areas. The opposite relationship between book longevity and content also raises new questions, as does the link between book size and number of area statements.

The exploitation of temporal data was able to highlight generational trends in some cases. The two first classifications of the restricted corpus, based on quantitative data, produced groups of works with a clearly chronological

[10] On the heuristic aspect of quantitative methods, see (Teissier et al., 2018).

character, notably for the second, from institutional recognition to field recognition. This raises the question of the existence of fashions in the form of geometry textbooks over time. However, this evolution must be put into perspective by considering the nature of the sources, emanating from official organisations in the first half of the period, and from practicing teachers in the second half.

The main topic of my research project is the teaching of plane areas in French geometry textbooks. This quantitative study has produced results beyond this specific theme. But despite the obvious limitations of some of the results obtained, especially on the statements contained in the textbooks, the quantitative approach remains a remarkable tool, if it constantly evolves in parallel with the qualitative aspects and by following the questions that it opens.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, C. G., upon reasonable request.

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