The Individual and the Collaborative Nature of Metacognitive Strategies and Their Unfoldings for Mathematical Modelling

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ABSTRACT

Background: Mathematical modelling has been pointed out as a means for teaching and learning mathematics in the classroom. **Objective**: To investigate consequences for the development of mathematical modelling activities arising from students' metacognitive strategies. **Design**: The research follows the guidelines of the qualitative approach. **Environment and participants**: The modelling activities were developed by students in the fourth year of a Mathematics degree course. **Data collection and analysis:** In classes of the discipline Perspectives on Mathematical Modelling, data were collected through recordings of classes held on Google Meet. The written records produced by the students and the reports delivered by them also make up the material for analysis. **Results:** The unfolding evidenced for the activities can be allocated into four groups: identification of the interaction between mathematics and reality; use of mathematical concepts and construction of models; validation of models and results; back-and-forth movements in mathematical modelling activities. **Conclusions**: Although the main agent of metacognition is the individual, in modelling activities, metacognitive strategies are not limited to the individual nature, there is also evidence of collaborative metacognition in the group. Some developments result from more of one metacognitive strategy than another. This signals that it is not an isolated strategy, but a set of them that enables actions in mathematical modelling activities.

Keywords: Mathematical modelling; Metacognitive strategies; Individual metacognition; Collaborative metacognition.

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A natureza individual ou colaborativa de estratégias metacognitivas e seus desdobramentos para a modelagem matemática

RESUMO

Contexto: A modelagem matemática vem sendo apontada como meio para o ensino e a aprendizagem da matemática na sala de aula. **Objetivo**: Investigar desdobramentos para o desenvolvimento de atividades de modelagem matemática decorrentes de estratégias metacognitivas dos estudantes. **Design**: A pesquisa segue orientações da abordagem qualitativa. **Ambiente e participantes**: As atividades de modelagem foram desenvolvidas por estudantes do quarto ano de um curso de Licenciatura em Matemática. **Coleta e análise de dados**: Em aulas da disciplina de Perspectivas da Modelagem Matemática, os dados foram coletados por meio de gravações das aulas realizadas no Google Meet. Também compõem material de análise os registros escritos produzidos pelos estudantes e os relatórios por eles entregues. **Resultados**: Os desdobramentos evidenciados para as atividades podem ser alocados a quatro grupos: a identificação da interação entre matemática e realidade; o uso de conceitos matemáticos e a construção de modelos; a validação de modelos e de resultados; movimentos de ida e vinda em atividades de modelagem matemática. **Conclusões**: Embora o principal agente de metacognição seja o indivíduo, em atividades de modelagem, as estratégias metacognitivas não se limitam à natureza individual, havendo também evidências de metacognição colaborativa no grupo. Alguns desdobramentos decorrem mais de uma estratégia metacognitiva do que de outra. Isso sinaliza que não é uma estratégia isolada, mas um conjunto delas que viabiliza as ações em atividades de modelagem matemática.
Palavras-chave: Modelagem Matemática; Estratégias Metacognitivas; Metacognição Individual; Metacognição Colaborativa.

INTRODUCTION

Mathematical modelling has been pointed out as a means for teaching and learning mathematics in the classroom. This appointment is based on aspects idicating that it can help students understand the real world[1], enhance their learning (motivation, concept formation, understanding, retention) and develop skills (Blum & Ferri, 2009; Castro & Almeida, 2023).

Even though the importance of modelling in the classroom has been recognised, studies reveal that the cognitive demands it requires can act as blockages, either for the succesfuly procedures of students in these activities or

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[1]Per *real world*, like Galbraith and Holton (2018), we understand everything related to nature, society, or culture, including everyday life, school, or university -not necessarily mathematics- subjects.

in its contributions on learning (Galbraith & Stillman, 2006; Almeida, 2022; Blum, 2015).

The above is perhaps why there has been increasing interest in research that examines students' psychological processes as they engage in modelling problems. In particular, metacognition is considered an aspect that deserves attention in mathematical modelling, and and there have been signals of interactions between students' actions in modelling problems and metacognitive strategies (Yildirim, 2010; Stillman, 2004; Vorhölter, 2018; Vorhölter, 2019; Vertuan & Almeida, 2016; Vorhölter & Krüger, 2021).

According to Jou and Sperb (2006), metacognition as a research object opens up a new field of investigation. It even prompts a paradigm shift in which *cognition* must be differentiated from *metacognition*, and the feeling of knowing should be considered a product of the metacognitive function.

A precursor of research on metacognition in the educational field, Flavel (1976), defines metacognition as the knowledge that the individual has of their cognitive events and suggests that metacognitive activity includes personal variables, the tasks variables, and the strategies used.

Personal variables concern the individual's knowledge of their cognition, skills, and motivations. The task variables refer to the individual's knowledge of how to deal with information. "For example, people know that familiar information requires less attentional effort than completely new information, just as they know that it is easier to remember the central idea of a story than the exact words used" (Jou & Sperb, 2006, p.179). Regarding strategy variables, Flavell (1987) says that while cognitive strategies refer to the outcome, metacognitive strategies concern the evaluation of the efficiency of that outcome. "For example, to solve an addition, add one number to another. This is a cognitive strategy. Repeating the operation several times to be confident that the cognitive strategy used led to success is a metacognitive strategy" (Jou & Sperb, 2006, p.179).

Although incipient, studies on metacognition in mathematical modelling recognise that using metacognitive strategies is crucial for developing successful modelling activities (Blum, 2011; Stillman, 2011; Vorhölter, 2019). Such studies indicate that, peculiarly, modelling activities are generally developed in groups, and metacognition has been referred to as an individual attitude.

Recognising that an essential feature of modelling activities is their development in groups, the range of metacognitive strategies, from interacting

with the situation to validating the response, is not limited to the individual nature. Instead, collaborative metacognition becomes relevant. Thus, studies that explore this characteristic are relevant so that the role of metacoginition and the scope of metacognitive strategies in modelling activities can be identified.

On the other hand, Schukajlow and Leiss, (2011) and Vorhölter, (2019) suggest that modelling activities can either be influenced by metacognitive strategies or affect students' metacognition. Considering this possibility, Castro and Almeida (2022) examined the potential of mathematical modelling to promote metacognitive strategies. In this research, conversely, we are interested in investigating unfoldings for developing mathematical modelling activities resulting from metacognitive strategies mobilised by students.

For this purpose, we focus on the actions of a group of students of a mathematics degree course, attending the school subject Mathematical Modelling from the Perspective of Mathematics Education, when developing two modelling activities.

MATHEMATICAL MODELLING

Although different understandings regarding modelling in mathematics education are shared, there seems to be a consensus around Pollak's (2015) claim that the central idea is always to identify a problem situation, decide what to keep and what to ignore in the formulation of a mathematical model, make use of mathematics in the idealised situation from a real-world situation, and then decide whether the results are suitable for the problem.

In line with these inidcations, this article is based on an understanding shared by Almeida (2018) that a modelling activity begins in an initial situation (problem-situation) and can be said to be completed in a final situation (answer to the problem identified in the initial situation). The transition between these two points is lined with actions that, previously defined or that emerge during the journey, are relevant for studentes to solve the activity successfully.

The transition mentioned in this article brings us to reflect on strategies and possibilities of action, especially when modelling is incorporated into mathematics classes in which the students' actions not only produce a solution to the problem but are also associated with their learning along this transition.

What this path should include has been incorporated into the so-called mathematical modelling cycles, which aim to make explicit the likely –or

perhaps desirable–students' actions when developing a mathematical modelling activity. According to Almeida et al. (2021), a modelling activity developed in the classroom can be characterised by six steps: understanding the real problem; mathematisation, resolution, interpretation, and validation; preparation of a report and communication of results (Figure 1).

Figure 1

Steps of the development of a mathematical modelling activity. (Almeida et al., 2021, p. 386)
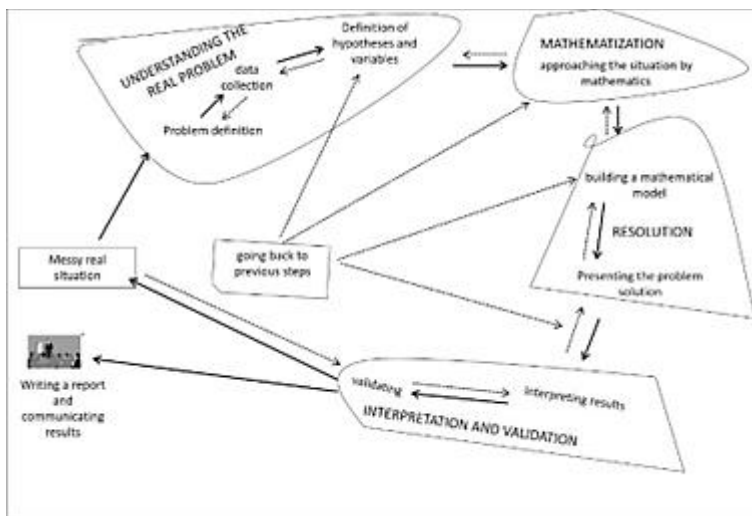
Understanding the real problem refers to the act or effect of finding out, of being informed about the situation to be studied; mathematisation means translating the real problem into a mathematical problem and implies using a mathematical language; in the resolution step, the students solve the mathematical problem; in the interpretation of results and validation, they interpret the results and validate the answer obtained for the real problem; and finally, in the classroom, the students share the results, defend their procedures and responses, and produce a report.

The dotted lines on the cycle are used to indicate the back-and-forth movement (the double meaning) that the modellers' actions may require, inferring a dynamic rather than a linearity between the different stpes of the

modelling. In other words, these arrows indicate the iterative refinement of the model and the solution in mathematical modelling activities of a real-world situation.

Students' actions during a modelling cycle require cognitive demands to face the association of a real-world situation and mathematical concepts or procedures. In this context, Stillman (1998) and Yildirim (2011) allude to the emergence of a metacognitive activity to permeate the students' actions in the different stages of the modelling activity.

METACOGNITIVE STRATEGIES IN MATHEMATICAL MODELLING ACTIVITIES

Metacognition has its principles structured by John Flavell, who, in the mid-1970s, conceptualised it as *the* knowledge of cognition itself, which can also be said as how *think about one's own thinking*. Psychologists such as John Dewey, Edmund Huey, and Edward Thorndike have been researching metacognition in different places, including educational environments, as a reference for studying how students learn. In line with the idea that the metacognitive activity includes the individual's variables, the task, and the strategies used, as we have already pointed out in the previous section, metacognition has been approached from two components: knowledge of cognition and regulation of cognition (Schraw & Moshman, 1995).

Knowledge of cognition occurs when one understands the key processes involved in learning, i.e., it is characterised by knowledge and awareness of cognitive processes, which can be controllable, stable, and, sometimes, fallible and late. It is evidenced by three knowledge strategies: declarative, procedural, and conditional.

Declarative knowledge refers to knowing about *what is known of things*. Procedural knowledge is associated with *knowing how* to employ procedures, strategies, or actions. Conditional knowledge implies *knowing why* to apply procedures, manifest skills, or use strategies.

The regulation of cognition happens when learning is regulated, i.e., it is related to controlling the learning process, making decisions about how to learn, organising the process, and assessing performance, which can trigger three main strategies: planning, monitoring, and evaluating.

Planning involves defining goals, objectives, and steps to follow, selecting appropriate strategies, making forecasts, processing information, and

allocating resources. Monitoring refers to awareness of learning and performance on specific tasks, identifying and correcting errors. The evaluation is related to the analysis of results and learning through reflecting and reassessing actions and verifying whether the objectives were achieved.

Studies on metacognition in mathematical modelling recognise the relevance of metacognitive strategies for the successful development of modelling activities (Blum, 2011, Stillman, 2011, Vorhölter, 2019; 2020; 2021). First, however, we must pay attention to the nature of these strategies, considering that they may also emerge collaboratively among students from the same group.

In this regard, we sought to identify manifestations of metacognition strategies, classify them according to their individual or collaborative nature, and observe the developments that they infer for the activity. Thus, we demonstrated each student's protagonism and autonomy and the collaborative resolution of problems and students' dialogical communication in the group, which raises the use of metacognitive strategies in the modelling process.

We understand a collaborative nature strategy to be one in which the reasoning processes are distributed among individuals, along with their tools, artefacts, and representations. In other words, the collaborative nature fosters cognition through various biases (Hollan et al., 2000) since the student needs to think both about their own cognition and that of their colleagues, which shows that the manifestation of metacognition can occur at different moments of the activity. Lai (2011) points out that there are recommendations for using collaborative or cooperative learning structures to stimulate the development of metacognitive strategies.

Magiera and Zawojewski (2019) suggest that organising students for collaborative work to solve complex problems, such as mathematical modelling, which require discussion and work in groups, can optimise the observation of metacognitive activity in practices in the school context. In other words, open investigation situations, resolutions of complex problems in which the subject is led to choose between several alternatives and anticipate the consequences of these choices, and the conduction of collaborative work are examples of aspects that can stimulate metacognition in modelling activities. Another significant mechanism pointed out by the authors is that interactions among individuals who work together require verbal tools that enable them to regulate or monitor the behaviour and thinking of the other.

The above suggests, from a Vygotskian perspective, that when seeking to monitor and evaluate the metacognitive activity initially directed to the thinking of others in social contexts, the individual becomes prone to internalise these social behaviours and self-monitor, evaluate, and tune their performance efforts. In fact,

> Considering the metacognitive functioning of individuals in social contexts is reconceptualised as a product of interactions between an individual or a group of individuals and a surrounding context. When goals and solutions are constructed collectively, and the desired product is socially shared cognition, group members regulate not only their thinking but also that of others and their collective problem-solving activity (Magiera & Zawojewski, 2019, p. 54).

Kim *et al.* (2013) and Vorhölter (2018) highlight that the performance of all group members towards a consensual goal is relevant to trigger individual strategies. Thus, we can consider that the group's interaction with the teacher encourages and triggers a process that allows them to detect errors and adapt thoughts, resolve obstacles and progress in solving a problem.

Along these lines, Iiskala *et al.* (2011) state that in contexts of work in collaborative groups, group metacognition seems to have more potential to incite actions than individual metacognition. Vorhölter (2019) shares this understanding while defending that, in metacognition of a social nature, the individual must make their thoughts available to others and discuss their assumptions, justifications, and conclusions.

METHODOLOGY

The investigation follows the guidelines of qualitative research. As suggested by Garnica (1977), qualitative research is often recommended as it constitutes a healthy exercise, especially in mathematics education, to enable the understanding of the phenomenon under study, considering its different nuances.

The qualitative approach to research, according to Bogdan and Biklen (1982), Lüdke and André (2013), and Godoy (1995b), involves getting descriptive data (people, places, interactive processes) obtained through direct contact between the researcher and the situation object of study. It emphasises

the process more than the product and is concerned with understanding and portraying the phenomenon from the participants' perspective.

In line with these specificities,[2] in this article, the empirical research involved 4th-year Mathematics degree students attending the school subject Mathematical Modelling from the Perspective of Mathematics Education. We bring to the discussion two mathematical modelling activities developed by three students (J_3, K_3 and L_3). The group was monitored by the class teacher (Prof) and researcher (Pesq), and the activities were developed in synchronous classes recorded on *Google Meet* and transcribed and organised into episodes. Together with the activity report and the questionnaires answered by the students, they provided the data on which our analysis is based.

During an activity, the researcher suggested the *Poker Game* (PG) as the tsubject for students to work on. The activity used three synchronous classes and extra-class assistance requested by the group. This topic arose from the situation in which a young Brazilian, Luis Garla, a resident of Londrina, participated in the final of the most important world poker tournament in 2020, facing the Greek player Alexandro Theologis. The problem was to determine *who would be more likely to win the game, Garla or Theologis, based on each one's starting pair of cards*. The resolution presented by the group is summarised in Figure 2.

In the activity *Vehicle Devaluation* (VD), the interest in the subject came from the students themselves, and the problem they want to solve is: *How to estimate the value of a vehicle after a few years of use?* The modelling process took six synchronous classes and some asynchronous meetings. Figure 3 presents a summary of the development presented by the group.

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[2]The data from this research are not part of a project submitted to the ethics committee. All participants signed the Informed Consent Form (ICF). The presentation of the data is the responsibility of the authors, and the journal Acta Scientiae is exempt from any responsibility, in accordance with Resolution N. 510, of April 7, 2016, of the National Health Council of Brazil. Full assistance and possible compensation for any resulting damage to any of the research participants is the responsibility of the authors.

Figure 2

Poker Game Modelling Activity

Figure 3

Vehicle Devaluation Modelling Activity

To identify students' metacognitive strategies, we used an instrument proposed by Castro (2022), aiming to capture the peculiarities of strategies of an individual nature (I) and a collaborative nature (C) in modelling activities and identify the unfoldings[3] for the implementation of the activity based on these strategies.

RESULTS

The data analysis indicated signalling elements of metacognitive strategies and their consequences for mathematical modelling activities. The analytical process was addressed to the group in each of the two activities developed.

Strategies of an individual nature (I) are manifested through students' speeches or actions, without explicit interference from others, when *thinking out loud* or exposing arguments or when they go over the group's steps to clarify the procedures assumed and/or executed. Strategies of a collaborative nature (C) refer to those in which the group acts on the metacognitive manifestation of a specific participant. In particular, such strategies derive from sources external to the individual, whether from colleagues in the group or from the teacher. They may, sometimes, sound like a warning for possible errors or omissions during the procedures required by the activity and, at other times, as *feedback* to someone else's thinking, confirming or validating their assertions.

In Table 1, we can see students' metacognitive strategies in the activity *Poker Game*, and in Table 2, those related to the *Vehicle devaluation* activity. In both figures, the strategies are characterised in each of the elements of metacognition (knowledge and regulation of cognition) and the development of these strategies for students' *to do* in the modelling activity.

From the students' metacognitive strategies in the two activities, we can characterise four groups of unfoldings for the activity: the same unfolding resulting from different strategies and under different natures (individual and collaborative); the same unfolding resulting from a strategy of an individual nature only; the same unfolding resulting from a strategy of only a collaborative nature; different unfoldings resulting from the same strategy of the same nature (individual or collaborative).

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[3]The term *unfoldings* refers here to possible consequences for the development of the modelling activity, resulting from the students' metacognitive strategies.

Table 1

Developments of students' metacognitive strategies for the activity Poker Game

Cognition knowledge strategies	I	C	Unfolding
Declarative knowledge		X	Promotes the relationship between mathematical aspects and specificities of the reality situation.
Procedural knowledge	X		Encourages the construction of a mathematical model for each poker player.
		X	Leads to the analysis of the constructed model.
	X		Enables model generalisation for both players
Conditional knowledge		X	It favours the use of technological resources (Excel, CurveExpert) for model validation
	X		Leads to the identification of potentially useful mathematical procedures in the construction of mathematical models
Planning		X	Enables model generalisation for both players
		X	Favours the use of technological resources (Excel, CurveExpert) for model validation
	X		Confirms the identification of potentially useful mathematical procedures in the construction of mathematical models
Monitoring		X	Encourages the construction of a mathematical model for each poker player.
		X	Favours the use of technological resources (Excel, CurveExpert) for model validation
		X	Leads to ways of verifying the mathematical model.
Evaluation		X	Enables the generalisation of the mathematical model

Table 2

Unfoldings of students' metacognitive strategies in the Vehicle devaluation activity

Cognition knowledge strategies	I	C	Unfolding
Declarative knowledge		X	Encourages students to choose the topic
	X		Promotes planning the procedures to be used

	Individual	Collaborative	Description
Procedural knowledge	X		Indicates mathematical procedures
		X	Indicates mathematical procedures
		X	Promotes the definition of variables and mathematical content
Conditional knowledge		X	Leads to the use of mathematical techniques and procedures to obtain the answer
		X	Leads to the use of technological resource (software)
	X		Leads to model verification
Planning		X	Guides the collection of information about the situation of reality
		X	Offers mechanisms for planning the construction of the mathematical model
		X	Leads to simplification in collected data
	X		Leads to simplification of the situation
Monitoring		X	Provides elements for problem delimitation
	X		Leads to the construction of the mathematical model
	X		Guides the formulation of hypotheses
		X	Leads to the complementation of the resolution, building a new mathematical model.
Evaluation	X		FavoUrs verification of the mathematical model and results
		X	Conducts identification of means for validating the response obtained

1st) The same unfolding resulting from different strategies and under different natures (individual and collaborative).

In the *Poker Game* activity, constructing a mathematical model for each player seems to result from the procedural knowledge strategy of an individual nature and the monitoring strategy of a collaborative nature. In Table 3, it is possible to observe how this happens in the activity.

Table 3

Metacognitive strategies of different natures in the same activity

Excerpts	Strategy	Unfolding
J₃: I also spoke with A₁ *[group 1]*, he told me that they calculated the probability of each hand using Excel.	Monitoring (C): Exposes strategies to build the model, establishing comparisons with what	Encourages the construction of a

K₃: Is this the winning probability we are trying to find using the model? **J₃: Yes, but then, we will have to do it for each hand and the probability, calculate it *barehandedly*, without any formulas, and then look for one that explains all of them. I'm wondering whether we can use the *Curve*.**	colleagues or the teacher suggested.	mathematical model for each poker player.
K₃: Because if we guarantee that such a player makes a certain move, we have to guarantee that of the five cards on the table, some are specific cards, and then it reduces or anulls the probability of the other player making a certain move.	Procedural knowledge (I): Declares that the construction of the mathematical model is based on the data collected and on the procedures defined in the mathematisation of the situation	

In the *Vehicle devaluation* activity, the interaction between declarative knowledge (individual nature) and procedural knowledge (collaborative nature) strategies triggers the mathematical resolution of the situation, as illustrated by the dialogue shown in Table 4.

Table 4

Metacognitive strategies of different natures in the same activity

Excerpts	Strategy	Unfolding
L₃: To get data on the subject, we searched for articles on internet sites and called some dealerships. They informed us of the current price of some cars, but they said that it would not be possible to inform us about the devaluation due to factors such as the pandemic, crises, among others.	Declarative knowledge (I): remembers, organises, and collects information about the situation of reality,	Mathematical solution to the situation

K₃: Remembering that the value of the car, in general, does not vary during the months of the same year, only from one year to another. **J₃: For the other years, according to our information and hypotheses, the devaluation is 5% per year, so we consider it more appropriate to use the largest integer function in which [t] is the largest integer less than or equal to t.** $V(t) = \begin{cases} 57.240 & \text{se } 0 < t \leq 1 \\ 57.240 \cdot 0{,}935^{[t-1]} & \text{se } 1 < t \leq 10 \end{cases}$	Procedural knowledge (C): declares that the construction of the mathematical model is based on the data collected and the hypotheses formulated

2nd) The same unfolding resulting from strategies of an individual nature only

Different strategies, but all of an individual nature, were the motto for student action in the activity. An example of this situation occurred in the *Poker Game* activity. In this case, identifying potentially useful mathematical procedures for constructing the mathematical model seems to result from the monitoring and planning strategies, both of an individual nature, as presented in Table 5.

Table 5

Metacognitive strategies of an individual nature in the Poker Game activity

Excerpts	Strategy	Unfolding
J₃: Hold on, let me think for a bit. Can we do an example with Theologis making three of a kind? Probability of t(7) is equal to probability of t(7). If he flips a "2" he already has three of a kind and that doesn't help Garla at all, because even if the other cards come in his favour, he still loses. So, only if Garla makes another three of a kind, because Theologis'	Monitoring (I): Presents analogous examples or assumes colloquial language to explain resolution strategies or make more appropriate choices for the activity. Planning (I): Searches, in its cognitive structure, elements to	Identification of potentially useful mathematical procedures in the construction of the mathematical model

Excerpts	Strategy	Unfoldings
three of a kind will be the lowest in the game. So, the model would be $P(T_n)= P(T_n) – P(G_n)$. Following this pattern, we can analyse the other moves to see if something different will happen.	mathematise the situation.	

Another example of unfolding that results from strategies of an individual nature, in this case, conditional knowledge and evaluation strategies, is the verification of the mathematical model in the *Vehicle devaluation* activity (Table 6).

Table 6

Metacognitive strategies of an individual nature in the Vehicle devaluation activity

Excerpts	Strategy	Unfoldings
J$_3$: Yeah, then, we calculate how much this car - which was brand new ten years ago- costs today! Then, we saw that the model works.	Conditional knowledge (I): evaluates whether its procedures produce adequate results.	
L$_3$: We constructed this by finding a linear regression and using the graph from June 2009 to June 2019. The difference between our model and the value given in the Fipe table was 1.919,20. Validating in the Fipe table the values of all the years analysed [2009-2019], we saw that the devaluation was small (1%) from one year to the next. But even with these oscillations, we can reach a value very close to the real. So we chose to check using the *Curve Expert*. We found an exponential and calculating for ten years, we found R\$16.872,75. We used the Curve for comparison.	Evaluation (I): checks whether your final results match the conditions of the problem.	Verification of the mathematical model

3rd) The same unfolding resulting from a strategy of a collaborative nature only

In some cases, the unfoldings seem to be associated only with metacognitive strategies of a collaborative nature, such as the "construction of the mathematical model" and the "Use of technological resources (Excel, CurveExpert) for model validation". In the *Poker Game* activity, these unfoldings result from conditional knowledge and monitoring strategies.

Table 7

Metacognitive strategies of a collaborative nature in the Poker Game activity

Excerpts	Strategy	Unfolding
J₃: I also spoke with A1 [group 1], he told me that they calculated the probability of each hand using Excel. Then they plotted the results as points on the Curve to get the functions. But it looks like they also got to intervals that gave a negative number, which I think you can get in a piecewise function.	Monitoring (C): construction of the model establishing comparisons with peers' suggestions.	Construction of the mathematical model
K₃: To validate, we can compare the results with the other group. There [in the group] there are the boys who understand the game well and can see whether that's right or we missed something. **J₃: We can run the regression in Curve Expert, and see how close the r index, how close the curve is to our points.**	Conditional knowledge (C): adequately justifies using mathematical concepts and methods.	Use of technological resources (Excel, CurveExpert) for model validation

4th) Different unfoldings arising from the same strategy of the same nature (individual or collaborative)

Some unfoldings seem to be related to the use of the same metacognitive strategy, which, however, is mobilised at different moments in the development of the activity. An example of this situation occurs in the *Vehicle Devaluation* activity, in which formulating hypotheses and constructing

a second mathematical model are two unfoldings resulting from the monitoring strategy under individual nature mobilised twice by a student (Table 8).

Table 8

Monitoring strategy and its different unfoldings for the Vehicle devaluation activity

Excerpts	Strategy	Unfolding
J3: We need to define some hypotheses. We found that in the 1st year, the vehicle suffers a 10% devaluation of the total value. So we hypothesised that the devaluation would be 10% in the first year.	Monitoring (I): admits that it is necessary to formulate hypotheses.	Formulation of hypotheses
J$_3$: As we could not validate the model for 2031 and, talking to the teacher and the group, I had the idea of considering a car from the year 2009 of the same 1.0 model that we used in our model and analysing its value using data from the Fipe table. We then verified that in the Fipe table, the value of the car in 2009 would have been R\$30.950. Then, we started building a new model.	Monitoring (I): exposes strategy to build a model.	Construction of a second mathematical model

The unfoldings for the development of the activity identified as resulting from metacognitive strategies, whether of an individual or collaborative nature, can be grouped according to four purposes in the activity: *interaction between mathematics and reality; use of mathematical concepts and the construction of mathematical models; validation of models and results; and the back-and-forth movements between phases of a cycle of modelling activities*.

The unfoldings regarding the *interaction between mathematics and reality* relate to the evidence of interlocution between aspects, information, and

knowledge about the problem situation and the translation or interpretation of this situation into mathematical language. The unfoldings also indicate the mathematical work oriented to meet the characteristics of the problem situation of the reality in focus and which required complementation in data collection, the definition of hypotheses and simplifications, for example.

Regarding *mathematical concepts and the construction of mathematical models*, the unfoldings highlighted are the application or manipulation of mathematical concepts that focus on mathematical resolution and the construction of the mathematical model. In this group, we can mention, for example, calculating, using technological resources, and identifying relevant mathematical content.

About the unfoldings regarding the *validation of the model and results,* metacognitive strategies led to the verification and validation of the mathematical resolution, the mathematical model, the mathematical result, or the answer to the problem situation of reality. An example of unfoldings that fall within this group is the use of technology resources, such as Curve Expert and Excel, to validate the model and answer.

The *back-and-forth movements in modelling activities* stem from strategies that allow us to perceive and use flexibility in the procedures required in the different phases of the development of the modelling activity, bringing to the activity a dynamic, as already indicated by the cycle in Figure 1. For example, the construction of a second mathematical model, a further simplification of the situation, when the student in the resolution phase needs to go back to the problem, when, in the construction of the model, it was necessary to define new hypotheses, or when the validation implied the resumption of the information used.

CONCLUSIONS

In this research we direct attention to students' metacognitive strategies in mathematical modelling activities, considering both their individual and collaborative nature. By recognising that an essential characteristic of a modelling activity is that it is carried out in a group, the students' set of metacognitive strategies, from interaction with the situation to validation of the obtained answer, is not limited to the individual nature.

Strategies of an individual nature are usually shown in students' monologues or speeches, in arguments that they appear to construct

independently, without interaction with other students. Strategies of a collaborative nature come from interactions with colleagues or the teacher who, in this case, act as sources that encourage students to follow, verify, or develop their process of thinking and understanding. At the same time, they can also lead them to detect and correct mistakes.

Regarding the unfoldings for the development of mathematical modelling activities arising from metacognitive strategies activated by students, it is possible to conclude that they drive students to ways of acting in the activity. Particularly, the students' actions and these strategies seem to be connected in different ways: the same unfolding stems from different strategies that have different natures (individual or collaborative); the same unfolding stems from a strategy of an individual nature only; the same unfolding stems from a strategy that is only collaborative; different unfoldings stem from the same strategy.

From the metacognitive strategies used by the students, we identified unfoldings for mathematical modelling activities related to different aspects of a modelling activity: interaction between mathematics and reality; use of mathematical concepts and construction of a model; validation of models and results; definition of characteristic back-and-forth movements of the actions indicated in a mathematical modelling cycle.

The characterisation of these unfoldings and the identification of the nature of the metacognitive strategies associated with them are aspects that previous research has explored minimally. These insights can guide the implementation of modelling activities in the classroom, taking into account the potential impact of these strategies on students' performance, with the goal of enhancing their success in modelling activities.

Thus, the research concludes that, although the leading agent of metacognition is the individual, in modelling activities, the metacognitive strategies are not limited to the individual nature, and there is evidence of collaborative metacognition in the group. In this sense, the results of the present research complement what Vorhölter (2019) and Vorhölter and Krüger (2021) point out regarding the characterisation of collaborative metacognition in the groups involved in a modelling activity, specifying the nature of the strategies and their action on the modelling activity being developed.

We conclude that some unfoldings result more from one metacognitive strategy than from another, which suggests that it is not an isolated strategy but a set of them that enables actions in mathematical modelling activities.

Exploring similarities and dissimilarities between the activities developed by various groups of students at different educational levels, and expanding discussions about how these groups exhibit metacognitive behaviour in distinct activities could be the focus of future research. Another possibility is to investigate how this behaviour occurs on an individual or collaborative basis.

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AUTHORSHIP CONTRIBUTION STATEMENT

LMWA was the professor of the subject in which the mathematical modelling activities were developed. Collaboratively, the three authors appropriated the theoretical foundations of the article and collected and analysed the data. The text was also drafted in a joint and collaborative manner.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, LMMA, upon reasonable request.

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