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Characterising the Didactic-Stochastic Knowledge of Future Mathematics Teachers: The Case of Chile

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ABSTRACT

Background: Stochastics' teacher education has become an important research topic since mathematics teachers are usually responsible for the teaching of stochastics (statistics and probability) in schools. However, the emergence of new theoretical approaches has resulted in the identification of a problem: organizing and describing the necessary professional knowledge to teach stochastics. **Objectives**: The aim of this research is to characterize the didactic-stochastic knowledge needed for pre-service mathematics teachers, considering Chile as a case study. **Design**: Following a qualitative perspective, through a content-analysis. **Setting and Participants:** The Pedagogical and Disciplinary Standards for Mathematics Teaching Programs. **Data collection and analysis:** The national guidelines on the stochastic education of pre-service' teachers were analysed to collect text fragments describing professional knowledge expected in pre-service teachers. **Results**: We obtained a set of 37 indicators organized according to the Didactic-Mathematical Knowledge Model, which includes disciplinary aspects (stochastic content), the knowledge of students and their learning (cognitive content) and interests (affective content), instruction processes (interactional and mediational content) and their link with the educational context and other knowledge areas (ecological content). **Conclusions**: We hope that the identified indicators become a useful tool to organize and evaluate stochastic education programs for pre- and in-service teachers. Moreover, we highlight the replicability of the method uses and the possible adaptation of results to other educational contexts.

Keywords: Professional knowledge; Pre-service teacher education; Stochastics Education.

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Caracterizando o conhecimento didático-estocástico dos futuros professores de matemática: O caso do Chile

RESUMO

Contexto: A formação de professores estocásticos tornou-se um tópico de pesquisa importante, uma vez que os professores de matemática são geralmente responsáveis pelo ensino de estocásticos (estatísticas e probabilidade) nas escolas. Entretanto, o surgimento de novas abordagens teóricas resultou na identificação de um problema: organizar e descrever os conhecimentos profissionais necessários para ensinar os estoquásticos. **Objetivos:** O objetivo desta pesquisa é caracterizar o conhecimento didático-estocástico necessário para os futuros professores de matemática, considerando o Chile como um estudo de caso. **Método:** Seguindo uma perspectiva qualitativa, através de uma análise de conteúdo de directrizes nacionais sobre a educação estocástica de futuros professores, recolhemos fragmentos de texto descrevendo os conhecimentos profissionais esperados nos futuros professores. **Resultados:** Obtivemos um conjunto de 37 indicadores organizados de acordo com o Modelo de Conhecimento Didático-Matemático, que inclui aspectos disciplinares (conteúdo estocástico), o conhecimento dos estudantes e sua aprendizagem (conteúdo cognitivo) e interesses (conteúdo afetivo), processos de instrução (conteúdo interativo e midiático) e sua ligação com o contexto educacional e outras áreas do conhecimento (conteúdo ecológico). **Conclusões:** Esperamos que os indicadores identificados se tornem uma ferramenta útil para organizar e avaliar programas educacionais estocásticos para professores pré e em serviço. Além disso, destacamos a reprodutibilidade dos usos do método e a possível adaptação dos resultados a outros contextos educacionais.

Palavras-chave: Conhecimento profissional; Educação de professores em pré-serviço; Educação Estocástica.

INTRODUCTION

Researchers have shown an increased interest in teachers' stochastics (statistics and probability) education in recent years (Salcedo y Díaz-Levicoy, 2022; Tauber y Pinto, 2021). On the one hand, stochastics has become a necessary cultural component for every citizen to be able to function effectively in an information society (Ruz et al., 2020). This situation has motivated a reform in the mathematics curriculum, which has incorporated elements of stochastics within the compulsory school trajectory of a great number of countries, including Chile (MINEDUC, 2009; 2021). Consequently, the need arises to prepare mathematics teachers to teach stochastics according to the current demands and needs. However, the latter has been reported as systematically deficient in recent years (Batanero et al., 2011; Groth y

Meletiou-Mavrotheris, 2018; Ruz, 2021). For this reason, it is paramount to research on the professional knowledge that a future mathematics teacher must develop to teach stochastics in the present reality, with the aim of identifying key aspects to reinforce and/or reform in this learning process.

Mathematics teacher education is a complex process, which involves the mastery of the mathematics that will be taught, being competent in its teaching, and learning from practice (Strutchens et al., 2017). Although a certain degree of consensus exists on the main aspects to consider, it has been observed that there is no collectively accepted agreement on how to characterize the professional knowledge of teachers to teach mathematics (Giacomone, 2018; Mason, 2016).

In the field of statistics education, the situation is similar, as the trend has been to characterize this knowledge from two perspectives: content and the teaching of content (Burgess, 2012; Callingham et al., 2016; Callingham y Watson, 2011; Groth, 2007; 2013; Watson, 2001), including in the most modern proposals the perspectives' interaction with the teaching technologies for stochastics (Huerta, 2018; Lee et al., 2016; Lee y Hollebrands, 2008; Wassong y Biehler, 2010, 2014). Despite the variety of perspectives to conceptualize teachers' professional knowledge and their similar components, there are important conceptual differences between these perspectives. Groth y Meletiou-Mavrotheris (2018) assert that it would be a good idea to take advantage of the different conceptualizations of the nature of stochastics knowledge for teaching, as these can become starting points to compare and contrast different viewpoints. Thus, this research considers the theoretical framework called Teacher's Didactic-Mathematical Knowledge Model (referred to here as CDM from Spanish, *Modelo de Conocimiento Didáctico-Matemático*) (Godino, 2009; Pino-Fan et al., 2018) as a referent, which will be applied to stochastics. The approach extends and incorporates the conceptualization of pedagogical content knowledge (Shulman, 1987), mathematical knowledge for teaching (Ball et al., 2008), and the notion of proficiency in the teaching of mathematics (Schoenfeld y Kilpatrick, 2008). From this perspective, the teacher's CDM is characterized as follows:

(1) mathematical knowledge, related to the knowledge of content, which allows the teacher to solve mathematical problems which will implemented in the classroom, linking them to those mathematical objects arising in subsequent grades.

(2) didactic (or pedagogical) knowledge, which corresponds to a category reinterpretation of Ball et al.'s model (2008), made up of six facets or

contents about the teacher's specialized knowledge to teach mathematics. That is, given a mathematics task, the teacher must be able to mobilize the diverse meanings and concepts at play (epistemic knowledge) and must be also able to solve the task using different procedures, showing various justifications and explanations (mediational and interactional knowledge), or adapt the task to the knowledge (cognitive content) and interest (affective content) of students, in a specific context (ecological content) (Godino et al., 2017), and

(3) meta-didactic knowledge, including the knowledge required for teachers to reflect about their own practice, with the aim of evaluating and detecting possible improvements in the mathematics' teaching process.

Consequently, we would like to highlight that the CDM model refers to mathematics education and that it needs to be specified for the teaching of stochastics due to the differences between both disciplines (Rossman et al., 2006). In that regard, efforts have been made across the globe to establish a pre-service teaching curriculum, which usually translates into guidelines or standards that become official documents defining which aspects must be mastered by a future teacher in regards to the teaching of stochastics.

In Chile, the main compulsory curricular document for teacher education is called Standards for Pre-service Teacher Education (MINEDUC y CPEIP, 2021). This document conditions and guides the content of the study programs promoted by Chilean universities, since at the end of the process, those graduating from these programs must take the National Diagnostic Test for Pre-service Teachers (www.diagnosticafid.cl/). Presently, the results of this test are only used as reference and have a formative purpose. However, taking the test is compulsory for obtaining a teaching degree. This coincides with several countries that have also defined standards for pre-service teacher education, which could be considered both a way to improve the teaching profession and a way to control teachers' practices (Flores, 2016). Thus, the standards are an essential source of information to understand teacher development; however, the former tend not to be aligned with theoretical frameworks explicitly.

Therefore, the main goal of this manuscript is to characterize the didactic-stochastic knowledge of future Chilean mathematics teachers based on the CDM model. In doing so, we consider the Chilean reality as a case study allowing us to answer the following: Which types of knowledge characterize the teaching practices of mathematics teachers that will teach stochastics at this time? To achieve that objective, we have two partial goals: a) systematizing the Chilean requirements about stochastics in teacher education according to the

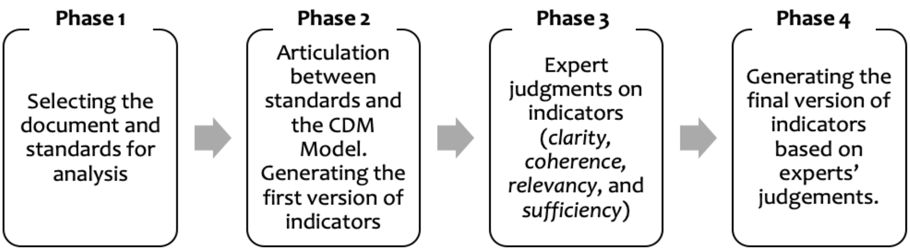
CDM model; b) validating by expert reviews an indicator system about the didactic-stochastic knowledge types of future Chilean mathematics teachers. In this manner, it is hoped that the obtained results become a useful tool for those responsible for stochastic teacher education not only in Chile but also in other countries, both for planning and evaluating stochastics teacher education programs.

METHODOLOGY

As researchers, we apply a qualitative approach, analyzing our data through content analysis. Content analysis implies coding, categorization, comparison of pre-existing categories, and the creation of links between the generated categories. This is done to finally be able to draw theoretical conclusions from the analyzed text (Cohen, 2007). Figure 1 illustrates a diagram with this research's methodological design stages, which are later described in detail.

Figure 1

Methodology phases



Phase 1. Selecting the document and standards for analysis

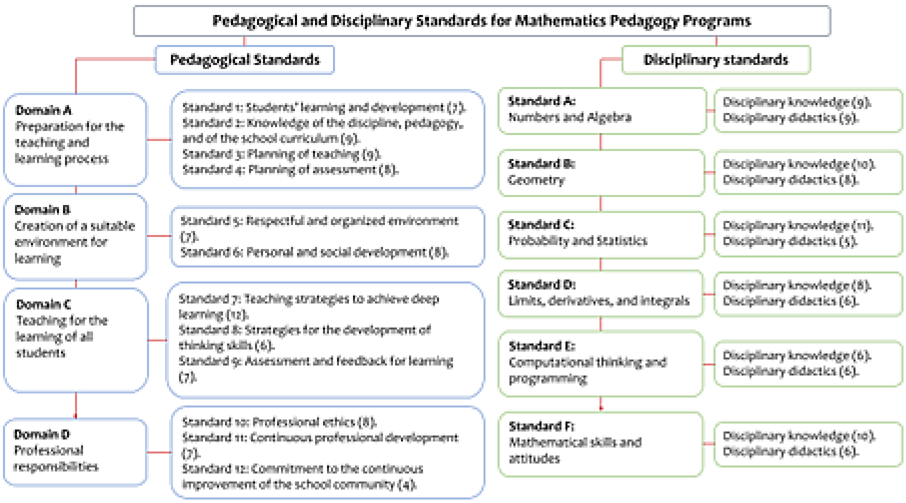
The analyzed document is the Pedagogical and Disciplinary Standards for Mathematics Teaching Programs (MINEDUC y CPEIP, 2021). This document is used as a referent for the accreditation for teacher education programs in Chile, and it is the basis for the National Diagnostic Test for Pre-service Teachers (ENDFID). Applying the latter and being accredited is a legal obligation for all the teaching programs in Chile, while the Pedagogical and

Disciplinary Standards (MINEDUC y CPEIP, 2021) is the document guiding all teacher education in Chile, which is why it was selected.

Figure 2 shows the components of this document, which is made up of pedagogical and disciplinary standards. The former is common to all pedagogy programs, whereas the latter are discipline-specific. Pedagogical standards consist four domains, where each one contains standards and their corresponding descriptors (quantity shown in parenthesis). On the other hand, disciplinary standards consist of five standards related to mathematical content, while the sixth standard refers to mathematical skills and attitudes. Each of these contains descriptors (their quantity in parenthesis) for both the disciplinary and the didactic-disciplinary content. For this research, we analyzed a total of 123 descriptors corresponding to domains A, B, C and D of the pedagogical standards and standards C and F from the disciplinary section. These descriptors were selected since a) the pedagogical standards are common to all teaching programs; and b) for disciplinary standards we were interested in those relating to the disciplinary and pedagogical knowledge of stochastics, as well as its associated skills and attitudes. Thus, the unit of analysis corresponds to each descriptor, described as text or sentence.

Figure 2

Components of the Pedagogical and Disciplinary Standards for Mathematics Pedagogy Programs (Adapted from MINEDUC and CPEIP (2021))



Phase 2. Development of indicators according to the CDM model

This phase consists of three stages. The first stage consisted of classifying the descriptors under the different theoretical content of CDM; the second stage consisted of writing the indicators, while the third stage consisted of contrasting the current indicators developed by Ruz et al. (2019), to be coherent with previous works developed in the field.

- Stage 1: Based on the CDM model (Godino 2009; Pino-Fan et al., 2018), two authors of this article separately classified the 123 descriptors into the contents of the CDM model. The authors agreed to make a primary and secondary classification of each descriptor, considering the primary classification as the one predominating in the descriptor, while the secondary one could be present partially. Once the classification was carried out individually, all the authors met to verify the degree of coincidence in the descriptors' classification.
- Stage 2: Afterwards, all the descriptors corresponding to the same key concept were grouped, and then an indicator was generated that enabled the description of such key concept and that reflected the descriptors. The result of this process was checked by another author that made suggestions for improvement or changes in the proposed indicators. Finally, all together, the authors came up with a final proposal of indicators for each key concept (see example in Figure 3, column 2).
- Stage 3: The third stage of this process began with reading and contrasting the indicators emerging from stage 2 and the indicators present in the Didactic Suitability Assessment Guide for the Instruction Process during the Teaching of Statistics (in Spanish, *Guía de Valoración de Idoneidad Didáctica de procesos de Instrucción en Didáctica de la Estadística* (GVID-IDE)) proposed by Ruz et al. (2019). This guide was designed based on the guidelines contained in documents of international consensus guiding teacher statistics (Franklin et al., 2015) and specific for Chilean reality (MINEDUC y CPEIP, 2012). Thus, as observed in Figure 3, in the second column we place the indicators generated in phase 2, and in the third column, the indicators present in the

reference guide; in both cases, they refer to the same key concept. Based on the comparison, we generated a fourth column with a list of indicators that include characteristics present in column 2 and 3. To finish, the final list of indicators was checked together by all the authors to then generate a first version that was sent to the experts.

Figure 3

Example of the process of inference and contrast of indicators.

Classification in components of the ecological content	Proposed indicator	GVID-IDE indicators	Merged indicators
Curricular adaptation	Adapts the school curriculum and its pedagogical practice based on the students' context, in order to respond to the characteristics and educational needs of each student.	The school disability integration guideline is taken into account.	Adapts the school curriculum and their teaching practices based on the students' context, in order to respond to the characteristics and educational needs of each student.
	Knows, uses, and updates a range of diverse teaching strategies so that their students develop the skills characteristic of stochastics.	The pedagogical and disciplinary approaches of the curriculum and their instruments to organize teaching are known.	Knows, uses, and updates a range of diverse teaching strategies so that their students develop the skills characteristic of stochastics.
	Knows and uses different strategies or procedures to monitor and assess their students' learning, in order to identify gaps in the expected and achieved goals, as well as biases that could reflect inequity in the access to learning opportunities.		Knows and uses different strategies or procedures to monitor and assess their students' learning, in order to identify gaps in the expected and achieved goals, as well as biases that could reflect inequity in the access to learning opportunities.

Phase 3. Analysis by Experts

To validate the content of the indicators which would potentially integrate the final indicator set, an analysis was carried out by experts. We telematically contacted eight Ibero-American researchers specialized in the didactics of stochastics and teacher education. All the participating experts hold a Doctorate's degree and work in pre- and in-service teacher education. They have between 5 and 21 years of research experience in the field of stochastic teacher education. Moreover, each of them was sent a document with the study purpose and a guide to assess the different theoretical contents considered in the developed model. Four criteria were considered: (1) *Clarity:* the indicator's syntax and semantics are appropriate and understandable; (2) *Coherence:* the indicator has a logical and consistent relation to the content it describes; (3) *Relevance*: the indicator is important and/or essential to describe the content; and (4) *Sufficiency*: the indicators making up a content are sufficient for its proper description.

Regarding the assessment, for the first three criteria (*clarity, coherence*, and *relevance*) a four-step Likert scale was used, starting from 1 (“Not met”) to 4 (“Met fully”). On the other hand, *sufficiency* was assessed qualitatively in the experts’ justifications, as they were consulted explicitly on the extent the considered indicators were enough to describe the theoretical content upon which they were described.

Finally, regarding the analyses carried out, we began an exploration within each content, by using the mean scores between each of their indicator. For example, for the epistemic content (C1) thirteen indicators are considered, so initially we analyzed the mean score between them, repeating this process with the rest of the contents. After that, we used Aiken’s V index (Aiken, 1980) to quantify the degree of agreement or concordance between the experts’ assigned scores. In practice, Aiken (1985) recommends as adequate or acceptable V point values above 0.7; when considering estimations in intervals at 95% reliability (Penfield y Giocobbi, 2004), thresholds above 0.5 in lower limits are accepted (Charter, 2003).

Phase 4. Filtering and creating the final proposal of indicators

Finally, based on the results of the analysis described in Phase 3, we checked those indicators that obtained point values below 0.7, which were reformulated taking into account the experts’ suggestions. Moreover, this process led us to reduce the number of indicators to reduce redundancies in the model. For each indicator to respond to a single content of the CDM model, all the indicators were checked, and if needed, they were rewritten, merged, or eliminated. Thus, based on this reduction of indicators, the final model was generated (see next section).

RESULTS

First indicators set (pre-experts’ review)

The first version of the indicators follows the structure of Ruz et al. (2019), that is, each indicator is associated to a content of the CDM model. These indicators align with that of the GVID-IDE (Ruz et al., 2019), which emerged from the different curricular guidelines on the statistics teacher education, both at the international level (Franklin et al., 2015) and in Chile (MINEDUC y CPEIP, 2012). Their basis is the proposal developed by Godino et al. (2013). Table 1 shows the key concepts for each content and in parenthesis are the number of indicators—both of the reference guide and the first set of indicators of the current study.

Table 1

Number of indicators according to key concept as a comparison between the GVID-IDE and the first set of current indicators

Didactic-statistic contents	**Key concepts** **GVID-IDE (Ruz et al., 2019)**	**First version current model**
Epistemic content	Definitions, propositions and procedures (8); Languages (4); Arguments (3); Problem situations (2); Relations (6)	Concepts and procedures (7); stochastic language (2); Argumentation (2); Problem situations (2)
Cognitive content	Previous knowledge (2); Curricular adaptations (3); Learning (3)	Previous knowledge (2); Curricular adaptations (3); Learning (4)
Affective content	Interests and needs (3); Attitudes (2); Emotions (1)	Interests (3); Attitudes (3); Emotions (3); Diversity (2)
Interactional content	Autonomy (1); Formative assessment (4); Teacher-Student Interaction (4); Interaction between Students (1)	Autonomy (1); Formative assessment (6); Teacher-Student Interaction (3), Interaction between Students (3) Interaction between Teachers (2); Planning (2)
Mediational content	Material resources (2); Classroom conditions (2); Time for teaching (1)	Material resources (3); Classroom conditions (2); Time for teaching (1)

Ecological content	Curricular adaptation (2); Didactic innovation (1); socio-cultural and professional adaptation (1); Value education (2); Connections (2)	Curricular adaptation (3); Didactic innovation (1); socio-cultural and professional adaptation (4); Citizenship education (2)

There are some differences between the GVID-IDE (Ruz et al., 2019) and the first set of current indicators. The latter contains six more indicators compared to those of the GVID-IDE, reflecting new qualities for current teacher education. Moreover, the latter stands out by a modern perspective of the discipline, going beyond statistics and considering its interaction with probabilities in what we conceptualize as stochastics. On the other hand, we observe that in comparison with the GVID-IDE (Ruz et al., 2019), the first version of the indicator set reduces the number of indicators belonging to the epistemic content, which is in turn reflected in the increase of indicators in other content categories. In regard to the key concepts, the first current version adds key concepts such as diversity in the affective content and interaction between teachers and planning in the interactional content, in order to meet the present teaching demands.

Results of the experts' review

We begin by exploring the experts' score distributions, according to the six *content* types of teachers' didactic-stochastic knowledge and the evaluation *criteria* considered (clarity, coherence, and relevance). In this vein, regarding all the content types considered, percentile 25 of scores referred to *coherence* and *relevance* reached a 4-point score, while for *clarity* percentile 40% reached the same score. That is, more than 60% of the total scores by experts was with the highest score in the three *criteria* referred to the six *content* types, which reflects a high degree of agreement with the content of the indicators as reviewed by experts.

Table 2 presents the descriptive statistics of the experts' scores. Regarding *clarity*, mean scores oscillated between 3.50 (epistemic content) and 3.69 points (mediational content). The most heterogeneous score behavior was observed in the ecological content with the highest Coefficient of Variation (CV) of 20.49%. In terms of *coherence*, the mean scores were higher than those of the previous criterion, varying between 3.69 (ecological content) and 3.86 points (epistemic content), also showing less dispersion between the experts. Relevance showed the highest and less dispersed scores across the six content types, as their mean scores oscillated between 3.85 (affective and mediational content) and 3.91 points (interactional content).

Table 2

Descriptive statistics of experts' scores.

	Clarity		**Coherence**		**Relevance**	
	M	**CV**	**M**	**CV**	**M**	**CV**
Epistemic	3.50	19.50%	3.86	11.09%	3.86	10.49%
Cognitive	3.68	16.96%	3.81	14.39%	3.89	11.05%
Affective	3.53	18.18%	3.78	14.69%	3.85	12.78%
Interactional	3.59	19.34%	3.83	13.16%	3.91	9.33%
Mediational	3.69	15.98%	3.81	13.96%	3.85	13.10%
Ecological	3.55	20.49%	3.69	18.10%	3.74	16.35%

Note: CV: Coefficient of variation

Then we determined Aiken's V index for each indicator making up the six didactic-mathematic contents considered, in order to quantify the degree of agreement between the experts' scores and identify those cases needing some adjustment based on the comments included in the review. On average, the global indexes were 0.86 (clarity), 0.93 (coherence) and 0.95 (relevance) and according to the six content types, scores were higher than 0.83 in all cases in the clarity criterion and higher than 0.9 in coherence and relevance. Table A1 in the Annex shows the point values of the V index, accompanied by their corresponding asymmetric confidence interval (CI) at 95%, for the three criteria considered, where only three cases were below the usual criterion of 0.7. For this reason, we took a stricter position and signaled with an * those cases with a V point lower than 0,8 in the identification of cases that must be checked.

In regard to the *stochastic content*, we highlight four indicators with a descended index in terms of clarity (see indicators 1.1, 1.4, 1.5 and 1.6 in Table A1 in the Annex, while indicator 1.13 was below the 0,8 range in the criteria of coherence and relevance. Concerning the *cognitive content*, we identified two indicators with a lower degree of agreement, in terms of coherence (2.1) and clarity (2.7). In the *affective content,* we detected a higher number of indicators below the accepted limit in terms of clarity (3.2, 3.4, 3.5, 3.7 and 3.9), as well as coherence and relevance for indicator 3.4. On the other hand, in the *interactional content*, it should be noted there is an indicator to be improved in terms of clarity (4.3) and another one in terms of coherence (4.14); moreover, indicator 4.11 must be checked in terms of those two criteria. *Mediational content* did not show descended indicators, whereas in the *ecological content* four cases stand out as low in clarity (6.1, 6.2, 6.4 and 6.9) in addition to indicator 6.8, which must be analyzed in terms of coherence and relevance. Therefore, it is recommended that all indicators previously identified be checked and potentially improved according to the qualitative analysis of the reports associated to each expert's score.

Final set of indicators (post experts' review)

After the qualitative check of the indicators with the least agreement, we adapted and rewrote them. For example, the stochastic content indicator 1.13 "promotes and uses historical problems that generated stochastics" was merged with indicator 1.2, which states "the understanding of the principles and historical-epistemological meanings of stochastics." In addition, we addressed a generalized comment of the experts, which concerned the model's extension and the fact that some indicators had similarities, so they could respond to different content types (Annex Table A2). Since our objective is for this model to be operational, the number of indicators per content were reduced by half, making sure that each indicator responded only to a single content. The final version of the Didactic-Stochastic Knowledge Model is presented below.

Epistemic Content

1.1-F: Understands the characteristics of the statistical models describing data variability in their context. 1.2-F: Understands stochastics' principles and historical-epistemological meanings. 1.3-F: Critically evaluates the use of descriptive and inferential procedures to solve problems in different knowledge areas. 1.4-F: Links descriptive and inferential statistics using data as evidence and expresses conclusions with a certain degree of uncertainty. 1.5-F: Communicates stochastic ideas consistently and effectively using oral or written language. 1.6-F: Articulates different data representations, being able to build them both manually and with technology. 1.7-F: Critically evaluates the validity of conclusions emerging from a stochastic analysis process. 1.8-F: Links the process of stochastic problem solving with stages associated with empirical research.
Cognitive content 2.1-F: Understands theories of human learning and their relation to the teaching of stochastics. 2.2-F: Builds, selects, and adapts assessments that are coherent with the stochastic learning methodologies used. 2.3-F: Considers the difficulties and erroneous conceptions of all students to (re)organize the learning experiences. 2.4-F: Understands the value of digital tools in the stochastics' learning processes. 2.5-F: Sequences learning objectives of stochastics, coherent with the curriculum, and the students' previous knowledge and skills. 2.6-F: Applies gradual approximations, from informal to formal, to introduce the understanding of stochastic topics of greater difficulty.

Affective Content 3.1-F: Applies motivation theories to promote engagement, persistence, and self-efficacy of students in the learning of stochastics. 3.2-F: Considers contexts and situations of interest for students in the modeling of stochastical phenomena. 3.3-F: Promotes positive attitudes towards stochastics and its own skills such as research, communication, and critical thinking. 3.4-F: Promotes willingness and commitment of all students towards the learning of stochastics. 3.5-F: Promotes the development of socioemotional competencies for decision-making and awareness of context in the learning of stochastics. 3.6-F: Promotes students' self-esteem and academic self-efficacy when learning stochastics. 3.7-F: Generates strategies for an equitable and active participation of all students, valuing diversity in all its expressions.
Interactional Content 4.1-F: Guides their students to move from guided work to an autonomous one, reinforcing their metacognitive skills in the learning of stochastics. 4.2-F: Analyzes data and evidence contributed by assessments to improve the techniques used. 4.3-F: Monitors the students' level of stochastic understanding before, during, and/or after the class. 4.4-F: Establishes respectful and inclusive interaction rules, coherent with the dynamics of a stochastics class. 4.5-F: Promotes an interactive teaching style, centered in real problems where the stochastic research process is valued. 4.6-F: Generates instances between students that allow them to model and reason stochastically to make decisions about a problem. 4.7-F: Promotes collaborative work between peer teachers in the critical evaluation of didactic strategies used in the teaching of stochastics.

Mediational Content 5.1-F: Promotes virtual and in-person learning opportunities fostering stochastic competencies. 5.2-F: Promotes a welcoming and stimulating class environment during the stochastics' teaching and learning process. 5.3-F: Integrates digital environments in different formats to solve stochastic problems. 5.4-F: Articulates different didactic, material, and digital resources included in the curriculum, and suggested by research in the teaching of stochastics. 5.5-F: Optimizes lecture time in mathematics lessons to address the teaching of stochastics.
Ecological Content 6.1-F: Renews their teaching strategies based on educational research in the field of stochastical education and the curricular updates. 6.2-F: Promotes the link of stochastics with other disciplines in which data intervene and uncertainty exists. 6.3-F: Promotes the use of stochastics in decision-making based on data present in modern democracies. 6.4-F: Considers the determining factors and restrictions of their students' social environment in the processes of teaching and learning of stochastics.

DISCUSSION AND CONCLUSIONS

In this work, we have addressed the issue of characterizing the professional knowledge that a future mathematics teacher must develop to teach stochastics, considering the Chilean reality as a case study. As a result, we have obtained a final set of 37 indicators, organized according to professor Godino's Didactic-Mathematical Knowledge Model (CDM) (Godino, 2009). In this vein, we highlight the method used to obtain an updated, more synthetic, and consistent version of the desirable characteristics for whom will teach stochastics in Chilean schools. Moreover, regarding the first approach proposed by Ruz et al. (2019), in this case the interactions between theoretical contents are avoided, and there is no differentiation between statistics and probabilities, including both under the umbrella of *stochastics* to highlight the unbreakable link between both disciplines.

Consequently, we consider the initially established goals as achieved, as we obtained a set of indicators that were properly validated in terms of their content. Thus, we can position the resulting indicators within the family of instruments of that nature (e.g., Godino, 2013; Godino et al., 2013; Godino et al., 2012; Ruz et al., 2019) whose use depends on the educational stakeholder who utilizes it. For example, in our case, the final indicators cover the didactic-stochastic knowledge of future teachers in relation to their students, while in the case of teacher trainers, these facets will enable them to guide, adapt, and develop study processes of the teaching of statistics in teacher education programs. Moreover, this work presents a methodology that enables the articulation of theoretical models, such as CDM, with other types of documents or content organization through the validation of experts (Figure 1).

On the other hand, regarding the projection of our results, from a practical point of view, we consider that the generated knowledge model can be a useful tool for both the assessment and design of stochastic subjects for both pre-service teacher education and professional development programs. Therefore, it is hoped that the intersection between indicators belonging to different contents enables the articulation of the different knowledge types required for the teaching and learning of stochastics. This intersection of indicators, and thus, contents, could become concrete through the design of an assessment rubric for teacher education programs, which could also guide the design and improvement of both existing and in-construction programs.

At the same time, from a theoretical perspective, this guide establishes guidelines for a future didactic-stochastic knowledge model for pre- and in-service teachers. The list of indicators enables the characterization of the knowledge types at play during the stochastics' teaching and learning processes in different scenarios. Moreover, the list could guide the reflection on which knowledge types are necessary for this discipline. Finally, a constant update of teacher education programs is needed, since the school curriculum constantly incorporates new education perspectives—even more so in the current era, where data and uncertainty play a fundamental role in citizens' daily activities. Thus, this knowledge guide hopes to be a contribution and a foundation on which to work on when thinking and reflecting on teacher education.

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AUTHORS' CONTRIBUTIONS STATEMENTS

FR and FMU conceived and conceptualize the presented idea. All authors adapted the methodology to this context, and collected the data. FR and FMU analysed the data. All authors actively participated in the discussion of the results, reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, F.R., upon reasonable request.

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ANNEXES

Table A1

Indexes and CI (95%) for Aiken's V according to value content and criterion

Content	Ind. Nº	Clarity		Coherence		Relevance	
		V	IC(95%)	V	IC(95%)	V	IC(95%)
Epistemic	1.1	0,67*	[0.47 - 0.82]	0.88	[0.69 - 0.96]	0.96	[0.8 - 0.99]

	1.2	0.88	[0.69 - 0.96]	0.83	[0.64 - 0.93]	0.88	[0.69 - 0.96]
	1.3	0.88	[0.69 - 0.96]	1	[0.86 - 1]	0.96	[0.8 - 0.99]
	1.4	0.75*	[0.55 - 0.88]	1	[0.86 - 1]	0.96	[0.8 - 0.99]
	1.5	0.79*	[0.6 - 0.91]	0.96	[0.8 - 0.99]	0.92	[0.74 - 0.98]
	1.6	0.75*	[0.55 - 0.88]	1	[0.86 - 1]	1	[0.86 - 1]
	1.7	0.88	[0.69 - 0.96]	1	[0.86 - 1]	1	[0.86 - 1]
	1.8	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	1.9	0.83	[0.64 - 0.93]	0.96	[0.8 - 0.99]	0.96	[0.8 - 0.99]
	1.10	0.83	[0.64 - 0.93]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	1.11	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	1.12	0.83	[0.64 - 0.93]	1	[0.86 - 1]	1	[0.86 - 1]
	1.13	0.92	[0.74 - 0.98]	0.79*	[0.6 - 0.91]	0.75*	[0.55 - 0.88]
Cognitive	2.1	0.92	[0.74 - 0.98]	0.79*	[0.6 - 0.91]	1	[0.86 - 1]
	2.2	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]

	2.3	0.96	[0.8 - 0.99]	0.96	[0.8 - 0.99]	0.96	[0.8 - 0.99]
	2.4	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]	0.92	[0.74 - 0.98]
	2.5	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	2.6	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	2.7	0.79*	[0.6 - 0.91]	0.92	[0.74 - 0.98]	0.88	[0.69 - 0.96]
	2.8	0.88	[0.69 - 0.96]	0.88	[0.69 - 0.96]	0.92	[0.74 - 0.98]
	2.9	0.83	[0.64 - 0.93]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	3.1	0.88	[0.69 - 0.96]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	3.2	0.79*	[0.6 - 0.91]	0.88	[0.69 - 0.96]	1	[0.86 - 1]
	3.3	0.96	[0.8 - 0.99]	0.96	[0.8 - 0.99]	0.92	[0.74 - 0.98]
Affective	3.4	0.75*	[0.55 - 0.88]	0.75*	[0.55 - 0.88]	0.75*	[0.55 - 0.88]
	3.5	0.79*	[0.6 - 0.91]	0.88	[0.69 - 0.96]	0.96	[0.8 - 0.99]
	3.6	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	3.7	0.79*	[0.6 - 0.91]	0.88	[0.69 - 0.96]	0.92	[0.74 - 0.98]

	3.8	0.88	[0.69 - 0.96]	1	[0.86 - 1]	0.92	[0.74 - 0.98]
	3.9	0.75*	[0.55 - 0.88]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	3.10	0.83	[0.64 - 0.93]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	3.11	0.96	[0.8 - 0.99]	1	[0.86 - 1]	1	[0.86 - 1]
Interactional	4.1	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	4.2	0.83	[0.64 - 0.93]	1	[0.86 - 1]	1	[0.86 - 1]
	4.3	0.75*	[0.55 - 0.88]	1	[0.86 - 1]	1	[0.86 - 1]
	4.4	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	4.5	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	4.6	0.92	[0.74 - 0.98]	1	[0.86 - 1]	1	[0.86 - 1]
	4.7	0.88	[0.69 - 0.96]	1	[0.86 - 1]	1	[0.86 - 1]
	4.8	0.88	[0.69 - 0.96]	0.88	[0.69 - 0.96]	0.92	[0.74 - 0.98]
	4.9	0.92	[0.74 - 0.98]	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]
	4.10	0.83	[0.64 - 0.93]	0.92	[0.74 - 0.98]	0.88	[0.69 - 0.96]

	4.11	0.67*	[0.47 - 0.82]	0.67*	[0.47 - 0.82]	0.83	[0.64 - 0.93]
	4.12	0.83	[0.64 - 0.93]	1	[0.86 - 1]	1	[0.86 - 1]
	4.13	0.88	[0.69 - 0.96]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	4.14	0.83	[0.64 - 0.93]	0.79*	[0.6 - 0.91]	0.92	[0.74 - 0.98]
	4.15	0.88	[0.69 - 0.96]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
	4.16	0.96	[0.8 - 0.99]	1	[0.86 - 1]	1	[0.86 - 1]
Mediational	5.1	0.88	[0.69 - 0.96]	0.88	[0.69 - 0.96]	0.92	[0.74 - 0.98]
	5.2	0.92	[0.74 - 0.98]	0.83	[0.64 - 0.93]	0.83	[0.64 - 0.93]
	5.3	0.83	[0.64 - 0.93]	1	[0.86 - 1]	1	[0.86 - 1]
	5.4	0.96	[0.8 - 0.99]	1	[0.86 - 1]	1	[0.86 - 1]
	5.5	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]	0.96	[0.8 - 0.99]
	5.6	0.88	[0.69 - 0.96]	0.96	[0.8 - 0.99]	1	[0.86 - 1]
Ecological	6.1	0.79*	[0.6 - 0.91]	0.83	[0.64 - 0.93]	0.92	[0.74 - 0.98]
	6.2	0.71*	[0.51 - 0.85]	0.92	[0.74 - 0.98]	1	[0.86 - 1]

6.3	0.88	[0.69 - 0.96]	0.92	[0.74 - 0.98]	0.92	[0.74 - 0.98]
6.4	0.79*	[0.6 - 0.91]	0.96	[0.8 - 0.99]	0.96	[0.8 - 0.99]
6.5	0.88	[0.69 - 0.96]	0.92	[0.74 - 0.98]	0.92	[0.74 - 0.98]
6.6	0.96	[0.8 - 0.99]	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]
6.7	0.92	[0.74 - 0.98]	0.88	[0.69 - 0.96]	0.83	[0.64 - 0.93]
6.8	0.96	[0.8 - 0.99]	0.79*	[0.6 - 0.91]	0.71*	[0.51 - 0.85]
6.9	0.79*	[0.6 - 0.91]	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]
6.10	0.83	[0.64 - 0.93]	0.92	[0.74 - 0.98]	0.96	[0.8 - 0.99]

Table A2

Initial indicators sent for experts' reviews and final indicators obtained after the assessment of experts.

Content	Initial Indicators	Final Indicators
Epistemic	1.1. Promotes the work with statistical models describing data variability (data = structure+ variability).	1.1-F. Understands the characteristics of the statistical models describing data variability in their context.
	1.2. Understands stochastics' historical-epistemological principles, acknowledging their value as a tool to study natural and social phenomena.	1.2-F. Understands stochastics' principles and historical-epistemological meanings.

1.3. Understands and applies procedures of descriptive statistics (central tendency, position, and dispersion measures) through the exploratory data analysis.	ELIMINATED
1.4. Understands and applies procedures of inferential statistics for the critical analysis of information present in different areas such as social sciences, health, and education.	1.3-F. Critically evaluates the use of descriptive and inferential procedures to solve problems in different knowledge areas.
1.5. Understands and calculates probabilities from different approaches (classical, frequency, and Bayesian) besides applying the notion of Independence.	ELIMINATED
1.6. Understands the value of context and the variability throughout the process of stochastic problem-solving.	ELIMINATED
1.7. Links descriptive and inferential procedures using data as evidence to generalize beyond their description and express conclusions with a certain degree of uncertainty.	1.4-F. Links descriptive and inferential statistics using data as evidence and expresses conclusions with a certain degree of uncertainty.
1.8. Communicates stochastic ideas, through written or oral stochastic language, consistently and effectively for different audiences.	1.5-F. Communicates stochastic ideas consistently and effectively using oral or written language.

	1.9. Understands and uses different data representation (graphs, tables, statistics summary, etc.) through manual construction and/or technology.	1.6-F. Articulates different data representations, being able to build them both manually and with technology.
	1.10. Promotes discussion and argumentation of data-based decisions to solve stochastics problem-solving.	ELIMINATED
	1.11. Evaluates and critiques the feasibility of conclusions coming from a stochastic analysis process.	1.7-F. Critically evaluates the validity of conclusions emerging from a stochastic analysis process.
	1.12. Models social and natural phenomena through problem-solving guided by the stochastic research cycle (problem, data, analysis, and results).	1.8-F. Links the process of stochastic problem solving with stages associated with empirical research.
	1.13. Promotes and uses historical problems that originated stochastics.	MERGED WITH INDICATOR 1.2
	2.1. Understands the main theories of learning and their relation with the teaching of stochastics according to students' characteristics.	2.1.-F. Understands theories of human learning and their relation to the teaching of stochastics.
Cognitive	2.2. Builds, selects, and adapts criteria and instruments of evaluation that are coherent with learning goals, including self- and co-evaluation processes, which provide timely information regarding the learning process of each student.	2.2.-F. Builds, selects, and adapts assessments that are coherent with the stochastic learning methodologies used.

2.3. Understands the educational needs of all their students, adapting their pedagogical practices to provide support according to students’ learning pace and individual characteristics.	ELIMINATED
2.4. Monitors students’ level of understanding and willingness before, during and/or after class so as to detect difficulties and strengthen the learning of new knowledge.	ELIMINATED
2.5. Identifies difficulties and erroneous conceptions of stochastics and considers them to (re)organize the learning experiences that enable their discovery and correction.	2.3-F. Considers the difficulties and erroneous conceptions of all students to (re)organize the learning experiences.
2.6. Understands the value of digital tools in the stochastics’ learning process.	2.4-F. Understands the value of digital tools in the stochastics learning processes.
2.7. Communicates assessment results in a timely manner and provides descriptive feedback on the degree of learning achieved by each student, establishing strategies able to overcome the identified gaps.	ELIMINATED
2.8. Selects, formulates, and sequences learning objectives that are coherent with the curriculum according to students’ characteristics, previous knowledge, and skills, ensuring everyone’s participation and understanding.	2.5-F. Sequences learning objectives of stochastics, coherent with the curriculum, and the students’ previous knowledge and skills.

	2.9. Considers informal approaches to introduce the understanding of stochastic topics of greater difficulty for students.	2.6-F. Applies gradual approximations, from informal to formal, to introduce the understanding of stochastic topics of greater difficulty.
	3.1. Understands and applies the main theories of motivation to promote engagement, persistence, and self-efficacy of their students in stochastics' teaching and learning processes.	3.1-F. Applies motivation theories to promote engagement, persistence, and self-efficacy of students in the learning of stochastics.
	3.2. Models stochastic phenomena that consider contexts of interest for their students and that are relevant for their comprehensive education.	3.2-F. Considers contexts and situations of interest for students in the modeling of stochastical phenomena.
Affective	3.3. Analyzes daily situations to identify students' ways of thinking, feeing, and acting.	ELIMINATED
	3.4. Promotes the integral education of their students (knowledge, skills, attitudes) that enable them to establish constructive relations for a healthy coexistence.	ELIMINATED
	3.5. Understands, demonstrates, and promotes the skills and attitudes characteristic of stochastics, such as research, questioning, communication, and critical thinking.	3.3-F. Promotes positive attitudes towards stochastics and its own skills such as research, communication, and critical thinking.
	3.6. Promotes deep learning, the commitment, and the positive disposition of all their	3.4-F. Promotes willingness and commitment of all students towards the learning of stochastics.

students towards the learning of stochastics.	
3.7. Understands and implements strategies for their students to develop socioemotional competencies fostering self-knowledge, self-regulation, awareness of their environment, and responsible decision-making.	3.5-F. Promotes the development of socioemotional competencies for decision-making and awareness of context in the learning of stochastics.
3.8. Observes and understands their students' emotional state and changes to establish meaningful links during the stochastics' learning process.	ELIMINATED
3.9. Develops and implements strategies to strengthen and protect their students' self-esteem and academic self-efficacy through the clear communication of expectations about the learning of stochastics.	3.6-F. Promotes students' self-esteem and academic self-efficacy when learning stochastics.
3.10. Recognizes and values their students' and communities' diversity, in all its expressions, discouraging discrimination based on gender, sexual orientation, religion, among others.	ELIMINATED
3.11. Generates strategies for an equitable and active participation of all the student body, with the aim of eradicating prejudice or bias	3.7-F. Generates strategies for an equitable and active participation of all students, valuing diversity in all its expressions.

	about the ability to learn stochastics.	
	4.1. Supports and guides their students to move from guided work to an autonomous one, where students can set their own learning goals in a reflexive manner, reinforcing their metacognitive skills.	4.1-F. Guides their students to move from guided work to an autonomous one, reinforcing their metacognitive skills in the learning of stochastics.
	4.2. Values and guides their students in the self- and co-evaluation processes that enable them to determine achieved outcomes and those requiring improvement.	ELIMINATED
Interactional	4.3. Offers their students descriptive, timely, and effective feedback, so as to reflectively set strategies allowing them to overcome the detected learning gaps.	ELIMINATED
	4.4. Designs formative and summative assessments, which are diverse in techniques and instruments, at different times in the learning process, using fair, rigorous, and transparent criteria for students.	ELIMINATED
	4.5. Analyzes the data and the evidence provided by assessments, improving the techniques used and addressing the effect those results may have on students.	4.2-F. Analyzes data and evidence contributed by assessments to improve the techniques used.

4.6. Communicates objectives, assessment criteria, and grades obtained clearly, making sure all students understand them.	ELIMINATED
4.7. Monitors during class their students' understanding through discussions or questions to identify reasoning levels, difficulties, or errors in order to reorient teaching.	4.3-F. Monitors the students' level of stochastic understanding before, during, and/or after the class.
4.8. Promotes setting up respectful and inclusive coexistence rules that are clear, coherent and consistent.	4.4-F. Establishes respectful and inclusive interaction rules, coherent with the dynamics of a stochastics class.
4.9. Establishes responsibilities and clear instructions that their students must have regarding their learning and their education process, verifying these are understood by everyone.	ELIMINATED
4.10. What is promoted is a teaching style that is interactive and centered in real problems, where the process of stochastic research is valued.	4.5-F. Promotes an interactive teaching style, centered in real problems where the stochastic research process is valued.
4.11. Understands and promotes the development of socioemotional skills, such as self-knowledge, respect for others, honesty, and equity in interaction in and outside the classroom.	MOVED TO AFFECTIVE CONTENT
4.12. Promotes and organizes interactions between students, groups, and the whole class that enable them to describe,	4.6-F. Generates instances between students that allow them to model and reason stochastically

	explain, ad predict phenomena, make decisions, and provide supported claims about a problem.	to make decisions about a problem.
	4.13. Understands the importance of collaborative work by participating with peers in several instances of mutual support, such as the adaptation of assessment strategies and procedures.	4.7-F. Promotes collaborative work between peer teachers in the critical evaluation of didactic strategies used in the teaching of stochastics.
	4.14. Solves problems using technology individually and collaboratively, supporting mathematical ideas in front of their peers and critically evaluating the strategies used.	
	4.15. Considers teaching strategies to promote original critical and creative thinking aligned with the disciplinary and across-subject learning objectives, and the students' diversity.	ELIMINATED
	4.16. Designs coherent, progressive, and sequenced lesson plans according to the different moments of the class, responding to students' pace and characteristics.	ELIMINATED
Mediational	5.1. Promotes virtual and in-person learning opportunities that engage students so they use their knowledge, skills, and attitudes about stochastics.	5.1-F. Promotes virtual and in-person learning opportunities fostering stochastic competencies.

	5.2. Solves problems using mathematical language and digital resources, individually and collaboratively in a welcoming and stimulating environment.	5.2-F. Promotes a welcoming and stimulating class environment during the stochastics' teaching and learning process.
	5.3. Knows a range of stochastics teaching resources that encourage the participation of all students, without a gender-bias.	ELIMINATED
	5.4. Plans and creates class contents that incorporate digital environments in different formats to solve stochastic problems.	5.3-F. Integrates digital environments in different formats to solve stochastic problems.
	5.5. Analyzes and articulates the different resources and documents complementing the curriculum.	5.4-F. Articulates different didactic, material, and digital resources included in the curriculum, and suggested by research in the teaching of stochastics.
	5.6. Optimizes lecture time, use of classroom space, educational resources, and digital technologies to achieve learning objectives.	5.5-F. Optimizes lecture time in mathematics lessons to address the teaching of stochastics.
Ecological	6.1. Analyzes and articulates the foundations, skills, attitudes, and historical development of stochastics according to the curricular progression for the teaching and learning of this discipline in school.	CONSIDERED IN EPISTEMIC CONTENT

6.2. Knows, uses, and updates a range of diverse teaching strategies so that their students develop the skills characteristic of stochastics.	ELIMINATED
6.3. Knows and uses different monitoring and assessment strategies or procedures for their students’ learning, so as to identify gaps between the expected and achieved goals as well as biases that could reflect inequity in the access to learning opportunities.	CONSIDERED IN INTERACTIONAL CONTENT
6.4. Renews and updates diverse teaching strategies that are effective and challenging for the students in order to promote both across-the-curriculum skills and those that are stochastic-specific.	6.1-F. Renews their teaching strategies based on educational research in the field of stochastical education and the curricular updates.
6.5. Knows, through different data collection techniques, their students’ individual, family, cultural, and social characteristics.	ELIMINATED
6.6. Promotes communication and collaboration between family, parents, guardians, and the teacher with the aim of improving and supporting their students’ learning of stochastics.	6.2-F. Promotes the link of stochastics with other disciplines in which data intervene and uncertainty exists.
6.7. Knows and applies the current educational guidelines, including students’ rights, legal framework, ad policies regulating the teaching	ELIMINATED

profession and the use of digital technologies, among others.	
6.8. Promotes coexistence relations that are respectful and inclusive, clarifying and respecting the norms for healthy coexistence in and out of the classroom.	MOVED TO INTERACTIONAL CONTENT
6.9. Promotes learning experiences in which the students apply stochastics to act effectively in a democracy regarding social phenomena where data intervene.	6.3-F. Promotes the use of stochastics in decision-making based on data present in modern democracies.
6.10. Generates and implements strategies for students' active and equitable participation in the learning of stochastics, with no gender bias, promoting critical and creative thinking.	ELIMINATED
	6.4-F. Considers the determining factors and restrictions of their students' social environment in the processes of teaching and learning of stochastics.