ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7402

Study of Round Bodies: Conceptions and Praxis of a Didactic Sequence in Light of Guy Brousseau's Theory

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Received for publication 18 Oct. 2022. Accepted after review 16 Nov. 2022
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Guy Brousseau's theory of didactic situations has been considered an apparatus for the methodological structuring of didactic sequences configured in didactic and adidactic situations. **Objectives**: To investigate the contributions of the theory of didactic situations to the study of round bodies, focused on a praxis that meaningfully consolidates knowledge for the students. **Design**: We propose solving problem situations concerning the calculus of surface areas and volumes of round bodies using Cavalieri's principle and Pappus's theorems. **Setting and participants**: The research was conducted with two high school third-grade classes of a state institute of education located in the municipality of Júlio de Castilhos, RS, Brazil, with the participation of 25 students. **Data collection and analysis**: It was carried out through activities developed in the classroom and feedback given through Google Classroom. It was subsidised by the documental transcription of students' records. **Results**: The research indicated that the didactic sequence development favoured intellectual autonomy and meaningful learning about the object of knowledge. **Conclusions**: The theory of didactic situations provided important subsidies for didactic organisation and analysis of the knowledge consolidation process involving the study of round bodies, indicating its application in the study of other mathematical objects in high school and higher education.

Keywords: Round bodies; Theory of didactical situations; Cavalieri's principle; Pappus' theorem; Areas and volumes.

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Estudo dos Corpos Redondos: Concepções e Práxis de uma Sequência Didática à Luz da Teoria de Guy Brousseau

RESUMO

Contexto: A Teoria das Situações Didáticas de Guy Brousseau tem sido considerada um aparato para a estruturação metodológica de sequências didáticas configuradas em situações didáticas e adidáticas. **Objetivos**: Investigar quais contribuições a Teoria das Situações Didáticas pode fornecer acerca do estudo dos corpos redondos e que estejam voltadas a uma práxis que consolide os saberes de forma significativa para os alunos, é o objetivo deste estudo. **Design**: Propomos a resolução de situações problemas acerca do cálculo de áreas de superfícies e de volumes de corpos redondos com a utilização do princípio de Cavalieri e dos teoremas de Pappus. **Ambiente e participantes**: A pesquisa ocorreu em duas turmas do terceiro ano do Ensino Médio em um Instituto Estadual de Educação localizado no município de Júlio de Castilhos, RS, com a participação de 25 alunos. **Coleta e análise de dados**: Foi realizada por meio de atividades desenvolvidas em sala de aula, bem como das devoluções efetivadas através do *Google Classroom* e subsidiada por meio da transcrição documental de registros dos alunos. **Resultados**: Os resultados da pesquisa sinalizaram que o desenvolvimento da sequência didática favoreceu a autonomia intelectual e a aprendizagem significativa acerca do objeto do conhecimento em pauta. **Conclusões**: A Teoria das Situações Didáticas forneceu subsídios importantes para organização didática e análise do processo de consolidação dos saberes envolvendo o estudo dos corpos redondos, sendo indicado sua aplicação no estudo de outros objetos matemáticos presentes no Ensino Médio e, também, no Ensino Superior.

Palavras-chave: corpos redondos; Teoria das Situações Didáticas; princípio de Cavalieri; teorema de Pappus; áreas e volumes.

INTRODUCTION

As Wheeler (1981, p. 352) points out: “better than the study of space, geometry is the investigation of the ‘intellectual space’ since, although it begins with the vision, it moves towards thinking, going from what can be perceived to what can be conceived”. Considering this idea of intellectual space, spatial geometry undoubtedly appears as a fruitful locus for developing the capacity for abstraction, formalisation, generalisation, and creativity, which is immediately sensitive when we refer to the geometric proficiency intended in high school.

Another irrefutable possibility of spatial geometry, based on knowledge historically constructed by man, is the perception of the harmony of forms through the notion of depth that space provides (Brasil, 2018).

Thus, this object of knowledge should be intended to, through teaching and learning situations, significantly promote the acquisition of consolidated knowledge in an educational praxis that a dynamic and emancipatory methodology must support.

Studies that address themes of spatial geometry carried out within the scope of the Professional Master's Degree in Mathematics in National Network - PROFMAT, were developed, for example, through concrete materials and mathematical modelling (Cunha, 2019; Souza, 2021), through technologies (Cardoso, 2020; Dantas, 2018; Silva, 2017) and the resolution of exercises based on the interpretative processes of geometry (Moser, 2020). Those studies emphasise that concrete materials, mathematical modelling, and technological tools are interesting pedagogical strategies that bring aspects of geometric visualisation to the classroom and help students better understand various situations that may be imposed on them. It may also arouse students' attention and curiosity, fostering autonomy in knowledge acquisition.

It is also surprising that geometric concepts strictly linked to their representations and geometric skills necessary to solve problem situations in the context of their apprehensions require an interpretive reading of the geometry around them.

From this perspective, this research approaches the solids of revolution inserted in the curricula of spatial geometry with the treatment of round bodies, concretely the figures of cylinders, cones, and spheres, the objects of study in the Curriculum Reference Matrix for High School of Rio Grande do Sul (Rio Grande do Sul, 2018) [Matriz Referencial Curricular Gaúcha para o Ensino Médio], prepared according to the skills and competencies of the National Common Core Curriculum - BNCC (Brazil, 2018).

The complexity of the concepts, content fragmentation, and decontextualisation from reality determine the students' reduced interest and barriers to learning.

We know that improving education quality requires multiple competencies that can impact the learner and help them break paradigms and take an autonomous and protagonist posture. To this end, this research sets up a didactic sequence contemplating didactical and adidactical situations, following Brousseau (2008), on the study of round bodies, making students responsible for the apprehension of this knowledge.

Given the above, we seek to investigate the contributions the theory of didactical situations can provide for the study of round bodies and which are focused on a praxis that consolidates knowledge in a meaningful way for students.

We submit the didactical and adidactical situations of the sequence proposed here by applying mathematical modelling resources, such as using acrylic solids and experimental activities to determine the centroids of plane figures and simulation of solids of revolution. Based on concepts developed in high school, we present Cavalieri's principle and Pappus's theorems for calculating surface areas and volumes of such solids. We also propose constructing models using physical and virtual resources from silos, a type of construction quite evident in the municipality under study. All the activities developed here have gone through the analysis of each of the moments that characterise the dialectic of the theory of didactical situations: action, formulation, validation, and institutionalisation (Brousseau, 2008) and thus consolidate a praxis in which the pedagogical relations between the student, teacher, and knowledge are resignified.

In this work, we present an excerpt of Guy Brousseau's theory of didactical situations and describe the methodological alternatives used in this proposal. Next, we illustrate some data collected in the classes in which we applied the didactic sequence with the analysis and results of the data obtained. Finally, we present the final considerations and findings on the validity of the praxis in this research.

ABOUT THE THEORY OF THE DIDACTICAL SITUATIONS

The theory of didactical situations, proposed by Guy Brousseau in 1986, is one of the fundamental theories in the didactics of mathematics. Its objective is to characterise the teaching and learning process in the classroom, reflecting on how we can conceive and present mathematical content to students. Its central object is the didactical situation.

The students' whole context is considered a didactical situation, including knowledge, the teacher, and the educational system. According to Brousseau (2008), an interaction becomes didactic "[...] if, and only if, one of the subjects demonstrates the intention to modify the knowledge system of the other (the means of decision, the vocabulary, the forms of argumentation, cultural references)" (Brousseau, 2008, p. 53).

From this perspective, Brousseau (2008) criticises the practice of seeing, in teaching situations, only what he calls the traditional didactical triangle, where only the relations between the teacher and student systems are considered. The author points out this scheme as inconvenient because it reduces the didactic environment to the teacher's action, in addition to omitting the relationships established between the subject and the adidactical environment.

Brousseau (2008) considers that the teacher teaches to make students appropriate knowledge and that it has functionality in contexts other than just situations with didactical purposes (exercises or proposed problems). Therefore, the student must acquire autonomy for which the theory of didactical situations effectively indicates the use of the *milieu* as an indispensable component in the learning process

The didactical situation occurs when the teacher and students are present. However, it starts much earlier, during lesson planning. When planning, the teacher must consider the epistemological, didactic, and cognitive obstacles to the mathematical object (content) with which the group will deal. It is also necessary to know the curriculum guidelines and framework and the textbooks (Brousseau, 2008 apud Reges, 2020) indicated for their students' level of knowledge.

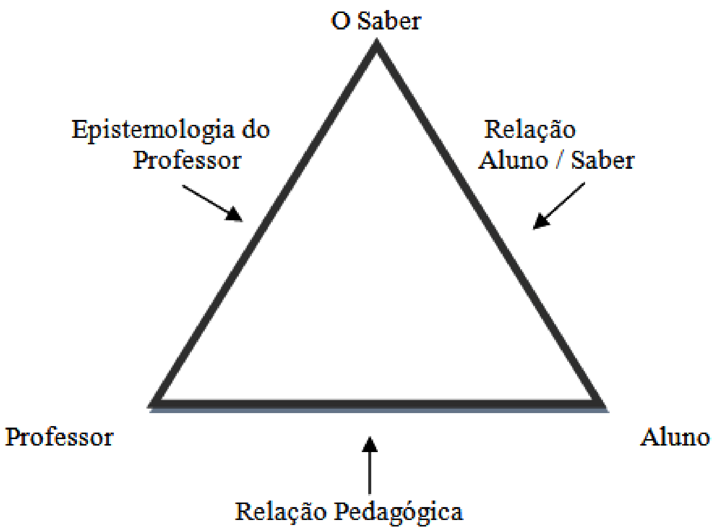
At this moment, the teacher must reflect and decide on the *milieu* that will be used to challenge the student to face the problem and find the answer. There are also moments when the students begin to reflect on the *milieu* that the teacher should have structured, in the sense that they become responsible for their knowledge construction process and appropriation of socially constructed knowledge.

To Almouloud (2007), when theorising about the phenomena linked to those interactions, seeking the specificity of the taught content, Guy Brousseau considers as fundamental the structure formed by the minimal system: *stricto sensu* didactic system, considering the interactions between teacher and students, mediated by knowledge in teaching situations, as illustrated in Figure 1.

This didactic triangle relates the elements that make up its vertices: knowledge, the teacher, and the student. In this triad, the knowledge-teacher relationship includes didactics, information management, and the teaching process; in the teacher-student relationship, the teacher's epistemology interacts with the student in favour of their formation; and in the student-knowledge relationship lies the student's construction of knowledge, i.e., the learning process.

Figure 1

Didactic system. (Adapted by Almouloud, 2007).



The theory of didactical situations is based on three assumptions, as presented below:

1. The student learns by adapting to a *milieu* which is a factor of difficulties, contradictions, and imbalance, similar to what happens in human society. This knowledge, the result of the student's adaptation, is manifested by new answers, which are proof of learning [...].
2. The *milieu* not equipped with didactic intentions is insufficient to allow learners' acquisition of mathematical knowledge. For this didactic intention to exist, the teacher must create and organise the *milieu* in which the situations likely to provoke this learning will be developed.

3. [...] this one *milieu* and these situations must strongly engage the mathematical knowledge involved in the teaching and learning process. (Almoloud, 2007, p. 32-33)

The didactical and adidactical situations are the fundamental elements of the theory of didactical situations. They play the leading role in the analysis and construction of situations for mathematics teaching and learning.

We should note that in the theory of didactical situations, the student's work is similar to a researcher's work, i.e., they test conjectures, formulate hypotheses, demonstrate, and build models, concepts, and theories. In this way, the teacher should carry out not the simple communication of knowledge but the devolution of a good problem. A student's error is a valuable source of information for developing good questions or new situations that can better meet the desired objectives.

The central object of the theory of situations is the didactical situation defined as:

> [...] the set of relationships established explicitly and/or implicitly between a student or a group of students, a specific *milieu* (possibly containing instruments or objects) and an educational system (the teacher) so that these students acquire knowledge constituted or in the process of being constituted (Brousseau, 1986 apud Almoloud, 2007, p. 33).

In the adidactical situation, as an essential part of the didactical situation, the student works independently, having to perceive the characteristics and patterns that will help them to understand new knowledge. The teacher should only be a mediator/observer, just giving feedback on the problem.

For Brousseau,

> When the students become capable of putting into operation and using by themselves the knowledge they are building, in a situation not foreseen in any teaching context and also in the absence of any teacher, what can be called an adidactical situation is occurs (Brousseau, 1986 apud Pais, 2011, p. 68).

In the adidactical situation, the teacher chooses activities so that the students can accept them, leading them to act, speak, reflect, and even evolve by themselves.

According to Almouloud (2007), adidactical situations enhance students' ability to act, reason, and transform previous beliefs and conceptions into knowledge closer to universal knowledge. Adidactical situations bring with them the characteristic that "the intention to teach is not revealed to the learners, but was imagined, planned and constructed by the teacher to provide them with favourable conditions for the appropriation of the new knowledge he/she wants to teach" (Almouloud, 2007, p. 33).

To analyse the learning process, the theory of didactical situations observes and decomposes this process into four distinct phases: action situation, formulation situation, validation situation, and institutionalisation situation. These phases are called situations and dialectics.

Table 1 was prepared following the descriptions established in Brousseau (2008) and Almouloud (2007). It exposes the component phases of the didactic situations, comprising a description for each phase. Those phases comprise the moments experienced by teacher and student, using a challenging *milieu* that generates learning of previously selected knowledge.

Table 1

Typology of didactic situations.

Status	Description
of an action	It occurs from the moment the students assume as their own the activity the teacher proposed. The dialectics of action, as the nomenclature already indicates, corresponds to a typical action situation. This is more experimental and intuitive than theoretical knowledge, as the student finds the solution to the problem but cannot make explicit the arguments they used in its elaboration.
of formulation	Here, the student already approaches some explicit theoretical models or schemes, i.e., they use more appropriate language to help solve the activity. In this situation, the student explains the procedures performed, but the validity of the knowledge used is not to be judged.
of validation	It occurs from the explanation by the person about what they did, how they did it and why they chose a specific way to carry out the activity. It is time to

	confront ideas and create arguments to defend them. The teacher should encourage students to express their ideas in the form of convictions. It is time to prove the validity of their arguments. It is time to advocate the adequacy of their answer, and even if more than one was accepted, they could argue about the economy in terms of effort and resolution time, that is, being the shortest way to the solution.
of institutionalisation	It occurs when the teacher again assumes leadership in the classroom context and develops the discussion of the results of the students' work, articulating the knowledge developed with universal knowledge. In this situation, the teacher tries to help the student transfer knowledge from the individual and particular level to scientific knowledge's historical and cultural dimensions

For Brousseau (2008), the teaching and learning process occurs through devolution and institutionalisation. In the feedback, the teacher gives the student a part of the responsibility for learning, whereas institutionalisation is aimed at establishing social conventions, and the teacher's intention is revealed. Institutionalisation is where the teacher reassumes part of the responsibility given to the students and formalises and generalises the objects of study.

According to Brousseau (2008), devolution is essential to the didactic contract. Considering the *milieu*, the teacher must carry out the process of convincing the group to take on the task of effective learning. The student is, therefore, expected to develop an active role in their learning, accepting the challenge posed by the teacher and starting to act, formulating hypotheses, and validating their arguments about the content in question.

In this phase, students are encouraged to act against problem situations, mobilising basic strategies and previous knowledge to perform the operations of selection, organisation, and interpretation of information, representing them in different ways and making decisions. In this way, mathematical knowledge building effectively occurs; consequently, the students make meaning of what they have done (Pommer, 2013 apud Reges, 2020).

During this process, the teacher relinquishes his role in conducting the investigation and knowledge construction process, avoiding direct interference in the students' construction, each within their condition.

The didactic situation should include moments of independent work for the students, in which it is possible to debate and exchange ideas, providing opportunities for their discoveries, based on a problematising situation.

The interaction of teacher, student, and environment (*milieu*) presupposes rules, which Brousseau (2008) calls a didactic contract. The didactic contract refers to the study of the rules and conditions that determine the operation of school education and concerns the most immediate and reciprocal obligations that are established between teachers and students. Some characteristics of mathematical knowledge, such as formalism, abstraction, and rigour, shape some implicit rules of the didactic contract.

Brousseau (2008) establishes three examples of didactic contract. In the first, the emphasis is on the importance of content. In this case, the teacher considers that he/she has a monopoly on knowledge; the student does not know anything about what is going to be taught. The teacher imposes a single method of organisation and presentation of content (linear sequence of axioms, definitions, theorems, demonstrations, and exercises). The student must pay close attention in class, take notes, repeat the classic exercises, study and take tests. Generally, the students think that the level of demand for the tests is higher than the level of the classes. As a result, there may be a conflict between students and teachers, and the assessment can be used as a control instrument.

In the second example, the emphasis is on the relationship between the learner and knowledge. Here, the student is the one who effectively must learn, and it is not the teacher who has the power to transmit knowledge. The teacher proposes group work and intervenes little to avoid "disturbance" (non-directive education). In this case, the teacher stops analysing the concepts in formation as if school learning were a spontaneous activity (everyday knowledge), and the traditional idea of curriculum is essentially modified.

In the last example, the emphasis is on the student's relationship with knowledge. In this case, the teacher is not considered a source of knowledge but tries to establish a level of intervention. The teacher plans the didactic situations (problems, games, activities, and research papers, among others), and the students can work individually, in small groups or even with the whole class. In this example of a didactic contract, there is a greater appreciation of problem-solving, leading the student to act actively in preparing mathematical concepts.

METHODOLOGY

In this study, we opted for a qualitative approach, and systematised our activities based on the study of round bodies, according to the typology of Guy Brousseau's theory of didactical situations in its five phases: action, formulation, validation, institutionalisation, and devolution.

Qualitative research is a research modality that investigates phenomena in all their complexity, with questions that essentially aim at understanding behaviour from the perspective of the subjects. This type of research collects data from in-depth contact with subjects in their natural contexts (Bogdan & Biklen, 1994).

According to Creswell (2014), "qualitative research begins with assumptions and the use of interpretive/theoretical frameworks that inform the study of research problems, addressing the meanings that individuals or groups attribute to a social or human problem" (p. 49 -50). To the author, in qualitative research, data collection takes place in the natural context of the participants, data analysis establishes standards, and the final report includes the voices of the participants, the reflection, description, and interpretation of the problem by the researcher, in addition to their contribution for literature.

Based on these characteristics, we developed this study as a valid instrument for observing students in their teaching and learning locus, their experiences in the construction of knowledge and their attitude towards the stages of the proposed didactic situations. The didactic contract established in the development of didactic situations focuses on the student's relationship with knowledge.

The research[1] included twenty-five high school 3rd-graders of a state institute of education in Júlio de Castilhos, Rio Grande do Sul. The students will be named A1 through A25.

The activity was developed in the blended teaching modality, mediated by the Google Classroom app of the Google suite for Education. For better cohesion in reading the results, the documental analysis considered the students of the face-to-face modality, as we understand that the analysis of students'

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[1] The participants were informed about the development of the research and agreed to participate, but the Free and Informed Consent Term was not signed, since the activities were developed in class as part of the contents of the Mathematics subject in which one of the authors is regent in the two classes with which the research was developed.

direct observation can enrich the research results. The action occurred during the third quarter of the 2021 school year, in compliance with the provisions of the Reference Matrices for the 2021-RS Blended Teaching Model, in which the skills and competencies listed for the development of mathematical contents were prepared according to the National Common Core Curriculum (BNCC).

The didactic and adidactic situations elaborated for the development of the didactic sequence were divided into four blocks:

- BLOCK 1 – Planning of solids of revolution;
- BLOCK 2 – Calculating the areas of the surfaces of the solids of revolution through its unfolding;
- BLOCK 3 – Getting to know Pappus's theorems;
- BLOCK 4 – Solving the problem situation involving the calculus of surface areas and volumes of round bodies in different contexts.

Each block is composed of activities to be developed by the students and includes didactic and adidactic situations, as described by Brousseau (2008).

For this article, we selected seven of the nine activities the students developed:

Activity 1 – Round bodies as solids of revolution

Activity 2 – Planning solids of revolution through a simulator

Activity 3 - Rocket

Activity 4 - Determining the centre of gravity of plane figures

Activity 5 – Rotating a triangle

Activity 6 – Spinning top

Activity 7 – Models

These activities are presented with the analysis and results in the next section.

Concepts and results about areas and volumes of solids of revolution/round bodies, involving Cavalieri's principle and Pappus's theorems, can be found in Lima et al. (2013) and Neto (2020a, 2020b, 2020c).

ANALYSIS AND RESULTS

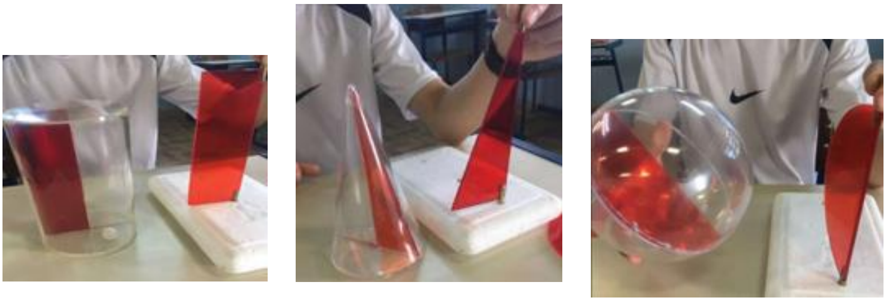
We present here the analysis of the seven activities proposed to the research participants and discussions and results around the data obtained and their relations with the theory of didactical situations.

Activity 1 – Round bodies as solids of revolution. It is a didactic situation in which students must use acrylic materials to observe that the cylinder, the cone, and the sphere can be obtained by rotating plane figures around an axis.

In Figure 2, we see one of the students manipulating the available material and identifying the region that generates each round body.

Figure 2

Handling of acrylic material.



During this activity, students answered the questions proposed in Activity 1.

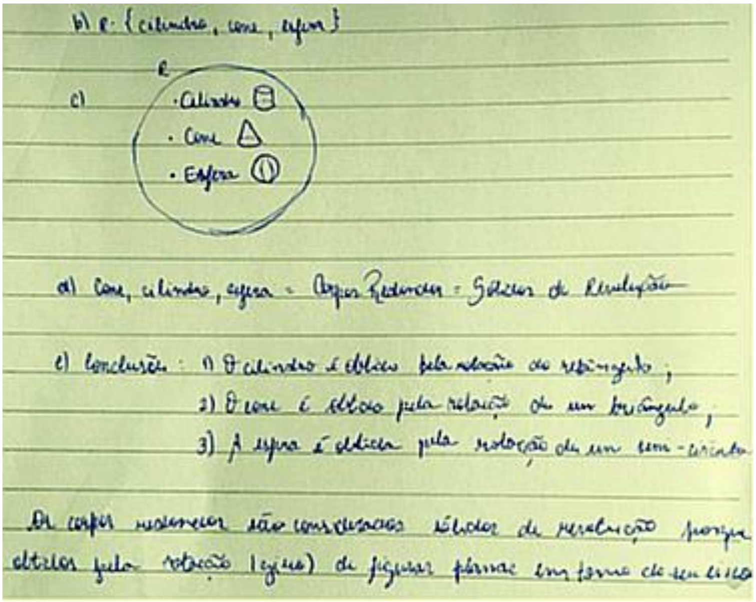
Activity 1 – Round bodies as solids of revolution
a) Fit each acrylic object into the styrofoam base and rotate. Can you see the solids determined by this spin? b) Among the other objects on the table, identify those that resemble those obtained by the rotation performed in the previous item. Call this set R. c) Represent through a diagram the geometric solids that form the elements of the set R. d) Use proper nomenclature to name the elements of the set R.

e) Draw conclusions based on the proposals above.

The answer prepared by one of the students is shown in Figure 3.

Figure 3

Student A1's answer to Activity 1.



In his conclusions, student A1 clearly identifies that the cylinder, the cone, and the sphere can be obtained by rotating a rectangle, a triangle, and a semicircle.

Regarding the typology of the didactic situations that occurred in this activity, we can consider the following moments: In the phase that characterises the dialectic of action, the students participating in the small groups set up their strategies, elaborating the hypotheses for the resolution of the activity, in which they related the acrylic material according to the solid of revolution to be generated. Then, in the following dialectic, formulation, they solved items (b), (c), (d) and (e). Finally, in the phase that represents the dialectic of institutionalisation, the teacher appreciated the students' results and conclusions, using appropriate nomenclature for the solids of revolution obtained from the rotation of plane figures and their constituent elements.

This activity was critical due to students' perception that round bodies are solids of revolution since Pappus's theorem used in later activities to determine areas and volumes is based on solids of revolution.

Activity 2 – Planning solids of revolution through a simulator, aims to lead the student to realise that the surfaces of solids of revolution generated by plane figures can be flattened and, through flattening, students are encouraged to recognise that each of the surfaces corresponds, respectively, to a plane figure. This activity is also configured as an adidactical situation.

To carry out this activity, we used a simulator of solids of revolution, a projection screen and figures such as triangles, rectangles, squares, trapezoids, and semicircles whose edge was made with coated wires.

It is noteworthy that before class, the research teacher posted a theoretical resource on the generation of solids of revolution and some spatial visualisation on Google Classroom. Such material intervened as a stimulus for three students to make a simulator model of solids of revolution. Hence, the students involved said they removed a mini electric motor from an old radio with a cassette player, which served as a device because of its rotation constant speed. Once the mini-motor power wire was isolated, they connected the wires to a 12V source and 2A current to plug them into the socket. Next, with a pen with holes in both parts to maintain balance, they assembled a rod that would serve as the axis of rotation of the flat figures (Figure 4).

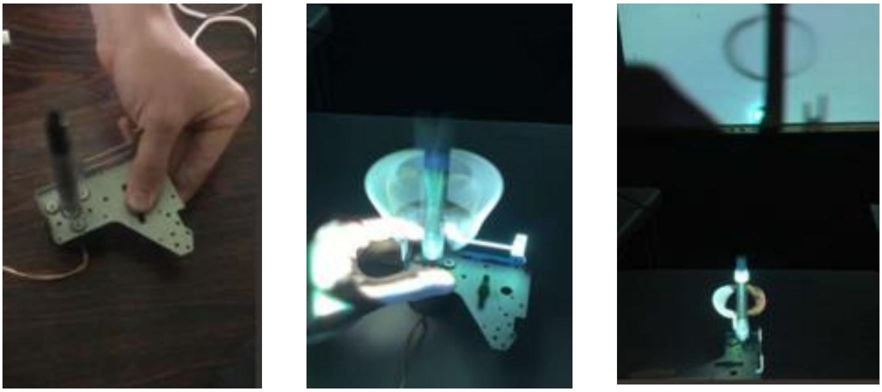
Initially, the teacher guided the class to form small groups, to which copper wires were distributed, which served as moulds to make the flat figures. Then, randomly, the groups chose a plane figure to start the simulation of solids of revolution and afterwards, each group presented it to the other colleagues, naming their respective elements in the plane and space.

The students effectively participated in the execution of the activity. They conjectured about the formed solids and the distinction between solid and surface of revolution. Technically, viewing the cylinder from rotating a rectangle was more difficult because, when moulding them, the edges must be perfectly straight and orthogonal.

In this activity, we list the following moments according to the typology of didactic situations. In the action dialectic, students in small groups set up their strategies and made their choices; in the phase following the formulation, there was intense communication of ideas about the simulation of solids of revolution. In the subsequent phase, validation, based on the communication of ideas initiated in the previous stage, the students demonstrated through the simulator that flat surfaces enhance solids of revolution, identified the generatrix of these solids and realised the notable difference between surface and solid of revolution. Finally, institutionalisation, a stage in which knowledge manifests itself in its universal character, took place with the interference of the teacher, who formalised the geometric concepts involved during the simulation of the solids of revolution. He also counted on the intervention of the class's physics teacher, who explained the phenomenon of retinal persistence and its relationship with the vision of images of solids generated through the simulator.

Figure 4

Making the simulator and carrying out Activity 2.

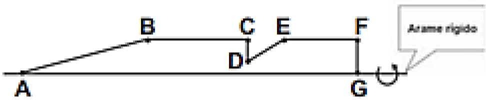


The simulations aimed to elucidate concepts of surfaces of revolution generated by plane figures and solids of revolution through a spatial view. Thus, the simulator was crucial as a stimulus and motivation factor for learning.

Activity 3, Rocket, deals with a question taken from Blue Test – ENEM 2010 – PPL reapplication (Available at: https://www.gov.br/inep/pt-br/areas-de-atuacao/avaliacao-e-exames-educacionais/enem/provas-e-gabaritos).

Activity 3 - Rocket

Numa feira de artesanato, uma pessoa constrói formas geométricas de aviões, bicicletas, carros e outros engenhos com arame inextensível. Em certo momento, ele construiu uma forma tendo como eixo de apoio outro arame retilíneo e rígido, cuja aparência é mostrada na figura seguinte:



Ao girar tal forma em torno do eixo, formou-se a imagem de um foguete, que pode ser pensado como composição, por justaposição, de diversos sólidos básicos de revolução.

Sabendo que, na figura, os pontos B, C, E e F são colineares, AB = 4FG, BC = 3FG, EF = 2FG, e utilizando-se daquela forma de pensar o foguete, a decomposição deste, no sentido da ponta para a cauda, é formada pela seguinte sequência de sólidos:

Ⓐ pirâmide, cilindro reto, cone reto, cilindro reto.

Ⓑ cilindro reto, tronco de cone, cilindro reto, cone equilátero.

Ⓒ cone reto, cilindro reto, tronco de cone e cilindro equilátero.

Ⓓ cone equilátero, cilindro reto, pirâmide, cilindro.

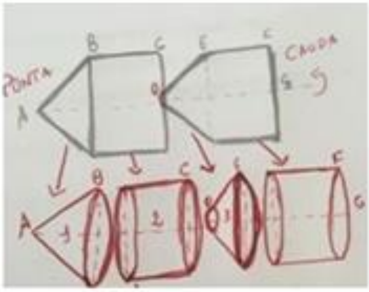
Ⓔ cone, cilindro equilátero, tronco de pirâmide, cilindro.

In solving this activity, we noticed an initial difficulty in interpreting the question; some students needed several re-readings until they found a suitable strategy. Figure 5 shows the correct resolution of Activity 3, performed by student A3.

All students, including student A3, solved the question through geometric representation. Although they initially had difficulties interpreting the question statement, they showed geometric perception skills and correctly recognised the relationship between the radius and the generatrix of a cone and a cylinder to identify them as an equilateral cone and an equilateral cylinder, respectively. However, we observed that three of the 25 answers did not mark the correct answer (c), with two students marking alternative (e) and one student marking alternative (d). Moreover, they did not justify these choices.

Figure 5

Student A3's answer to Activity 3.



c) cone reto, cilindro reto, tronco de cone e cilindro equilátero

In the dialectic of action, we observed that the students created strategies to solve the question, sometimes by drawing, simulating the rocket's rotation, and sometimes by conjectures. As for the formulation, the students exchanged information with their colleagues about how the rocket could be generated; particularly, in this case, we noticed the very intense formulation phase. In the following dialectic, validation occurred when the students established the relationship between the plane figures and the solids of revolution obtained from the decomposition of the rocket. Finally, during institutionalisation, the teacher explained the resolution of the activity, highlighting the concepts employed.

Activity 4 - Determining the centre of gravity of flat figures aims, through a simple test, to make students identify the centre of gravity of flat figures and relate them to their barycenter. For this activity, we used clippings of triangles, squares, rectangles, trapezoids, and circles in cardboard, a string and a base with a tripod and pendulum available in the school's physics laboratory. This activity includes an experimental activity for determining the centre of gravity of plane figures, a valuable definition in the application of Pappus's theorems.

The steps for this activity are as follows:

i) Drill several holes on the edges of the triangle; in one of these holes, introduce a string, suspend the triangle at the base of the pendulum, trace the first median, and repeat the procedure for the other three holes tracing the respective medians.

ii) Identify the meeting point of the three medians.

iii) Repeat the previous process with the rectangle and square; in these cases, draw the diagonals.

iv) Repeat the process with the trapezoid and the semicircle.

v) Graphically represent your findings.

To execute Activity 4, students gathered into small groups and received pieces of paper and cardboard. A metal tripod with a hook on the upper base and a small circular metal plate measuring a quarter of a circumference containing a suspended needle were also used (Figure 6).

Figure 6

Students' resolution of Activity 4.

The teacher verbally guided the steps of the experiment as they happened. The students effectively participated in the proposed activity; for some, the notion of the centre of gravity of plane figures was unknown; even knowing how to answer, for example, that the barycenter of a triangle is where the medians meet, they did not establish any analogy with its centre of gravity.

During the development, we noticed that, intuitively, the students were tracing good strategies and successfully executing the foreseen steps.

Thus, during the phase that characterises the dialectic of action, students established strategies to determine the centre of gravity of triangles, some quadrilaterals, and semicircles experimentally and through the medians. In the subsequent phase, formulation, they debated the subject and confronted the hypotheses and conjectures about the process to determine the centre of gravity, also considered the barycenter. Next, the teacher encouraged the students by asking them to find the centres of gravity of the figures under analysis and determine their measurements. The students then verified the veracity of their hypotheses, thus determining the centre of gravity of the figures when validation took place. Finally, the research teacher reassumed and formalised mathematical knowledge –the institutionalisation stage– presenting the relationships for the calculus of the centre of gravity or barycenter of plane figures, highlighting that determining these variables will be paramount for the theorems' applications.

Activity 5 – Rotation of a triangle, deals with a problem about solids of revolution available at the OBMEP Mathematics Portal (Brazilian Public School Mathematics Olympiad): [https://portaldaobmep.impa.br/uploads/material/5phlpk0gta0w0.pdf .](https://portaldaobmep.impa.br/uploads/material/5phlpk0gta0w0.pdf)

Activity 5 – Rotating a triangle

O triângulo ABC sofre uma rotação sobre o eixo s da figura. Determine o volume do sólido gerado.

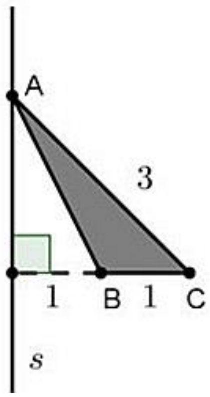


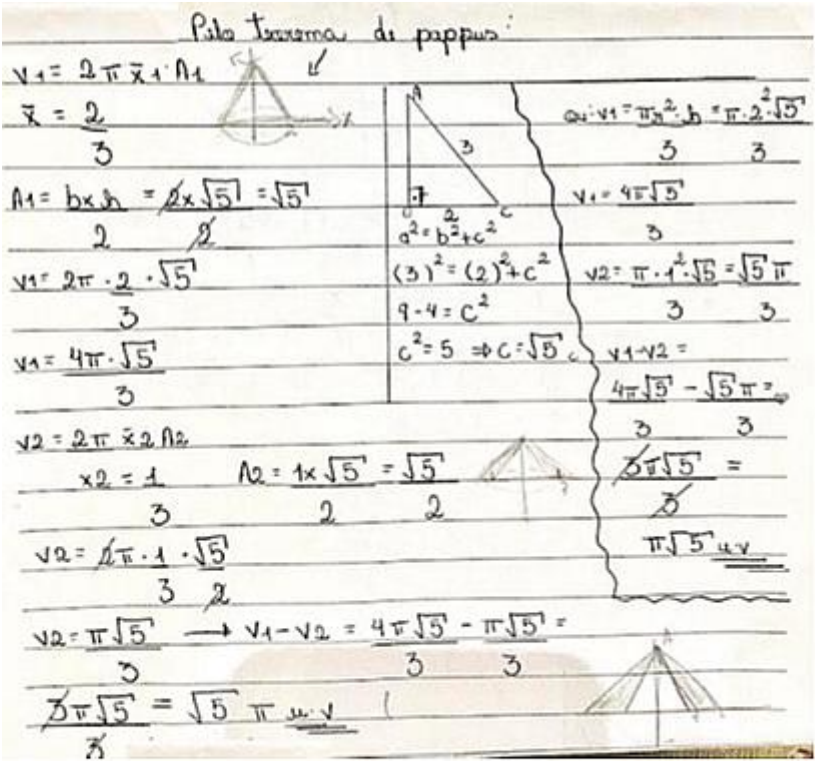
Figure 7 shows student A8's resolution of Activity 5.

We observe that student A8 solves the problem first using Pappus' theorem, in which the volume of a solid of revolution is calculated using the formula$V = 2\pi \cdot \bar{x} \cdot A$, being $\bar{x}$ the distance from the barycenter to the axis of rotation and A the area of the region to be rotated around this axis. Afterwards, he uses Cavalieri's principle, with the formula for the volume of the cone$V =$

$\frac{1}{3}\pi \cdot r^2 \cdot h$, with r as the radius of the base of the cone and h, its height. We also observed that, in both cases, the student realises that the volume to be obtained in the question represents the difference in the volumes of the external cone and the internal cone, formed by rotations around the axis$s$, of triangles OAC andOAB, respectively.

Figure 7

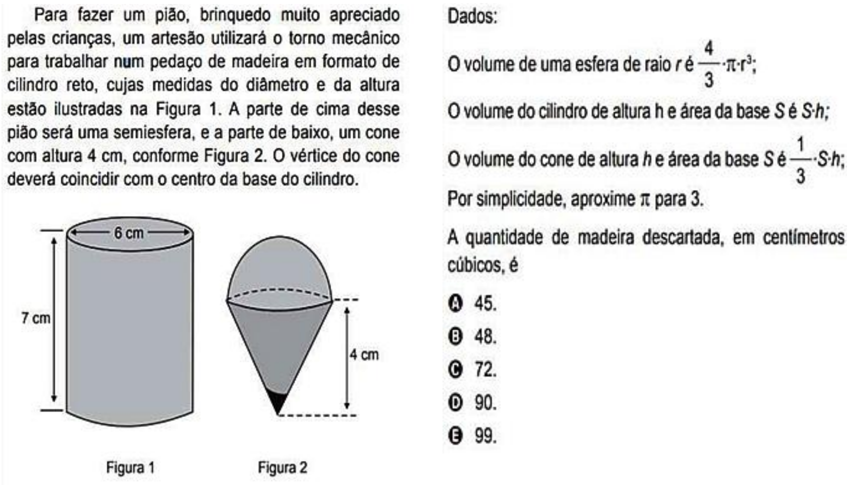
Student A8's resolution for Activity 5



In general, then, we appreciate the successive moments of the typology of didactic situations in their dialectics. In sequence, we have the action when the students prepared the resources to solve the situation in this context. After rotating the triangle to obtain the solid, most students applied Pappus's theorem to calculate the volume of this solid; the formulation stage occurred when they debated the question hypotheses; and validation happened when students put the previous moments into practice, thus resolving the issue. Institutionalisation happened when the teacher formalised knowledge by presenting a commented resolution of the issue.

Activity 6 – Spinning Top, deals with an ENEM/2014 question (Question 166/Gray test; available at: [https://www.gov.br/inep/pt-br/areas-de-atuacao/ 655 valiação-e-exames-educacionais/enem/provas-e-gabaritos)](https://www.gov.br/inep/pt-br/areas-de-atuacao/avaliacao-e-exames-educacionais/enem/provas-e-gabaritos) and involves the calculus of the cylinder, cone, and sphere volumes.

Activity 6 – Spinning top


Para fazer um pião, brinquedo muito apreciado pelas crianças, um artesão utilizará o torno mecânico para trabalhar num pedaço de madeira em formato de cilindro reto, cujas medidas do diâmetro e da altura estão ilustradas na Figura 1. A parte de cima desse pião será uma semiesfera, e a parte de baixo, um cone com altura 4 cm, conforme Figura 2. O vértice do cone deverá coincidir com o centro da base do cilindro.

Dados:

O volume de uma esfera de raio r é $\frac{4}{3}\cdot\pi\cdot r^3$;

O volume do cilindro de altura h e área da base S é $S\cdot h$;

O volume do cone de altura h e área da base S é $\frac{1}{3}\cdot S\cdot h$;

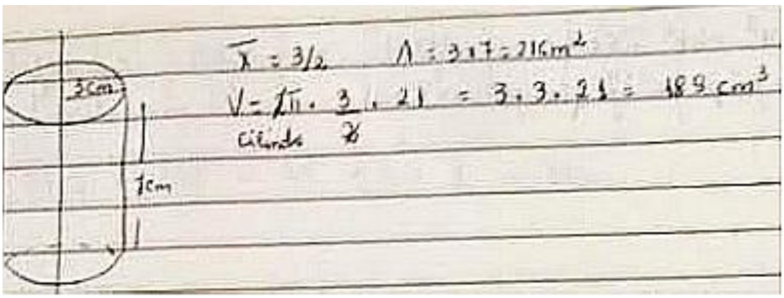
Por simplicidade, aproxime π para 3.

A quantidade de madeira descartada, em centímetros cúbicos, é

A 45.
B 48.
C 72.
D 90.
E 99.

Figure 8

Resolutions presented by students A1 and A13 to Activity 3.



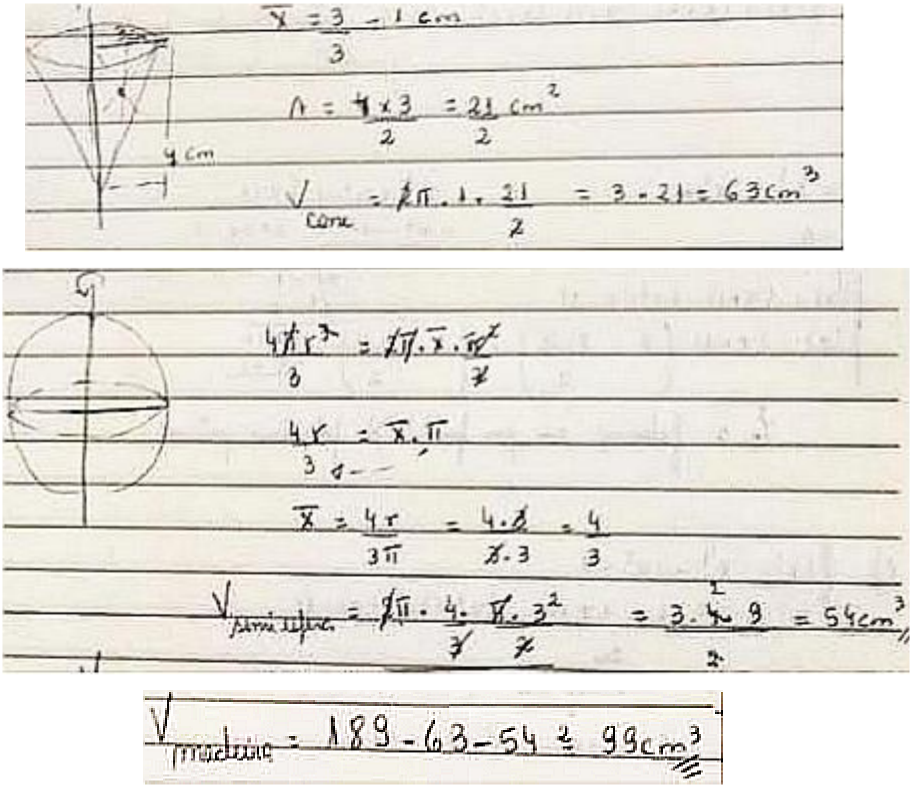


Figure 8 shows the resolutions presented by students A1 and A13. In it, A1 uses the Pappus Theorem and A13, Cavalieri's principle, to obtain the answer to the question.

As seen in Figure 8, students A1 and A13 calculated the volume of the cylinder, cone, and sphere and then subtracted the volumes of the sphere and cone from the volume of the cylinder, thus obtaining the amount of discarded wood. The resolution presented by the students demonstrates that the moment of validation was consolidated since they obtained an adequate answer to solve the problem.

Under the light of the theory of didactic situations, we witnessed the following moments of their dialectic: in action, the students elaborated their hypotheses according to the viability of the variables presented in the question statement, observing that, to solve the problem, they had to calculate the volumes of the cylinder, cone, and semisphere. During formulation, students discussed and confronted conjectures and hypotheses about how to solve the question, with some opting to use Pappus's theorem and others Cavalieri's principle. We then noticed the validation when the students put into practice their elaborations extracted from the previous moments, making the calculations to obtain the answer to the question. Finally, the researcher professor presented the commented resolution of the question for the research subjects, aiming that mathematical knowledge became an object of appropriation of knowledge. It was how institutionalisation occurred.

Activity 7 – Models, articulates an adidactical situation, with the skill EM13MAT201 – "Propose or participate in actions suited to the demands of the region, preferably for their community, involving measurements and calculations of the perimeter, area, volume, capacity, or mass" (Brasil, 2018, p. 534) from BNCC. In this activity, the students were asked to carry out an action involving the construction of models of silos in the municipality of Júlio de Castilhos or its surroundings and that were preferably part of their itineraries. Also, they were requested to elaborate on a problem situation involving the studied theme.

Figure 9

CESA unit: photo and model - G2 group.

In this activity, the students were divided into 11 groups (G1 through G11) and each group, when choosing a silo, took a photo, and obtained measurements from the owners/company to make the model. After making them, the groups held a virtual exhibition on the class page, describing details about the construction of the models and significant aspects of the work carried out. In Figure 9, we can observe the photo and physical model presented by the G2 group. This group chose as a model the silos of the former unit of CESA (State Company of Silos and Warehouses), currently incorporated into COTRIJUC (Wheat Cooperative of Júlio de Castilhos).

The problem situation elaborated by the G2 group is shown in Figure 10.

Figure 10

Problem situation - G2 group.

A capacidade de armazenagem da unidade da CESA representada é de 38500 t. Supondo que o peso específico é de 0,75t/m³ para soja e milho e a compactação é de 5%.

a) Determine o volume equivalente em m³ de soja que pode ser armazenado.

$0{,}75 = \frac{38500}{V}$

0,75V= 38500

V= 38500/0,75= 51333,33m³

V= 51333,33 x 5%=256,66

V= 51333,33-256,66=51076,67 m³

b) Se a unidade da CESA é representada por seis silos, com altura da base cilíndrica de aproximadamente 15m, determine a medida do raio da base. Use $\pi \cong 3$

$51333,\ 33/6 = 3.R^2.15 \Rightarrow R^2 = 8555{,}55/45 \Rightarrow R = \sqrt{190.123} \cong 13{,}7\text{m}$

As we can see, the students in group G2 used the value of the specific mass of soybeans (the problem refers to the specific weight), to find the capacity of the silos in cubic meters without compaction of 5% and, with this information, given the height, calculated the radius of the silo. We observe that $V = 51333{,}33\text{m}^3$ corresponds to the volume, in m^3, associated with the capacity given in tons and considering the specific mass, while $V_c = 256{,}66\text{m}^3$ and $V_f = 51076{,}67\text{m}^3$, correspond, respectively, to the compacted volume and the final volume of the soybean stored in the silo. Students used the same letter$V$ to indicate those volumes.

The G9 group created a virtual model (Figure 11) of a silo located on a farm in the countryside of the municipality on the street Val de Serra; to this end, the group used SketchUp software, suitable for 3D modelling, and Lumion for rendering (photorealism). The data presented were obtained informally and through a visit to the site.

Figure 11

Silo in Val de Serra: photo and 3D modelling - G9 group.

With the simulated data, the group elaborated the problem situation presented in Figure 12.

The problem elaborated by the G9 group asks for the volume of the cylindrical part of the silo whose representation was carried out by the model. As we see, the students used Pappus's theorem to calculate the volume.

Regarding the typology of didactic situations and the moments of their dialectic, especially in this situation of adidactic nature, we see the following. In activity 4 – Models, the action occurred during decision-making about the silo that would be chosen for the groups' construction of the models and distribution. At this phase, non-mathematical knowledge emerged as basic models. The groups' strategies were explained verbally; thus, formulation occurred. Third, validation occurred when the groups expressed their arguments in constructing the proposed model, justified the calculations, and developed a problem situation about the theme. Finally, institutionalisation occurred when all the procedures adopted, from action to validation, were duly organised with the teacher's help. In this context, institutionalisation was also experienced when the teacher issued considerations related to the presentation of models and problem situations elaborated by the subjects in their roles.

Figure 12

Problem situation - G9 group.

Determine o volume do silo, correspondente a parte cilíndrica com altura de 10m e raio da base 7m. Use para π uma aproximação de 3.

$V = 2\pi\bar{x}A$ $\bar{x} = \frac{R}{2} = 3,5$ $V = 2 \cdot 3 \cdot 3,5 \cdot 7 \cdot 10 = 1470\mathrm{m}^3$

CONCLUSIONS

This research sought to investigate, based on Guy Brousseau's theory of didactic situations and following its paths, the contributions a didactic sequence can bring to the construction of significant know-how about the object of solid knowledge of revolution with emphasis on round bodies for high school. Thus, abstracting from our experience according to the praxis developed, we infer, in the narrative that follows, some considerations about this trajectory.

By proposing didactic situations in the form of questions extracted from ENEM assessments, the OBEMEP Mathematics Portal, and university entrance exams, based on the precepts of skills and competencies of the BNCC, we found, faced with the devolutions, the effective participation of the subjects involved. In addition, going through the careful analysis of each of the moments of the dialectic that constitute the typology of didactic situations observed from the moment of action, subsequent formulation and validation, and institutionalisation, we observe that the favourable aspects overlapped the obstacles of learning. In addition, the sequence presents a narrative, contract, and challenge privileged devolution.

To promote the context mentioned above, we experienced adidactical situations, among which we used manipulative resources, such as acrylic solids, an experiment to simulate solids of revolution, and an experimental activity to determine the centre of mass of plane figures. These resources fostered an understanding of the object to be taught. The students responded by getting engaged and participating with autonomy, being capable of conjecturing, elaborating hypotheses, and finally, validating their actions.

To calculate the surface areas and volumes of solids of revolution, we showed students another possibility through the interesting classical Pappus's theorem, which is not usually presented in high school textbooks. This approach was different from current pedagogical practices, which usually favour formulas. In this practice, we used acrylic solids and an experiment in which students could determine the barycenter of flat figures. This fact aroused the students' interest and curiosity. The resources helped them search for the solutions to the problem situations, evidenced in the actions, formulations, and validations, and the satisfactory data found in the devolutions.

The construction of physical and virtual models reproducing the silos, a prevalent type of building in Julio de Castilhos and its surroundings, which aggregate round bodies in its architecture, was a reason for students' interest and contagious participation. This didactical situation built a bridge between the geometry that deals with round bodies studied in class and the geometry that inspires the formats and gives personality to their paths.

Thus, we can consider that the research objective was feasible with the praxis developed. Google Classroom platform was an important ally in complementarity between the physical and virtual spaces. The perspective of an articulated environment provided the involved with productive moments of interaction with the theoretical aspect of the taught object.

As professors and researchers, we agree with Brousseau and modestly believe that we have fulfilled his suggestion that: "The teacher must (...) simulate a scientific microsociety in the classroom if they want knowledge to be an efficient way for posing good questions and resolving debates and if they want that languages are used to dominate situations of formulation and that demonstrations are proof" (Brousseau, 1996, p. 37).

Finally, we hope that the didactic sequence, the result of this research elaborated based on the theory of didactical situations, serves as a suggestion for teaching practice and new constructs about the study of solids of revolution with emphasis on round bodies intended for high school.

AUTHORSHIP CONTRIBUTION STATEMENT

ALVP and JR conceived the research topic. ALVP performed the activities and collected the data. ALVP, JR, and LDP discussed and analysed the data. All authors actively participated in the discussion of the results and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, ALVP, upon reasonable request.

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