ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.8099

Students' Analytical Thinking Process in Solving Sets Problems in Terms of The Characteristics of Pseudo Thinking

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ABSTRACT

Background: In solving mathematical problems, analytical thinking is an important type of thinking. Several studies show variations in students' levels of analytical thinking, so it is necessary to characterize analytical thinking to determine students' conditions in solving mathematical problems. **Objectives:** This study contributes to analyse the characteristics of students' analytical thinking processes in solving sets of problems. **Design:** The type of research used in this study is case-study qualitative research to analysing the students' analytical thinking processes using the Anderson framework in terms of the pseudo characteristics thinking using the Vinner and Subanji framework. **Setting and participants:** The selected subjects are students who have studied the set of material. They are mathematics undergraduate students at the State University of Malang, Indonesia. **Data collection and analysis:** The instruments in this study were a set of problem tests and interview guidelines. The technique of checking the validity of the data in this study was carried out using source triangulation. **Results:** The results showed that 40 research subjects, when solving set problems, had gone through analytical thinking stages, namely differentiating, organizing, and attributing. Then, in addition to these results, there are also four characteristics of students' analytical thinking process in solving a set of problems based on possible answers that occur to them. **Conclusion:** The author's analysis shows there are four characteristics of analytical thinking process are thinking real true, true-pseudo, false-pseudo, and real false. Based on study results, this paper also presents recommendations for problem solutions related to four characteristics of analytical thinking.

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Keywords: analytical thinking; characteristics; pseudo; sets.

Processo de pensamento analítico dos alunos na resolução de problemas de conjuntos em termos das características do pseudo-pensamento

RESUMO

Contexto: Na resolução de problemas matemáticos, o pensamento analítico é um tipo importante de pensamento. Diversos estudos mostram variações nos níveis de pensamento analítico dos alunos, por isso é necessário caracterizar o pensamento analítico para determinar as condições dos alunos na resolução de problemas matemáticos. Objetivos: Este estudo contribui para analisar as características dos processos de pensamento analítico dos alunos na resolução de conjuntos de problemas. **Objetivos**: Este estudo contribui para analisar as características dos processos de pensamento analítico dos alunos na resolução de conjuntos de problemas. **Design**: O tipo de pesquisa utilizada neste estudo é a pesquisa qualitativa de estudo de caso para analisar os processos de pensamento analítico dos alunos usando a estrutura de Anderson em termos do pensamento pseudocaracterístico usando a estrutura de Vinner e Subanji. **Ambiente e participantes**: Os sujeitos selecionados são alunos que estudaram o conjunto da matéria. Eles são estudantes de graduação em matemática na Universidade Estadual de Malang, na Indonésia. **Coleta e análise de dados**: Os instrumentos deste estudo foram um conjunto de testes de problemas e orientações para entrevistas. A técnica de verificação da validade dos dados deste estudo foi realizada por meio da triangulação de fontes. **Resultados**: Os resultados mostraram que 40 sujeitos da pesquisa, ao resolverem problemas definidos, passaram por etapas de pensamento analítico, nomeadamente diferenciação, organização e atribuição. Então, além desses resultados, há também quatro características do processo de pensamento analítico dos alunos na resolução de um conjunto de problemas com base nas possíveis respostas que lhes ocorrem. **Conclusões**: A análise do autor mostra que há quatro características do processo de pensamento analítico: pensar real, verdadeiro, verdadeiro-pseudo, falso-pseudo e real falso. Com base nos resultados do estudo, este artigo também apresenta recomendações para soluções de problemas relacionados a quatro características do pensamento analítico.

Palavras-chave: características; conjuntos; pensamento analítico; pseudo.

INTRODUCTION

Thinking is a mental activity that occurs within a person when faced with a problem and requires a person to solve it (Ruggiero, 2003). In solving problems, a person carries out cognitive activities by connecting the knowledge they already have and what is understood about the problem (Allsop, 2016). Furthermore, thinking involves mental operations including induction,

deduction, classification, and reasoning to analyze, criticize, and reach good conclusions (Arends, 2012). Meanwhile, thinking can also be seen from the practices and evidence carried out, which cannot be accessed directly (Lam & Chan, 2020). From these several opinions, it provides an understanding of the definition of thinking broadly, so that it can be concluded that thinking is a mental activity if someone is faced with a problem and is required to solve the problem so that it involves the action of processing knowledge within a person. Then, Slavin (2005) stated that information processing in the mind goes through a process that begins with external stimulation in receiving information until a response is produced to the stimulus received through the information processing stages. Information is received by the human senses, then filtering of information occurs so that there is information that is passed on and information that is ignored into short-term memory (working memory). In short-term memory, information processing is carried out by repetition and encoding. Furthermore, information that is frequently recalled (repetition) will be stored in long-term memory. If one day the information stored in long-term memory is needed, it will be done by retrieval (Slavin, 2005)

Thinking is done by using abilities rather than their own abilities which are necessary for all activities, from carrying out daily tasks to making decisions (Güner & Erbay, 2021). Thinking that occurs in students' cognitive domains can be seen in students' behavior in solving problems so that we can understand the process. Students who tend to be able to solve problems are students who are able to coordinate operations in solving various problems, including using the thinking and reasoning that students have in solving problems (Shin et al., 2020). Furthermore, mathematics is a science that is synonymous with problem solving (J. Kilpatrick, Jane Swafford, Mathematics Learning Study Committee, 2001). In addition, problem solving in mathematics can create and develop students' thinking patterns (Anderson et al., 2001). This is also in accordance with the policy of *the National Council of Teachers of Mathematics* which states that problem solving must be the focus of the curriculum in learning (NCTM, 2000)

In solving problems, someone will describe the process of solving the problem and require analytical skills. The ability to analyze is needed because it is one aspect of cognitive learning which is the goal of learning. Analyzing itself is closely related to thinking (Anderson et al., 2001). Several indicators in the analytical thinking process can be used as a reference to find out whether someone is capable or used to thinking analytically. These indicators are: *differentiating*, *organizing* and *attributing*. The analytical thinking process of students in solving mathematical problems by changing limited information

questions, namely the cognitive process of students sorting or distinguishing relevant or irrelevant and important or unimportant information from the problem structure (differentiating), organizing systematic and coherent relationships between relevant information or important (organizing), and providing attributes for the problem by combining information (attributing), in the case of organizing students change/transfer the information then proceed to change the question to obtain the final solution (Ad'hiya & Laksono, 2018; Anderson et al., 2001; Annizar et al., 2021; Karenina et al., 2019).

Based on the stages proposed by previous research, researchers describe the stages of analytical thinking as follows:

a. Differentiating

At the differentiating stage, there is a process of identifying and sorting out the important parts of a problem structure. Differentiating occurs when students do the following things:

- Understand the relevant and important parts of a structure. This can be known when students are able to tell in their own language about the parts in a given set problem.
- Find relevant and important parts of a structure. This can be known when students are able to find out what is known and what is asked in set problems.
- Sort out the relevant and important parts of a structure. This can be known when students are able to distinguish the important parts in a set problem, including what is known and what is asked.

b. Organizing

At the organizing stage, there is a process of preparing a resolution plan by connecting the parts of the plan that have been prepared and then operating the parts of the plan that have been linked to get a problem solving solution. Organizing occurs when students do the following things:

- Develop a resolution plan. This can be seen when students are able to state or describe problems in mathematical form or models.
- Connect the parts of the plan that have been prepared. This can be known when students are able to determine the relationship between the parts of the plan that has been prepared.

- Operate the parts of the plan that have been linked. This can be known when students are able to operate the parts of the plan that have been linked.

c. Attributing

At the attribute giving stage, there is a process of determining the meaning of the solution and concluding the solution that has been interpreted, so that the conclusions obtained can be a solution to the matter in question. Providing this attribute occurs when students do the following things:

- Determine the solution from the results of the operation. This can be known when students are able to determine a solution based on the results of the operations that have been carried out
- Interpret or interpret the solution that has been determined. This occurs when students are able to interpret the meaning of the results of predetermined operations.
- Concluding the interpreted solution. This happens when students are able to conclude solutions to problem solving based on the results of operations that have been interpreted.

Solving problems in mathematics requires a thinking process and allows students to have several characteristics in solving problems. Then, in characterizing the thinking process, researchers used the pseudo-thinking theory proposed by Subanji (2011). When solving problems, there are several possible answers that occur to students. For students who give correct answers and are able to provide justification, this means that the answer is "real true", this is normal. On the other hand, students who show the correct answer, but are unable to provide justification for the answer, then the truth of the answer is only "true-pseudo". Meanwhile, students who show false answers and after reflection still produce false answers, it means that the student's thinking process is indeed "real false". Another possible behavior is that students give false answers, but after reflecting they are able to correct them so that they become the correct answer. Subanji calls it “false-pseudo” thinking, and false truth as pseudo-true thinking (Subanji, 2011).

Subanji (2011) states that students whose thinking processes are "pseudo" will relate to problems that they consider to be the same. Pseudo thinking is false thinking so that the correct answer is not necessarily the result of a correct thought process and the wrong answer is not necessarily the result of a wrong thought process. Furthermore, Vinner (1997) explains that students

are forced to study certain topics and solve certain problems but do not exercise control over what they think (Vinner, 1997). Therefore, pseudo thinking is not the result of students' actual thinking processes, but rather comes from pseudo or vague thinking processes (Wibawa, 2016). So, adapting the thinking character proposed by Subanji (2011), student thinking character is the way students think about solving problems by looking at the student's possible answers. There are 4 characters in thinking based on students' possible answers, namely:

1. Real true, that is, the student gives the correct answer and is able to provide justification,

2. True-Pseudo, namely the student shows the correct answer, but is unable to provide justification for the answer,

3. False-Pseudo, that is, the student gives a wrong answer, but after reflecting he is able to correct it so that it becomes the correct answer

4. Real false, namely students who show wrong answers and after reflection still produce wrong answers.

Thus, in this research pseudo thinking is pseudo thinking so that the correct answer is not necessarily the result of a correct thought process and the wrong answer is also not necessarily the result of a wrong thought process because a person does not control what he thinks.

One of the basic materials in mathematics is set material. The set cannot be defined. A set can be intuited as a collection of things or objects that can be clearly defined (Morash, 1987). Then the point of view to see the process of solving mathematical problems in set material has been carried out by several researchers. First, research on associations uses a learning perspective during the Covid-19 pandemic (Latifah & Sutirna, 2021). The two association problems were also researched using a *self-efficacy perspective* (Loviasari & Mampouw, 2022). Third, the association problems were researched using a difficulty analysis perspective (Mursalina et al., 2019). Fourth, the association problem is researched using an analysis perspective of critical thinking skills (Lestari & Roesdiana, 2021). Fifth, set problems are researched using the perspective of conceptual and procedural error profiles (Natsir et al., 2016). The large number of studies related to set material show that set material is an interesting topic to research, besides that set material is material in the study of basic mathematics courses which is the basic science mastered by mathematics students.

Then as a preliminary study, researchers examined 3 students to solve set problems. The results of the preliminary study show that there are differences in students' thinking processes in each stage of analytical thinking when solving set problems. Therefore, researchers will determine the possible characteristics of students' thinking processes in each stage of analytical thinking.

Based on analytical thinking studies, research studies on sets, and preliminary study phenomena, this research will describe the characteristics of students' analytical thinking in solving set problems.

METHODOLOGY

This research includes qualitative case study research. Qualitative research is research that investigates human problems or social phenomena with research findings obtained not through statistical procedures. Researchers create complex images, examine wording, detailed reports from the research subject's point of view, and conduct studies in natural situations (Creswell, 2013). This research will present an analysis of the characteristics of students' analytical thinking in solving group problems. In this research, researchers act as planners, implementers, data collectors, data interpreters, and reporters of research results. Activities carried out by researchers include collecting direct observation data, interviews, recording student activities in solving group problems, collecting data in the form of test results, making conclusions and research reports.

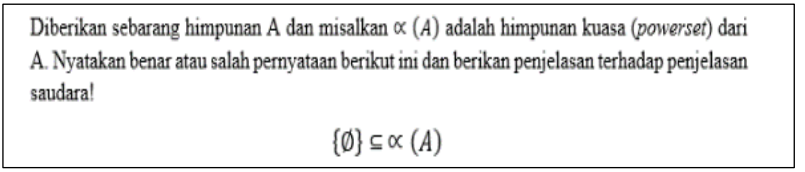
The subjects used in this research were undergraduate Mathematics students who had studied set material. In qualitative research, the research subject is determined by the researcher (purposive sampling), namely selecting the best people or places that can help us understand a phenomenon (Creswell, 2013). The subjects selected were 40 Mathematics undergraduate students from the State University of Malang. Of the 40 subjects, researchers took 4 subjects based on the purposive sampling method. These 4 samples are samples that are able to communicate well, are willing, and are representative of the group (40 subjects).

Research instruments are tools used by researchers to collect data (Creswell, 2013). The instruments used in this research were a set problem test sheet and an interview guide. The set problem test was carried out to get an overview of students' analytical thinking in solving set problems in each stage of analytical thinking. Meanwhile, the interview guide is used as a guide in

conducting interviews to find out more about students' thinking processes and data that cannot be seen directly. The following are the set problem test instruments used in the research:

Figure 1

Sets Problem Test Instrument


Diberikan sebarang himpunan A dan misalkan $\propto (A)$ adalah himpunan kuasa (*powerset*) dari A. Nyatakan benar atau salah pernyataan berikut ini dan berikan penjelasan terhadap penjelasan saudara!

$$\{\emptyset\} \subseteq \propto (A)$$

Translation: *Given any set A and let ∝(A) be the power set of A. State whether the following statement is true or false and provide an explanation for your explanation!*

$$\{\emptyset\} \subseteq \propto (A)$$

The set problem test instrument above is a test development from questions related to subsets in the book *Bright to Abstract Mathematics* written by Morash, RP (1987). Next, the problem set test is validated by expert validators. The expert validator is a Universitas Negeri Malang postgraduate mathematics lecturer with a minimum doctoral educational qualification.

Data collection in research consists of test result data, interview data, observation data and audio recording results. This research data analysis uses qualitative data analysis techniques which consist of 6 stages, namely

1. Prepare and organize data.

Researchers prepare data for analysis. The data prepared is all data collected by researchers during the data collection process, namely test result data and interview data. The test result data is in the form of student work, while the interview data is in the form of students' verbal statements regarding the student's process in completing the given set of tests.

2. Exploring and coding data.

Once the data is ready to be analyzed, researchers explore and code the data. After the researcher observed the interview transcripts and students' written answers, they coded the data based on stages of analytical thinking. Data coding is one way to reduce data. Data reduction aims to select important and relevant data, as well as simplifying the data to explain what is the target of analysis. Based on the results of tests and interviews, researchers select appropriate data to answer research problems.

3. Describe data.

Researchers transcribed data from students' analytical thinking processes obtained from test results and interviews. Next, the researcher describes the results of the student's work in writing and describes the results of the interview descriptively.

4. Present and report findings.

Researchers present their analytical thinking process data descriptively. Apart from that, researchers also report research findings in narrative form which includes an explanation of the students' analytical thinking character in solving group problems.

5. Interpret research findings.

Researchers interpret research findings by comparing the findings obtained with theories or previous research results in order to obtain valid conclusions. Making conclusions is the process of taking the essence of the data description. The conclusions obtained are answers to the problem formulation, which is related to students' analytical thinking processes in solving set problems.

6. Validate the accuracy of the findings.

The accuracy of the findings was validated using triangulation techniques and external audit (Creswell, 2013).

RESULTS AND ANALISES

Based on the results of the data analysis carried out, we identified several characteristics of students' analytical thinking in solving set problems. Descriptions related to these characteristics are presented based on research subjects. There are four research subjects whose analytical thinking will be described in terms of what aspects of analytical thinking are involved in the

problem-solving process and how these aspects play a role in the problem-solving process.

Subject 1 (S1)

In the interview session, see *transcript 1 turn 2*, S1 identified three important pieces of information in the statement given, namely that there is a set A, the power set of set A ($\alpha(A)$, a set whose members are the empty set ($\{\emptyset\}$), and a mathematical statement $\{\emptyset\} \subseteq \propto (A)$. This can be interpreted that S1 identifies and differentiates (*differentiating*) important and relevant information (*keywords*) provided in the given problem structure.

Table 1

Transcript 1

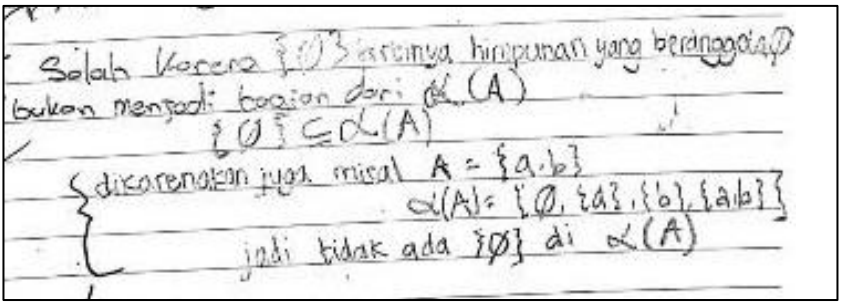
Turn		*Transcript 1*
1	P	From the problems presented, what important parts were found?
2	S1	There is a set A and a power set of A, then a set whose members are the empty set is not a subset of the power set of A
3	P	Continue trying to read the statement$\{\emptyset\} \subseteq \propto (A)$
4	S1	The set whose members are the empty set is a subset of the powerset A
5	P	So what is the solution and explanation of the solution?
6	S1	The answer is that the statement is false, because the set containing the empty set is not a subset of powerset A
7	P	Are you sure the answer is like that? Is there a difference between subsets and elements?
8	S1	Yes, I think so. Maybe it's almost the same, I forgot.

Based on S1's written answer, see Figure 2, S1 justified that the statement given in the problem was a false statement. S1 argues that this statement is false because the notation $\{\emptyset\}$means that the notation of a set whose members are $\emptyset$. And according to S1, the set is not a subset of the set $\alpha(A)$. This statement was also confirmed by S1 in the interview session, see *transcript 1 turn 6*. This can be interpreted as S1 justifying or attributing

regarding the truth of the statement given, in this context S1 claims that the statement is a false statement, which is based on important information and the relationship between this information.

Figure 2

S1 Answers in Solving Set Problems



Translation: *False, because {∅} turns out that the set whose members are ∅ is not part of α(A). ∅⊑α(A). Because the set* $A = \{a, b\}$, $\alpha(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$*and writing that* $\{\emptyset\}$*it is not in the set* $\alpha(A)$

In S1's written answer, Figure 2., S1 also added a case example, namely by assuming the set $A = \{a, b\}$, $\alpha(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$and writing that $\{\emptyset\}$it is not in the set $\alpha(A)$. It can be concluded that S1 combining the information provided on the problem with relevant information, namely case examples to provide and the relationship between each of these important pieces of information (*organizing*). In this context, the relationship we want to show is that the set whose members are the empty set ($\{\emptyset\}$) is not in the power set of set A, $\alpha(A)$. The word "none" in the written answer, S1 means ($\{\emptyset\}$) is not a subset of $\alpha(A)$, see *transcript 1 turn 6*.

If you look at the written example, S1 wrote that symbolically it is written $\{\emptyset\} \notin \alpha(A)$. It can be assumed that S1 concludes that "for example the set $A = \{a, b\}$, $\alpha(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$", it can be concluded that S1 can determine the power set of set A. Based on the concept of power sets, it is true that $\{a\} \sqsubseteq \alpha(A)$, $\{b\} \sqsubseteq \alpha(A)$, $\{a, b\} \sqsubseteq \alpha(A)$, and $\emptyset \sqsubseteq \alpha(A)$and it is also true that $\{\emptyset\}$it is not a member of $\alpha(A)$, or $\{\emptyset\} \notin \alpha(A)$. If we look at S1's claim that the statement $\emptyset \sqsubseteq \alpha(A)$is false, the reason is because S1 was not careful in looking at the symbols listed and S1 did not understand the difference in meaning and concept between symbols $\sqsubseteq$ and $\in$, see *transcript 1 turn 8*. Based on S1's analytical thinking stages in solving the problem, S1 has the character of real false thinking because he makes the decision that the statement $\{\emptyset\} \sqsubseteq$

$\alpha(A)$is a false statement so that the solution given is false, then S1 also gives a false understanding of the concept $\sqsubseteq$ and $\in$.

Subject 2 (S2)

In the interview session, see *transcript 2 turn* 2 and *turn* 4, S2 identifies that there are two important pieces of information in the statement given, namely that there is an empty set and a mathematical statement $\{\emptyset\} \subseteq \propto (A)$. This can be interpreted as S2 identifying and differentiating *important and relevant* information given in the given problem structure.

Table 2

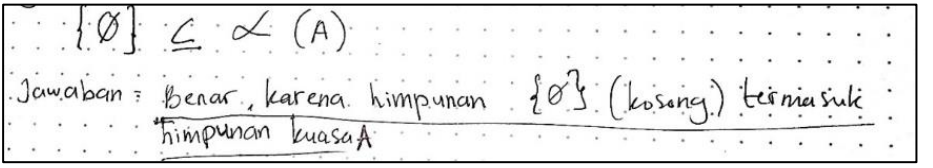
Transcript 2

Turn		*Transcript 2*
1	P	From the problems presented, what important parts were found?
2	S2	There is an empty set, that's what's in the statement
3	P	Keep trying to read the statement$\{\emptyset\} \subseteq \propto (A)$
4	S2	The empty set is a subset of the power set A
5	P	So, what is the solution and explanation of the solution?
6	S2	The answer is that the statement is correct, because every empty member is included in the power set, if a member means a subset too.
7	P	So, are you sure that's the reason? What is the difference between a subset and an element? Or the same?
8	S2	Yes, I'm sure, it's the same in my opinion

2's written answer, see Figure 2, S2 justifies that the statement given in the problem is a true statement. S2 argues that this statement is false because every empty member is included in the power set. According to S2, the set, because it is a member of the power set, is also a subset of the set $\alpha(A)$. This statement was also confirmed by S2 in the interview session, see *transcript 1 turn 6*. This can be interpreted that S2 justifies or provides attributes (*attributing*). regarding the truth of the statement given, in this context S2 claims that the statement is a true statement, which is based on important information and the relationship between this information.

Figure 3

S2 Answers in Solving Set Problems



Translation: *True, because {∅} (empty) including power sets A*

In the written answer S2, Figure 2., S2 immediately give an answer with a short reason, namely that the statement is true because the empty set is a power set. It can be concluded that S2 directly gives attributes to the written answer which states the conclusion that the statement is true (*attributing*). Then *differentiating* and *organizing* in S2 can be revealed in the interview process according to *transcript 2*. S2 identify the empty set as a relevant part of the problem. Next, S2 justified that the statement given in the problem was a true statement. S2 argues that the statement is true because every empty member is included in the power set A, so it is also a subset of the power set A. If we look at S2's claim that the statement $\emptyset \sqsubseteq \alpha(A)$is true, the reason is because S2 considers symbols $\sqsubseteq$ and $\in$ have the same meaning and concept, see transcript 2, *turn* 8. Then S2 is still not correct in reading $\{\emptyset\}$, namely the empty set because it should be a set whose members are empty sets, see transcript 2, *turn* 4. Based on S2's analytical thinking stages in solving the problem, then S2 has the character *pseudo* thinking is correct because it provides a decision that the statement $\{\emptyset\} \sqsubseteq \alpha(A)$is a true statement so that the solution given is correct, but S2 also provides a false understanding in understanding the concept $\sqsubseteq$and $\in$interpreting it $\{\emptyset\}$.

Subject 3 (S3)

In the interview session, see transcript 3 *turn* 2 and *turn* 4, S2 identifies four important pieces of information in the statement given, namely there is an empty set, set A, *powerset* A and a mathematical statement $\{\emptyset\} \subseteq \propto (A)$. This can be interpreted as S2 identifying and differentiating *important and relevant* information given in the given problem structure.

Table 3

Transcript 3

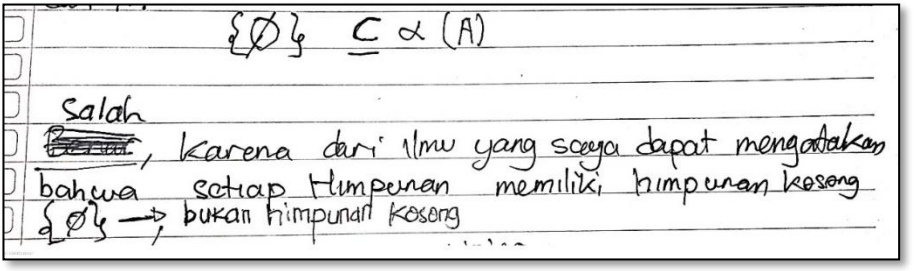
Turn		*Transcript 3*
1	P	From the problems presented, what important parts were found?
2	S3	I look directly at the statement, there is an empty set, then from set A there is powerset A. The empty set is always present in every set, but it is not simply an empty set, so I think the statement is false
3	P	Keep trying to read the statement$\{\emptyset\} \subseteq \propto (A)$
4	S3	The set containing the empty set is a subset of the power set A
5	P	Are you really sure that this statement is false? What is the reason
6	S3	Not sure yet, bro, wait a minute... a set consisting of an empty set is different from an empty set. What is in the powerset is an empty set, not a set consisting of empty sets. But that's a subset, not an element
7	P	Okay. So, what's next?
8	S3	Oh, yes, yes, sir. It's different between subsets and elements
9	P	So how?
10	S3	Oh yes, if it is a subset of the power set, it means $\{\emptyset\}$it is included.
11	P	Certain? Why is that?
12	S3	Sure sir. That's if the powerset already exists $\emptyset$ then it's a subset of powerset A$\{\emptyset\}$
13	P	OK bro, are you sure? Don't want to change your mind again?
14	S3	Sure sir, that's my answer

3's written answer, see Figure 4, S3 justified that the statement given in the problem was initially a false statement. S3 argued that the statement was false because of the notation$\{\emptyset\}$ If you read a set whose members are the empty set, it is not a subset of *the powerset* of a set, but what is in the *powerset* is an empty set, see *transcript 3 turn* 6. However, upon reflection, according to S3, notation$\{\emptyset\}$ read as a set which consists of an empty set if seen from the way of giving meaning to $\sqsubseteq$ and $\in$then it is different, therefore S3 changes the decision, because the empty set is a member of *powerset* A then the set which

has the members of the empty set becomes a *subset* of the set $\alpha(A)$so that the statement that's true. This statement was also confirmed by S3 in the interview session, see *transcript 3 turn 10* and *turn 12*. This can be interpreted that S3 justifies or provides attributes (*attributing*). regarding the truth of the statement given, in this context S3 after reflection claims that the statement is a true statement, which is based on important information and the relationship between this information.

Figure 4

S3 Answers in Solving Set Problems



Translation: *False, because because from the knowledge I got that every set has an empty set.* $\{\emptyset\}$ *not empty set*

In the written answer S3, picture 4 ., S3 initially gave the answer that the statement was false because S3 initially paid little attention to the symbols $\sqsubseteq$ and $\in$. Then after reflection, S3 states that $\sqsubseteq$ and $\in$that are different, so that the set consisting of the empty set is a subset of power set A. So, the final solution given by S3 is that the statement is true because $\{\emptyset\}$ This $\alpha(A)$. *subset* can be concluded that S3 combines the information provided on the problem with relevant information, namely case examples to provide and the relationship between each of these important pieces of information (*Organizing*). In this context, the relationship we want to show is that the set whose members are the empty set ($\{\emptyset\}$) does not exist in $\alpha(A)$but is a subset of $\alpha(A)$.

Based on S3's analytical thinking stages in solving this problem, S3 has the character of *pseudo-* false thinking because it makes a decision that the statement $\{\emptyset\} \sqsubseteq \alpha(A)$ is initially false but after reflection states that the statement is correct so the solution given is correct. This is because S3 completes the problem solving steps and also provides a false understanding of the concept $\sqsubseteq$ and $\in$ meaning $\{\emptyset\}$.

Subject 4 (S4)

In the interview session, looking at *transcriptS4 turn 2* and *turn 4*, S4 identified four important pieces of information in the statement given, namely there is an empty set, set A, *powerset* A and a mathematical statement $\{\emptyset\} \subseteq \propto (A)$. This can be interpreted as S4 identifying and differentiating *important and relevant* information given in the given problem structure.

Table 4

Transcript 4

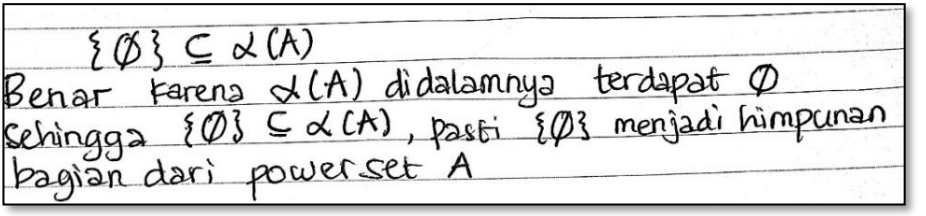
Turn		Transcript 4
1	P	From the problems presented, what important parts were found?
2	S4	The important parts are the set A and its power set, after that in the statement there is a set consisting of the empty set, subset, and power set of A
3	P	Keep trying to read the statement$\{\emptyset\} \subseteq \propto (A)$
4	S4	The set whose members are the empty set is a subset of the power set A
5	P	So, what is the solution and explanation of the solution, sis?
6	S4	The answer is that the statement is correct, because the powerset of A must have an empty set. This means, the set containing the empty set is a subset of powerset A
7	P	Are you sure that's the answer? Understand between subsets and elements? Is it the same or different?
8	S4	Yes, I'm sure sis. That's different, bro. A subset is a subset while an element is a member. If the empty set is the powerset of A, the set containing the empty set is a subset of the powerset of A

4's written answer, see Figure 5, S 5 justified that the statement given in the problem was a true statement. S4 argues that this statement is true because the notation $\{\emptyset\}$means that the set containing the empty set is a subset of the power set A. S4 is able to differentiate between concepts $\sqsubseteq$ and $\in$ is accompanied by a detailed explanation, see *transcript 4 turn 8*. According to S4,$\{\emptyset\}$ is definitely *a* subset of $\alpha(A)$. This statement was also confirmed by S4 in the interview session, see *transcript 1 turn 6*. This can be interpreted as S4 justifying or providing attributes (*attributing*). regarding the truth of the statement given, in this context S4 claims that the statement is a true statement,

which is based on important information and the relationship between this information.

Figure 5

S4 Answers in Solving Set Problems



Translation: *True, because $\alpha(A)$ contains ∅, so {∅}⊑α(A), {∅} must be a subset of the power set A*

In the written answer S4, figure 5, S4 wrote that $\alpha(A)$there is an empty set in it. S4 provides an argument that the empty set clearly exists in the power set A. So, the set containing the empty set is a subset of *the power set* A. This can be concluded that S4 combines the information given in the problem with relevant information, namely providing an explanation of the relationship. between each of these important pieces of information (*organizing*).

Based on S3's analytical thinking stages in solving this problem, S4 has the character of real correct thinking because it provides a decision that the statement $\{\emptyset\} \sqsubseteq \alpha(A)$is correct so that the solution given is correct. This is also reinforced by justification and correct understanding of S4 understanding the concept $\sqsubseteq$ and $\in$ interpreting it $\{\emptyset\}$.

Discussion

Analytical thinking process is a thinking process with the characteristics of selecting information or an important part of information from materials and determining the relationship of material parts with wholistic materials. Someone' analytical thinking process can be seen when solving a problem. The students' analytical thinking process in solving mathematical problems can be seen based on the critical thinking stages: differentiating, organizing, and attributing. The differentiating stage is carried out by identifying and sorting out the relevant and important parts of a problem. The organizing stage is carried out by preparing a settlement plan by connecting the parts of the plan

that have been prepared and then operating the parts of the plan that have been linked to get a solution. The attribute stage is carried out by determining the meaning of the solution and concluding the interpreted solution so that the conclusions obtained can be a solution to the matter at issue. The initial research results show that in solving the sets problems, the students analitically think by presenting the differentiating, organizing, and attributing stages. Those are shown through the students' answers when solving the sets problems. The analysis results on the students' answers in the previous studies showed that the students' answers wrre various in solving the sets problems. Based on those various answers, the researcher used the theory of Subanji (2011) related to students' thinking characters.

From the results of the research conducted, there are differences in students' thinking processes in each stage of analytical thinking. Why is it different? because each person in their thinking process produces new schemes and has their own characteristics (Shodikin et al., 2021). Then, from these differences, 4 characteristics of student thinking emerged based on possible student answers in solving set problems. This is in line with previous research which found that in solving mathematical problems, 4 thinking characteristics emerged based on students' possible answers (Subanji, 2011). The characteristics found from each subject are:

1. Subjects with Real True Thinking Characteristics.

In this characteristic, the subject gives the correct answer and is able to provide justification, this is found in S4. The subject has fully carried out the *differentiating* analytical thinking stage. The subject is correct in reading {∅} so that the subject is correct in reading the statements presented, then in *organizing* the subject is able to plan and carry out plans to solve the set's problems. So that at the end of the analytical thinking stage, namely *attributing*, the subject provides the meaning of the solution and provides a conclusion that answers the problem set presented. This is in line with previous research, that subjects with good analytical thinking characteristics mean that the subject has correct understanding (Wibawa, 2016b).

2. Subjects with Pseudo- True Thinking Characteristics.

In this characteristic, the subject shows the correct answer, but is unable to provide justification for the answer. This is done by S2. The subject has not fully carried out the *differentiating* analytical thinking stage. The subject is not

correct in reading {∅} so that the subject is not correct in reading the statement presented, then in *organizing* the subject is able to plan and implement the plan to solve the set's problems even though it is not correct. So that at the end of the analytical thinking stage, namely *attributing*, the subject provides the meaning of the solution and provides a conclusion that answers the problem set presented. This is in line with previous research, that subjects whose analytical thinking character is not good means those subjects have incorrect understanding (Agustin et al., 2019).

3. Subjects with Pseudo-False Thinking Characteristics.

In this characteristic, at first the subject shows the false answer, but is able to provide justification for the answer so that in the end the subject is able to get the correct answer. This is done by S3. The subject initially did not fully carry out the *differentiating analytical thinking stage*. At first the subject was not correct in reading {∅} so the subject was not correct in reading the statements presented, but after reflection the subject was able to provide justification and correct the answers given. Then in *organizing* the subject is able to plan and carry out the plan to solve the set's problems accompanied by corrections from the subject after reflection. So that at the end of the analytical thinking stage, namely *attributing*, the subject provides the meaning of the solution and provides a conclusion that answers the problem set presented. This is in line with previous research, that subjects with false analytical thinking characteristics mean that the subject has the correct understanding after reflection (Indri & Widiyastuti, 2018).

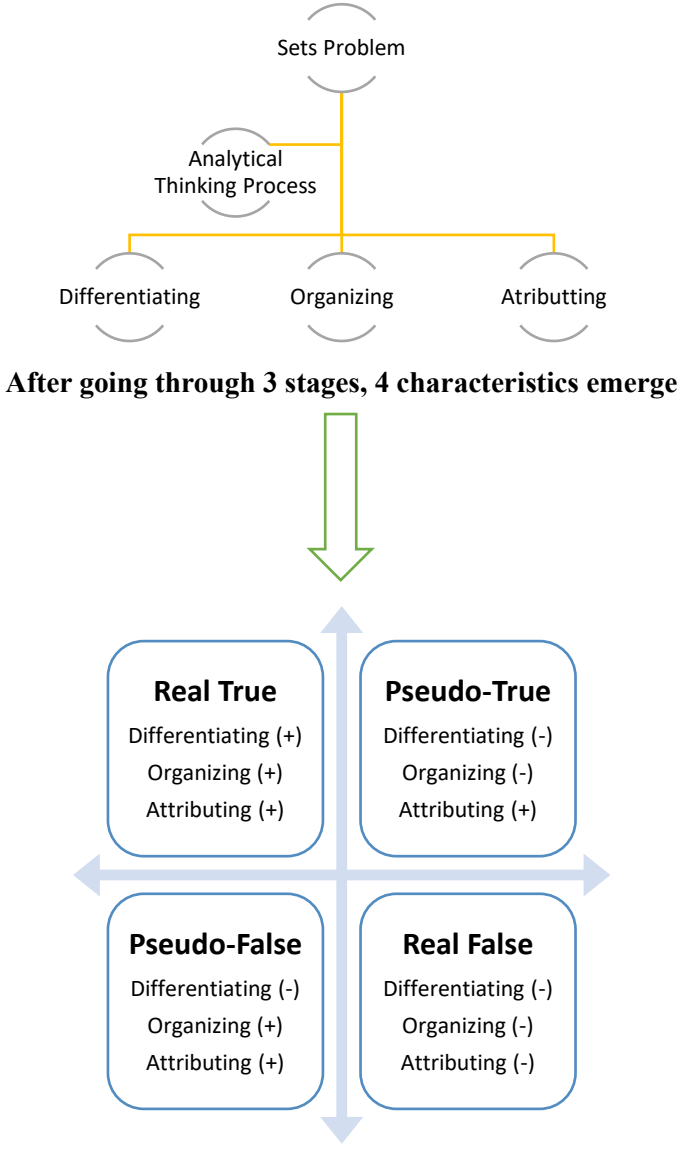
4. Subjects with Real False Thinking Characteristics.

In this characteristic, the subject shows the false answer, but is unable to provide justification for the answer so that after reflection, the subject still gives the false answer. This is done by S1. The subject has not fully carried out the *differentiating* analytical thinking stage. The subject is not correct in reading {∅} so that the subject is not correct in reading the statement presented, then in *organizing* the subject is able to plan and implement the plan to solve the set's problems even though it is not correct. So that at the end of the analytical thinking stage, namely *attributing*, the subject provides the meaning of the solution and provides a conclusion that answers the problem set presented. This is in accordance with previous research, that subjects with low thinking abilities are in line with their analytical thinking abilities, so that subjects still provide poor understanding even though they have gone through reflection (Vinner, 1997b).

So, authors can visualizations related to research results can be seen in the following graph:

Figure 6

Visualizations Research Results



From the graph above we can see that in the thinking process, the subject carries out different stages of analytical thinking, both at the differentiating, organizing and attributing stages. This difference can be seen from the characteristics of pseudo thinking which consists of 4 types of characteristics. Subjects with real true characteristics can carry out all stages of analytical thinking completely at the differentiating, organizing and attributing stages. In the analytical thinking process, the subject is able to optimally solve set problems. Subjects with true-pseudo characteristics are not yet able to completely solve set problems with 3 stages of analytical thinking. At the differentiating and organizing stages, the subject is still not optimal in carrying out these stages, even though at the attributing stage the subject is able to carry out these stages. Subjects with false-pseudo characteristics are not yet able to completely solve set problems with 3 stages of analytical thinking. At the differentiating stage, the subject is still not optimal in carrying out these stages, even though at the organizing and attributing stages the subject is able to carry out these stages. Apart from that, after reflection, the subject is also able to independently correct mistakes at the differentiating stage. Subjects with real false characteristics have not been able to completely solve set problems with 3 stages of analytical thinking. At all stages of analytical thinking, the subject is still not optimal in carrying out these three stages.

This difference in stages is in accordance with previous research which states that a thinking process allows new schemes or different characteristics to emerge for each subject (Shodikin et al., 2021).

CONCLUSION

The conclusion of the research which aims to analyze the characteristics of students' analytical thinking processes in solving set problems is that there are 4 characteristics of students in analytical thinking in solving set problems. From 40 subjects, 4 characteristics of analytical thinking were obtained, namely characteristics of thinking that were real true, pseudo-true, pseudo-false, and real false thinking. Each student solves set problems through stages of analytical thinking, but whether they are complete or not can be seen from each of the thinking characteristics found. So, the researcher also concluded that when students solved the set problem, there were differences in students' thinking processes in each stage of analytical thinking.

Suggestions that researchers can give based on research results are regarding solutions for subjects who are incomplete in carrying out this stage of analytical thinking. The problem of incomplete stages of analytical thinking in the characters of pseudo-true, pseudo-false, and real false thinking can be

overcome by creating scaffolding (assistance to students). Thus, it is recommended that further research can be carried out by applying scaffolding which makes students' analytical thinking stages more optimal.

AUTHORS' CONTRIBUTIONS STATEMENTS

AM carried out conceptualisation, developing theory, writing the original draft, and visualisation; TH was responsible for reviewing, editing, and developing theory; LA was responsible for the review, formal analysis, methodology; AW was editing and proofreading the article; DW was editing and proofreading the article.

DATA AVAILABILITY STATEMENT

Anyone making a reasonable request to the first author of the article, AM, will be provided with the supporting data for the research findings.

FUNDING STATEMENT

This research was funded by the Educational Fund Management Institution (LPDP).

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