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Hybrid Relationship Mediated by Digital Technologies in the Continuing-FormAction of Teachers Who Teach Mathematics

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ABSTRACT

Context: Existing studies on the use of digital technologies in mathematics education do not problematize the symmetry in the relationship between humans and non-humans, placing only humans as protagonists. **Objective:** To map the mediations constituted between the actors in a continuing education course for mathematics teachers, mediated by all entities that compose the sociotechnical network. **Methodological design:** The theoretical-methodological assumptions of Actor-Network Theory, where humans and non-humans can associate and be protagonists of a certain action, assuming the post-humanist paradigm. **Setting and participants:** We analyzed the sociotechnical network through the mapping of associations and mediations established between humans (30 mathematics teachers) and non-humans (computer, internet, among others) in an extension course. **Data collection and analysis**: Data were collected during remotely conducted meetings, transmitted and recorded using the resources of the Google Meet platform. For data analysis, we used the principles of agnosticism, generalized symmetry, and free association as a reference to describe the mediation relationships between the actors during the course. **Results:** The results indicate that the mediations established between the entities contribute to the continuity of the sociotechnical network, promoting transformations and contextualization of mathematical concepts. **Conclusions:** Mediators do not limit themselves to humans and intermediaries to objects, but both can take actions, promoting continuity in the continuing education of mathematics teachers.

Keywords: Sociotechnical Network; Continuity; Continuing Professional Development; Mathematical Concepts; Digital Technologies.

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Relação Híbrida Mediada pelas Tecnologias Digitais na FormAção-Continuada de Professores que Ensinam Matemática

RESUMO

Contexto: A pesquisa foi motivada pela percepção de que os estudos existentes com o uso de tecnologias digitais no ensino de matemática, não problematizam a simetria na relação entre humanos e não humanos, colocando apenas os humanos como protagonistas. **Objetivo:** Este artigo tem por objetivo mapear as mediações constituídas entre os *actantes* em um curso de formAção-continuada para professores que ensinam matemática, agenciadas por todos os entes que compõem a rede sociotécnica. **Desenho metodológico:** Neste estudo, inspiramo-nos nos pressupostos teórico-metodológicos da Teoria Ator-Rede, em que humanos e não humanos podem se associar e ser protagonistas de determinada ação, tomando como pressuposto o paradigma pós-humanista. **Ambiente e participantes:** Analisamos a rede sociotécnica por meio do mapeamento das associações e mediações constituídas entre humanos (30 professores que ensinam matemática) e não humanos (computador, internet, entre outros) em um curso de extensão. **Coleta e análise de dados:** Os dados foram coletados durante os encontros promovidos de forma remota, transmitidos e gravados com os recursos da plataforma *Google Meet*. Para a análise dos dados, utilizamos os princípios do agnosticismo, da simetria generalizada e da associação livre como referência para descrever as relações de mediação entre os *actantes* durante o curso. **Resultado:** Os resultados apontam que as mediações estabelecidas entre os entes contribuem para a continuidade da rede sociotécnica, promovendo transformações e contextualização de conceitos matemáticos. **Conclusões:** Os mediadores não se atêm aos humanos e os intermediários aos objetos, mas ambos podem protagonizar ações, promovendo continuidades na formAção-continuada de professores que ensinam matemática.

Palavras-chave: Rede sociotécnica; Continuidade; FormAção-continuada; Conceitos Matemáticos; Tecnologias Digitais.

INTRODUCTION

In these introductory notes, we invite you, the reader, to analyse the cartoon in Figure 1 and reflect with us.

Figure 1

Comic strip about the modernisation process (from Question 100913 (2013))

Why does Mafalda, in the last frame of the comic strip, look so astonished after her friend says she is going to buy a knitting machine when she grows up? Would cut and sew affect Mafalda and her friend in the first frame, so a proposal for a change of attitude? Does the fact that the current generation is guided by technology, the space age, and electronics, among other areas, as pointed out in the second frame, guarantee the performativity of new practices? Does entering science, in the third frame, mean abandoning old actions?

Between the first and last frames, we went from one extreme to another. At first, the friend believes she is a determined woman, capable of mastering and abandoning the cutting and sewing that had been passed down for generations due to the possibilities that modernisation and technological advances offered them. Ultimately, she cannot free herself from this practice, showing to be dominated by past conduct, to the point that Mafalda becomes frustrated when she sees the advancement of technology and the proposition of reproducing old practices with new machines (Question 100913, 2013).

Seeking to connect the reflection of the comic strip, which shows that humans are not in control of all actions as we imagined, with our object of study, we risk saying that teachers who teach mathematics also do not have total control as, many times, non-humans can also play the leading role in classroom practices. Given this, we will problematise the continuing education of those professionals based on the footprints left by digital technologies today, as we saw in the comic strip presented. We understand continuing education as a flowing action linked to other formative moments and current legislation, as proposed by Santana (2023).

To elucidate some gaps and agendas still open in research in mathematics education with digital technologies, this work considers them agents capable of promoting transformations, movements, and differences and being protagonists in specific actions. In this direction, we will investigate the relationship between humans and non-humans.

For almost two decades, the relationship *man-technologies-media* in Brazil (Borba; Villarreal, 2005) and the human and non-human aspects are highlighted in connection, which allows a different and non-dichotomous look at digital technologies in the construction of knowledge (Bairral, 2015). However, this model does not problematise the symmetry in the relationship between humans and non-humans, placing the former as protagonists in visualisation, observation, experimentation, and content creation in mathematical learning. Technologies continue as assistants to humans and not as producers with humans. Faced with this gap, we propose a rupture and a symmetrical look at the relationships between subjects and objects in the continuing education of teachers who teach mathematics. We emphasise that our objective is not to prioritise the ANT in relation to the concept of Human-Beings-With-Media, but to follow the flow of the network, enabling reflections, analyses, and new discussions.

To do this, we took as our object of study the associations established in a continuing education course for teachers who teach mathematics in the remote mode, legitimised in the context of the COVID-19 pandemic.

This controversial scenario led us to investigate how the relationship between humans (participants) and non-humans (digital technologies) can contribute to a continuing education course in the remote mode. To meet the purpose of this study, we sought to map the relationships established between humans and non-humans in a continuing education course for teachers who teach mathematics, performed through digital technologies and participants.

In the next section, we mobilised concepts from the actor-network theory (ANT) developed by Bruno Latour. Next, we represent the research objective in theoretical terms.

MEDIATORS AND INTERMEDIARIES CONSTITUTING THE SOCIOTECHNICAL NETWORK IN THE CONTINUING EDUCATION OF TEACHERS WHO TEACH MATHEMATICS

Inspired by Latour (2012, 2019), we will seek to focus on the continuities and discontinuities performed by humans and non-humans when they act and manage what the author considers a sociotechnical network. For Latour (2012), the notion of a sociotechnical network relates to the associations between the most varied types of *actants* and is defined by its blended actions, i.e., combined between humans and non-humans. For the ANT, *actants* promote action and, from there, lead several others, humans or not, to act, analysed with the same degree of importance and producing transformations in the meaning of what they supposedly carry. At the same time, associations are seen as a relationship between actions promoted by distinct *actants* that come together temporarily when they act (Latour, 2012).

The notion of a sociotechnical network thus designates a series of associations that help those investigating to see the discontinuities necessary to generate actions. Thus, "the network is not just a technical device, as it could be a railway network, a water supply network, a sewage network or a mobile telephone network" (Latour, 2019, p. 38). The network designates a series of associations revealed through a test (that of surprises in ethnographic research), which makes it possible to understand which series of small discontinuities should be passed to obtain a certain continuity.

This investigation will use Bruno Latour's concept of mediation as a parameter to map the constitution of the sociotechnical network that tends to form. For Latour (2012), mediation is the action of mediators, those who, in action, lead several others to act and promote differences; that is, transformations in what they transport, in what makes transport possible and in what they circulate. Inspired by this concept, Lemos (2013) points out that mediation is any action an *actant* takes toward others in the search for strategies and interests for the future stabilisation of the sociotechnical network. In the same direction, Praude (2015) reports that every action promoted results from mediation without necessarily placing humans at the centre of intentionality. Santaella and Cardoso (2015, p. 168) note that "the concept of mediation requires that the social be seen as the product of an association between human and non-human actors, functionally symmetric in the actor-network theory." In turn, Salgado (2018, p. 107) says: "in mediation, *actants* make others *actants* act and produce changes in the agents themselves

and the meanings produced." Thus, we understand mediation as a hybrid action that does not simply transport meanings through a specific medium but transforms, displaces, recreates, modifies, and translates other actions.

Thus, mediators "make things happen" by translating what they transport, redefining and unfolding continuities in action. In this transportation, the environment is transformed, along with what is transported (Latour, 2012). It thus differs from intermediaries. For Latour (2012), intermediaries are only responsible for transporting without promoting changes in what they transport, the environment, and themselves. They do nothing but convey and displace traits aimed at nature and society. This transported content does not mobilise those intermediaries.

By way of example, we will mobilise two examples of non-humans that connect to the definitions of mediator and intermediary in a mathematics class. When the overhead projector is used in a specific class to display slides with systems of equations, it acts as an intermediary, transporting the image to the student without promoting any transformation. On the other hand, when we use software such as Geogebra to associate specific equations in the input panel, it acts as a mediator, transforming the equations into graphs and allowing reflections, analyses, and animations.

However, there is a rupture that Latour (2019) defines as a double click, the leaps from one pole to another without showing the sociotechnical network performed, which reduces the importance of mediation. For example, we can mention double-clicking on a computer folder, which opens and shows the files inside. However, we do not see the entire mediation process with the algorithm –the interface with the operating system behind the action of opening the folder. Double-clicking is the demon that tends to kill mediation. It discloses the essence and keeps it away from mediation (Latour, 2019).

Returning to the concept of mediation and intermediation proposed by Latour (2012) and rekindled by other researchers, we can conjecture that digital technologies have the potential to behave as mediators and/or intermediaries in mathematics teaching. Over time, many discussions have arisen about incorporating digital technologies into mathematics education due to their contributions to different areas of knowledge. However, until the last decade, studies showed that these resources were still little used or used poorly in mathematics teaching-learning, even with teachers seeking continuing education (Stormowski et al., 2015).

Those debates allow us to discuss teachers' continuing-formAction (*formAção-continuada*, in Portuguese) during remote teaching developed during the COVID-19 pandemic, with the help of digital technologies. Taking Santana's (2021) studies as a parameter, we use the term *formAction* (*formAção*, in Portuguese) to highlight the action promoted by *actants* that connect, mediate, and promote transformations in the flow of the sociotechnical network. The author uses *continuing-formAction* –with the hyphen– to highlight that the network is continuous and must be disentangled, considering the associations of the *actants* that compose them.

In this scenario of discontinuity, teachers' continuing-formAction also had to adapt, being performed in "new" networks. In these, human beings did not fully control practices and actions. Instead, they were supported by overlapping, in which objects and things also participate in them and contribute to their formation (Silva et al., 2020). In this sense, the modes of existence can be expanded, and there may be others, as beings must pass through other beings to exist. This is a scientific, political, and even religious perspective of the recognition and the intertwining of existences (Latour, 2019).

Discussions about the use of digital technologies in education and teacher continuing-formAction took on broader dimensions during the pandemic due to the challenges and tensions imposed on working methods. We are experiencing the era of digital culture, as digital technologies are promoting a revolution today. They lead us to embody habits generated by a "pedagogy of urgency," restructuring and personalising the pedagogical work of all education professionals and placing a fundamental value on the ability to use technologies (Reis; Negrão, 2022). This fact reveals the potential of *actants*' "make-do", creating new ways of studying, working, and existing. For Latour (2019), existing is continuing a trajectory by overcoming discontinuity, acting, and being acted upon during transformations.

Now, we can represent the objective in theoretical terms. We will map the mediations between the actants in a continuing-formAction course for teachers who teach mathematics, managed by entities that comprise the sociotechnical network.

The following section will present the method adopted to consolidate the research object based on the methodological tools that Bruno Latour and collaborators defended.

METHODOLOGY

To meet the objective of this study, we rely on the assumptions of the post-humanist paradigm, which makes us go beyond the idea of the uniqueness of (human) subjects and embrace a material world that also values the interference of other (non-human) agents in actions, to carry out empirical research (Monteiro et al., 2020). In this scenario, not only can humans connect in a sociotechnical network promoting actions and continuities, but all things embody capacities to act, associate, and be protagonists in each action. Space is open for other modes of existence, and the challenge is to create an environment for trajectories without dissociating subjects and objects (Latour, 2019). This perspective allows us to track the associations and mediations existing between subjects (educator and participants) and objects (Google Meet, Google Forms, mathematical concepts, Geogebra) that mobilise, relate, affect, and promote continuities and discontinuities in the constitution of the sociotechnical network organised in the continuing-formAction course in virtual environments.

To continue the flow of actions, we will consider what Nobre and Pedro (2010) proposed, considering the network a "methodological tool" to map the associations and mediations between *actants*. To do this, we will seek to follow a few steps: (i) follow the tracks left by *actants* to identify messages that mediate in the construction of mathematical concepts; (ii) translate and record messages seeking to highlight how mediations affect *actants* during the construction of mathematical concepts; and (iii) go through the flow of the sociotechnical network in search of possible stabilisations, which materialised the construction of concepts through mediation. We will seek to organise a debate and highlight the importance of opening the possibility for other ways of seeing the world, understanding different ways of existing, and giving space to different modes of existence (Latour, 2019).

The data were collected from observing activities developed in a continuing-formAction course, a research laboratory. To track mediation, the meetings were recorded in Google Meet. To analyse the data, we followed the principle of generalised agnosticism, which presupposes listening to the voices of all *actants* intertwined in the controversy or controversies. We also intend to follow the generalised symmetry that proposes using the same discourses for all human or non-human *actants* in the controversy. Finally, we start from the principle of free association, which advocates the non-separation between humans and non-humans, nature, and culture, but rather

associations as hybrids, i.e., the connection of everything that exists and is around us.

In the next section, we will continue mapping and describing the associations and mediations promoted by *actants* in the continuing-formAction course in mathematical research in the remote mode. We will emphasise mediations, as they are the only ones capable of establishing the continuity of the sociotechnical network (Latour, 2019). It is worth mentioning that, for ANT, a good description is self-explanatory (Latour, 2012).

DESCRIBING THE LABORATORY: TRACKING THE MEDIATIONS OF THE ACTANTS

The work activities were developed in an extension course promoted by the Collaborative Group in Mathematics and Education (Grupo Colaborativo em Matemática e Educação - GCMEduc), with the title of Mathematical Investigations in Virtual Environments (Investigações Matemática em Ambientes Virtuais). In this course, we count on the participation of humans (teachers who teach basic education mathematics linked to different schools and segments, undergraduate and postgraduate students at the UEFS, and students enrolled in the master's degree course in scientific education, inclusion, and diversity at UFRB) and non-humans (computers, the Internet, Google Meet, Google Docs, Geogebra software, systems of equations, among others).

The course took place remotely in the second half of 2021 due to the impacts caused by the COVID-19 pandemic. The actions promoted during this period had 30 participants during six meetings with different themes. The course was composed of synchronous and asynchronous activities and totalled 40 hours. The moments of synchronous activities were conducted and recorded in the virtual platform Google Meet.

To enter the meeting room, participants needed to access the Internet and click on a link available weekly on the Google Classroom platform. This space was also used to post scheduled asynchronous activities. These resources allowed virtual meetings and scheduled activities to be carried out in different geographic spaces, allowing participants to exchange messages via chat, activate microphones and cameras, and socialise links to integrate Google Forms, Google Docs, and Google Drive, among other resources.

Below, we will present a description of the laboratory practices of one of the meetings of the continuing-formAction program. The theme of this meeting was related to the challenges and perspectives on curriculum management in mathematics with investigative tasks and aimed to discuss the solutions of systems of equations with two variables.

In the laboratory description, emphasis was given to the mediations created between humans and non-humans during the analysis of the solutions of the systems of equations, seeking to track the associations triggered in the creation of sociotechnical networks. We will use the term *participants* to refer to all the entities present at the meeting. We did not identify the teacher educator so as not to hierarchise, as Latour (2012) proposes.

Mediation between participants and an investigative task in the Google Docs

Mediations between participants and an investigative task on the possible solutions of a system of equations with two variables began when we made one link available on the Google Meet chat, giving access to a task in Google Docs. In this task, there were three systems of equations (Figure 2). Participants should analyse the task, solve the problem, and present their solutions to everyone who followed the course in real time. Thus, we can say that Google Meet was a mediator and played the leading role in the meeting. As the participants were in different geographic spaces, the meeting would not have occurred without the platform or another resource in which we could make a videoconference.

We then asked participants to solve the systems using previously known methods (such as addition, substitution, comparison, and scaling). This movement could also be presented to students in the final years of elementary school, even if they did not know the content. Therefore, students sought strategies to discover solutions by establishing relationships with other *actants*.

Course participants remained with their cameras and microphones deactivated. They interacted just by typing their answers into the Google Docs document, and everyone could see them (Figure 3).

Figure 2

Systems of equations with two variables

i. $\begin{cases} 3x - y = 10 \\ 2x + 5y = 1 \end{cases}$

ii. $\begin{cases} x - 2y = 5 \\ 2x - 4y = 2 \end{cases}$

iii. $\begin{cases} 2x - 6y = 8 \\ 3x - 9y = 12 \end{cases}$

Figure 3

Solutions presented by participants

Solução i:	Solução ii:	Solução iii:
x = 3 ; y = - 1 (Participante A)	Sistema Impossível (Participante B)	Sistema Impossível (Participante B)
x = 3, y = -1 (Participante B)	Sistema Impossível (Participante F)	Sistema possível e indeterminado (Participante E)
(3, - 1) (Participante C)	Sistema Impossível (Participante E)	
(3, -1) (Participante D)		
x = 3; y = -1 (Participante E)		

In the first resolution movement, the answers of the systems remained the same (systems i and ii). The above indicated that the participants did have prior knowledge of the content. However, in a class with students without this knowledge, this would possibly not occur, giving rise to an investigation scenario for constructing a method, a concept. Furthermore, through actants' mediation (participants-Google Docs), we could perceive that this resource can create a collaborative scenario in class, as students can put their solutions in the document and write how they understand the resolution of a specific task.

At this moment, all colleagues could collaborate and verify the solutions as if there were a chart with the socialisation of responses, thus promoting a mobilisation for the continuity of the sociotechnical network. It is worth highlighting that the access link and the Google Docs behave as mediators to transform action, promote movements, and manage relationships and collaboration between entities, also playing a leading role in the "make-do".

After the mediation moments, the following question was raised: "*Do you think a student can construct a resolution method, a concept within this open question with the teacher's mediation?*" Faced with the question, participant (A) argues:

> For students who may not have prior knowledge, in this first system, generally, they would try to build, obtain the appropriate solution. In the second and third systems, they would also possibly try it, and those who have prior knowledge tried to use one of the methods, but generally, the students say: "Ah! Teacher, it is difficult; it's not possible." They don't have this idea of the impossible.

Advancing in this scenario of associations, we notice that the answers "impossible system" or "possible and indeterminate system" are not usual in the final years. We conjecture that the basic education student could, through socialisation and attempts, seek strategies to manage ways of solving an equation system. Based on the movement of strategies students use to solve the system, teachers' socialisation and mediation, and/or other resources, such as applications, we can build the formulation, the definition of an addition method, and the presentation of a substitution method.

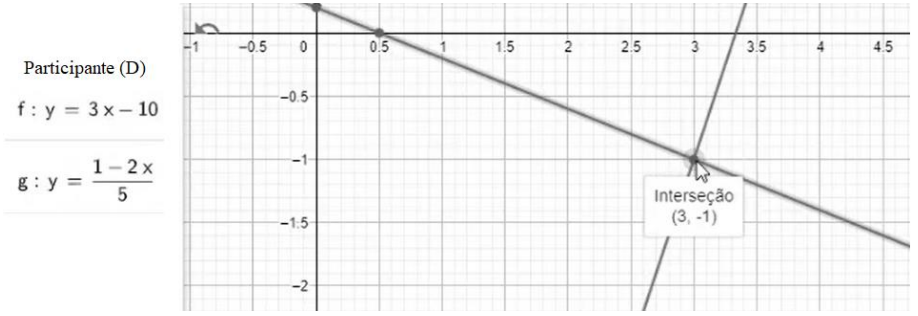
To expand the flow of the sociotechnical network and arrange continuities for the investigative task, we proposed a second phase associating systems of equations with functions because one characteristic of this type of task is that it can be used in different teaching years or stages based on students' prior knowledge and the possibility of continuing with this discussion.

Mediation between participants and an investigative task using Geogebra

Mediations between participants and an investigative task associating the solutions of a system of equations with the content of functions occurred when participants were asked to find the function that corresponded with the equations of each previously presented system. As soon as one of them transformed system (i), we introduced Geogebra, which is online and free, on Google Meet. Then, the functions found in the input field of the software were typed, and the program transformed those equations into graphs, thus creating a mediation capable of redefining the collective and promoting new links for the analysis of the straight lines that now represent each equation of the system (Figure 4).

Figure 4

Representation of system (i) in GeoGebra



At that moment, there was a leap in the process; the software transformed the equations on the graph, but we cannot tell how the agency occurred. Regardless, the relationship between humans and non-humans contributes to better representation, visualisation, and contextualisation, ensuring the continuity of the investigation and construction of mathematical concepts. Furthermore, it was possible to notice that the change in some of the signs of the function affects the system solution and graph construction.

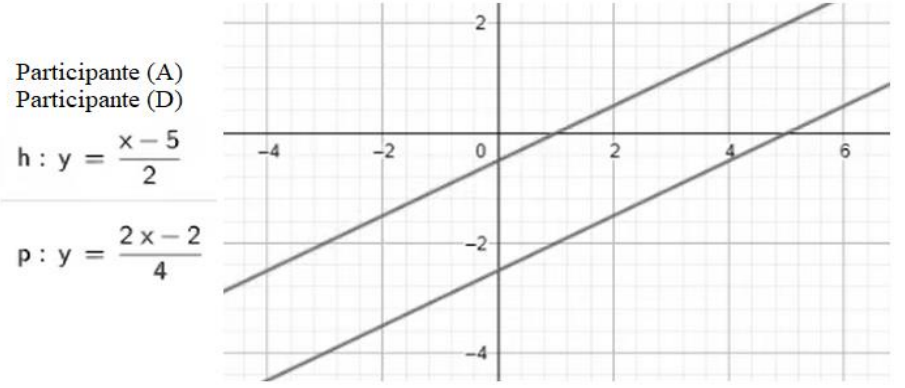
On this occasion, with the continuity of the sociotechnical network, the participants began associating by activating microphones and cameras, conducted analyses to construct the concept of a possible and determined

system, and realised that the system has a single solution. Given this, the straight lines connected to the graph gave clues to the collective, showing themselves as competitors, touching only at a single point. In this context, both human and non-human participants mediated, seeking to promote transformation and movement in the flow of the sociotechnical network, outlining, in digital technologies, the promotion and possibility of constructing concepts related to systems of equations, expanding them to link them with function content.

After creating the graph of the first system of equations, participants were asked to find the functions associated with system (ii). Thus, we continued with the graphic construction of this system (Figure 5).

Figure 5

Representation of system (ii) in GeoGebra



At this point, the participants showed that in system (ii), despite not being able to observe a solution, there are functions associated with such equations. Graphic analysis in GeoGebra revealed that the straight lines that make up the graph are non-coincident parallel, i.e., they do not touch each other at any point. Thus, the *actants* mediate in the research activity for constructing and maintaining the mathematical concept.

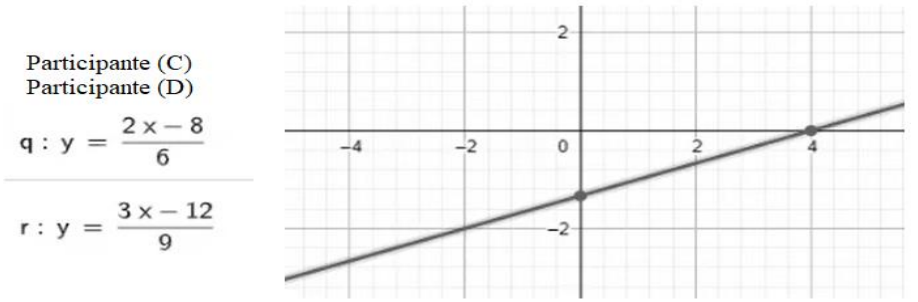
On this occasion, participants associated the construction of the concept of an impossible system (which has no solutions) with a graph of functions performed by non-coincident parallel lines, highlighting this fact.

Thus, the parallel lines arranged by the GeoGebra mediated, as they promoted actions to stabilise the sociotechnical network, transported meaning, and recreated traces to translate a fact to mobilise their protagonism.

Finally, participants were asked to find the functions of system (iii). At this point, they were already more comfortable socialising information through the microphone or the chat in which they participated. These two instruments were very important for actions and associations to occur; without them, spoken and written communication would not happen. Most cameras were open, allowing the perception of gestures and expressions and demonstrated a more interactive scenario for the moment of investigation. After defining the functions, the continuity of the sociotechnical network took place through the agency of the graph in the software (Figure 6).

Figure 6

Representation of system (ii) in GeoGebra



After constructing the graph of the functions represented in system (iii), the participants agreed on a different characteristic concerning system (ii), as the straight lines mobilised at that moment were linked to changeable lines; they revealed to be parallel and coincident, i.e., they had all the dots in common. Hence, the scenario of uncertainty led to a rupture that this system is not impossible, as initially identified by Participant (B); instead, it is a possible and indeterminate system (which has infinite solutions), as mentioned by Participant (E). In this way, the relationship between humans and non-humans contributed to better visualisation, contextualisation, transformation, and understanding of the task, in addition to promoting continuities for the mediation and expansion of the sociotechnical network. It is worth highlighting that mediations were only possible due to

intermediaries, such as the computer, the Internet, Google Meet, and systems of equations, which are often not noticed and/or are not considered in a given action.

Going through these associations, impressions began to be mobilised about the investigative task with the possibility of expanding the final grades of elementary education to secondary education and even higher education, as can be seen in these statements from the participants:

> **Participant (D)**: *I confess that I tried to imagine my eighth-grade students, one of the grades in which I work in the system. I was in disbelief that it would be possible without them knowing this content, but I tried to visualise that space, with the presentation of activities open as this one, the questions arising, and mediation happening, and, in fact, it is possible. We don't have control over how things will happen; from what I captured, that's the purpose of the activity. But it is possible, they will look for different strategies, as participant (A) pointed out, try to replace letters with numbers to check equality and try to escape the algebraic calculation a little. I believe this will happen.*
>
> **Participant (E)**: *An activity of this nature also helps the student attribute meaning to the mathematical content. And this meaning does not necessarily need to be something of the day-to-day, from the student's reality, but a mathematical meaning, a geometric representation of the function's equation, for example. So, I think we end up winning in a very positive way. Of course, we cannot control all the variables, as Participant (D) put it, perhaps the result will be very positive.*
>
> **Participant (A)**: *It is also important to highlight that in an activity like this, for example, for second-grade students who already know matrices and determinants, they could associate these systems with these contents, trying to use the rule of Cramer to discover the solutions or to reach the conclusion that the system was possible determined, possible indeterminate, and impossible.*

> **Participant (N)**: *And if we are in higher education, I can say more; we can talk about the derivative, look at each equation, transforming it into a function and seeing its derivative.*

At that moment, it became clear to these participants that the investigative task has an open nature, and that the teacher can associate them in different ways, depending on the audience; the most varied concepts can be performed with students without the need for exposure, without the mechanism of having it ready. Therefore, there is a range of agency possibilities, as the investigative task can potentially construct conjectures and mathematical concepts.

Given the associations created so far, humans and non-humans have been affecting each other. They contribute to expanding the flow of the sociotechnical network and the continuity of new branches; they mediate the emergence and stabilisation of controversies.

DISCUSSION

In the previous section, we presented two practices that explain existing associations between *actants* of different natures, promoting mediations in the composition of the sociotechnical network flow. Our purpose was to map those mediations constituted between the *actants* in a context of continuing-formAction for teachers who teach mathematics, managed through digital technologies and all the entities that make up this sociotechnical network in the remote mode. To this end, we sought to observe how humans and non-humans relate, how they affect each other, and how the heterogeneous relationship can contribute to the continuity and/or discontinuity of the network in the search for stabilisation in the expansion of the investigative task, the creation and analysis of mathematical concepts.

Inspired by Latour (2012, 2019), when describing the practices constituted in the course, we bring associations that make visible the discontinuities necessary to generate an action, promoting differences and transformations through digital technologies, "making-do" the mathematics investigation in remote mode. By mapping the tracks left by *actants* in the continuity of the sociotechnical network in search of mediations, we participated in the dynamics of its movements regarding continuing-formAction. However, our objective was not to interfere in the associations or stabilisation; we just took on the role of detectives, looking for traces and clues left by the *actants*, as proposed by Latour (2012).

Mediations between participants and an investigative task on the possible solutions of a system of equations with two variables moved through the relationship between humans and non-humans. They promoted an expansion in the flow of the sociotechnical network based on movements outlined by Google Docs, managing convergences, divergences, and uncertainties. According to Latour (2019), if there is some discontinuity or hiatus (small interruptions in the sociotechnical network), which explains a particular type of continuity, seeking to trace a trajectory, it can be characterised as mediation. In the broad sense of the term *mediation*, we consider it to seek to promote actions so that *actants* act, transport, move, and transform the sociotechnical network in search of stabilisation (Latour, 2012). In this sense, we can say that the network performed resulted from mediations promoted by the human participants in the course and all the entities that compose the sociotechnical network (Praude, 2015).

Advancing in this scenario of continuity, the task gained characteristics for investigation and expansion. Furthermore, discussions were taking place around its application in the classroom with students in their final years of elementary school and high school. In this sense, mediators (participants, Google Docs, investigative tasks on systems) "made-do" the translations of what they transported, redefining and reconstituting the continuity of actions. This movement ends up transforming the environment itself, along with what is transported (Latour, 2012). During these associations, some participants behaved as intermediaries, remaining apathetic to interaction, without promoting actions or linking traits aimed at nature or society, as well as Google Meet, which only transported the actions without promoting any changes (Latour, 2012; Salgado, 2018).

Mediations between participants and an investigative task associating the solutions of a system of equations with the content of functions promoted the expansion of the sociotechnical network to transform, modify, and translate facts with digital technologies (GeoGebra), which can streamline and personalise communication for different purposes (Bairral, 2015). At that moment, the software promoted actions capable of transforming, moving, recreating, modifying, and translating other actions, for example, into graphic representation. The enigmatic problems that made the systems' solutions invisible and abstract acted as a spokesperson (Latour, 2019). Given this, it is clear that the mediation proposed by Latour (2012, 2019) is not only associated with humans. Non-humans can also participate in those actions. Therefore, the relationships are hybrid, and both the subject and the objects

can be mediators in constructing and analysing mathematical concepts (Praude, 2015; Silva et al., 2020).

For Latour (2019), the sociotechnical network designates a series of associations that allow us to see the discontinuities necessary to generate an action. Such discontinuities are neither straight nor curved; they move in their own way, without sticking to the results, seeking to access new beings and trace new paths. In general terms, we highlight that the associations between *actants* contributed to a better presentation, personalisation, and contextualisation of mathematical concepts. Just as it promotes mediations in expanding and stabilising the flow of the sociotechnical network, given that the transformations linked during the actions mobilise other *actants* and encourage continuity in the continuing-formAction of teachers who teach mathematics. Thus, we can affirm that research activities and the relationship between humans and non-humans continue to perform new modes of existence in the continuing-formAction of teachers who teach mathematics.

SOME CONSIDERATIONS

Given the associations described here, we sought to map the mediations constituted between the *actants* in a context of continuing-formAction for teachers who teach mathematics, managed through digital technologies and other entities that compose the sociotechnical network. The results indicate that the mediations between the participants and an investigative task in Google Docs and online GeoGebra performed by the relationship between humans and non-humans contribute to the continuity of the sociotechnical network and promote transformations, displacements, translations, and the contextualisation of mathematical concepts.

Observing the dynamics of its movements, we conclude that the sociotechnical network does not freeze in the face of discontinuities but that these are necessary to promote action, driving the search for entities that favour possible stabilisation. Furthermore, it became evident that mediators do not necessarily focus on humans and intermediaries on non-humans, but that both can lead, mediate, and intermediate actions to promote continuities in the stabilisation of the sociotechnical network, in teachers' continuing-formAction, and the construction of mathematical concepts.

After expanding the flow of the sociotechnical network, reflections on and analyses of the solutions of systems of equations with two variables emerged, starting from the mobilisation of the relationship linked by the

actants. Given the associations, connections, and clues mapped, we conclude that we can associate an open-ended mathematical investigation task with more than one specific content and with more than one grade or school segment, depending on students' prior knowledge and the form of teachers' guidance. At the same time, we conclude that digital technologies have the potential to promote continuity in mathematical research activities; they expand, transport, transform, recreate, and contextualise horizons and can act as mediators in the process. Given this, we propose using *Mathematical Mediations* as a concept to highlight the association between humans and non-humans and symmetrically emphasise the protagonism of both in the transformation and recontextualisation of mathematical knowledge in the research scenario.

Therefore, this sociotechnical network is not isolated, centred only on human participants, but is aimed at all human or non-human entities that promote actions and movements of continuity and/or discontinuity. Those movements can contribute to new practices being investigated in mathematics education.

As an implication, we highlight the need to map the *Mathematical Mediations* not just of humans in research in mathematics education. Considering that, for ANT, the network would be acentric or multi-centred, we must insert the agency of technical objects into the game. We consider that the *actants* act and, in action, they promote movements that can be tracked, transforming, redefining, and reconfiguring new associations for the construction of mathematical knowledge, as well as for the continuing-formAction of teachers who teach mathematics.

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AUTHORS' CONTRIBUTION STATEMENTS

Both authors conceived the idea of the study to constitute one of the chapters of a master's thesis. DGA developed the entire research during the course completion work, in partnership with their advisor, conducted literature review, conducted data collection and analysis and interpretation of the results, as well as led the writing of the manuscript, and developed the conclusions and final considerations. FCMS led the thorough examination of the theory contributing to the critical review of the content and guided throughout the process and contributed to the elaboration of this article.

DATA AVAILABILITY STATEMENT

Once data collection was completed, audio and video recordings were downloaded to a local electronic device (flash drive and computer) and continue to be stored on Google Drive. We emphasize that the data is archived under the research group's domain and will be made available if needed.

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