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Comparative Study of Elementary School Mathematics Textbooks between Singapore and Indonesia: The Case of Fractions

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ABSTRACT

Background: Mathematics textbooks are one of the tools that link the expected goals of the mathematics curriculum and its practice in mathematics classes. Thus, reviewing mathematics textbooks between countries is one way to reflect and improve based on good practice. **Objectives**: To examine the structure and order of contents presented in Indonesian and Singaporean mathematics textbooks, as well as the differences between the two. **Design**: This research uses textbook comparison study and focuses on how mathematical tasks are presented. The two textbooks compared are Senang Belajar Matematika (Indonesia) and My Pals Are Here (Singapore). **Data collection and analysis**: We analyzed it by conducting coding based on the adaptation of Gracin's framework which includes five dimensions of analysis: content, mathematical activity, level of complexity, answer form, and contextual features. The validated coding results are used to report research results. **Results**: Some aspects of competence for each textbook tend to have the same proportions. In the mathematical activity aspect, both provide operation and calculation activities that are more dominant than the others. However, Singaporean mathematics textbooks offer more opportunities for argumentation and reasoning, and interpretation activities, which interpretation activities are not available in Indonesian mathematics textbooks. We also found that both textbooks offer a variety of task activity types, with the Indonesian textbook offering more types. **Conclusions**: The two textbooks' structure and order of content are significantly different, with the distribution of the number of tasks for each dimension being more even in Singaporean textbook.

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Keywords: textbook research; textbook comparison; fractions; elementary school; mathematical task.

Estudo comparativo de livros didáticos de matemática do ensino fundamental entre Cingapura e Indonésia: o caso das frações

RESUMO

Contexto: Os livros didáticos de matemática são uma das ferramentas que ligam os objetivos esperados do currículo de matemática e sua prática nas aulas de matemática. Assim, a revisão de livros didáticos de matemática entre países é uma forma de refletir e melhorar com base em boas práticas. **Objetivos**: Examinar a estrutura e a ordem dos conteúdos apresentados nos livros didáticos de matemática da Indonésia e de Cingapura, bem como as diferenças entre os dois. **Design**: Esta pesquisa usa o estudo comparativo de livros didáticos e se concentra em como as tarefas matemáticas são apresentadas. Os dois livros comparados são Senang Belajar Matematika (Indonésia) e My Pals Are Here (Cingapura). **Coleta e análise de dados**: Nós o analisamos realizando uma codificação baseada na adaptação do framework de Gracin que inclui cinco dimensões de análise: conteúdo, atividade matemática, nível de complexidade, forma de resposta e características contextuais. Os resultados de codificação validados são usados para relatar os resultados da pesquisa. **Resultados**: Alguns aspectos de competência para cada livro tendem a ter as mesmas proporções. No aspecto da atividade matemática, ambos fornecem atividades de operação e cálculo que são mais dominantes que as demais. No entanto, os livros didáticos de matemática de Cingapura oferecem mais oportunidades para argumentação e raciocínio e atividades de interpretação, atividades que não estão disponíveis nos livros didáticos de matemática indonésios. Também descobrimos que ambos os livros oferecem uma variedade de tipos de atividades de tarefas, com o livro indonésio oferecendo mais tipos. **Conclusões**: A estrutura e a ordem do conteúdo dos dois livros didáticos são significativamente diferentes, com a distribuição do número de tarefas para cada dimensão sendo mais uniforme no livro didático de Cingapura.

Palavras-chave: pesquisa de livros didáticos; comparação de livros didáticos; frações; escola primária; tarefa matemática.

INTRODUCTION

Mathematics textbooks became important in students' learning achievement in mathematics as it include activities that guide the students on how to learn mathematics and influence teacher instruction (Purnomo et al., 2019; Purnomo, Shahrill, et al., 2022). That is certainly makes sense as textbooks are tools to connect intended and implemented curriculum (Valverde et al., 2002). Besides that, mathematics that are thoughts in class is affected by

the provided textbooks because most of the teachers use them as their main learning resources (Kilpatrick et al., 2001). Therefore, critical analysis of mathematics textbooks became something urgent and significant to give descriptions for improvement and development of textbooks that are based on relevant research.

In the last two decades, mathematics textbook analysis has become a topic that increases its attention in mathematics education research (Fan, 2013; Purnomo, Julaikah, et al., 2022; Trouche & Fan, 2018). Both individual mathematics textbook analysis that is compared with a certain standard and a series of textbooks that is compared based on a certain characteristic, for example, the characteristic of the nation of the textbook compared (Bütüner, 2019; Hwang et al., 2020) or comparing between the old curriculum and the current curriculum (Johansson, 2003; Rahmawati et al., 2020). This article reports our study that uses the comparative study of elementary mathematics textbooks from Indonesia and Singapore that focuses on how fraction tasks are presented and what are the requirements that is provided in the tasks.

The main objective of textbook comparative study is not to find the weakness of one's side that is being analyzed, but to find what are the best practices that can be adopted, both in constructing textbooks, and in a larger context, such as patterns and demands in the curriculum. Textbook analysis between countries also describes similarities and differences in how each student obtains the opportunity to learn. For that reason, some researchers pick out textbooks from countries that get high achievements in international surveys, such as PISA and/or TIMSS.

Textbook comparative study about fractions has been done by picking out some developed countries as a comparison, such as America, Japan, China, Singapore, and many more. Some of the researchs are focusing to compare one or more fraction operations, like addition and subtractions (Charalambous et al., 2010; Hwang et al., 2020), multiplication (Kar et al., 2018), and division (Bütüner, 2019; Li et al., 2009). There are also some that focus on the whole fraction subject matter (Yang, 2018; Yang et al., 2010). Nevertheless, other than those research, comparative study that comprehensively focus on analyzing mathematical task for one particular topic of fractions is not being done a lot, especially between two countries that have similar cultural family, but have different education and technology development, that is our research, Singapore and Indonesia. Comprehensive analysis on the wholeness of a certain content allows us to discover the consistency of how that content is handled by the analyzed textbooks. Therefore, this research wants to describe children's

learning opportunities in learning fractions between the two countries, that is Indonesia and Singapore.

Research Questions

This study aims to describe the opportunities to learn fractions between Indonesian and Singaporean mathematics textbooks. To direct the objectives of this study, the research questions were specifically made as follows.

1. How is the structure and order of content presented in Indonesian and Singaporean mathematics textbooks?
2. What are the differences in mathematical activities, complexity levels, answer forms, and contextual features of fractional tasks between the mathematics textbooks in the Indonesia and Singapura?

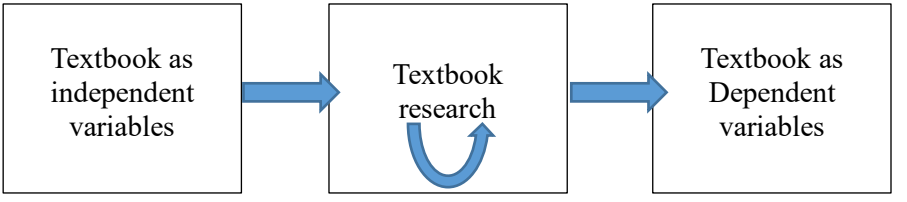
THEORETICAL BACKGROUND

The focus of textbook research in the context of this study

Studies related to textbook research have become a topic of concern for mathematics education researchers in the last two decades (Purnomo, Shahrill, et al., 2022). According to Fan (2013), there are three frameworks to identify and understand textbook research issues. The three frameworks can be visualized in Figure 1.

Figure 1

Textbook research framework



Based on Figure 1, frameworks related to textbook research issues are classified into textbooks as independent variables, as intermediate variables, and as dependent variables. This framework is not only looks at textbooks as

study subject, not only what the textbooks is like or the quality of the textbooks, but also how they are affected by other elements (independent variables) and how they affect other aspects (dependent variables).

The first type, textbooks as an independent variable, namely issues related to how textbooks affect other factors; In other words, textbooks are treated as independent variables (compared to other factors as dependent variables). The issue can be exemplified by the focus of the research that examines how and to what extent textbooks influence learning behavior and teaching mathematics; what is the impact of textbooks on students' mathematics achievement; etc.

The second type, textbooks as dependent variables, namely issues related to how various factors affect textbook development and production, or what makes a textbook into a textbook. The issue can be exemplified by the focus of the research that examines: What role do socio-cultural or political values have in the development of mathematics textbooks in different educational systems? What are the challenges of creating mathematics textbooks using contemporary technologies?

Lastly, issues related to the textbook itself, namely the textbook as a research topic. This issue can be exemplified by several research questions: How do textbooks describes mathematical knowledge? To what extent do textbooks reflect (if any) curricular standards? How do textbooks represent a particular pedagogy (e.g., cooperative learning and constructivism)? How can textbooks represent the importance and development of cultural and societal values?

Based on the three frameworks above, this study focuses on textbooks as research subjects. Fan and his colleagues (Fan, 2013; Fan et al., 2013) use the term "textbook research" which includes the 3 frameworks above, however, we think that "textbook research" is more suitable in the usage of textbooks as research subjects only. Moreover, Fan et al. (2013) conducted a literature study and summarized four categories of textbook research focus, those are the role of textbook, textbook analysis and comparison, textbook use, and other categories. Nonetheless, from several identified studies, not all textbook analyses are textbook comparison studies. Moreover, Fan's study also did not find the role of the textbook category. Fan et al. (2013) argues that this type is often not presented as an individual research study, which instead, is placed in the discussion section or as a pilot study of the main study. Therefore, we classified the textbook categories into four different categories from those proposed by Fan et al. (2013), namely textbook analysis, textbook comparison,

textbook use, and other areas (which are very likely to be included in the role of the textbook). As for this study, we use a textbook comparison study.

Analysis Framework in Textbook Research

Some textbook research has a different focus than the dimensions analyzed, such as studies that focus on the instructional content presented or studies that focus on presenting mathematical tasks. This study focuses on how mathematical tasks are presented and what cognitive demands are offered. However, we also conducted a vertical analysis of the general description of the textbook, both physically, content order, and general information regarding the number of task items in the textbook.

Some researchers have several reasons why they focus on studying mathematical tasks (Bingolbali, 2020; Gracin, 2018; Purnomo et al., 2019; Purnomo, Shahrill, et al., 2022; Wijaya et al., 2015). First, because this section provides opportunities for students to learn independently, both in class, and when doing homework. Second, providing opportunities for students to be engaged in the class. Third, as a way to easily get feedback from the knowledge and skills obtained.

This study adopts five analysis dimensions from Gracin (2018), including content, activity, level of complexity, answer form, and contextual features. He developed the five-dimensional framework based on Austrian standards and the work of Yan and Lianghuo (2006). He uses this framework to analyze "tasks" in two series of mathematics textbooks that are often used in grades 6, 7, and 8 in Croatia. The following is an explanation of the five-dimensional Gracin framework.

Mathematical Activities

There are four main mathematical activities classification, namely Representations and modeling (H1), calculation and operation (H2), interpretation (H3), and argumentation and reasoning (H4). Representation is the translation of mathematical data presented to other mathematical representations, while modeling involves seeing relevant mathematical relationships from a given situation and representing the same problem in mathematical form (symbolic, graphic, etc.). The calculation is performing computational operations using concrete or general numbers, while operations are performing concrete, sensible, and efficient computational steps or referring

to transforming units of measurement, transforming mathematical expressions, completing calculations, estimating results, approximations, and performing constructions. Interpretation refers to the ability to see the relationship and relevance of the presented data interpretation in a context, including reading graphical representations or mathematical symbols. The argumentation is describing the mathematical aspects of making pro or con decisions that require the relationship and characteristics of appropriate mathematical implementations, mathematical rules, and the use of appropriate mathematical language, while reasoning is the use of arguments to get conclusions. Table 1 shows examples of each of these mathematical activities.

Table 1

Examples of each mathematical activity (Gracin, 2018)

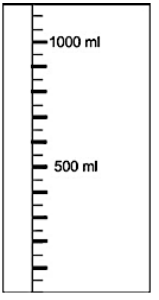
Mathematical Activity	Example
Representations and modelling	Half of a cylindrical barrel is filled with water. The barrel stands 82 cm tall. The tube's base is 82 centimeters in diameter. Make the base at a size of 1:100.
Calculation and operation	Half of a cylindrical barrel is filled with water. The barrel stands 82 cm tall. The tube's base is 82 centimeters in diameter. Calculate the volume of water in the cylinder in liters
Interpretation	Half of a cylindrical barrel is filled with water. The barrel stands 82 cm tall. The tube's base is 82 centimeters in diameter. What is the formula expressing?
Argumentation and reasoning	Half of a cylindrical barrel is filled with water. The barrel stands 82 cm tall. The tube's base is 82 centimeters in diameter. Is the surface area of the tube affected by the height of the water in the tube? Express your thoughts.

Level of Complexity

There are three levels of complexity of tasks, namely reproduction (K1), connections (K2), and reflection (K3). Reproduction involves applying basic knowledge and skills about concepts, rules, procedures, and representations in mathematics. Making connections refers to a more complex level of tasks where a combination of concepts and activities is required to solve a problem. Reflective thinking involves the reflection of mathematical ideas that can‘t be read directly from the given problem which requires them to use knowledge creatively about mathematics. Table 2 shows examples of each of these complexity levels.

Table 2

Examples of each level of complexity (Gracin, 2018)

Complexity level	**Example**
Reproduction	Write the decimal 0.75 as a fraction.
Connections	Paijo measures out 1/8 liter of milk in the container shown below. In the measuring jug, mark the height of the milk. 
Reflection	Sometimes it's better to express rational numbers as fractions and other times as decimals. Give two instances when people would use fractions and two instances when they would use decimals. Explain your response.

Answer form

The answer forms are divided into three, namely closed answer (A1), open answer (A2), and multiple choice (A3). Closed answers are tasks with one correct answer, while open answers are tasks with several or many correct answers. Examples of each answer form can be seen in Table 3.

Table 3

Examples of each answer form (Gracin, 2018)

Answer Form	Example
Closed answer	Find the solution to 2x + 6 = 12!
Open answer	Write a real-world problem that involves calculating the equation 2x + 6 = 12.
Multiple choice	What is the answer to the equation 2x + 6 = 12? A. x = 6; B. x = 3; C. x = -3; D. x = -6

Contextual features

This aspect is used to see the relationship between real-world experiences that are combined in textbook tasks. This contextual feature is divided into three, namely intra-mathematical context (C1), realistic (fictitious) context (C2), and authentic context (C3). Intra-mathematical context is a problem that is not related to the real world. Realistic (fictitious) context is a fictitious problem that contains artificial data from the textbook authors, while authentic context is a problem that involves real-world situations, or the data is obtained by students themselves from their daily lives. An example of each contextual feature can be seen in Table 4.

Table 4

Examples of each contextual feature (Gracin, 2018)

Contextual Feature	Example
Intra-mathematical context	Write the fraction 3/4 in percentage form.
Realistic (fictious) context	On Monday between 3 and 4 p.m., a student counted 12 cars, 3 trucks, 4 motorbikes, and 1 bus passing in front of his school. Display this information in the form of a table of relative frequencies.
Authentic context	Make a table of the relative frequency of motorbikes, cars, buses, and trucks in front of your school.

METHODOLOGY

Textbooks selection

In this study, Indonesian and Singaporean textbooks were compared based on mathematical tasks (worked example and exercise) on fractions. The Indonesian textbook that became the object of this study was the revised 2013 curriculum of the 5th-grade elementary school students published by the Ministry of Education and Culture of the Republic of Indonesia entitled Senang Belajar Matematika– SD/MI Kelas V (Fun to Learn Mathematics– Grade 5 Elementary School). The textbook was chosen because it has been provided by the government and is used as a mandatory reference in elementary schools in Indonesia. On the other hand, the Singapore textbook which is the object of this study is Pupil's Book My Pals Are Here! Math 5a published by Marshall Cavendish Education. This textbook was chosen because 60% of schools in Singapore use this textbook (Yang et al., 2010).

Analytical Coding and Data Analysis Framework

Data analysis is classified into two, namely vertical and horizontal analysis. Horizontal analysis focuses on physical characteristics and general

description of textbooks, while vertical analysis focuses on fractional tasks in both textbooks. We analyzed Indonesian and Singaporean mathematics textbooks by adapting the Gracin framework. The following is a research instrument based on the Gracin framework.

Table 5

Research Instruments based on the Gracin Framework

Dimension	Question	Details and Code
Mathematical Content	What mathematical content do students need to know to perform a particular task?	What is mathematical content presented in the textbook?
Fraction content	What fractional content do students need to know to do a particular task?	What is fraction content presented in the textbook?
Mathematical activities on fractions	What mathematical activities do students need to do in fractions?	• Representation and Modeling (H1) • Operation and Calculation (H2) • Interpretation or interpretation (H3) • Argument (H4)
The level of complexity	What is the level of complexity of the knowledge and activities that students need to do the task?	• Application of knowledge and skills of fractional material (K1) • Problem solving related to fractions (K2) • Reflection or application of reflective knowledge (K3)
Form of answer	What kind of answer does the task require on fractions?	• Closed answer (A1) • Open answer (A2) • Multiple choice (A3)
Contextual features	Is the task in the fractions contextual?	• Intra-mathematical context (C1) • Realistic context (C2)

- Authentic context (C3)

Three researchers (second author, third author, and fourth author) coded both Indonesian and Singaporean mathematics textbooks with Table 1 as the reference. After each author conducts the coding, they then peer check, and the agreed results are used for presentations in focus discussion group (FGD) sessions. FGD was chosen because it is one of the discussions that can be run systematically and directed in discussing a problem, can be run in a relaxed or informal atmosphere, and also can improve and strengthen findings if there are doubts. The coding results were validated by three other researchers (first author, fifth author, and last author). The agreed coding results are used to report research results that are presented descriptively.

RESULTS AND ANALISES

Textbook and Content Overview

The Indonesian textbook has 258 pages with 42 pages containing fractional content, while the Singapore textbook has 176 pages with 49 pages of fractional content (including the subchapter title page). Physically, the Indonesian textbook is smaller than the Singaporean textbook.

By modifying the Gracin framework, the researcher analyzed the content in the form of a description. Based on the table of contents, the mathematics content presented in each textbook can be shown in Table 6.

Table 6

Mathematics Contents Presented in each Textbook

Indonesian Textbooks	Singaporean Textbooks
1. Operasi Hitung Pecahan (Fractions Operation) **a. Penjumlahan dan Pengurangan Pecahan (Addition and Substraction of Fractions)**	1. Whole Number 2. Operations of Whole Numbers 3. Fractions and Mixed Numbers a. Lesson 1 - Fractions and Division

b. Perkalian dan pembagian pecahan dan desimal (Multiplication and Division of Fractions and Decimals)	b. Lesson 2 - Addition of Mixed Numbers
	c. lesson 3 - Subtraction of Mixed Numbers
2. Kecepatan dan Debit (Speed and Debit)	4. Multiplication of Whole Numbers, Fractions, and Mixed Numbers
3. Skala (Scale)	a. Lesson 1 - Product of a Fraction and a whole number
4. Bangun (Shapes)	b. Lesson 2 - Product of Two Fractions
5. Pengumpulan dan Penyajian Data (Data Collection and Presentation)	c. Lesson 3 - Product of a Mixed Number and a Whole Number
	5. Fractions: Word Problems
	Lesson 1 - Solving Word Problems
	6. Area of a Triangle
	7. Ratio
	8. Volume of Cubes and Cuboids

As shown in Table 2, for Indonesian mathematics textbooks, fractions are the first content taught in grade 5, while grade 5 students in Singapore study fractions after studying whole number and operations. The fractions content presented in the Indonesian textbook is only in one chapter, while in the Singapore textbook, the fractions content is divided into three chapters. We also identified that Singaporean mathematics textbooks present material about the meaning of fractions that is being connected to the meaning of fractions as division, whereas Indonesian mathematics textbooks directly discuss addition operations in fraction.

Mathematical Task Overview

Both Indonesian and Singaporean mathematics textbooks use special sections to present mathematical tasks. Each section has its own criteria which can be seen in Table 3. In Indonesian mathematics textbooks, mathematical tasks are grouped into 11 sections with a total of 462 task items in one fraction chapter, while the Singaporean mathematics textbooks consist of 9 sections for each chapter with a total of 186 fraction task items spread into three chapters, namely 86 questions in chapter 3, 69 questions in chapter 4, and 31 questions in chapter 5.

In Indonesian mathematics textbooks, the task activity with the most questions are the “Fun to Try” activity with 351 questions. The activity contains practice questions so that students can find concepts from the existing exercises. On the other hand, Singaporean textbooks have fewer learning activities, namely 9 activities. Most questions are in the "Try" activity, which is 72 questions. The activity is an exercise that is conducted by students where the questions still refer to the previous "Learn" activity.

Table 7

Task activities in Indonesian and Singaporean mathematics textbooks

Indonesian Textbooks		**Singaporean Textbooks**	
Task Activity	**Description**	**Task Activity**	**Description**
Ayo Amati (Let's Observe)	Contains questions before students learn the material	Before You Learn	Contains questions before students learn the material
Aktivitas (Activity)	Contains steps for student activities	Learn	Contains sample questions and the steps to do it
Asyik Mencoba (Fun to Try)	Contains guided exercises that expect students to find concepts from existing exercises with fun	Try	Always located after "Learn", contains practice questions similar to the example questions in "Learn"
Asyik Bereksplorasi	Contains activities that train the students to think creatively and innovatively	Hands-On Activity	Contains steps for student activities

(Fun to Explore)	(students answer open-ended questions)		
Uji Kompetensi (Competence test)	Contains practice questions that cover basic competencies and material in chapters	Chapter Review	Contains practice questions that cover the material in a chapter
Asyik Berlatih (Fun to Practice)	Contains questions that involve problem solving and HOTS	Chapter Wrap-Up	Contains summary and example questions
Tugas Proyek (Project Task)	Is the application of basic competencies; can cover other subjects	Put On Your Thinking Cap!	Contains practice questions
Berpikir Kritis (Critical thinking)	Contains critical questions that can stimulate students' imagination to think critically (including students' ability to ask critical questions)	Math Sharing	Contains students' activities
Belajar Bersama Orang Tua (Learning with Parents)	Contains activities that aim to show the role of parents (participation) in the learning process	Recall	Contains sample questions based on the previous chapter
Tantangan (Challenge)	Located after "Ayo Mencoba", contains one practice question that is similar to the example questions in "Ayo Mencoba"		
Contoh soal disertai jawaban (Worked example)			

Table 7 shows that the specific sections of Indonesian mathematics textbooks are more comprehensive than Singaporean mathematics textbooks.

In the Indonesian Mathematics textbook, there are activities involving parents and sample problems, which the Singaporean mathematics textbook does not explain explicitly.

Although quantitatively the number of task sections and the number of items is higher, in terms of quality based on each dimension in the Gracin framework, the distribution of cognitive billing for Singaporean mathematics textbooks is more proportional than Indonesian mathematics textbooks. In general, these conclusions are shown in Table 8 and Table 9. In detail, the comparison of each of these dimensions is reviewed in the following subsection.

Table 8

Number of items distribution based on analysis results for Indonesian Textbook Grade 5

Task Activity	Total	H1	H2	H3	H4	K1	K2	K3	A1	A2	A3	C1	C2	C3
Ayo Amati (Let's Observe)	3	0	3	0	0	0	3	0	3	0	0	0	3	0
Aktivitas (Activity)	4	0	4	0	0	0	4	0	4	0	0	3	1	0
Asyik Mencoba (Fun to Try)	351	0	351	0	0	316	34	1	331	20	0	344	7	0
Asyik Berekspl orasi (Fun to Explore)	2	0	2	0	0	0	2	0	0	2	0	2	0	0
Uji Kompete nsi (Compete nce test)	27	0	27	0	0	14	12	1	6	1	20	16	11	0
Asyik Berlatih	30	0	30	0	0	0	30	0	30	0	0	0	30	0

Task Activity	Total	H1	H2	H3	H4	K1	K2	K3	A1	A2	A3	C1	C2	C3
(Fun to Practice)														
Tugas Proyek (Project Task)	1	0	0	0	1	0	0	1	0	1	0	0	0	1
Berpikir Kritis (Critical thinking)	1	1	0	0	0	0	1	0	0	1	0	0	1	0
Belajar Bersama Orang Tua (Learning with Parents)	1	0	0	0	1	0	0	1	0	1	0	0	0	1
Tantangan (Challenge)	2	0	2	0	0	1	1	0	2	0	0	2	0	0
Worked example	40	0	40	0	0	34	6	0	40	0	0	34	6	0
Total	462	1	459	0	2	365	93	4	416	26	20	401	59	2
% (Percentage)	100	0.22	99.35	0.00	0.43	79.00	20.13	0.87	90.04	5.63	4.33	86.80	12.77	0.43

Table 9

Number of items distribution based on analysis results for Singaporean Textbook Grade 5

Task Activity	Total	H1	H2	H3	H4	K1	K2	K3	A1	A2	A3	C1	C2	C3
Before You Learn	8	2	5	0	1	3	5	0	8	0	0	3	5	0
Learn	28	7	21	0	0	20	8	0	28	0	0	13	15	0
Try	72	17	55	0	0	64	8	0	72	0	0	64	8	0
Hands-On Activity	8	4	3	0	1	5	3	0	7	1	0	3	2	3
Chapter Review	44	12	32	0	0	37	7	0	44	0	0	37	7	0
Chapter Wrap-Up	15	3	12	0	0	10	5	0	15	0	0	10	5	0
Put On Your Thinking Cap!	9	0	8	1	0	5	4	0	8	1	0	6	3	0
Math Sharing	1	0	1	0	0	0	1	0	1	0	0	1	0	0
Recall	1	1	0	0	0	0	1	0	1	0	0	0	1	0
Total	186	46	137	1	2	144	42	0	184	2	0	137	46	3
% (Percentage)	100	24.73	73.66	0.54	1.08	77.42	22.58	0	98.92	1.08	0	73.66	24.73	1.61

Mathematical Activities

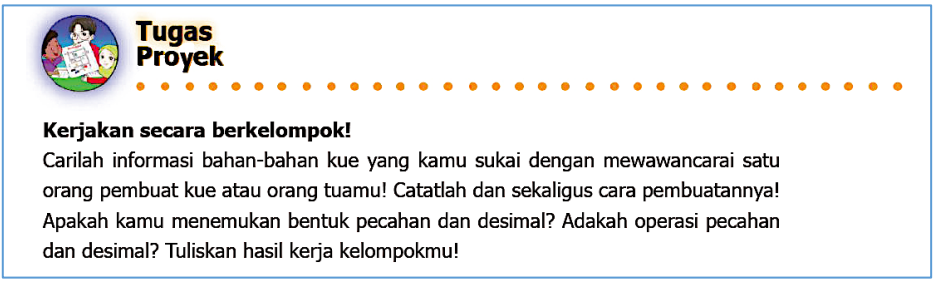
Four categories of mathematical activity, namely representation and modeling (H1), operation and calculation (H2), interpretation (H3), and argumentation & reasoning (H4), are calculated based on the parts of the lesson. Table 8 shows the distribution of mathematical activities based on these categorizations.

Table 8 shows that of the 462 tasks in Indonesian mathematics textbooks, most of them are classified as operations and calculations which

consist of 459 items (99.35%). The rest are 2 items for argumentation & reasoning activities, each of which is located in the "Tugas Proyek" (Project Task) and "Belajar Bersama Orang Tua" (Learning with Parents) sections, 1 item for the representation and modeling activity that is located in the "Berpikir Kritis" (Critical Thinking) section, and interpretation activities are not identified in this textbook. The part that displays the most H1 activities in this textbook is the Asyik Mencoba (Fun to try) which is 351 items and the rest is spread into several other parts, except for Tugas Proyek (Project Task), Berpikir Kritis (Critical thinking), and Belajar Bersama Orang Tua (Learning with Parents).

Figure 2

Argumentation and reasoning activities in Indonesian textbooks



Translation:

Work in a group!

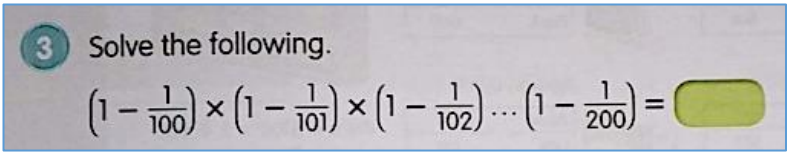
Find the ingredients of your favorite cake by interviewing a cake maker or your parents! Write it down with the step-by-step procedure! Did you find any fractions and decimals? Is there any fraction and decimals operation? Write the results!

Different findings for Singaporean mathematics textbooks are shown in Table 5. The distribution of each activity dimension in this textbook is slightly more proportional than in Indonesian textbooks, even though both provide operation and calculation activities that are more dominant than the others. Of 186 items, 137 items (73.66%) of them are operations and calculation activities. As for Indonesian textbooks, interpretation activity is the lowest compared to other activities (0.54%) and then followed by argumentation & reasoning activity (1.08%). The interpretation activity in Singapore textbooks

can be exemplified in Chapter 4, page 85, in the Put on Your Thinking Cap section shown in Figure 3.

Figure 3

Interpretation activity in Singaporean textbook



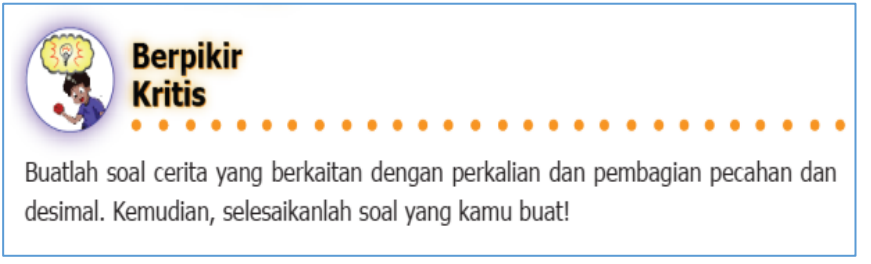
Complexity Level

The level of complexity is sorted and grouped into three categories, namely reproduction (K1), connection (K2), and reflection (K3). Table 4 shows the distribution of the complexity level in each lesson in the Indonesian textbook, while in Singaporean textbook can be seen in Table 5.

Indonesian mathematics textbooks provide fractional task items which are dominated by the reproduction level, namely 365 of the 462 (79%) task items. The most distribution is in the Asyik Mencoba (Fun to try) section, then followed by a competency test and sample questions with answers. The next portion for other complexity level is the connection level which provides 93 task items (20.13%) and the last is the reflective knowledge level which includes 4 items (0.87%). One of the reflective items is exampled in Figure 4.

Figure 4

Reflective level in Indonesian mathematics textbooks.



Translation:

Make story problems related to multiplication and division of fractions and decimals. Then, solve the problem you created!

The task item of the complexity level in the Singapore mathematics textbook is not much different from that of the Indonesian mathematics textbook. The highest distribution is also at the reproduction level, which is 144 out of 186 (77.42%) and followed by the connection level, which is 42 questions (22.58%). Meanwhile, the level of reflective knowledge cannot be identified in this textbook.

Answers Form

The answers are categorized into three categories, namely closed answers (A1), open answers (A2), and multiple choice (A3). The difference between the two, in the answer form of the mathematical tasks, is that there is no multiple choices form for Singapore mathematics textbooks. As for Indonesian mathematics textbooks, there are all three.

Both Indonesian and Singaporean mathematics textbooks provide a closed answer form (A1) that takes the largest portion. In Indonesian mathematics textbooks, there are 90.04% of the 462 task items, while Singapore's 98.92% of the 186 task items. For the open answer form (A2), there are more in Indonesian mathematics textbooks, namely 5.63% than those found in Singaporean mathematics textbooks, which are 1.08%. Tasks in the form of multiple choice (A3) in Indonesian mathematics textbooks can only be found in the competency test section, which is 20 items (4.33%) and cannot be found in Singaporean textbooks.

Contextual Features

Gracin divides contextual features in textbooks into three aspects, namely intra-mathematical context, realistic context, and authentic context. Based on Tables 4 and 5, although both textbooks are more dominant in intra-mathematical features, the distribution of contextual features for Singapore textbooks is more even.

There are 86.80% of the 462 questions in the Indonesian textbook which are questions on intra-mathematical features and clustered in the Asyik

Mencoba (Fun to Try) section, while in the Singaporean textbook there are 69.35% of the 186 questions with intra-mathematical features. In other words, the portion for realistic features and authentic features in Singapore mathematics textbooks is more evenly distributed than in Indonesian mathematics textbooks. The following are examples of realistic and authentic features both taken from Singapore math textbooks.

Figure 5

Realistic Features in Singaporean Textbook

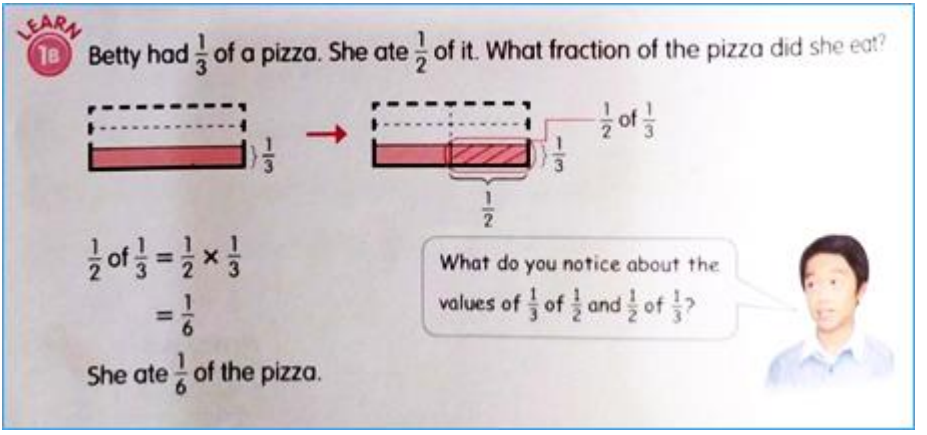
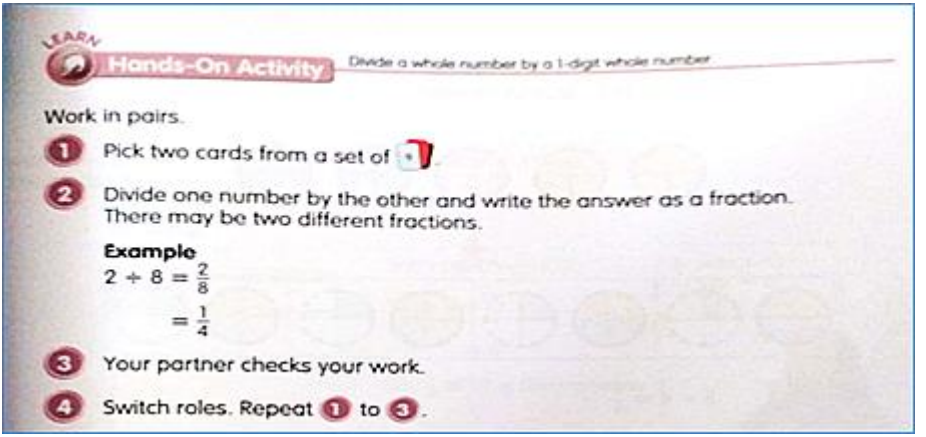


Figure 6

Authentic Features in Singaporean Textbook.



The realistic feature as exemplified in Figure 5 is one of the 15 task items that are all contained in the Learn section of the Singapore textbook. Meanwhile, Figure 6 is one of the 3 task items which are all contained in the Hands-on Activity section.

DISCUSSION

This study aims to compare the description and distribution of fraction materials presented and what are the competence requirements in the Indonesian and Singaporean mathematics textbooks, especially in the mathematical task section. We took a sample of mathematics textbooks in Indonesia, namely Senang Belajar Matematika Kelas V SD (Fun to Learn Mathematics– Grade 5 Elementary School) and Pupil's Book My Pals Are Here! Math 5a for the Singaporean mathematics textbook.

Our findings show that the structure and order of the content of the two textbooks are significantly different in Indonesian textbooks, fractions are the first material presented in grade 5, which includes a discussion of fractional arithmetic operations consisting of addition, subtraction, multiplication, and division of fractions in 1 chapter. In contrast to the presentation in the Singapore textbook, their fraction material is divided into 3 chapters, after they learn the whole number and its operations. Uniquely, they linked their understanding of

fractions and division at the beginning of their lesson. As stated in the study by (Purnomo, Arlini, et al., 2021), the context of fractions as division is very familiar to students and helps them to develop the concept of partition. Some researchers agree that the meaning of fractions, other than as part of the whole, needs to be presented and adjusted to students' preparation level. However, empirical studies state that fractions in Indonesian textbooks are only introduced as part of a whole (Rahmawati et al., 2020). The presentation is also general and traditional, so students often experience misconceptions in doing parting in fractions, they only focus on the number of parts of the whole, yet they do not interpret that the parts must be congruent (Purnomo et al., 2014).

Our findings also show that in general, the type of task activities for Indonesian mathematics textbooks is more comprehensive than in Singaporean mathematics textbooks. All types of task activities in Singaporean textbook have been accommodated in Indonesian textbooks' task activities. In Indonesian mathematics textbook, there are activities involving parents and sample questions, which the Singaporean mathematics textbook does not explain explicitly. Parent involvement in their child's learning process is highly recommended both cognitively, affectively, behaviorally, and pedagogically to increase student involvement, students' self-concept, and their learning outcomes (Purnomo, Apriyanti, et al., 2022; Purnomo, Safitri, et al., 2021).

In general, although some aspects of competence for each textbook tend to have the same proportions, it is more evenly distributed in Singaporean textbooks than in Indonesian textbooks. For example, in mathematical activities, their operation and calculation activities are more dominant than the other activities. However, Singaporean mathematics textbooks provide more opportunities for argumentation and reasoning activities, as well as interpretation activities, which interpretation activities are not available in Indonesian mathematics textbooks. This finding is in line with the results of international surveys that show unequal differences between the two countries in terms of mathematical literacy problem solving (Wijaya, 2017).

Argumentation and reasoning activities, as well as interpretation activities, are important to be offered more in textbooks. It's because these competencies provide learning opportunities to practice higher-order thinking skills and are useful for students. It's not only can be used for routine solving, but also in multi-context problem solving and become skills that are more needed in their respective jobs in the future.

We also highlight the portion between dimensions of the mathematical activity which is dominated by operations and calculation activities. These

activities are indicated with an emphasis on procedural rather than conceptual aspects of knowledge. The emphasis on this activity, without a solid conceptual knowledge base, often causes students to make errors in computing, stagnate when they forget the procedures, and be less likely to develop number sense (Byrnes & Wasik, 1991; Purnomo et al., 2014), and in turn devalued mathematics. This dominance in the portion of operation and calculation activities is parallel to the analysis results for complexity level and answer form task item. Both textbooks are dominated by the reproduction level, which involves the basic knowledge application and skills about concepts, rules, procedures, and representations in mathematics. This level is in line with the dominance of answer form, that is in the form of closed answers, for both textbooks. Only 1 task item is included in the reflective thinking level, namely in Indonesian mathematics textbooks. Our findings are in line with previous studies, which found that most mathematics textbooks in various countries were dominated by closed questions (Yang et al., 2017; Yang & Sianturi, 2022). These questions require lower-order thinking skills than higher-order thinking skills (Gracin, 2018).

The parallel results between the dominance in operation and calculation activities, being at the reproduction level, and the closed answers form are also in line with the dominance of intra-mathematical context questions. Intra-mathematical context questions require more basic knowledge and skills, which are generally in the form of operations and calculations, especially in fractions. This finding is in line with the findings from several previous studies that are related to the use of context-based problems, which rarely contained in mathematics textbooks, including mathematics textbooks in Indonesia (Purnomo et al., 2019; Wijaya et al., 2015; Yang & Sianturi, 2022).

Although Singaporean mathematics textbooks are also dominated by intra-mathematical features, their contextual features are more proportional and more comprehensive in providing learning opportunities by including all three features. Meanwhile, Indonesian mathematics textbooks, with 3 times the number of tasks, still display fewer realistic and authentic features. Without overriding the role of intra-mathematical context features, realistic and authentic features are very important for students to better interpret what they learn (Wijaya et al., 2018), improve mathematical disposition, increase student involvement in learning mathematics, and appreciate the beauty of mathematics. This competency is also relevant to multi-context-based mathematical literacy which is expected to be used in solving problems of everyday life as well as to prepare them with skills for work.

Another important note from our findings regarding contextual features is that both of them still provide few authentic features, which is a feature that is more directed to the use of students' real experiences compared to parable real-world situations. The authentic feature needs to get a more proportional portion so that students feel the presence of mathematics in their problems to handle. Of course, it suits the nature of mathematics itself that mathematics originates from and is used for social and cultural needs (Ernest, 1991; Hersh, 1997; Shapiro, 2000).

CONCLUSION

This study describes the learning opportunities of fractions covered in Indonesian and Singaporean mathematics textbooks, particularly analyzed with Gracin's five frameworks. Our findings indicate that the two textbooks have significant differences in content structure and sequence. We also found that both textbooks offer a variety of task activity types, with Indonesian textbooks offering more types. The dominance of operation and calculation activities, reproduction level, closed question forms, and intra-mathematical context features can be found in both textbooks. However, the distribution of the number of tasks for each dimension is more even in Singaporean textbooks.

AUTHORS' CONTRIBUTIONS STATEMENTS

YWP plaid a role in conceptualization and methodology. YMA, AA, FM, and IWP collected the data. YMA, AA, and FM analysed the data. YMA, AA, FM, and IWP performed writing the original draft. YWP and FMF performed writing, reviewing & editing. All authors actively discussed the results and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by the corresponding author, YWP, upon reasonable request.

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