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Integration of theory and practice in mathematics education for early childhood teachers

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ABSTRACT

Background: The growing interest in improving the mathematics preparation of preschool teachers, combined with the lack of clarity regarding the educational devices that affect teaching practice, motivated the design of a scaffolding system that articulates theory and practice through a theoretical module and a practical workshop intended for pre-service teachers**. Objective**: To investigate the impact of a workshop using lesson study with video analysis on the practice of the pre-service teacher. **Design**: Instrumental study of multiple cases. Mixed approach. **Participants**: 20 third-year pre-service teachers of the early childhood education program participated in the practical workshop. **Data collection and analysis**: A practice assessment guideline, a Likert scale, and an interview were administered at the beginning and end of the practice workshop. The data were analysed with descriptive statistics and content analysis. **Results**: Following the practical workshop, half of the cases increased their percentage of achievement in accordance with the practice guideline. Ninety-five per cent of participants stated that lesson study benefited their learning. **Conclusions**: Teaching practice is a complex dimension and resistant to transformation. It is necessary to continue researching the mathematics preparation of pre-service early childhood teachers.

Keywords: Mathematics; Early childhood education; Initial teacher education; Practice; Teacher.

Integración Teoría-Práctica en la Formación en Matemáticas del Profesorado de Infantil

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RESUMEN

Contexto: El creciente interés por mejorar la preparación en matemáticas del profesorado de infantil en formación y la falta de claridad sobre los dispositivos educativos que inciden en la práctica docente, motivó el diseño de un Sistema de andamiaje que articula teoría-práctica, a través de un módulo teórico y un taller práctico destinado a profesores en formación**. Objetivo**: Investigar la incidencia que tiene un taller que utiliza el Estudio de Clase con análisis de videos sobre la práctica del profesor en formación. **Diseño**: Estudio instrumental de casos múltiples. Enfoque mixto. **Participantes**: 20 estudiantes de tercer año del programa de profesorado de infantil que participaron en el taller práctico. **Recolección de datos y análisis**: Se aplicó una pauta de evaluación de la práctica, una escala Likert y una entrevista al inicio y término del taller práctico. Los datos fueron analizados con estadísticos descriptivos y análisis de contenido. **Resultados**: Después del taller práctico, la mitad de los casos aumentaron el porcentaje de logro en la pauta de práctica. El 95% de los participantes manifiesta que el Estudio de Clase favoreció su aprendizaje. **Conclusiones**: La práctica docente es una dimensión compleja y resistente a la transformación. Es necesario continuar investigando la preparación en matemáticas del profesorado de infantil en formación.

Palabras Claves: Matemática; Educación infantil; Formación docente inicial; Práctica; Profesor.

INTRODUCTION

One of the purposes of research in initial teacher education is to gather evidence that will answer two essential questions: What do pre-service teachers need to know to be effective teachers? And how do they transform that knowledge into classroom practices that benefit student learning? (Avalos, 2011).

The answers to these questions are relevant to the education of early childhood teachers in general and, in particular, to the preparation they receive in mathematics, given: i) the impact of early childhood teaching on mathematics learning (Jain & Brown, 2020, Cerezci, 2020); ii) evidence of insufficient knowledge and practices for the teaching of mathematics in future early childhood teachers (Pincheira and Alsina, 2022a, Goldrine et al., 2021, Goldrine et al., 2015, Samuel et al., 2018, Esen et al., 2012; iii) the need to renew early childhood teacher education (Jain & Brown, 2020, Mir & Ferrer, 2014, Alsina, 2013); and iv) the scarce research on the formative devices used in the preparation of early childhood teachers to teach mathematics (Parks & Wager, 2015), which motivates a growing interest in increasing research on the education of early childhood teachers in mathematics (Beisly et al. 2024, Alsaeed & Mohammad, 2023, Julio & Zanetti, 2022).

It is well known that learning mathematics is not only important in the early years but also a predictor of future academic achievement in mathematics and language (Kilday & Kinzie, 2009), highlighting the significance of mathematics education in early childhood and underscoring the importance of teacher education in fulfilling this role. However, some studies show that pre-service teachers do not have sufficient preparation (Pincheira & Alsina, 2022b), presenting, for example, weaknesses in understanding children's thinking when solving mathematical tasks (Samuel et al., 2018) and insufficient knowledge about mathematical concepts, which can lead to the implementation of inappropriate activities for children (Esen et al., 2012).

These findings motivate research into the formative devices used in the education of early childhood mathematics teachers. Parks and Wager (2015) report a deficiency in education since the courses that prepare teachers do not necessarily include the content, methodologies, and practices appropriate for the early childhood, maintaining that there is little evidence to clearly establish which are the ideal formative devices for the preparation of pre-service teachers to teach mathematics in early childhood. Hidalgo-Méndez et al. (2024) describe the panorama of mathematics education in early childhood at Spanish universities, finding that the number of subjects and their duration are insufficient to provide pre-service teachers with the necessary teaching skills. Julio and Zanetti (2022) warn of the need to clarify what mathematical content needs to be included in the education of early childhood teachers. Similarly, Horm et al. (2013) warn that teacher education programs have limited practicum, which makes it difficult for pre-service teachers to associate theoretical courses with classroom practices.

In this context, the lack of clarity regarding the formative mechanisms that influence the practice of pre-service teachers presents a problem. The article proposes (i) the ability to teach mathematics (ATM), encompassing dimensions of knowledge and teaching practice, and (ii) a scaffolding system comprising a theoretical module and a practical workshop designed for early childhood teachers. The research question that emerges is: Does a formative device that integrates theory and practice have a positive impact on the practical dimension of pre-service preschool teachers' ability to teach mathematics? The objective is to investigate the impact of the scaffolding system workshop on the capacity to teach mathematics in a practical dimension.

THEORY-PRACTICE SCAFFOLDING SYSTEM TO DEVELOP THE ABILITY TO TEACH MATHEMATICS OF PRE-SERVICE TEACHERS

In response to the question, 'What do teachers need to know to generate learning opportunities?' the theoretical construct of the ability to teach mathematics (ATM) is proposed to conceptualise the knowledge and practice of early childhood teachers (Olfos et al., 2022b). This construct forms the basis of a formative device, a theory-practice connecting scaffolding system, which is operationalised through a theoretical module and a practical workshop (Olfos et al., 2022a). The device utilises the lesson study methodology (Isoda & Olfos, 2009) and incorporates video analysis (Leavy & Hourigan, 2018). This system focuses on teaching numbers as curriculum content, given that number sense, counting skills, and basic arithmetic operations constitute a fundamental domain in early childhood education (Clements & Sarama, 2011).

The ability to teach mathematics in early childhood education

The ATM construct is a theoretical development that captures the body of knowledge, beliefs, and practices that early childhood teachers mobilise to teach (Olfos et al., 2019). The construct encompasses Shulman's (1987) components of teacher knowledge: content knowledge, pedagogical knowledge, and pedagogical content knowledge. Starting from Shulman (1987), Ball et al. (2008) later proposed a mathematical knowledge for teaching (MKT) model, which Carrillo et al. (2018) subsequently refined into a model of mathematics teachers' specialised knowledge (MTSK) that encompasses the subdomain of knowledge related to mathematical practice. Based on these references, Alsina and Delgado (2022) propose a knowledge model for mathematics teaching in early childhood education that includes teaching management. The ATM broadens the scope, incorporating both the knowledge dimension and the teaching practice dimension.

Several studies have researched the pedagogical content knowledge (PCK) of early childhood teachers (Lee, 2017; McCray & Chen, 2012; Platas, 2015), finding that the following elements are constitutive of teacher knowledge and, therefore, should be promoted from initial education: i) knowledge of mathematical concepts suitable for early childhood, ii) deep understanding of the development of logical-mathematical thinking in the early years, iii) ability to interpret the mathematical content and processes present in children's activities and plays and iv) knowledge of the representations and methodological strategies appropriate for early childhood.

The ATM, as a human capacity, involves cognition, praxis, and the value disposition associated with beliefs about mathematics and its teaching in early childhood. It is composed of the dimensions of knowledge and teaching practice, together with the components [1] Content knowledge (CK) and [2] Pedagogical content knowledge (PCK) referring to teacher knowledge about child mathematical thinking (PCK - CMT) and PCK referring to knowledge for teaching (PCK-T) (Table 1). The dimensions of the construct refer to conceptual knowledge (teaching knowledge) and know-how (teaching practice) that interact dialectically in teaching to generate opportunities for learning mathematics in early childhood education.

Table. 1

Early childhood teachers' ability to teach mathematics (Olfos et al., 2019)

Component		Dimension	
		Teaching knowledge	Teaching Practice
	CK	Mathematical concepts and semiotic representations suitable for early childhood education	Teaching mathematical concepts suited for children. Mathematical objects and representations
PCK	CMT	Stages in learning mathematical notions. Interpretations that lead children to difficulties in understanding	Teacher mediation against common strategies, questions, and misinterpretations in children
	T	Approaches to teaching mathematics. Sequence of mathematical tasks. Resources for manipulation and representation. Beliefs about teaching and learning in early childhood	Organisation of teaching. Elaboration and management of learning opportunities. Selection of resources.

CK refers to the knowledge that teachers express verbally or mobilise through concrete, pictorial, or symbolic representations that they present to the

students. It is practical knowledge put into action as a teaching skill. The PCK-CMT provides knowledge about learning mathematics in the early years. The PCK-T covers knowledge of the mathematics curriculum in early childhood, as well as the organisation of teaching in early childhood.

Teachers promote enriched mathematical learning when they enable children to access: i) play-based mathematical activities, ii) manipulative resources and representations, iii) problem solving, and iv) activation of mathematical thinking skills. A mathematics education that presents these conditions offers access to a quality curriculum (Sarama & Clements, 2009). In this sense, the ATM aims to characterise knowledge and practice suitable for children.

Theory and practice connecting the scaffolding system

A key element in teacher education programs is to have a learning model that includes practicum opportunities that allow pre-service teachers to connect theory and practice in authentic situations (Alsaeed & Mohammad, 2023). In the education of early childhood teachers, articulating theory and practice is essential (Mir & Ferrer, 2014), to such an extent that an insufficient connection between theoretical and practical education could be a critical factor in teacher preparation. To address this possible disconnect between theoretical knowledge and teaching practices, the theory-practice connecting scaffolding system (SSt+p) is proposed as a teacher education device to strengthen ATM (Olfos et al., 2022a).

The SSt+p addresses disciplinary and didactic content, aiming to influence the dimensions of teaching knowledge and teaching practice. It is organised in two stages (Table 2). In the first, a theoretical module is applied, and in the second, a practical workshop, both of which focus on teaching and learning the concept of numbers in early childhood under a problem-solving approach (Olfos et al., 2022a).

Table. 2

Contents and methodologies of the scaffolding system

Stage 1. Theoretical module			:age 2. Workshop Stage 2. Practical workshop	
CK	PCK-T	PCK-CMT	PCK-T / PCK-CMT	
Logic Number Ordinal Cardinal	Problem solving	Learning paths associated with numbers	Analysis of own videos Productive discussions	Lesson study Planning-implementation-analysis cycle

Count Represent quantity Numerical composition	Design of teaching situations Manipulative resources and representation	Counting principles and strategies	about teaching and learning	and reformulation of teaching situations

The theoretical module comprises a sequence of seven sessions that explore the development of the concept of numbers in early childhood, drawing on class logic (Kamii, 1985) and counting practices (Gelman & Gallistel, 1978). Disciplinary and didactic knowledge is studied, and professional tasks are carried out. Table 3 shows the second and sixth sessions of the theoretical module.

Table. 3

Theoretical module sessions

	CK	PCK-CMT	PCK-T	PROFESSIONAL TASK
2	Classification	Understanding logical thinking skills	Identify an activity with a problem-solving focus	Observe the classification in children's play
6	Number Numerical range	Identify the numerical range according to the age range: 3-4, 4-5, 5-6 years	Identify the notion of number in the objectives of the early childhood curriculum	Design a teaching activity for cardinal numbers

The practical workshop consists of 12 sessions, during which two cycles of lesson study are conducted. Lesson study is defined as "a collaborative, reflective, and iterative professional development practice that engages teachers in the process of systematically examining their teaching to become more effective teachers" (Jain & Brown, 2020, p. 157).

In the lesson study, pre-service teachers, organised into subgroups, collaboratively plan a math activity with a problem-solving focus. A pre-service teacher implements the activity in a preschool classroom, filming it on video, after which the subgroup watches the video and reflects on the lessons learned to improve the initial planning. Then, another pre-service teacher re-

implements the improved activity in a different course, filming it again and analysing it collaboratively to refine it to a final version. Table 4 presents the practical workshop sessions.

Table. 4

Practical workshop sessions

S	Moments of the lesson study	(classroom implementation)
1	Recalling the concepts covered in the theoretical module	
2	The first approach to a mathematical activity for children: choosing the mathematical content	
3	Designing an activity according to the mathematical content	
4	Elaborating the first planning: moments of the experience	
5	Elaborating the first planning: teacher mediation and manipulative resources	
		Student 1 Classroom 1
6	Watching the video of classroom 1. Joint reflection. Identifying strengths and weaknesses	
7	Watching the video of classroom 1. Joint reflection. Identifying improvements	
8	Elaborating the first planning: moments of the experience	
9	Elaborating the first planning: teacher mediation and manipulative resources	
		Student 2 Classroom 2
10	Watching the video of classroom 2. Joint reflection. Identifying improvements	
11	Preparing the final planning	
12	Students present conceptual and practical learning achieved in the workshop	

The results obtained by Jain and Brown (2020) demonstrate that the lesson study, with its iterative and reflective phases, has a positive impact on the effectiveness of teaching and the quality of lessons for pre-service early childhood teachers. The SSt+p with the theoretical course and the practical workshop works under the premise that the key connections between theory and practice are conceived, in Schön's terms (1983, 1987), as "knowledge in action" and "reflection from action." Reflection on practice allows articulating the dialectic between theory and practice (Lave, 1991), which strengthens teaching capacity. For this reason, the inclusion of reflection is a crucial component of teacher education programs (Jain & Brown, 2020).

The processes of reflection on practice constitute the basis of the scaffolding system to the extent that they integrate the theoretical dimension with the practical dimension of ATM, particularly in the practical workshop, thanks to the use of the lesson study methodology, which promotes collaborative reflection that increases individual reflection skills for the improvement of teaching (Jain & Brown, 2020).

METHODOLOGY

Design

Mixed-approach research was carried out. We employed an instrumental multiple case study (Stake, 1999) with pre-service teachers participating in an early childhood teacher education program, measuring their progress before and after the practical workshop.

Participants

The participants were 20 third-year pre-service teachers enrolled in an early childhood teacher education program, all of whom were female, with an average age of 22 years, who agreed to participate in the research by signing an informed consent form.

Instruments

Three instruments were applied:

a) PECEMPra. It serves as a guideline for assessing the practical ability of pre-service early childhood teachers to teach. It has 23 indicators to assess the practical dimension of ATM: five for CK, six for PCK-CMT, and twelve for PCK-T. It has a content validity of 0.97 (Aiken V coefficient) and a reliability of 0.80 (Cronbach's alpha coefficient) (Olfos et al., 2022b). Examples. Indicator 3: "The pre-service teacher implements the activity through a problem

situation that facilitates the learning of the mathematical notion" (CPK-T). Indicator 19: "Verbally express to children the relationship that the activity has with the mathematical notion" (CK). Indicator 21: "It offers children—according to their age range—opportunities to describe and/or discuss the procedures, connections, and/or strategies used when tackling mathematical activities" (PCK-CMT). The ATM component they measure is indicated in parentheses.

b) Semi-structured interview on the math activity that the student implemented in a preschool classroom. The interview was designed and applied by the researchers. The protocol includes 11 questions related to the practical dimension of ATM, as well as three questions about SSt+p. For example, what mathematical concept did you address in this activity? (CK) Do you think this activity creates a situation that encourages problem solving? (CPK-T), and do you think this activity promotes children's mathematical thinking? (PCK-CMT)

c) Likert scale, developed by the researchers, with 36 statements to collect the pre-service teachers' opinions on the practical workshop and the SSt+p. For example, whether they think it helped their learning, such as making pedagogical decisions with classmates to plan a math activity, analysing the activity implementation video to identify areas that need improvement, and planning-implementing-analysing-improving-reimplementing cycle.

Data collection and analysis procedures

The pre-service teachers first attended the SSt+p theoretical module, then the practical workshop. At the beginning and end of the workshop, each pre-service teacher planned a math activity that was filmed in a preschool classroom with 5- to 6-year-old students. We collected videos from ten pre-service teachers, which were observed by two external experts: one international specialist in early childhood mathematics teaching and one national specialist in early childhood teacher education, who attributed them scores by applying the PECEMPra. There was an 80% agreement between both judges. The data collected were analysed using descriptive statistics.

The interview was conducted at the beginning and end of the practical workshop, using the video of the student's math activity as a reference. The interview was conducted with ten students. The data were subjected to content analysis.

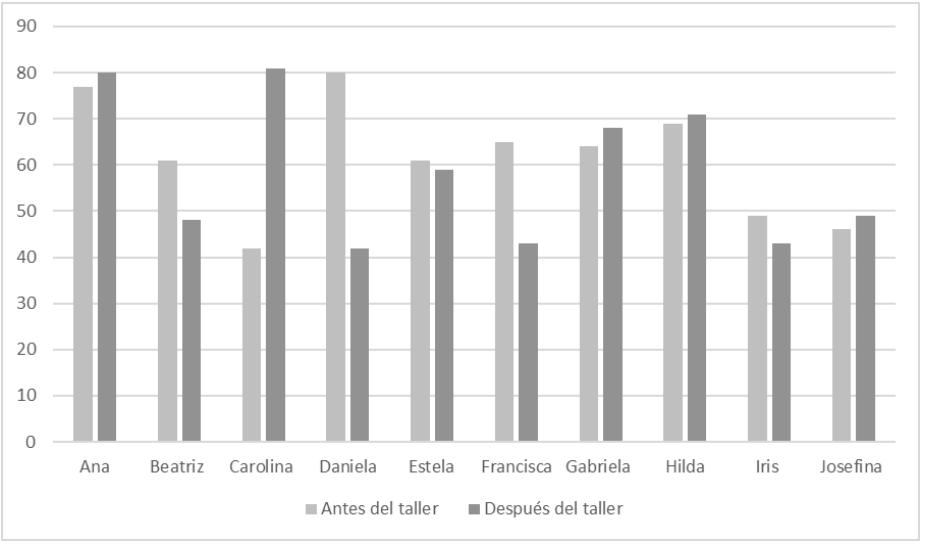
The Likert Scale was administered to 20 students at the end of the practical workshop. The data collected were analysed using descriptive statistics.

RESULTS

Figure 1 illustrates the ten cases. Each respondent was assigned a fictitious name, and the figure shows the percentage of achievement in PECEMPra at the beginning and end of the practical workshop.

Figure. 1

Percentage of achievement in PECEMPra at the beginning and end of the practical workshop



The percentages yielded a median of 62% and 54% at the beginning and end of the workshop, respectively. Of the ten cases, five increased their percentage in PECEMPra, corresponding to Ana, Carolina, Gabriela, Hilda, and Josefina. One case, Estela's, maintained the percentage. Four cases showed a decrease: Beatriz, Daniela, Francisca, and Iris.

When comparing the students' practical performance at the beginning and end of the workshop, based on the 23 indicators of the guideline, we noted that some students (between four and seven) demonstrated these performances from the beginning of the workshop. This is the case with indicators 1, 6, and 13, which refer to implementing a mathematical activity by presenting it orally,

visually, and/or kinaesthetically, accommodating children and organising resources to facilitate the learning of mathematical concepts (PCK-T).

Other performances, at the beginning of the workshop, were performed by one or two students, and at the end, were demonstrated by four students. They correspond to the performances related to the combined use of representations (indicator 11, PCK-T) and time management prioritising the mathematical task (indicator 14, PCK-T).

On the other hand, some performances are not present in students either before or after the workshop. For example, indicator 8: "Offers children the opportunity to use different procedures" (PCK-T); indicator 10: "Identifies in the classroom the mathematical difficulty that the child shows when facing the task" (PCK-CMT); and indicator 21: "Offers children— according to their age range—opportunities to describe the strategies used when facing mathematical activity" (PCK-CMT), which refer to the mobilisation of mathematical reasoning processes in students.

Carolina, the case with the highest achievement at the end of the workshop (from 42% to 81%), in the first activity, promotes oral storytelling based on a story, performing a memorisation task. In contrast, the second activity incorporates a problem situation, encouraging students to mobilise their strategies. Her teaching performance improved significantly in the items associated with PCK-T and PCK-CMT.

In contrast, Daniela, who showed a decrease in the achievement percentage (from 80% to 42%), worked on the notion of "adding" in the first activity by combining representations and encouraging students to mobilise strategies. In the second activity, to associate numbers with quantities, she organised four work groups, each with six or seven children, who showed difficulties in giving instructions and conducting the course, affecting student participation and resulting in a lower PECEMPra.

When we noted a decline in the achievement percentage, we realised the difficulties were associated with two reasons. First, the structure of the activity does not provide a final framework for expressing mathematical ideas and strategies for solving, synthesising, and institutionalising what has been learned. Secondly, the activity fails to achieve active and autonomous participation of students in solving the mathematical task, with the student assuming a role that tends toward adult-centrism. These aspects are associated with CK, PCK-CMT, and PCK-T indicators, so their absence affects the achievement percentage.

In Table 5, extracts from the interviews conducted with the ten cases are displayed, with the researcher identified as “R” and the student as “S”.

Table. 5

Excerpts from the interview before and after the workshop

Before	After
Case: Ana R: Are there elements of the theoretical module that you took into account? S: Yes, pose a mathematical situation as a problem situation, not as an exercise.	R: Did you think it was a problem situation? S: I didn’t present them with a problem to solve, it was just like a game.
Case: Beatriz R: What did you think of your teaching mediation? S: I didn’t give the children the answer; they looked for it.	R: Are there elements of the workshop that you took into account? S: (…) Mathematical problems that can occur in everyday life.
Case: Carolina S: Children learn more by working with a story (…), than by sitting with a pencil and a piece of paper. The activity itself was not about solving a problem.	R: Did you think it was a problem situation? S: Yes, (…) it was through its resolution.
Case: Daniela R: Did you find the activity appropriate? S: Yes, because I had concrete material (…), the most skilled children were telling their strategies, and then they helped the other children	R: What did you think of your teaching mediation? S: Open questions were asked without giving the answers to the children but rather allowing the answers and strategies to emerge from them.
Case: Estela	R: What did you think of your teaching mediation?

R: What did you think of your teaching mediation? S: (…) I found that it was very directed (…), I gave them the path of how it had to be done	S: I think it was better than the first one, now, I was hoping that the strategy to use would emerge from them.
Case: Francisca R: What did you think of your teaching mediation? R: I think that, at times, it was too invasive.	R: What did you think of your teaching mediation? S: It seemed appropriate to me (…), I tried to make them realise that they were missing elements
Case: Gabriela No interview	R: Did you think it was a problem situation? S: The truth is no, there was no problem.
Case: Hilda R: Did you think it was a problem situation? S: Maybe it wasn't a problem, but they still worked on mathematical thinking.	R: What did you think of your teaching mediation? R: (…) The first video was more like, do this, now it's like, why are you doing this? How many do you have?
Case: Iris R: What did you think of your teaching mediation? S: Some who are more shy do not participate in the activity, so in small groups, we could have offered mediation to those children.	R: What did you think of your teaching mediation? S: (…) I tried to divide them into small groups, so that they could work better, (…) I tried to ask them questions, not giving them the answers, but rather so that they could realise for themselves that there was an error.
Case: Josefina R: What did you think of your teaching mediation? S: I still have a hard time dominating the group (…), I tried not to answer them, asking them more questions, so	R: Did you think it was a problem situation? S: I didn't use problem solving, because I only presented them with a game.

that they would realise it on their own.

As observed in the interview excerpts, students were able to reflect on the activity they implemented in the classroom. They could identify whether they had raised a problem situation. They recognise the relevance or otherwise of their teaching mediation in supporting children in the independent resolution of mathematical tasks. However, when triangulating their answers with the PECEMPra results, we observed that the indicator: "Implements the activity through a problem situation that facilitates the learning of mathematical notions" (PCK-T) and the indicator: "Uses mediation strategies to help the child advance in the mathematical task, without indicating the resolution strategies or solutions" (PCK-T) are lower in the guideline which shows a lack of harmony between what was assessed in the scale and what students reported in the interviews.

Regarding the practical workshop, the ten students reported that it provided them with tools to plan, implement, and analyse mathematical experiences. They point out that reviewing their videos during the lesson study favours the detailed analysis of classroom experiences, identifying strengths, errors, and improvements, as Josefina explains: "The videos are a very important source for us because the fact of recording myself makes me realise the mistakes, that I neglected a child, I ask myself, didn't I consider that? It allows us to realise our errors and strengths."

Students acknowledge that SSt+p facilitates the integration of theoretical and practical education, as Daniela points out: "First, we had the theoretical course, which was last year, and we could put all of that into practice. Also, everything we could see at the beginning, the concepts we reviewed, were covered throughout the process. So, there was a real transition from theory to practice, and that was very beneficial because that's what's most difficult."

The results of the Likert Scale (n = 19) in Table 6 show that the students' answers are located between 95% and 100% in the categories "agree" (4) and "totally agree" (5), indicating a positive opinion regarding the methodology of the practical workshop.

Table. 6

Students' opinions on the practical workshop

I consider that it favours learning:	3	4	5
Making pedagogical decisions together with classmates to plan a mathematics activity	5%	11%	84%
Analysing the video of the implementation of the activity to identify aspects to improve		11%	89%
The plan-implement-analyse-improve-reimplement cycle		5%	95%

While most pre-service teachers (89%) believe that a course integrating a theoretical module and a practical workshop favours learning how to teach mathematics in early childhood, 11% suggest that only a practical workshop be carried out. No student agrees that only a theoretical module should be carried out. In the open section of the instrument, two students stated:

"When it comes to putting theory into practice, it's more significant for our professional work, providing a deeper understanding and thus improving teaching practice and reflection on mathematics in early childhood education."

"I found the course to be very good because there was ample opportunity to share knowledge with future colleagues, to engage in self-criticism of what one has mastered, both theoretically and practically, and also to work on implementation, analysis, improvement, and reimplementation."

In summary, after the workshop, five of the ten cases showed an increase in the percentage of PECEMPra achievement. In the interviews, students reflect on the mathematical activity performed in class, recognising whether or not they posed a problem situation and the relevance of teacher mediation. The students noted that the workshop provided them with tools to plan, implement, and analyse math experiences. Ninety-five per cent of students stated that the lesson study cycle benefited their learning. Around 89% believe that the theoretical module, followed by a practical workshop, helps students learn how to teach mathematics to preschoolers. However, the evaluation of the workshop and the lesson study reveals that the indicators that

decreased in PECEMPra show a lack of harmony between what the students report and their performance in classroom practice.

DISCUSSION AND CONCLUSIONS

Currently, there has been an increase in interest in analysing the effectiveness of mathematics courses for early childhood teachers due to insufficient preparation, and, therefore, the need to improve initial teacher education (Torbeyns et al., 2020; Blömeke et al., 2015; Cohrssen & Taylor, 2016). In this context, the impact of the scaffolding system on the practical dimension of ATM was investigated, providing empirical support to the body of knowledge on formative devices for early childhood mathematics teacher education.

The SSt+p practical workshop addresses the findings of other research on early childhood teacher education, which suggests that mathematics courses should not only deliver knowledge but also integrate practice opportunities (Chen et al., 2022; Gresham & Burleigh, 2019) that allow pre-service teachers to apply the contents of university courses to the real school context in authentic situations (Alsaeed & Mohammad, 2023). In this sense, a process similar to that reported by Jain and Brown (2020) was carried out in a formative course for preschool teachers. Initially, they reviewed the theoretical content and subsequently conducted a lesson study, implementing a mathematics activity in a real classroom, followed by peer analysis. The lesson study, due to its cyclical and collaborative nature, provides a pertinent methodology for promoting theory-practice integration, constituting an ideal approach for the education of early childhood teachers in mathematics (Jain & Brown, 2020).

On the Likert scale, more than 80% expressed a positive opinion about the workshop and the lesson study. In the interviews, they report that they integrated theory and practice, achieving, in the words of Maldonado-Ruiz and Soto Gómez (2021), the "theorisation of practice" and the "experimentation of theory." Meanwhile, they applied the knowledge acquired in the theoretical module and reinterpreted it through their teaching actions, reflecting jointly on them during the lesson study.

However, data derived from the PECEMPra application shows that half of the cases showed an increase in the score. While students report increased knowledge of early childhood mathematics teaching, this is not always reflected in their classroom practice. The practical workshop had a positive impact in half of the cases, not in all ten cases as expected.

Therefore, despite the reported integration of theory and practice by students, PECEMPra yields partial results. Students appear to begin the workshop with a practice that includes performances assessed in PECEMPra. However, in half of the cases, they do not show substantial improvement as a result of the workshop. Although pre-service teachers report that they developed knowledge and practices associated with planning and implementing mathematics activities, this acquired knowledge does not necessarily correspond to the achievements obtained in PECEMPra. Teachers' tendency to assume an adult-centric role and their lack of classroom experience may be explanatory factors.

The ATM as a theoretical construct refers to a dimension of conceptual knowledge of teaching and a dimension of know-how in teaching practice, postulating that both dimensions interact dialectically in teaching. However, it appears that knowledge about teaching mathematics in early childhood does not necessarily translate into satisfactory practice at this educational level. Previous studies (Reyes-Santander et al. 2023; Goldrine et al., 2021) have already warned of the complexity associated with configuring teaching practice in initial education, showing that teacher education courses do not have the same impact on student diversity, and that, in particular, practice tends to resist transformation. In this same sense, Leavy and Hourigan (2018), who also used lesson study as a university methodology, at the end of the course found that, although pre-service teachers had knowledge in general, this was not necessarily connected to the classroom contexts, so they concluded that it is necessary to continue offering them more opportunities to refine their knowledge in response to the challenges of classroom practice.

Although the literature highlights the need for pre-service teachers to have practicum opportunities (Chen et al., 2022; Gresham & Burleigh, 2019; Horm et al., 2013), it remains challenging to identify the characteristics that field experiences within mathematics courses should possess to influence their practical performance. The results of this work represent a challenge for the early childhood teachers' education, as they demonstrate what Blömeke et al. (2015) already warned about, that teaching competence is not characterised by a linear increase or decrease in one aspect, but by changes in the components of this professional competence.

Although students appreciated the connection between theory and practice achieved in the practical workshop thanks to the lesson study and the analysis of their videos, further research is needed to explore the characteristics that formative devices should possess to effectively contribute to the

development of optimal practices for early childhood teachers. In this sense, Hidalgo-Méndez et al. (2024) suggest that further research is necessary to educate early childhood mathematics teachers, identifying both the outstanding and weakest aspects that could be improved, thereby providing pre-service teachers with an optimal university education. Similarly, Alsina et al. (2024) highlight the need for early childhood mathematics teacher education programs to place greater emphasis on learning professional tasks.

One of the main contributions of this work is the evaluation of the pre-service teacher's on-site practice after completing a formative activity for teaching mathematics in early childhood education. Jain and Brown (2020) studied a peer analysis after a mathematics and science formative program in early childhood, but without having a specific instrument to measure it. In research by Alsaeed and Mohammad (2023), the study of teaching practice was also addressed after a formative process in mathematics in early childhood, but only through the analysis of classroom situations without conducting a specific on-site measurement of the practice. Alsina et al. (2024) and Pincheira and Alsina (2022b) analysed the planning of mathematical activities for children as a professional task, but without considering their implementation or assessment of on-site teaching practice. Unlike these works, this article provides a study on the impact of a formative activity on teaching practice, analysing the complexities involved in this relationship.

Another contribution of the article is that it addresses Alsina et al.'s (2024) suggestion for future research about the need to study mathematics teaching practices through research methods that include lesson study, analysis of class videos, and interviews with pre-service teachers, methodologies that were included in the present research to move from the measurement of teacher knowledge to the evaluation of practice. Likewise, we are attentive to Pincheira and Alsina's (2022b) suggestion that renders it necessary for research to go beyond the analysis of teaching planning to address the study of the on-site practice of pre-service preschool teachers in the teaching of mathematics. This aspect was the focus of this work.

Despite these contributions, the research was limited by the low number of subjects, a limitation that Jain and Brown (2020) also recognise in their work. Therefore, research with larger samples is required. In the future, it is necessary to continue researching the devices required by initial teacher education programs (Kim & Youn, 2016) to clarify the contents and methodologies that positively impact the practice of early childhood teachers.

AUTHORSHIP CONTRIBUTION STATEMENTS

TGG and ROA designed the research and carried out the on-site intervention. TGG and SE collected data. AVG analysed the data. TGG and AVG reviewed and approved the final version of the article.

DATA AVAILABILITY DECLARATION

The data supporting the results of this study are available from the corresponding author, TGG, upon request.

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