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Competence of Analysis and Reflection on the Management of Textbook Lessons by Pre-Service Teachers

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ABSTRACT

Background: Being able to assess what is happening in a teaching-learning process is one of the teacher's competencies. Teachers often must analyse and select the educational resources they consider relevant for their students. Since textbooks are an important tool for instructional design, the teacher must be able to analyse their suitability, identify limitations and make adaptations to overcome them. **Objectives**: This paper describes the design, implementation, and results of a training action with prospective primary education teachers, aimed at the development of competence for the analysis, identification of conflicts and proposals for the management of textbook lessons, particularly the content of proportionality. **Design**: The research is interpretative and exploratory, using content analysis to examine the participants' response protocols. **Setting and Participants**: The experience was carried out in the framework of the Primary Education grade; the sample consisted of 48 students. **Data collection and analysis**: Data were collected by observer/researcher annotations and participants' responses to the proposed assessment task. **Results**: The results show that trainee teachers make progress in identifying conflicts and make suggestions for improvement to increase didactic suitability in the different facets, but that they are not specific enough when describing effective conflict resolution proposals, especially in the epistemic and cognitive facets. **Conclusions**: For future teachers to become more proficient in the mode of use, it is necessary to reinforce their didactical-mathematical knowledge of proportionality.

Keywords: Teacher education; Didactical analysis and intervention; Textbook analysis; Proportionality.

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Competencia de análisis y reflexión sobre la gestión de lecciones de libros de texto por maestros en formación

RESUMEN

Antecedentes: Poder valorar lo que ocurre en un proceso de enseñanza-aprendizaje es una de las competencias del profesor. Este debe analizar y seleccionar los recursos educativos que considera pertinentes para sus alumnos. Dado que los libros de texto constituyen un importante recurso para el diseño instruccional, el docente debe ser capaz de analizar su idoneidad, identificar limitaciones y realizar adaptaciones que las solventen. **Objetivos**: Describimos el diseño, implementación y resultados de una acción formativa con maestros en formación, orientada al desarrollo de la competencia para el análisis, identificación de conflictos y propuestas de gestión de lecciones de libros de texto, particularizadas al contenido de la proporcionalidad. **Diseño**: La investigación es interpretativa de tipo exploratorio, y se aplica el análisis de contenido para examinar los protocolos de respuesta de los participantes. **Lugar y participantes**: La experiencia se realizó en el marco del grado de educación primaria; la muestra fue de 48 estudiantes. **Recogida y análisis de los datos**: La información se recopiló por anotaciones del observador/investigador y las respuestas de los participantes a la tarea de evaluación propuesta. **Resultados**: Los resultados muestran que los maestros en formación progresan en la identificación de conflictos y que plantean sugerencias de mejora para incrementar la idoneidad didáctica en las distintas facetas, pero que no son suficientemente específicos cuando describen propuestas efectivas de solución a conflictos, especialmente en las facetas epistémica y cognitiva. **Conclusiones**: Para que los futuros maestros adquieran mayor competencia en el modo de uso, es necesario reforzar sus conocimientos didáctico-matemáticos sobre la proporcionalidad.

Palabras claves: Formación de profesores; Análisis e intervención didáctica; Análisis de libros de texto; Proporcionalidad.

Competência de análise e reflexão sobre a gestão de aulas de livros didáticos por professores estagiários

RESUMO

Contexto: Ser capaz de avaliar o que acontece em um processo de ensino-aprendizagem é uma das competências do professor. Os professores devem analisar e selecionar os recursos educacionais que consideram relevantes para seus alunos. Como os livros didáticos são um recurso importante para o projeto instrucional, o professor deve ser capaz de analisar sua adequação, identificar limitações e fazer adaptações para as ultrapassar. **Objetivos**: Descrevemos a concepção, implementação e resultados de uma ação de treinamento com professores estagiários, orientada para o desenvolvimento da competência para a análise, identificação de conflitos e propostas para a gestão de lições em livros didáticos, particularmente no que diz respeito ao conteúdo da proporcionalidade. **Design**: A pesquisa é interpretativa e exploratória, utilizando análise de conteúdo para examinar os protocolos de resposta dos participantes. **Ambiente e participantes**: A experiência foi realizada no âmbito do grau do ensino primário; a amostra foi de 48 estudantes. **Coleta e análise de dados**: As informações foram coletadas das notas do observador/pesquisador e das respostas dos participantes à tarefa de avaliação proposta. **Resultados**: Os resultados mostram que os professores estagiários fazem progressos na identificação de conflitos e fazem sugestões de melhorias para aumentar a adequação didática nas diferentes facetas, mas não são suficientemente específicos ao descrever propostas eficazes de resolução de conflitos, especialmente nas facetas epistêmica e cognitiva. **Conclusões**: Para que os futuros professores adquiram maior competência no modo de uso, é necessário reforçar seus conhecimentos didático-matemáticos de proporcionalidade.

Palavras-chave: Treinamento de professores; Análise didática e intervenção; Análise de livros didáticos; Proporcionalidade.

INTRODUCTION

Many teachers identify textbooks as the institutional knowledge to be taught and learned; therefore, when planning instructional processes, they use them to decide which and how to present content or the methodology to follow (Rezat, 2012; Salcedo et al., 2018). However, how information is presented in textbooks can be an obstacle or an opportunity for students to learn the targeted content (Törnroos, 2005). For this reason, various studies focus on how teachers use these curricular materials (Remillard & Kim, 2017; Rezat, 2012; Shawer, 2017).

Authors such as Choppin (2011), Godino et al. (2017), and Thompson (2014) defend that teachers should acquire an analytical and critical position to, in first place, identify weaknesses or shortcomings in the instructional process proposed by the author of the book and second, reflect on how to solve them. However, the results of various research studies show that teachers have difficulties in carrying out these actions (Grossman & Thompson, 2008; Yang & Liu, 2019) and that, without tools to guide them in this task, their text analysis is often intuitive or partial and lacks a critical approach that would allow them to make reasoned decisions about how to manage its use (Beyer & Davis, 2012). Consequently, teacher education should consider the development of pre-service teachers' didactical competencies to analyse and manage textbooks by designing and implementing formative actions that incorporate specific instruments.

In this paper, we aim to study pre-service teachers' analysis of the suitability of mathematics textbook lessons on a particular topic and, based on their considerations, identify potential conflicts in the lesson and develop informed proposals on resource management.

As a theoretical framework, we use the onto-semiotic approach (OSA) of mathematical knowledge and instruction (Godino et al., 2007). Within this framework, a model of categories of didactic-mathematical knowledge and competencies (DMKC) has been developed (Godino et al., 2017), and theoretical-methodological tools have been devised to support the planning and implementation of formative activities that promote, among others, the competence of analysis and didactic intervention. This is oriented towards the design, implementation, and assessment of their own and others' learning sequences, which allows them make decisions about the changes that could improve the development of students' mathematical competence (Breda et al., 2017; Burgos & Godino, 2021; Burgos et al., 2020; Giacomone et al., 2018; Giacomone et al., 2019; Godino et al., 2017; Godino et al., 2018). Some of the specific interventions developed in the field of teacher education based on the DMKC model have used the didactic competence tool, its components and indicators (Godino, 2013) to organise teachers' systematic reflection on their own practice or that of others and to develop the competence to evaluate planned or implemented instructional processes (Breda et al., 2018; Font et al., 2018; Giacomone et al., 2018; Hummes et al., 2019; Morales-López and Araya-Román, 2020; Pino-Fan et al., 2013).

Some of these studies have dealt specifically with the mathematical content of proportionality (Burgos et al., 2018; Burgos et al., 2020; Castillo et al., 2021; Castillo & Burgos, 2022; Esqué & Breda, 2021). Despite the longitudinal and cross-cutting importance of this content in the primary and secondary curricula, as well as being the bridge to advanced mathematical thinking, proportionality is often not adequately addressed in school mathematics textbooks at both stages (Ahl, 2016; Burgos, Castillo et al., 2020; Shield & Dole, 2013). Student textbooks emphasise rote learning and avoid arguing about the conditions that characterise a situation of direct proportionality, which hinders the development of adequate proportional reasoning (Fernández & Llinares, 2011; Lamon, 2007; Riley, 2010). Furthermore, and not least, both pre-service and in-service teachers have difficulties in teaching concepts related to proportionality (Ben-Chaim et al., 2012; Berk et al., 2009; Buforn et al., 2018; Van Dooren et al., 2008). In this sense, authors such as Nicol and Crespo (2006) or Remillard and Kim (2017) suggest that it is possible to diagnose and correct these deficiencies by reflecting on the instructional processes envisaged in textbook lessons, generating meaningful learning in teachers.

Thus, Burgos and Castillo (2022) described the results of a training action with prospective teachers aimed at developing their competence to identify semiotic conflicts in a proportionality lesson from a primary school textbook. To assist with their analysis, we offered participants a guide for the analysis of proportionality lessons in textbooks (TLAG-Proportionality) (Castillo et al., 2022) based on the facets, components, and criteria of didactical suitability (Godino, 2013). As a result of its application, pre-service teachers identified conflicts related to the presentation of content, required prior knowledge and progression in learning, as well as modes of intended interaction and use of resources. However, other conflicts related to specific knowledge of proportionality went unnoticed by most pre-service teachers. Given that teachers' criticisms of textbooks may be related to their beliefs (Yang and Liu, 2019) and that these beliefs shape resource use (Lloyd, 2002), Burgos and Castillo (2022) posited the need to compare beliefs inferred from initial assessments (without having received specific training) with analysis, identification of conflicts in textbook lessons, and decision-making about textbook management following training.

In this paper, we describe the design, implementation, and results of a formative experience, which aims to promote in prospective teachers their competence for the didactic analysis of a textbook lesson on proportionality, the identification of its deficiencies and decision-making for its improvement and effective use, using the TLAG-Proportionality (Castillo et al., 2022a). The main objective is to analyse how this competence evolves in relation to the identification of conflicts and reflection on its mode of use. To achieve this, we set out to:

1. Identify and describe the shortcomings the participants indicated in an initial diagnostic task.
2. Analyse and evaluate the conflicts that were identified after the training and their proposals for resolving them.

THEORETICAL FRAMEWORK

Considering a textbook lesson as an instructional process (planned or foreseen) in which the author proposes a sequence of mathematical practices for studying a specific topic allows us to apply the different OSA theoretical tools to its systematic analysis (Godino et al., 2007). In this theoretical framework, mathematical objects emerge from the practices that are mobilised in response to a specific problem situation and a whole typology of objects (problem-situation, language, concepts, propositions, procedures, and arguments) is considered according to their nature and purpose (Godino et al., 2007). Moreover, these mathematical objects are not isolated; instead, they are related to each other, forming *configurations* that can be *epistemic* (mathematical objects and processes implemented by an institution to solve a mathematical problem) or *cognitive* (network of mathematical objects and processes used by students to solve a problem situation).

When planning a process of instruction on a mathematical object (e.g., proportionality) for students at a given educational level (e.g., sixth grade of primary education), the teacher must first delimit what this object represents for mathematical and didactic institutions. He/she will turn to the corresponding mathematical and didactic-mathematical texts, to the curricular guidelines and, in general, to what experts consider to be the operational and discursive practices inherent to the object whose instruction is being pursued. With all this, the teacher will determine the system of practices we designate as *the reference institutional meaning of the object* (by extension, mathematical content). On the other hand, the *intended institutional meaning* is given by the system of practices planned for a mathematical object for a particular instructional process (such as that provided by the author of a textbook in a lesson on proportionality aimed at sixth-grade students). OSA assumed that learning implies students' appropriation of the intended institutional meanings through participation in the community of practices generated in the classroom. Thus, the system of practices that the student manifests regarding the resolution of mathematical tasks in which the object is involved determines the *personal meaning achieved* by the student.

To explain the difficulties and limitations of the learning and teaching implemented, OSA introduces the term *semiotic conflict*, understood as every "disparity or mismatch between the meanings attributed to the same expression by two subjects –people or institutions– in communicative interaction" (Godino et al., 2007, p.113). When the mismatch occurs between institutional meanings (e.g., between the reference meaning and the meaning implemented in a textbook lesson or by a teacher), it is said to be an *epistemic conflict* whereas, if the disparity occurs between the meaning expressed by a subject and the reference meaning, it is said to be a *cognitive conflict.*

In this paper, we adopt the mathematics teacher's didactic-mathematical knowledge and competencies model (DMKC) (Godino et al., 2017) developed within the OSA framework. The two key competencies of the mathematics teacher are *mathematical competence* and the *competence of* *didactic analysis and intervention,* which, in essence, consists of "designing, applying and assessing one's own learning sequences, and those of others, through didactic analysis techniques and quality criteria, in order to establish cycles of planning, implementation, assessment, and improvement proposals" (Breda et al., 2017, p. 1897).

To develop this competence, the teacher needs, on the one hand, the knowledge to describe and explain what has happened in the teaching and learning process. On the other, the teacher needs the knowledge to make reasoned evaluative judgements of what has happened and to propose critical decisions for future implementation. This global competence of analysis and didactic intervention of the mathematics teacher is articulated through five sub-competencies associated with conceptual and methodological tools of OSA: *global meaning analysis competence* (identification of problem situations and operational, discursive and normative practices involved in their resolution); *onto semiotic analysis competence of practices* (recognition of the web of objects and processes involved in practices); *competence of management of didactic configurations and trajectories* (identification of the sequence of interaction patterns between teacher, student, content and resources); *competence of normative analysis* (recognition of the network of norms and meta-norms that condition and support the instructional process); *competence of analysis of didactical suitability* (assessment of the instructional process and identification of potential improvements). A detailed description of all these sub-competencies can be found in Godino et al. (2017).

The didactical adequacy of an instructional process is understood as the degree to which such process (or a part of it) meets specific characteristics that allow qualifying it as optimal or adequate to achieve the adaptation between the personal meanings achieved by students (learning) and the intended or implemented institutional meanings (teaching), taking into account the circumstances and available resources (environment) (Breda et al., 2017). It involves the coherent and systemic articulation of the facets involved in the teaching and learning processes (Godino et al., 2007): epistemic, ecological, cognitive, affective, interactional, and mediational. For each of these dimensions, a system of components and general empirical indicators is identified, constituting a guide for systematic analysis and reflection, aiming at the progressive improvement of teaching and learning processes. These suitability indicators must be enriched and adapted according to the specific mathematical content to be taught (Breda et al., 2017), but also to the type of instructional medium, understanding that both constrain aspects of the different components in didactic suitability.

Thus, in Castillo et al. (2021), the system of didactical suitability components and indicators of Godino (2013) is revised and particularised to develop a textbook lesson analysis guide for proportionality (TLAG-Proportionality) as a tool to guide the reflection of pre-service and in-service teachers on the instructional processes planned in textbook lessons on proportionality. This guide includes specific indicators based on an exhaustive review of research results and expert judgments assumed by the academic community (Breda et al., 2017), which allow identifying didactic-mathematical knowledge to be considered in the evaluation, redesign or management of proportionality teaching processes.

METHOD

This work is framed within an exploratory interpretative research approach. The design research method (Cobb et al., 2003) is applied in a real classroom context based on the planning, implementation, and retrospective analysis of an intervention. In addition, content analysis (Cohen et al., 2011) is applied to examine the responses of the prospective primary school teachers (PTs onwards) who intervened in the formative experience.[1]

Research context and participants

As part of the research, formative cycles involving the design of tasks, their actual implementation, and the retrospective analysis of each experience were planned. We implemented a first cycle as a pilot test in 2020 with a group of students in the third grade of primary school (Castillo et al., 2022b). A second cycle was implemented in 2021 with 48 students, also with primary school third-graders. In both cases, the formative experience was developed within the framework of the subject Design and Development of the Primary Mathematics Curriculum at a Spanish university. In this course, two weekly classes are given: the first (two hours long) is theoretical, in a large group, and the second (one

[1] The Informed Consent Form (TCLE) was not signed, because the identity of the participants is not revealed. In any case, we exempt Acta Scientiae from the consequences derived from it, including comprehensive assistance and eventual compensation for any damage resulting from any of the research participants, according to Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

hour long) is practical, in small groups, in which students work collaboratively (teams of 4 or 5 members).

During their degree studies, students have received specific formation on epistemic, cognitive (mathematical learning, errors, and difficulties), instructional (tasks and activities, materials, and resources) and curricular aspects so that by the time the experience is developed, participants are expected to be able to put into practice the knowledge acquired to solve, design and sequence mathematical tasks according to specific content, in our case, proportionality. In the subject of Curriculum Design and Development, the use and analysis of the textbook as a resource in the mathematics classroom and the planning and assessment of teaching and learning processes are considered.

In this paper, we analyse the information collected in the second cycle from the observer/researcher's notes and the written responses of the PTs to the evaluation task proposed at the end of the course. These were examined by the research team through content analysis and considering as categories the components and associated indicators for the different facets proposed by the OSA.

Design and implementation

Initial exploration activity

In order to involve the PTs in a reflection on the need for theoretical and methodological tools to guide teachers in the systematic assessment of teaching practice, they were asked, as a voluntary activity prior to the training session on textbook analysis, to carefully read the lesson "Proportionality and percentages" by González et al. (2015) and answer the following questions: What did you think of the lesson you have just analysed? Did you identify any errors or elements that could limit students' learning?

Introduction to didactic analysis of textbook lessons

The training of PTs on the didactic analysis of textbook lessons took place over two class sessions (2 hours each) delivered via the Google Meet platform. The first session was devoted to describing a methodology for the analysis of textbook lessons:

1. *Description* of the lesson and division into elementary units of analysis (didactic configurations).

2. *Onto-semiotic analysis*. For each of the units of analysis or didactic configurations into which the lesson is divided:

a) detail the mathematical practices proposed;

b) identify the mathematical objects (concepts, procedures, propositions, arguments, and languages) involved in them,

c) describe the main mathematical processes.

3. Assessment of the didactic suitability of the lesson, identifying the epistemic, cognitive, and instructional conflicts observed.

The analysis of a lesson was exemplified using the text “Percentages and proportionality” from the book by Ferrero et al. (2015) for the sixth grade of primary school. After this formative session, the corresponding practice session of the course was devoted to the first stage of the lesson analysis (general description and onto-semiotic analysis of the different configurations into which the lesson is divided) of proportionality for primary school sixth graders by González et al. (2015).

In the second formative session, didactical suitability was presented as a global criterion for assessing a planned, scheduled or implemented instructional process (or part of it). Reflection on the didactical suitability of a textbook lesson on a specific topic requires considering both the analysis of previous practices, objects and processes and the didactic-mathematical knowledge about that content (in our case, proportionality).

Assessment of the competence of didactic suitability analysis

After the second formative session, the PTs worked individually on the didactical suitability analysis and identification of conflicts in the proportionality lesson by González et al. (2015). This is the same lesson considered in the collaborative work session to analyse practices, objects, and processes. To do so, we gave them the tables that make up the TLAG-Proportionality, adapted from Castillo et al. (2022a). Then, we asked them to answer the following questions: What changes would you introduce in the teaching and learning process to resolve the conflicts you have identified and improve the study process set out in the textbook lesson? Based on the conflicts identified, specify in a concrete and justified way how you would solve at least four of them.

RESULTS

To analyse whether the training improves the PTs' competence to identify conflicts in the lesson and guides them in making decisions about material management, in this section, we show which shortcomings the participants indicated in the diagnostic task, which ones they mostly recognised after the analysis, and which ones they proposed to solve them.

Initial identification of conflicts in the lesson

To show the progress in identifying conflicts, we must consider what aspects the participants specified in the initial exploration task. Of the 48 participants, 19 (i.e., 39.58%) explicitly indicated that they did not see any errors or elements that could be a constraint to learning on the part of the learners. The remaining participants identified conflicts that we have classified according to the dimension they affect:

Epistemic-ecological conflicts (22 observations)

a) The problems are not suitable or are decontextualised ("*the decontextualisation of some of the exercises [...] some of the exercises are set out in a complicated way*", PT14).

b) Representations are unsuitable or require explanation ("*lack of analogies with pictorial representations*", PT13).

c) Fundamental content about proportionality is missing or not sufficiently explained. Here, for example, the "*inadequate explanation of proportional quantities*" (PPT40) or "*of proportional relationships using tables*" (PPT35) are detailed.

d) There is no explanation of "*what percentages are for*" (PT29), percentages are not connected to fractions and "percentage increases and decreases are not studied" (PT2).

e) Some fundamental procedures are not explained: the rule of three, calculation of distances by applying scale, "*there is no explanation of reduction to unity*" (PPT43).

Cognitive-affective conflicts (ten observations)

a) Lack of attention to prior knowledge of fractions, decimal numbers, units of measurement and conversion between them.

b) Difficulty of problems. It is considered to include "*overly complex examples*" (PT32), and "*some problems are difficult and could frustrate students because they cannot solve them*" (PT26).

c) Mathematical reasoning is not encouraged ("*a strategy explained as an example is followed for the various problems that are posed, mathematical reasoning is slowed down by inducing students to follow patterns*", PT11), and the flexibility of strategies to solve the problems is not encouraged (PT15).

d) Failure to motivate students ("*everything is given as "done", it does not motivate students to learn by discovery*", PT24).

Instructional conflicts (13 observations)

a) Mechanised exposure that does not engage students (PT3) and limits their autonomy ("*a single way of explaining content impedes students' learning achievement*", PT48).

b) The presentation of the topic is not adequate: some fundamental concepts are not explained, or insufficient explanation could lead to difficulties for students ("*to get the fourth term in the rule of three, to have difficulties in understanding proportional magnitudes, to understand and use the scale of a map*", PT27; "*with the terms VAT, discount*", PT30).

c) Some problem statements or conditions are confusing.

d) No manipulative activities are included.

e) The sequencing of content is not suitable ("*should explain percentages after proportional magnitudes*", PT23).

Although few PTs could make these limitations explicit, except for those who consider the complexity of the tasks to be excessive, they are adequate.

Identification of conflicts after the didactic analysis of the lesson

In the final evaluation task, all participants indicated several conflicts that had been previously categorised by the research team. A total of 324 observations were recorded in the epistemic-ecological aspect, 181 in the cognitive-affective aspect and 178 in the instructional aspect. Of the 48 participants, 30 detailed and justified the conflicts encountered, while 18 reported them as an absence or bias in the degree of compliance with the associated indicator in the TLAG-Proportionality. For each dimension, we highlight in Tables 1, 2, and 3, respectively, the most representative significant conflicts detected (by at least ten of the 48 PTs). Next, we show prototypical examples of their identification by the participants.

Table 1

Epistemic-ecological conflicts identified by PT and frequency (Freq.)

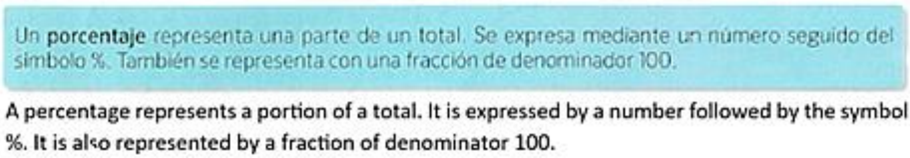
Component	Descriptor of the conflict	Freq.
Problems	Students are not encouraged to formulate problems on proportionality and percentages.	30
	No situations are presented in which multiplicative and additive comparisons can be distinguished, nor others in which both relations are worked on simultaneously.	12
	No mental arithmetic situations involving proportional reasoning are proposed.	17
Languages	Different types of representation of the relation of proportionality are not used, and pupils are not encouraged to interpret or translate between them.	18
Concepts	Fundamental concepts (constant of proportionality, covariance of quantities, invariance of ratio) are not clearly presented.	28
	The multiplicative nature of comparisons between proportional quantities is not clearly defined.	18
	The proportional nature of percentages is not sharply defined.	18
Propositions	The fundamental propositions of the subject do not appear clearly and correctly.	12

	Sufficient and necessary propositions do not appear to distinguish when the relationship between two magnitudes is of direct proportionality.	12
Procedures	The problems allow students to decide which proportionality procedures are most appropriate.	15
Arguments	Propositions and procedures are not justified, or the argument is always of the same type.	22
Argumentation process	No situations are proposed for the student to investigate, formulate conjectures, and argue about proportionality relationships.	18
Modelling/ Generalisation	No situations allow the student to use the mathematical model of the linear function to understand relationships between magnitudes or to generalise them.	23
Intra- and interdisciplinary connection.	The subject is not related to other cross-cutting content, the history of mathematics or other disciplines.	18

The PTs mainly highlight the lack of clarity in the definition of the percentage (Figure 1) or of scale ("*the author does not include a general definition of scale, but rather particularises it to the 1:50,000 case*", PT24) and the fact that both notions do not appear connected to the relationship of proportionality of magnitudes, which prevents procedures involving the calculation or application of scales from being justified based on the latter.

Figure 1

Definition of percentage. (González et al., 2015, p. 112).


Un **porcentaje** representa una parte de un total. Se expresa mediante un número seguido del símbolo %. También se representa con una fracción de denominador 100.

A percentage represents a portion of a total. It is expressed by a number followed by the symbol %. It is also represented by a fraction of denominator 100.

For example, PT23 states:

> *The definition of percentage is confusing. It is not clear whether or not it is synonymous with a fraction. It is not explained why the fractions obtained are equivalent. The relationship between the ratio "a of each b" and the fraction a/b is not made explicit. It is not explicit that only decimal fractions can be expressed as percentages and that not all fractions are*.

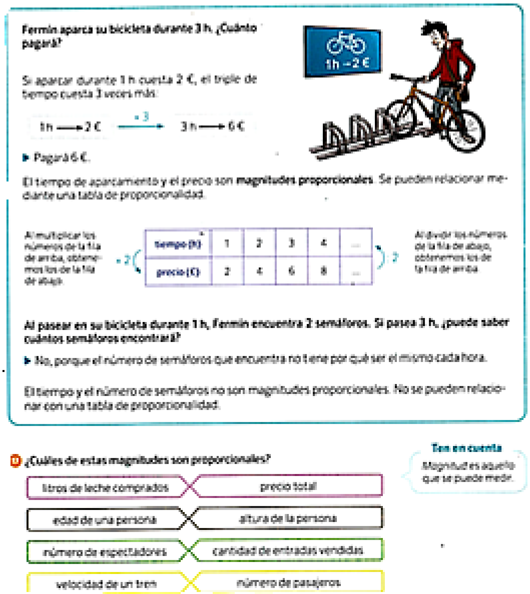
They also point out that the text does not include a clear and general definition of proportional magnitudes, and the description is limited to introductory examples linked to the representation through a proportionality table. For example, PT37 states that "*there is no definition of proportional quantities. The expression proportionality table is used as a synonym for a series of proportional numbers; tables with three or four columns are presented as such, without indicating that the series is unlimited*". Similarly, other participants point out, as PT33 does, that "*there is a conflict regarding the explanation of why two magnitudes are proportional. Only, it is justified that they are not proportional because they cannot form a proportionality table*" (Figure 2).

The role given to the tabular register, and the lack of explanation and justification for its use, can lead, as several PTs indicate, to "*thinking that the presence of a two-dimensional table implies the existence of a proportionality relationship*" (PT23). Furthermore, they mention the absence of an explicit definition of a constant of proportionality ("*it is not made explicit that the ratio between the quantities that correspond must always be the same: constant of proportionality*", PT5) and that the model of the linear function is not introduced or used ("*also leaving aside the relationship of proportionality with the mathematical model of the linear function*", PT45).

Although they indicate the lack of fundamental propositions, they do not specify what these are or which of the propositions included by the author are imprecise or incomplete. Only PT33 notes that "*the constant of proportionality is not specified, nor are the conditions of regularity that allow us to consider directly proportional magnitudes*".

Figure 2

Introduction to proportional quantities. (González et al., 2015, p.116).



Fermín aparca su bicicleta durante 3 h. ¿Cuánto pagará?

Si aparcar durante 1 h cuesta 2 €, el triple de tiempo cuesta 3 veces más:

1 h ⟶ 2 € —·3⟶ 3 h ⟶ 6 €

▸ Pagará 6 €.

El tiempo de aparcamiento y el precio son **magnitudes proporcionales**. Se pueden relacionar mediante una tabla de proporcionalidad.

Al multiplicar los números de la fila de arriba, obtenemos los de la fila de abajo. (· 2)

tiempo (h)	1	2	3	4	...
precio (€)	2	4	6	8	...

Al dividir los números de la fila de abajo, obtenemos los de la fila de arriba. (: 2)

Al pasear en su bicicleta durante 1 h, Fermín encuentra 2 semáforos. Si pasea 3 h, ¿puede saber cuántos semáforos encontrará?

▸ No, porque el número de semáforos que encuentra no tiene por qué ser el mismo cada hora.

El tiempo y el número de semáforos no son magnitudes proporcionales. No se pueden relacionar con una tabla de proporcionalidad.

¿Cuáles de estas magnitudes son proporcionales?

- litros de leche comprados – precio total
- edad de una persona – altura de la persona
- número de espectadores – cantidad de entradas vendidas
- velocidad de un tren – número de pasajeros

Ten en cuenta
Magnitud es aquello que se puede medir.

Fermín parks his bicycle for 3h. How much will he pay?

If parking for 1h costs 2€, parking for 3 times as long costs 3 times as much: 1h⟶ 2€ ×3⟶ 3h⟶ 6€. He will pay 6€.

Parking time and price are **proportional magnitudes**. They can be related by means of a proportionality table.

By multiplying the numbers in the top row, we obtain those in the bottom row.

By dividing the numbers in the bottom row, we get the numbers in the top row.

While riding his bicycle for 1h, Fermín finds 2 traffic lights. If he rides for 3h, can he know, how many traffic lights he will encounter?

No, because the number of traffic lights he encounters need not be the same every hour.

Time and number of traffic lights are not proportional quantities. They cannot be related to a proportionality table.

Keep in mind.

Magnitude is that which can be measured.

17. Which of these quantities are proportional?

Liters of milk purchased, total price.

Age of a person, height of a person.

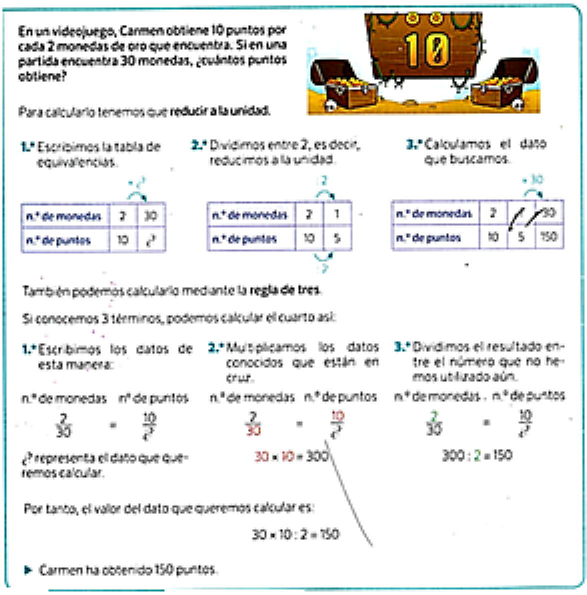
Number of spectators, number of tickets sold.

Speed of a train, number of passengers.

Regarding the absence of argumentation or justification of the procedures in the textbook, the participants indicate as PT5 that “*the procedure of unit reduction is not suitably explained in the textbook. It only states ‘we divide by two, i.e., we reduce to unity’*” (see Figure 3).

Figure 3

Reduction to the unit and rule of three. (González et al., 2015, p.118).



En un videojuego, Carmen obtiene 10 puntos por cada 2 monedas de oro que encuentra. Si en una partida encuentra 30 monedas, ¿cuántos puntos obtiene?

Para calcularlo tenemos que **reducir a la unidad.**

1.º Escribimos la tabla de equivalencias.

n.º de monedas	2	30
n.º de puntos	10	¿?

2.º Dividimos entre 2, es decir, reducimos a la unidad.

n.º de monedas	2	1
n.º de puntos	10	5

3.º Calculamos el dato que buscamos.

n.º de monedas	2	1	30
n.º de puntos	10	5	150

También podemos calcularlo mediante la **regla de tres.**

Si conocemos 3 términos, podemos calcular el cuarto así:

1.º Escribimos los datos de esta manera:

n.º de monedas n.º de puntos $\frac{2}{30} = \frac{10}{¿?}$

¿? representa el dato que queremos calcular.

2.º Multiplicamos los datos conocidos que están en cruz.

n.º de monedas n.º de puntos $\frac{2}{30} = \frac{10}{¿?}$

30 × 10 = 300

3.º Dividimos el resultado entre el número que no hemos utilizado aún.

n.º de monedas n.º de puntos $\frac{2}{30} = \frac{10}{¿?}$

300 : 2 = 150

Por tanto, el valor del dato que queremos calcular es:

30 × 10 : 2 = 150

▶ Carmen ha obtenido 150 puntos.

In a video game, Carmen gets 10 points for every 2 gold coins she finds. If she finds 30 coins in a game, how many points does she get?

To calculate it we must **reduce to the unit.**

1°. We write the equivalence table (number of coins, number of points).

2°. We divide by 2, that is, we reduce to the unit.

3°. We calculate the data we are looking for. We can also calculate it by rule of three.

If we know 3 terms, we can calculate the fourth one like this:

1°. We write the data like this:

number of coin number of points $\frac{2}{30} = \frac{10}{¿?}$

¿? represents the data we want to calculate.

2°. We multiply the data with the data that are in crosses. $\frac{2}{30} = \frac{10}{¿?}$, $30 \times 10 = 300$

3°. We divide the result by the number we have not used yet. $\frac{2}{30} = \frac{10}{¿?}$, $300 : 2 = 150$

Therefore, the value of the data we want to calculate is $30 \times 10 : 2 = 150$

Carmen has obtained 150 points.

Certainly, when the author of the text presents the rule of three procedure, he does not explain why the corresponding pairs of values are written as a fraction, why the fractions are equal, or why it is solved by multiplying ‘the known data that are in crosses’. Nor does he refer to the property: ‘in a proportion the product of means equals the product of ends’. In this sense, the participants consider as PT46 that “*the procedures of reduction to the unit and rule of three are not clear, and it may be the case that some students do not realise that, to apply them, the magnitudes must be proportional*”. Indeed, the lesson does not analyse or discuss which conditions must be fulfilled or which assumptions must be accepted for the direct proportionality model to be applicable. Furthermore, they indicate that these are the only procedures given and that the learner is not given the possibility to “*discuss or explain his or her point of view on other procedures for solving proportionality problems*” (PT46). Similarly, all PTs indicated some conflict in the cognitive-affective dimension, and all are correct.

Table 2

Cognitive-affective conflicts identified by PTs' *and frequency (Freq.)*

Component	Descriptor of the conflict	Freq.
Previous knowledge	Previous knowledge regarding fractions, the equivalence of fractions or the measurement of magnitudes is not considered.	20
Individual differences	The lack of diversity of strategies (progressive, additive, multiplicative construction) does not allow access and achievement for all students.	19
Progression in difficulty	The difficulty of the proposed situations is similar.	15
	No warning of possible errors (illusion of linearity, assumption of sufficient necessary conditions or use of wrong additive strategies).	22
Assessment	No self-assessment tools are proposed.	28
	The assessment methods are inadequate for the students to know their progress and acquire the expected knowledge and competencies.	12
Attitudes	Flexibility of learners to explore alternative methods of problem solving is not encouraged.	24
Emotions	No motivational elements, no logical or original reasoning.	17
Values	Students are not encouraged to value the usefulness of mathematics in their daily lives (unrealistic work).	10

Participants consider that the lesson should focus more on the prior knowledge needed to tackle the study successfully. For example, they indicate, as does PT12, that the lesson "*does not recall the concepts of magnitude, units of measurement, quantity, and the numerical value of measurements; if the student is not familiar with these concepts, he/she may struggle*". They also point out as a limitation the lack of flexibility in solving the proposed problem situations ("*on almost no occasion are diverse strategies promoted, so there is no individual support*", PT35).

Regarding the progression in difficulty, 15 participants consider the complexity of the tasks to be similar. Their justification is based on that the activities are "*all of the same type*" without considering the aspects related to the content that influence the difficulty of the tasks, for example, the presence of integers and non-integers, that divisibility relations are not always established between quantities and that the order of presentation of the data in the problems is not always the same (Fernández & Llinares, 2011; Van Dooren et al., 2009). They also consider it conflictive that students are not warned of possible errors ("*sometimes boxes appear that remind students of important elements of the processes to be followed, but they do not consider other types of errors or difficulties*", PT23; "*There is no mention of common errors or mistakes that can be made in order to try to avoid them beforehand and benefit the students*", PT45) and they place it together with the particularisation in the author's presentation of the proportionality ratio and the procedures of reduction to the unity and the rule of three, as potential use of erroneous additive strategies in some of the proposed tasks. For example, PT23 mentions that the student may use such strategies in a comparison problem (Problem 4 in Figure 4):

> *Another cognitive conflict can be found in exercise 4 because the student will look at each child's number of errors, choosing the younger child as the best result obtained, without considering the proportionality of successes and errors* (PT23).

Figure 4

Tasks proposed by González et al. (2015, p. 123)

4 Dos hermanos comparan sus resultados en un examen. El mayor ha respondido correctamente a 25 preguntas de las 30 que tenía su prueba, y el pequeño a 16 de las 20 que tenía la suya. ¿Quién ha obtenido mejores resultados?

4. Two brothers compare their scores on a test. The older brother has answered 25 of the 30 questions on his test correctly, and the younger brother has answered 16 of the 20 questions on his test correctly. Who has performed better?

7 Una fotografía mide 15 cm de ancho y 10 cm de largo. Si Fiona quiere ampliarla para un marco de 45 cm de ancho, ¿cuánto medirá el largo?

A. 30 cm B. 150 cm C. 50 cm D. 3 cm

7. A photograph is 15cm wide and 10cm long. If Fiona wants to enlarge it for a 45cm wide frame, how long will it be?

PT45 also considers that "*not noticing the errors*", together with the fact that "*the multiplicative nature of comparisons between proportional magnitudes is not explained in a suitability manner*", can lead to the incorrect use of additive strategies:

On the other hand, in exercise 7, a cognitive conflict arises because [the student] would see the difference between 45cm and 15cm, which would be 30cm, and add it to 10cm, resulting in 40cm, or simply looking at the 30cm and put that result, but without associating that it is a proportionality problem.

In addition, many PTs indicate the lack of self-assessment tasks or that the teacher's use of the problems in the lesson as assessment activities would not allow for correctly measuring the acquisition of mathematical knowledge and competencies. Finally, they mention the rigid nature of the lesson ("*they focus on a single method of solving without allowing for flexibility, there are no exercises in which students are allowed to investigate or explore new methods*" PT7) and that it does not offer students the possibility of assessing the usefulness of mathematics:

It explains the steps to be followed in the procedure of reduction to the unity and the rule of three, but it does not go beyond that and does not explain their usefulness. It does not create in the students the need to use them in another context or their real [context] (PT33).

Table 3

Instructional conflicts identified by PTs and frequency (Freq.)

Component	Descriptor of the conflict	Freq.
Author→student interaction	The presentation of the content by the author is not clear or well organised.	22
	No promotion of situations where consensus is sought based on the best argument.	26
	No use of a variety of argumentative resources to engage the learner.	18
Interaction between students.	Assignments do not encourage a dialogue between students where they question and argue about different points of view.	26
Autonomy	No opportunities for students to take responsibility for their studies.	11
Material resources	Using manipulative materials for the study of proportionality (scalimeter, pantograph,	37

	proportion compass, Geogebra, etc.) is not encouraged.	
Sequencing	The content and activities sequencing is inadequate, and insufficient space is reserved for more complex content.	15

As seen in Table 3, among the main instructional conflicts identified by the participants, the shortcomings in the interactional components and the scarcity of material resources in the lesson stand out. Similarly to PT23, the participants consider that:

> *The author does not underline key concepts such as symmetry of the proportionality relationship, the constant of proportionality; the difference between situations of proportionality, procedures for solving the situations and systems of representation used; and the percentage as an expression of proportions. There is no emphasis or encouragement of peer-to-peer dialogue in the sharing. The use of manipulative materials is hardly encouraged.*

About the sequence, they also argue that it is not suitable. Basically, they consider that no attention is paid to the progression in the different meanings of proportionality, that other procedures have not been incorporated before including the rule of three and that contents such as scales require further explanation. For example,

> *The sequence is not the best, as proportionality should have been explained with intuitive experiences concerning estimation; the rule of three should have been explained at the end of everything once the pupils acquired sufficient experience in this topic. Not enough space has been devoted to the content of scales which may be more complex for them* (PT13).

Proposals for improvement and conflict resolution

In addition to identifying deficiencies, making decisions about the use of materials is part of the didactic intervention competence that PTs must acquire. Thus, participants were asked to suggest changes to improve the study process in the textbook lesson and to elaborate concrete proposals for fruitful solutions to the conflicts they had indicated (at least four).

Although the suggestions for improvement they included were all meaningful and appropriate (second column in Tables 4, 5, and 6 below), we note that not all conflict resolution proposals are equally pertinent, so we propose the following categorisation:

- *not pertinent* if it does not provide a solution to the conflict;
- of *medium pertinence* if the proposed solution is not entirely suitable or lacks a didactic-mathematical basis;
- *pertinent* if it answers the conflict based on specific didactic-mathematical knowledge.

For example, as a conflict, more than half of the participants raise that some concepts are not presented in the lesson clearly, mainly those of proportional magnitudes, percentages, or scales. Faced with this conflict, they suggest definitions to be included in the unit. As shown in Figure 5, PT44 adequately appreciates that the lesson does not define proportional magnitudes. However, their proposed solution to this conflict is not pertinence, as the definition they propose is incorrect:

Figure 5

The proposed definition of non-pertinent proportional magnitudes (PT44)

The difference between proportional and non-proportional magnitudes is not clear. The book presents the following statements: "Time and price are proportional magnitudes. They can be related by means of a proportionality table". "Time and the number of traffic lights are not proportional quantities". Here, I would add the definition of proportional magnitudes, so that students understand what I means and can check it by means of the example.
Therefore, we could add:
"A proportion is an equality between two ratios. Two magnitudes are proportional when, as one increases or decreases, the other increases or decreases, respectively, by the same extent, for example: (Example given in the lesson)".

We consider the PT40 proposal to be of medium pertinence to the same conflict, given that, as can be seen in Figure 6, even though it includes a definition of the proportionality relationship, it is not entirely pertinent: it does not establish the functional relationship (it does not include the notion of the constant of proportionality) and it particularises the scalar multiplicative relationship.

Figure 6

The proposed definition of proportional magnitudes of medium pertinence (PT40)

To provide a solution to the [conflict with the] concept of proportional relationship and its multiplicative nature, I would define it as, "We say that two magnitudes are directly proportional if when the measure of one quantity of the first magnitude is doubled, tripled, etc., the measure of the other magnitude is also doubled, tripled, etc."

In Tables 4, 5, and 6, we summarise the suggestions for improvement and the frequencies of efficient conflict resolution proposals in the epistemic-ecological, cognitive-affective, and instructional dimensions, according to the components to which they are linked and their pertinence degree (NP: not pertinent, MP: of medium pertinence, P: pertinent).

Table 4

Proposal for improvement and solution to epistemic-ecological conflicts. Degree of pertinence

Component	Suggestions for improvement	Effective conflict resolution proposal			
		NP	MP	P	Total
Problems	– Include similarity problems, ratio comparison problems. – Introduce mental arithmetic activities in each configuration.	7	8	1	16
Languages	– Improve representations	2	1	0	3
Concepts	– Define proportional and non-proportional magnitudes. – Define percentage and scale, relating them to proportionality.	5	17	4	26
Proposals	– Emphasise scalar and functional relationships.	2	0	0	2
Procedures	– Promote flexibility in problem solving.	3	3	2	8

	– Explain in detail and separately the reduction to the unity and the rule of three.				
Arguments	– Justify all procedures.	2	0	0	2
Processes (Argumentation /Modelling)	– Ask students to justify all solutions. – Include linear function with Geogebra.	4	5	1	10
Curriculum	– Relate with other contents.	2	1	0	3
Total		27	34	8	70

The PTs have proposed solutions to 70 of the epistemic-ecological conflicts identified (i.e., 26.44% of those identified with some degree of representativeness). However, a high percentage of the proposals are considered not pertinent. Regarding the problems component, although most of the suggestions for improvement are related to the need to include problems that cover all the meanings of proportionality (Aroza et al., 2016) –in particular, they point out similarity or ratio comparison problems or that students pose proportionality problems (usually based on a "proportionality table")– most of the specific proposals for solving conflicts refer to mental arithmetic being considered in all configurations (proportional magnitudes, reduction to unity, rule of three, scales) and not only with percentages.

More than a third of the proposals address the need to include appropriate definitions for the fundamental concepts in the lesson: proportional magnitudes (Figures 5 and 6), percents, and scales. However, most are not entirely suitable or do not improve the shortcomings that motivate them. For example, in the case of proportional quantities, the definitions are either inadequate (based on the property in act, "more in A..., more in B") or only consider the scalar multiplicative relationship (Figure 6). In the case of percentages, they emphasise the percentage as a symbol, as a decimal fraction, and rarely try to link it to the proportionality relationship, as shown in Figure 7.

Figure 7

Proposed definition of percentage given by PT44.

To provide a solution to the conflict with the concept of percentage and its relation to proportionality, I would define the concept as: "A percentage is a mathematical symbol, %, which represents a quantity given as a fraction whose total (denominator) is 100. Percentages can be expressed as a decimal fraction of denominator 100. "It is used to define a relationship between two quantities, so that the percentage of a quantity refers to the proportional part that this number of units is with respect to each 100 of the quantity."

For scale, they propose defining it as "*a ratio of proportionality between the represented measure and the real measure, expressed in the same unit of measurement*" (PT33). In other cases, the proposals for solving conflicts are less specific, for example, asking them to justify the solution to the problems, including the linear function (through Geogebra) or cross-cutting themes and other content related to mathematical history.

Table 5

Proposal for improvement and solution to cognitive-affective conflicts. Pertinence degree

Component	Suggestions for improvement	Effective conflict resolution proposal			
		NP	MP	P	Total
Previous knowledge	– Include activities to detect and reinforce previous knowledge.	1	3	3	7
Individual differences	– Include extension and reinforcement activities.	3	2	0	5
Progression in learning	– Extend the degree of complexity of the tasks.	2	5	0	7
Assessment	– Change the "review" configuration to real self-assessment tasks.	6	17	0	23
Attitudes/ Emotions	– Propose situations where pupils conjecture and explore other methods. – Propose more engaging and interesting activities	5	2	3	10

Total	17	29	6	52

The participants put forward proposals for solutions to 31.14% of the cognitive-affective conflicts identified. Of these, 63.46% are considered to be of some pertinence. Most of them focus on including in each configuration, or globally at the end of the lesson, some self-assessment activities to be solved at the end of the student's book. In some of the proposals, the PTs also consider it a good time to include various solution strategies. Thus PT39 states:

> *At the end of the lesson, I would dedicate a section to ASSESSMENT and SELF-ASSESSMENT, with problems that include different ways of solving them ... This would avoid the need to carry out a separate evaluation and allow them to see their own evolution and their weaknesses and strengths*.

On the other hand, they propose to start the lesson with some introductory activities to detect students' prior knowledge of fractions, the equivalence of fractions, and the measurement of magnitudes. For example, PT4 states:

> *I would focus on finding out prior knowledge, such as the use of fractions or whether they know what a magnitude is, for which a questionnaire could be applied beforehand, as we often anticipate what the students know and are familiar with*.

To guarantee adequate progression in learning, they consider it necessary to increase the level of complexity. However, they do not specify the types of activities with which this increase would be achieved. Moreover, they propose warning pupils of possible difficulties with certain content or frequent errors:

> *I would solve this conflict by adding, before the author's explanation of the content to be explained in each configuration, warnings about the difficulties students may encounter in this section, what may be more complex for them, and the most common errors that students tend to make in their understanding* (PT21).

In the affective aspect, they suggest including "*activities that encourage curiosity" (PT16), "not so routine, more attractive activities*" (PT25). They also believe that it would improve the affective aspect of the lesson to propose situations where students conjecture and explore alternative ideas or methods (PT17), but they do not specify in what way.

Table 6

Proposal for improvement and solution to instructional conflicts. Pertinence degree

Component	Suggestions for improvement	Effective conflict resolution proposal			
		NP	MP	P	Total
Author→student interaction	– Expand and improve the explanation of each configuration.	7	7	0	14
Interaction between students	– Include group activities. – Ask learners to discuss and debate assignments.	2	11	0	13
Autonomy	– Encourage autonomous work.	1	1	0	2
Material resources	– Incorporate manipulative materials.	16	8	4	28
Sequencing	– Change the order of the configurations.	0	4	1	5
Total		26	31	5	62

In the case of the proposal for solving instructional conflicts (40% of all those indicated), although most of them suggest using material or computer resources, they do not usually explain what type of resources (except Geogebra or Excel) or how they would use them. These proposals do not take the form of activities to be implemented in the classroom, except for the construction of figures to a particular scale, the creation or interpretation of maps or taking measurements to design plans (Figure 8).

Figure 8

Proposed use of material resources (PT46)

Absence of the use of a variety of resources and materials: In the maps and scales configuration, I would use real maps and plans so that pupils must interpret them correctly. For example, we could do an activity in which pupils, in groups, must draw a map of the school and then calculate the scale at which they have done so, using their steps as measurement unit.

Others propose using Google Maps ("*For example, with Google Maps, calculate the distance from the school to each pupil's home using the scale offered by this tool*", PT12). In general, there is a lack of knowledge about materials that may be pertinent for teaching and learning proportionality ("*there is no need for very specific materials, because any material that can be divided can be useful for working on proportionality*", PT22).

As for sequencing, they believe that it would improve the process of study through the lesson to alter the order of presentation of the configurations, so that proportional magnitudes are presented before percentages, and to postpone the introduction of the rule of three ("*I would change the order of the configurations, so that the rule of three is tackled as the last content as indicated in the appropriate sequence of proportionality*", PT28).

In the interactional component, the participants propose to improve the presentation of the lesson ("*I would improve the explanation in each didactic configuration, which is too brief and particularised*", PT3), emphasising the fundamental concepts of the subject (constant of proportionality, symmetrical nature of the proportionality relationship, fundamentally), but they do not go so far as to make a sufficiently accurate description. They are more specific when their proposal is associated with one of the proposed tasks. For example, for PT33, the formulation of activity 17 (Figure 2) may give rise to a conflict and suggests the alternative statement shown in Figure 9.

Finally, to guarantee a suitable interaction between students, the participants propose to use some of the most significant tasks of each configuration to work on them collaboratively and use them to argue and justify their answers. In this sense, they also propose to add "*justify your answer*" to all the problems in the lesson.

Figure 9

Proposed reformulation of activity 17 (PT33)

A very clear conflict appears in exercise 17, this is since, in this activity, we are asked: Which of the following magnitudes are proportional?

It shows a set of boxes where a series of magnitudes are displayed. However, they are anchored in pairs and have the same colour, which can lead the child to understand that these pairs of magnitudes are proportional only because the two boxes have the same colour. I would change this and put all the squares in the same colour so that there is no confusion, and I would also not link the squares together so that each one is independent.

Activity 17 statement corrected: Which of the following quantities are proportional? Explain why they are.

Litres of milk purchased	Total price
Age of a person	Height of person
Number of spectators	Number of tickets sold
Speed of a train	Number of passengers

CONCLUSIONS

Teachers interpret and mediate the content of textbooks when they use them, so they must have the necessary knowledge and skills to use these resources (Kim, 2007) appropriately. Carrying out a critical analysis to guide how this resource is used is a professional teaching task that can be difficult and requires specific training (Beyer & Davis, 2012; Godino et al., 2017; Shawer, 2017). As Remillard and Kim (2017) suggest, this work is particularly important for primary school teachers, who usually do not have extensive preparation in mathematics and are, therefore, more inclined to rely on curricular resources, which implies that training should ensure that teachers are equipped with the tools to analyse and adapt curricular materials and use them adequately (Beyer & Davis, 2012).

In this paper, we have described the design, implementation, and results of a formative experience with PTs, aimed at developing their competence for the didactic analysis of a textbook lesson, the identification of deficiencies, and the ability to propose how to manage and improve them. As an instrument to guide this analysis, participants were provided with the TLAG-Proportionality

(Castillo et al., 2022a) based on the facets, components, and criteria of didactical suitability (Godino, 2013).

To assess the development of this competence in PTs, we initially proposed that they first read and assessed the lesson. In this case, a high percentage of the participants explicitly indicated that they did not observe any element that could hinder learning by potential students, although those who indicated a shortcoming did so promptly, showing traits associated with the different facets that affect the instructional processes, especially in the epistemic-ecological area.

The analysis of the assessment tasks shows that training in didactic analysis and lesson assessment based on the criteria of didactical suitability improves the PTs' competence in identifying conflicts in the lesson. However, in relation to informed decision-making on material management, the conflict resolution proposals were not entirely accurate or effective. Participants knew what they wanted to change but not so much how.

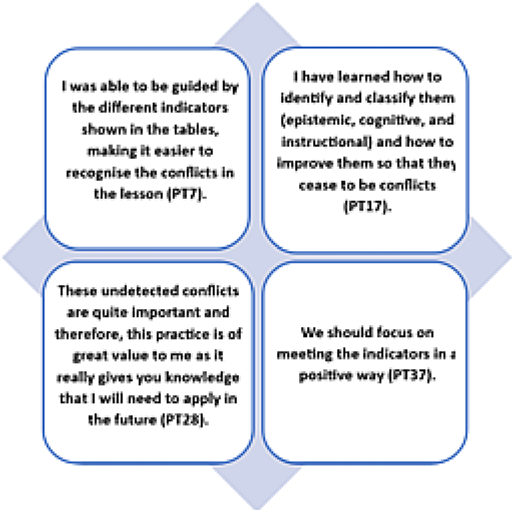
The fact that PTs propose solutions to some conflicts and not to others may owe to several reasons. On the one hand, their beliefs about the most important aspects to ensure a 'suitable' teaching and learning process. Suitability criteria are seen as standards that are principles (rather than norms that are rules) and are, therefore, incremental (Breda, 2020). As a priori consensuses, partial suitability criteria should be treated together, giving different relative weights to each criterion depending on the context. In the search for the adequacy of a teaching and learning process, i.e. the balance between the different partial suitability criteria, the greater weight given to some principles depending on the context or the needs of the learners tilts decisions in a certain direction (Breda, 2020).

On the other hand, their proposals are influenced by their didactic-mathematical knowledge and ability to deal with some aspects more confidently than others. For example, we have observed that the definitions they propose for directly proportional magnitudes, percentages or scales, which they qualify as deficient in the lesson, are not suitable either, or that they do not propose new situations to work on the different meanings of proportionality in an articulated way (Aroza et al., 2016), even though they had considered this to be a deficiency in the lesson. Although the PTs received training on factors affecting difficulty (Fernández & Llinares, 2011; Van Dooren et al., 2009) and the diversity of strategies in solving proportionality problems (Lamon, 2007), and identified conflicts in terms of attention to diversity and progression in learning, they did not design specific tasks to address them. Participants were more responsive to conflicts in the instructional aspect, and the less dependent on mathematical content, the more suitability was found.

In light of these results, we believe it is relevant to promote formative actions oriented towards knowledge and teaching competencies but focused on the importance of content when developing the mathematics necessary for teaching (Burgos et al., 2020; Davis, 2015; Esqué & Breda, 2021). PTs consider this guidance based on the didactic suitability criteria to identify and manage conflicts necessary (Figure 10).

Figure 10

PTs' reflections on the formative action



Moreover, it helps them to express their ideas in formal terms, using professional language ("*when identifying the failure, maybe I didn't know how to express it in didactical-mathematical terms*", PT29), which can be considered evidence of improvement in the participants' reflective competence (Ivars et al., 2018).

Textbooks and other digital resources serve teachers as a link between the intended curriculum and the very different and complex world of the classroom (Valverde et al., 2002), so they must be able to use them in a variety of ways, taking into account their students' needs (Lloyd, 2002). It is possible that the results of our research would have been different with practising teachers or if the PTs had been able to put the lesson and its improvement proposals into practice in a primary classroom, as in the work of Esqué and Breda (2021). It is, therefore, necessary to investigate the results of a formative intervention such as the one described in this paper when the proposals must be effectively implemented in a real classroom context.

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AUTHORS' CONTRIBUTIONS STATEMENTS

M.B participated in the conceptualisation, research, methodology definition, data processing, monitoring, validation, visualisation, writing, reviewing, and editing. M.J.C participated in the conceptualisation, data processing, validation, visualisation, reviewing, and editing. Both authors actively participated in the analysis and discussion of the results and reviewed and approved the final version of the paper.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, [M.B.], upon reasonable request.

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