

Navigating riverine mathematics: local orientation systems in Xingu River communities

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ABSTRACT

Approach: For the riverine communities, the Xingu River is not just a geographical feature; it is a vital part of their daily lives and cultural identity. The river influences social interactions and economic activities within these communities. Local knowledge and experience are essential for navigating the river, with expert pilots skilled in interpreting the changing conditions. This local expertise helps determine the location and orientation of the riverine communities, which in turn reflects their social structure. **Aim:** The objective is to identify the orientation and location systems of riverine communities along the Xingu River and their relationship with social structures from an Ethnomathematical perspective. **Design:** Using a qualitative ethnographic methodology and a social geometry approach, this research reveals how navigation, shaped by both natural and social factors, influences relationships and power structures within two communities in the Xingu River region. **Context and Participants:** Two riverine communities along the Xingu River have been selected for their accessibility, involving key informants, including adults and children. **Data Collection and Analysis:** Multiple periods of participant observation and unstructured interviews were performed. The qualitative analysis of the data involved identifying language games and family resemblances between local and academic orientation systems. **Results:** The results reveal similarities between local systems and formal mathematical language in how space is conceptualized. This suggests that the inhabitants of the region have a well-articulated understanding of the geometry and topography of their environment. The orientation and location systems developed by these communities are deeply connected to their knowledge of the natural environment and their cultural practices. Moreover, comparing the two analyzed communities highlighted the social and power dynamics established through local mathematical language games that organize the socio-demographic space. The school plays a central role in these communities, serving as the primary link to the state. In addition to providing education, the school reinforces the community's identities and their connection to the land, becoming a symbol of the

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struggle for fundamental rights. However, the research indicates that the school's role is threatened by the broader social and political context of the region.

Conclusions: This study explores the mathematical practices of orientation and location within riverine communities, contributing to a deeper understanding of the region's cultural, social, and ecological dynamics. It also examines how these factors influence mathematics and are influenced by it in return. The research highlights the challenges of conducting studies in remote riverine areas, emphasizing the importance of building trust with local communities and respecting their unique rhythms and cultures. Additionally, the study underscores the need for further research to understand how orientation and location systems are intertwined with other community aspects, such as education and environmental conservation.

Keywords: Space; Locate; Ethnomathematics; Language Games; Social Geometry.

Navegando por las matemáticas ribereñas: sistemas de orientación locales en las comunidades del río Xingú

RESUMEN

Planteamiento: Para las comunidades ribereñas el río Xingú es más que una característica geográfica; es una parte integral de la vida cotidiana y la identidad cultural de las comunidades. El río influye en las interacciones sociales y las actividades económicas. El conocimiento y la experiencia locales desempeñan un papel crucial en la navegación del río, con pilotos expertos que saben leer las condiciones cambiantes del río. Estos conocimientos locales plasman los sistemas de localización y orientación de las comunidades ribereñas que a su vez reflejan la estructura social de las comunidades. **Objetivo:** El objetivo es identificar los sistemas de orientación y localización de comunidades ribereñas del río Xingú y su relación con las estructuras sociales desde una perspectiva Etnomatemática **Diseño:** A través de una metodología cualitativa y etnográfica y un enfoque de geometría social, la investigación revela cómo la navegación, influenciada por factores naturales y sociales, da forma a las relaciones y estructuras de poder dentro de dos comunidades de la región del río Xingú. **Contexto y Participantes:** Dos comunidades ribereñas del río Xingú han sido elegidas por las posibilidades de acceso, involucrando informantes claves adultos y niños. **Recopilación y análisis de datos:** Se realizaron varios periodos de observación participante y entrevistas no estructuradas. El análisis cualitativo de los datos se realiza identificando juegos de lenguajes y semejanzas de familias entre sistemas de orientación locales y los académicos. **Resultados:** Los resultados destacan similitudes entre los sistemas locales y el lenguaje matemático formal en la conceptualización del espacio, lo que sugiere una comprensión articulada de la geometría y la topografía de su entorno por parte de los habitantes de la región. Los sistemas de orientación y ubicación desarrollados por estas comunidades están profundamente arraigados en su conocimiento del entorno natural y en sus prácticas culturales. Además, la comparación entre las dos comunidades analizadas permitió identificar y visibilizar las relaciones

sociales y de poder que se establecen a través de juegos de lenguaje matemático locales que organizan el espacio sociodemográfico. Emerge el papel central en estas comunidades de la escuela, ya que sirve como vínculo primario con el Estado. Más allá de brindar educación, la escuela refuerza las identidades de la comunidad, así como la conexión con su tierra y se convierte en un símbolo de la lucha por los derechos fundamentales. Sin embargo, la investigación sugiere que el papel de la escuela se ve amenazado por el contexto social y político más amplio de la región. **Conclusiones:** Al explorar las prácticas matemáticas de orientación y localización de las comunidades ribereñas, el estudio contribuye a una comprensión más profunda de las dinámicas culturales, sociales y ecológicas de esta región y cómo estas influyen y, a su vez, son influenciadas por las matemáticas. En el estudio emergen los desafíos de realizar investigaciones en comunidades ribereñas remotas. Generar confianza con las comunidades y respetar sus ritmos y culturas únicos es esencial. Se destaca la necesidad de realizar más investigaciones para comprender cómo estos sistemas de orientación y localización se entrelazan con otros aspectos de la vida de las comunidades, como la educación y la conservación del medio ambiente.

Palabras claves: Espacio; Localizar; Etnomatemáticas; Juegos de Lenguaje; Geometría Social.

INTRODUCTION

In the Brazilian Amazon context, the river is present in everyday life and is part of the identity, social, commercial, religious, and educational relationships (Silva & Souza Filho, 2002). Therefore, to be a riverine is not only to live next to the river, but to have an affective and effective relationship with it. This relationship is established, among other ways, by the way in which dwellings are organized along the river: some live on its banks, others a little further into the jungle, and they may be isolated houses or small communities.

Access to the riverine communities in the Brazilian Amazon primarily occurs via the river, using various types of boats. The type of boat—its shape, size, and material—is influenced not only by its intended use but also by the geographical features of the river in the area where it is utilized. Consequently, in the Amazon's rivers, you can find small, medium, and large boats made from wood, aluminum, or iron, powered by motors or oars. These boats transport people, food, and various materials.

This daily movement follows a logic rooted in the direct relationship with nature, influenced by the water cycle and various natural elements such as sand, trees, and rocks. The boats navigate the river in a specific manner, adapting to the changing conditions of the environment. These factors determine the "water paths" taken during transits, which are partially mapped out by local navigators. In particular, the boat pilots, who are natives of the area,

demonstrate greater expertise as they have learned to navigate through their direct relationship with the river.

This article is based on research conducted in a riverine region of the Brazilian Amazon, and within the context of Rural Education at a Brazilian university.

The aim of the research is to identify the orientation and localization systems utilized in several riverine communities along the Xingu River and their connection to social structures.

The term "orientation" refers to the process of positioning oneself within a specific geographical area, whereas "localization" pertains to identifying the geographical position of a particular place. Both actions are essential when traveling from one place to another.

THEORETICAL FRAMEWORK

Ethnomathematics and family resemblance in mathematics.

As a research program, Ethnomathematics requires abandoning the predominant idea of a single mathematical rationality and instead assumes that there are other rationalities mobilized by different groups. It proposes the appreciation and revaluation of these differences (D'Ambrosio, 1990, 2008).

These other rationalities are presented in different contexts and moments and are called ethnomathematics—in plural and lower case—being the ways, styles, arts, techniques (tics) that serve to explain, learn, and face (mathema) different environments (ethno), which can be natural, social, cultural, or imaginary (D'Ambrosio, 1990).

However, there are challenges to restructuring the conception of mathematics as a relative cultural construction, primarily due to the ingrained belief in a universal and culture-independent mathematics (D'Ambrósio, 2004). To address this issue, a philosophical alternative is proposed, based on Wittgenstein's (1999) ideas in his mature work. It suggests that mathematics develops through its practical use, as it is created through human discourse in response to the social, natural, and cultural contexts of those who utilize it. Each group possesses its own system of meanings related to how they approach concepts of quantity, relationships, and space, referred to as the QRS system.

So, rather than arguing about whether mathematical knowledge is certain or fallible, we should recognise that it is created in our talk. Thus mathematics is neither a description of the world,

nor a useful science-like theory. It is a system, the statements of which are “rules” for making sense in that system (Barton, 1999, p. 52).

Wittgenstein (1999) challenges the idea of a universal and homogeneous language by introducing the concepts of "family resemblances" and "language games." These terms imply that linguistic and mathematical activities are connected through their own specific rules, which develop in particular contexts and acquire their unique meanings. This perspective counters the notion of a single universal language and emphasizes the diversity and relativity of languages—and mathematics—along with their applications. Consequently, there are various "languages", each with its own rules and grammar, which may conflict with those of a universal language. This is equally applicable to mathematical languages. However, the rules and grammar present in a given language can exhibit similarities to the rules of other languages, which is what Wittgenstein refers to as family resemblance (Wittgenstein, 1999).

This study proposes to examine the language games mobilized by the research participants and their family resemblance with the language of official mathematics. This knowledge is produced from hegemonic thought loaded with formalism and abstraction, so the objective is to reveal alternative mathematical rationalities that can broaden our perspectives on understanding the world—essentially, to explore different language games (Barton, 1999).

The idea is that when different language games encounter one another, or when we strive to identify their family resemblances (for example, by describing a practice of one language using the terms of another), it leads to a reorganization of both language games and a deeper negotiation of their meanings. Therefore, this dialogue between language games and the quest for shared characteristics becomes a political process aimed at creating a hybrid language game that enhances our understanding of both local (possibly more than one) and universal processes (Parra-Sánchez, 2018).

Conceptions about space

Localizing is one of the six universal mathematical activities that generate mathematical ideas and concepts across different cultures (Bishop, 1999; D’Ambrósio, 2001). Conceptions of space are organized at three levels: physical, sociodemographic, and cosmological (Bishop, 1999). This research focuses on the organization of sociodemographic space. In the literature on ethnomathematics, two conceptions of space are identified: a static conception

based on position and reference, and a dynamic conception based on movement and velocity. The dynamic view of space, influenced by cultures such as the Navajo and the philosophy of yin and yang, demonstrates how mathematics develops differently under various cultural and linguistic influences (Barton, 2008).

In the mathematical language games used in formal education, a static perspective is predominant. Various reference systems exist to define the position of a point in space. In this study, we will focus on the plane, as socio-demographic space is commonly represented as a plane in most language games (such as maps and plans), including those considered in this research (Barton, 2008).

Among the reference systems introduced in school, the Cartesian coordinate system stands out. This system is defined by two perpendicular straight lines known as axes, to which a direction is assigned. The point where these axes intersect is called the origin. Additionally, it is important to establish a metric for measuring lengths, which will indicate the distances between two points. Each point in the plane can then be uniquely identified by a pair of Cartesian coordinates, which represent the distances of that point from each of the axes.

Coordinate systems defined by multiple points are less commonly taught in basic formal education, but they are often included in professional training because they underpin the operation of various technological devices. An example of this is the double distance coordinate system, which is defined by two points on a plane that serve as origins, along with a length metric. Other points on the plane are specified by the distances from each of the two origins. However, it is important to note that in this system, these coordinates may not always determine a single unique point.

When discussing localization in a geographical and sociocultural context, it's essential to consider the specific situation that necessitates localization and the cultural group involved. For instance, Polynesian languages take into account both the listener's and speaker's perspectives, along with the social dynamics between them. This approach enables to recognize family resemblances using the double distance coordinate system mentioned earlier (Barton, 2008). Furthermore, the language employed varies depending on the context and is influenced by local concepts (Bishop, 1999).

There are several ways to express distances, not only by using standard length units from the international system (such as meters and their multiples)

or unconventional units (like an arm's length or a city block), but also through other measurements. These can include time (such as a day's walk or five minutes by car) and volume (for example, the distance that can be traveled with a particular amount of fuel) (Gavarrete & Albanese, 2018).

A dynamic understanding of space is not typically found in the language of school mathematics. However, maritime navigation does incorporate this concept. Navigation by position relies on a geographic coordinate system, which is static and has family resemblances to the Cartesian system, although it is based on angles on a non-flat surface. In contrast, path navigation utilizes natural markers and signals, which reflects a dynamic conception of space (Barton, 2008). It is from this second perspective that family resemblances with other forms of localization in the riverine context of the Brazilian Amazon can be identified, distinguishing them from conventional static coordinate systems.

Social geometry

Social geometry is a concept in sociology that studies the structures of social space, describing the social relationships established among community members using mathematical and geometric language (Ferreira, 1955).

The term "social geometry" has come to describe the relationship between the organization of geographic space and the social issues that govern societies. For instance, it encompasses the mathematical modeling of how populations have historically distributed across geographic areas (Le Bras, 2000), as well as the impact of social interaction structures on the architectural features of human constructions (Fernández-Álvarez, 2006).

In this paper, we propose an inverse exercise, similar to the approach taken by Albanese and Mesquita (2022). Our goal is to identify and highlight the social and power relations established within the community by describing the local mathematical language games that shape the socio-demographic space.

METHODOLOGY

This research is inspired by critical ethnography, which aims not only to describe and analyze the ways of life within a specific culture, but also to understand how the knowledge generated in that context interacts with society, particularly concerning past and potential political actions. Consequently, the process of sharing personal narratives requires both the personal and intellectual engagement of the researchers, making it an ethical and political act (Thomas, 1993).

A field immersion took place from 2017 to 2023 in riverine communities along the Xingu River, specifically in Altamira (Cachoeira do Jabuti) and Porto de Moz (Nossa Senhora de Nazaré), located in the southwestern part of the state of Pará in the Brazilian Amazon. This is a case study focused on the two riverine communities along the Xingu River. The selection of two communities will facilitate a dialogue between their language games, allowing us to uncover additional family resemblances as a result of this interaction.

In the community of Cachoeira do Jabuti, both adults and children participated as key informants. In contrast, only adults were involved in the community of Nossa Senhora de Nazaré. The key informants, including both adults and children, signed the Free and Informed Consent Form and the Free and Informed Assent Form, respectively¹.

Rural riverine communities in the Brazilian Amazon are diverse and have different forms and organizations. Some communities consist of a large number of families, often with a level of kinship among them, sometimes led by a common patriarch or matriarch. The selection criteria for the communities studied were based on the ongoing research of the first author: one project related to his doctoral studies and another connected to his role as a professor in a Rural Education degree program.

Data collection techniques included participant observation and unstructured interviews, along with narrative records in field diaries, audio and video media, and photographs. The data collection process was cyclical or spiral (Albanese, 2015) and alternated with different phases of data analysis. The data were analyzed from a qualitative perspective, focusing on language games to understand how participants organized their sociodemographic space. This involved identifying reference points, directions, meanings, and personal or shared metrics, which allowed for the reconstruction of local reference systems. These systems were then interpreted to find family resemblances with the language games discussed in the theoretical framework.

¹ The authors did not submit the project to the Ethics Committee, considering that the research was conducted collectively with community residents (it was presented and approved at a community meeting) and personal data are not published, causing no harm to the residents. The authors assume all responsibility for the consequences, including full assistance and possible compensation for damages to any participant in the research, in accordance with Resolution 510 of April 7, 2016 of the Brazilian National Health Council.

RESULTS

Water ways on the Xingu River

The primary means of transportation in the region is via the Xingu River. Both communities being studied are situated along its banks, and access to each is only possible through water, using either larger passenger boats or smaller private boats. A relevant point of reference for navigation in this area is the nearest big city, which can be Altamira in some instances and Porto de Moz in others.

The community of Nossa Senhora de Nazaré is located downstream from the city of Porto de Moz, on the left bank of the river. In contrast, the community of Cachoeira do Jabuti is situated upstream from the city of Altamira. This information suggests a conceptual framework reminiscent of a Cartesian coordinate system defined by the river bed, which is not a straight line. The direction of travel from the city to the community indicates the flow of the river: for Nossa Senhora de Nazaré, the direction is in favor of the current, while for Cachoeira do Jabuti, it is against the current. These directions help define the left and right banks of the river for each of the communities being studied.

During the journey from the city to the community, passengers are informed about the expected time of arrival, which can vary depending on the time of year. For instance, a boat leaving Altamira at around 11:00 a.m. typically arrives at the community of Cachoeira do Jabuti between 1:00 and 2:00 p.m. This arrival time can also be influenced by the number of passengers and goods being transported, as well as seasonal conditions. The Xingu River experiences six months of full water and six months of drought, with its water dynamics differing from those of other rivers closer to the Brazilian Amazon coast, where tidal cycles are an important factor. In the Xingu River, the fluctuations are marked by flooding during the Amazonian winter rains, which last from November to mid-April, and a dry season that coincides with the Amazonian summer. The dry season begins in May, but the drought becomes more pronounced in September and October (Franco et al., 2015). These seasonal variations play a crucial role in how communities organize their activities throughout the year, as their daily routines are closely linked to the river (Formigosa & Giongo, 2022).

During the dry season, which coincides with the austral summer, river travel tends to be slower. In contrast, during the flood season, residents can navigate the river more easily and quickly. Pilots should be aware of certain

elements that can affect navigation, such as the presence of stones or rocks indicated by the backwaters that form nearby, particularly in the stretch of river upstream from Altamira. Additionally, pilots need to watch for sandbanks that are either exposed or barely submerged, especially in the stretch downstream from Porto de Moz (see Figure 1).

Figure 1

Rocks, sand banks and island in the river bed.



Downstream from Porto de Moz, small islands form on some sandbanks during the dry season. These islands are later uprooted by the strong currents during the rainy season. As a result, they begin to move down the river and can sometimes cause damage to ports and boats.

There are no maps for navigation, so pilots orient themselves on the river using distinctive landmarks. These can be buildings on the banks or notably tall and leafy trees that stand out from the others. In the river itself, pilots identify features such as rocks, islands, or beaches. The pilots are knowledgeable about "channels", which are paths or routes that allow boats to pass through safely due to greater depth, free of rocks, sandbanks, backwaters, or dangerous currents. The location of these channels can change throughout the year, depending on the river's flow.

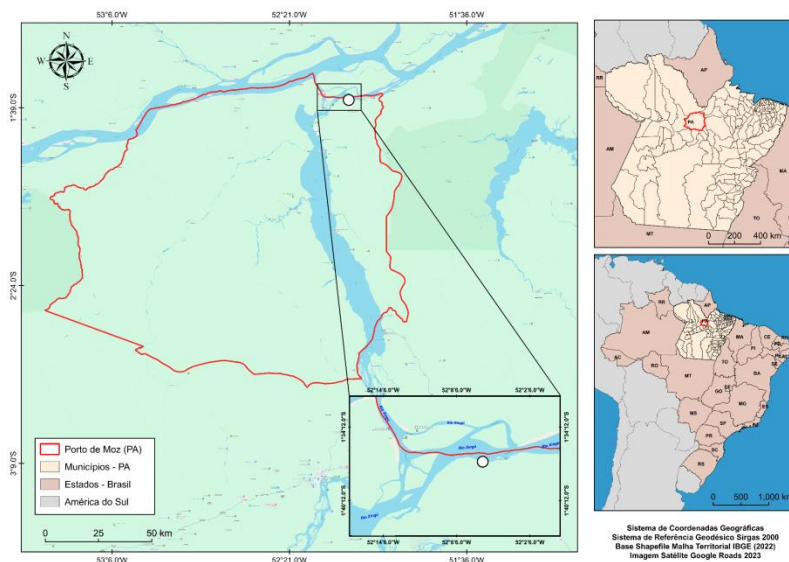
From a pilot's perspective, navigating rivers has family resemblances with path -or waterways- navigation. Additionally, traffic is organized so that boats traveling downstream keep to the right side of these channels, while those heading upstream stay on the left side, similar to rules for land routes.

Community of Nossa Senhora de Nazaré, in Porto de Moz

The community of Nossa Senhora de Nazaré, also referred to as Vila do Baiano, is named after its founding patriarch. He relocated from the state of Bahia in northeastern Brazil to work on the rubber plantations of the Amazon.

Figure 2

Community of Our Lady of Nazareth, image by Miguel Santos.

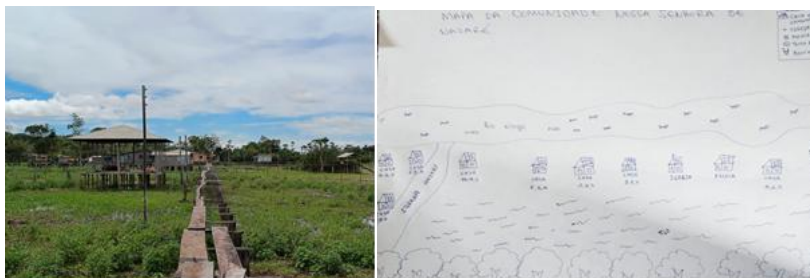


The community has three harbors, but during the dry season, boats coming from the city cannot reach them due to the formation of a sandy beach that obstructs the shore. In contrast, smaller boats owned by residents can get closer to the shoreline, so it's necessary to contact someone from the community for assistance with the transfer before disembarking.

To establish one's orientation within the community, it's essential to understand its layout. The houses in the Nossa Senhora de Nazaré community are all situated closely together on the same side of the river. They are connected by a single wooden bridge, which was constructed by the residents themselves (Figure 3).

Figure 3

Photo of the bridge that connects the houses and schematic of the distribution of the houses in the community, by Liliane Barbosa.



In this community, houses are not numbered. When you enter, you must ask the residents for the name or nickname of the person you are looking for. Typically, you can get this information from the person who assists with disembarkation using his/her own boat.

Indications within the community include hand signaling and identifying characteristics of each house, such as its shape, color, ornamental plants in front, any pets, or fishing equipment. The community is structured around a single trail, which has family resemblances with path navigation relying on various landmarks rather than a length metric.

Currently, there is no standard distance between houses. When the community was established, each family was allocated a plot measuring 500 meters by 1,000 meters to build their home. As some descendants started their own families, they often constructed their houses on their parents' land, usually next door. This has led to changes in the distribution of houses over time, resulting in more than one house sometimes being located on the same plot.

It should be noted that all residents of the community are related by blood, except for husbands and wives who have formed families. According to the residents, the community has few inhabitants because, without schools, children must move to neighbouring communities or the city to continue their education. Unfortunately, many of them end up never returning. As a result, the community gradually becomes emptier and may, at some point, cease to exist.

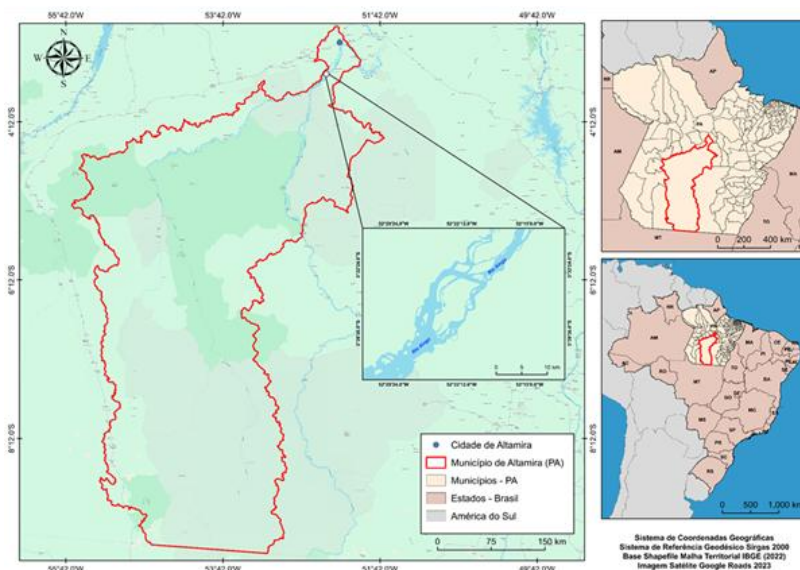
This lack of schools significantly impacts the maintenance of the community's way of life, an issue that will be addressed later.

Community of Cachoeira do Jabuti (Turtle Waterfall)

The name of the community is derived from a rock that appears during the dry months, shaped like a turtle, as well as from the small waterfalls formed by the river nearby. This rock serves as the community's most significant reference point. The Cachoeira do Jabuti community is situated upstream from the city of Altamira, as illustrated in Figure 4.

Figure 4

Community of Cachoeira do Jabuti, image by Miguel Santos.



The boat arriving from the city docks at the school's port, but not all the houses in the community can be accessed from there. Each house has its own private port, and very few are connected by land (Formigosa & Giongo, 2022). Once you reach the school, you should seek information from a community resident who lives nearby or from the children who attend the school. However, depending on whom you speak with, you may receive different directions. Adults in the community tend to use the waterfall as a reference point to orient themselves and locate places within the area. In contrast, children view the school as their main reference point. This difference can be illustrated through the example of Doña Isabel's home, a resident of the community.

Adult orientation

To reach Mrs. Isabel's house from the main port where the school is located, you need to travel by boat and then walk through the jungle. We will first describe the journey to her house along the river and then through the jungle. According to a local adult, Doña Isabel's house is located "above the waterfall, to the left of the river". Again, this is a language game with family resemblances to the Cartesian reference system.

This information contains the implicit determination of a reference system with family resemblances to the Cartesian reference system, centered at the Tortuga waterfall. The first direction in this system follows the course of the river, while the complementary direction is determined by a line orthogonal to that of the river. It is important to note that we are not speaking of a fixed axis since the river is not a straight line; rather, it is a winding trail that delineates a path. The river's direction is the same as navigation, specifically from the town of Altamira toward the community, moving against the current. Thus, locations that are "above" the waterfall are positioned upstream, whereas those that are "below" are downstream, closer to the river's mouth. The orthogonal direction, whether to the right or left bank, is also defined by the orientation of the navigator moving upstream. The metric used for measuring the journey from the school harbor to other harbors in the community is based on time. However, several factors can influence the duration of this journey, including whether one is traveling against the current or with it, the season of the year (which affects the river's water level), and the type of boat used.

This identification of the river's direction and its banks is contrary to academic geographical terminology, which defines the river's direction based on the flow downstream.

Residents travel within the community using small boats, typically made of wood, some equipped with oars. Rowing boats allow travel between houses located above or below the waterfall. However, navigating the waterfall itself is not possible with this type of boat as rowing upstream is prevented by the force of the water, while rowing downstream is risky due to the strong current that can lead to a loss of control.

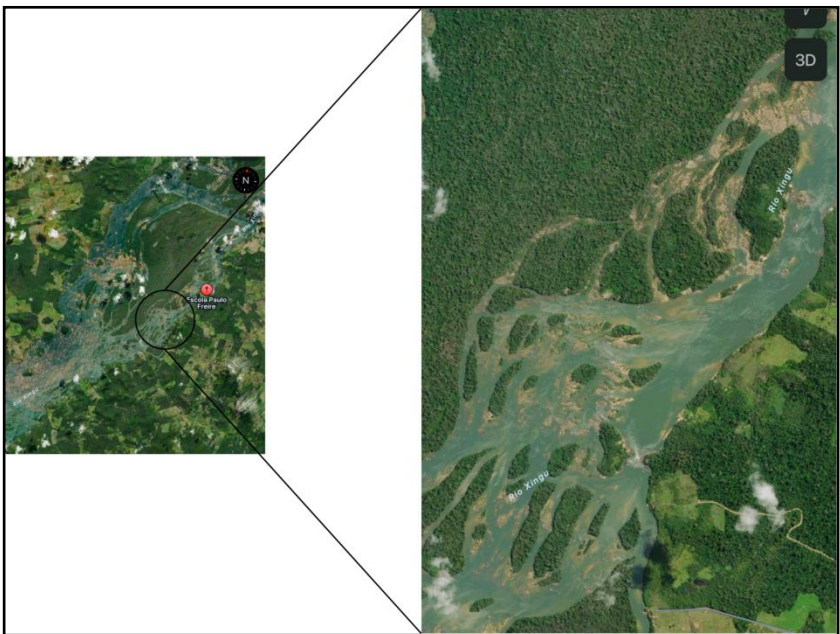
Motorized boats provide a means to travel between all the ports in the community. However, this is influenced by several factors, including the geographical features of the river, which can necessitate alternative routes and detours to avoid dangerous areas (as shown in Figure 5). Additionally, the seasonal climate plays a role; during the rainy season, strong currents and

hazardous eddies can pose risks. The economic resources of the community members also affect travel, particularly in terms of fuel costs and the challenges related to acquiring it.

As a result, travel is primarily undertaken for essential activities such as obtaining food, seeking medical care (since the community lacks health services), or participating in community events. From the perspective of the adults, moving around the community requires careful planning and resource allocation. Thus, travel is focused mainly on fulfilling these critical needs.

Figure 5

Channels formed between the islands.



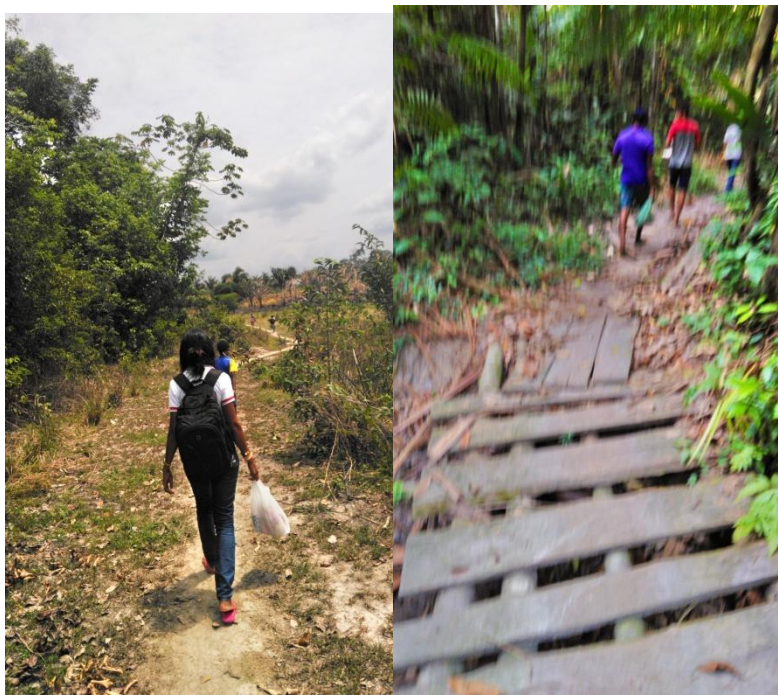
The symbolic role of the rock and the turtle waterfall serves as both a unifying element of identity for the community and, as the research has shown, a potential source of division. This dual role became evident during the ethnomathematical analysis of the community space.

From the ports, paths lead into the forest (Figure 6). These paths can be covered on foot or by bicycle and are marked by a variety of landmarks, both

natural and man-made, that help inhabitants orient themselves. These landmarks may include a century-old tree, an unusual rock, or a small building.

Figure 6

On the way to Doña Isabel's house.



These paths determine new reference systems, all starting from the harbor on the river where the boat docks. These reference systems have family resemblances with the language of path navigation, where the markings are primarily influenced by floral elements in the landscape or by features of the path itself.

Child orientation

In the community, the primary reference point for the children is their school, known as the Paulo Freire School, commonly referred to as the "escuelita" (little school). According to the reference system for adults discussed in the previous section, the school is situated below the turtle waterfall on the left bank.

The children are transported to school daily by a school bus owned by a community resident and funded by the state. The bus is wooden, covered, and motorized, capable of carrying up to fifteen people. When the river is full, the boatman begins picking up the children from their homes at 6:30 AM and arrives at school around 7:45 AM. However, when the river is dry, the trip takes much longer, requiring the children to leave their homes earlier.

To indicate where they live, the children first decide whether to go up or down the river, using the little school as their starting point. Depending on their chosen direction, they then specify if they are on the left or right bank of the river. In this new system of reference, two opposite directions are established for the river's flow: one direction goes 'upstream,' which is against the current and "after" the school, while the other direction goes downstream, which is with the current and toward the area "before" the school.

It is important to note that children orient themselves differently in relation to adults. This difference arises not only from a change in perspective but also from a different way of perceiving their environment. Consequently, instructions are formulated in a manner that emphasizes "before" and "after" rather than "up" and "down". This implies that, for places down the school, the left or right bank is identified oppositely to the way adults do.

To establish distances on the river, the children do not use a metric. Instead, they identify various reference points, such as large trees or specific docks, to determine where they need to dock in order to reach their homes. These reference points—like trees and rocks—while sometimes acknowledged by adults, hold unique meanings for the children, shaped by their imaginations. For instance, a tree with two open branches might be the *two antennae*, while leaves of a certain shape could be imagined as a *small dog*.

Crossing from one side of the Tortuga rock to the other necessitates a fuel-powered boat, which can be quite costly. As a result, adults only make the journey when absolutely necessary. This waterfall effectively serves as a barrier, delineating the territory and separating the people who reside there. In contrast, for children, this physical barrier poses no challenge, as they cross it daily using public transportation.

So, the role of the school is crucial as it serves not only social and educational functions but also acts as a bridge between the 'lower part' and the 'upper part' of the community. Attending school provides children with opportunities to transition between different spaces, allowing them to observe, identify, imagine, and learn how movement occurs within their environment.

However, rural schools face ongoing threats from the push for standardized and uniform educational practices, driven by external demographic and political changes.

DISCUSSION

After detailing the language games and the factors that influence them, as well as their family resemblances to mathematical conceptions of space, we compared the two communities under analysis. Our goal is to identify family resemblances between the language games of these communities, which will enable us to elevate the analysis to a social and political level.

Neighborhood and kinship relations vary from one community to another, influenced by their history and the geographical characteristics of their territories. This variation is evident in the arrangement of houses within each community. In Nossa Senhora de Nazaré, the houses are closely aligned, while in Cachoeira do Jabuti, they are more spread out along the river and into the jungle, resulting in a more distant relationship among residents.

It is important to note that, unlike Nossa Senhora de Nazaré, traffic to Cachoeira do Jabuti is restricted due to the geographical characteristics of the river, which limit the volume and intensity of boat traffic in the communities upstream of Altamira. In contrast, the community of Nossa Senhora de Nazaré is situated between the towns of Gurupá and Porto de Moz, surrounded by several nearby communities. The river in this area is more navigable, resulting in consistent boat traffic passing by the community. Additionally, there are a greater number of boats of various types and sizes, including wooden boats, docked in the harbors.

Figure 7

Ports along the Xingu River downstream of Porto de Moz.



In the community of Cachoeira do Jabuti, there is a perception that the river is sparsely populated. Unlike other rivers in the Amazon, we do not see boats frequently traveling on it. When we first encountered this community in 2017, we felt that the river was lifeless, partly influenced by narratives surrounding the negative impacts of the Belo Monte hydroelectric dam. However, over time, we have come to understand that the river has its own way of existing and resisting. The people who inhabit this area foster a unique relationship with nature (Alves & Toutonge, 2020) by respecting the elements that make up their environment. They move through their own time and space, which contrasts with the fast-paced lifestyle of urban life and escapes the homogenization commonly found in cities.

One key finding from the analysis is that the survival of the communities (current and future) is closely tied to the school. The school's symbolic, social, and political role becomes apparent through the ways in which the space is managed (Hage, 2014; Lopes & Parente, 2017).

Language games are influenced not only by the configuration of the territory but also by the lifestyles and social relationships within each community, as highlighted by the contributions of social geometry. In this context, the rural school plays a crucial role as a unifying and identity-building element for the community. It holds symbolic value that transcends its physical presence, representing the struggle for land (Lopes & Parente, 2017). Additionally, the rural school acts as a bridge that breaks down geographical barriers, facilitating encounters, one of the essential functions of education (Formigosa & Giongo, 2022).

Schools in these territories are at risk of closure due to low student enrollment and the government's claim of high maintenance costs. When closures occur, children are relocated to far-off schools known as nucleated schools, which receive students from different communities where schools have been closed. This can result in travel times exceeding two hours each way by school transport. Such long travels impact children's health, leading to complaints of headaches caused by the noise of the boat's engine, and can even jeopardize their safety due to the inadequacies of the transport system (Carmo et al., 2020). Without viable alternatives, some families are forced to withdraw their children from school. Many others opt to leave their communities in search of schools, either moving to neighboring areas or even to urban centers, leading to the gradual 'emptying' of these territories. This situation is currently unfolding in the Nossa Senhora de Nazaré Community, where the local school was closed in 2018.

In the Brazilian Amazon, many schools have been closed without a thorough technical study being conducted to assess the impacts on the local communities. Additionally, the opinions of these communities regarding the closures have been disregarded, contrary to Brazilian legislation (Novaes et al., 2021). The presence of a school in these communities is crucial for ensuring that families remain in the area and can maintain their way of life, as it provides children with access to a fundamental right: education. The struggle for this right has been a significant part of the history of these communities. In many cases, even when conditions are poor, the existence of the school serves as a symbol of the state's presence in the region. Losing the school, therefore, signifies a weakening of their social and community organization (Hage, 2014; Lopes & Parente, 2017).

In addition, communities play a vital role in conserving the region's biodiversity. By living in harmony with nature and respecting its natural cycles, these communities serve as guardians of the forest. However, when these territories are abandoned, other actors often seek to exploit the land in ways that disregard environmental protection.

During the research period, we observed the community of Cachoeira do Jabuti actively fighting for their empowerment in response to threats of school closure. In contrast, we found that there was no similar movement among the residents of Nossa Senhora de Nazaré for the reopening of their school.

CONCLUSIONS

This research has identified the systems of orientation and location used by some riverside communities along the Xingu River in the Brazilian Amazon. It highlights certain family resemblances between local language practices and formal mathematical language. Additionally, the analysis explores how social structures are influenced by or shape the organization of space within these communities.

The critical ethnographic methodology, combined with the approach of social geometry, has enabled us to identify the school as a central hub within these river communities. This school serves as the only connection to the state, ensuring access to education while also reinforcing families' ties to their land. As a result, it has become a symbol of the struggle for fundamental rights.

Expanding this research to other riverine communities would be interesting, but it comes with significant challenges. To conduct these studies, time, financial resources, and, most importantly, the establishment of trusting relationships with local communities are needed. Researchers must first build trust with community residents, who often fear sharing information that could be used against them, as has regrettably occurred in the past. Additionally, the research must respect the unique timing and dynamics of each community. The logistical costs of traveling along the Amazon rivers also present a significant financial burden, which limits the frequency and duration of the research efforts.

Amazonian riverine communities exhibit a wide variety of lifestyles and social organizations. This diversity is evident in their orientation and navigation systems. For instance, communities located near river mouths at the sea are significantly influenced by lunar and tidal cycles. Their daily lives are structured around these weekly changes. In contrast, communities further inland, such as those along the Xingu River, experience more distinct seasonal cycles, and their orientation systems are adapted to these specific conditions.

It would be valuable to explore how these factors impact the organization of schools, considering their role as spaces that promote diverse forms of learning and serve as centers of resistance and stability in territories historically associated with land struggles. Examining how schools interact with the social and political dynamics of riverine communities can catalyze a transformation in political discourse and reinforce the school's role as a site for empowerment and resistance in the fight for land.

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AUTHORS CONTRIBUTIONS

MMF and VA came up with the idea and developed the theoretical framework for this study. MMF adapted the methodology to suit this context, conducted the fieldwork, and collected the data. Both MMF and VA analyzed the data and engaged in discussions about the results. MMF drafted the manuscript, while VA reviewed and edited it. All authors reviewed and approved the final version of the paper.

DECLARATION OF DATA AVAILABILITY

Data supporting the results of this study will be available from the author MMF upon reasonable request.

REFERENCES

- Albanese, V. (2015). Desarrollo de una tesis doctoral en Etnomatemática: construcción de una investigación emergente. *Revista Latinoamericana de Etnomatemática*, 8(2), 381–397.
<https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/177>
- Albanese, V., & Mesquita, M. (2022). Unravelling mathematical practices and social geometry from and with the fishing community. *Revista Colombiana de Educación*, 87, 117–140.
<https://doi.org/https://doi.org/10.17227/rce.num87-11941>
- Alves, E. da S., & Toutonge, E. C. P. (2020). Os saberes das águas no ensino de ciências: intertrocas de conhecimentos e pessoas. *RIS - Revista Insignare Scientia*, 3(4), 286–302.
- Barton, B. (1999). *Ethnomathematics and Philosophy*. 54–58.
<https://doi.org/https://doi.org/10.1007/s11858-999-0009-7>
- Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. Springer.

- Bishop, A. J. (1999). *Enculturación Matemática*. Paidós.
- Carmo, E. S. do, Cunha, F. F. da, & Prazeres, M. S. C. dos. (2020). Transporte escolar na Amazônia: uma análise dos condicionantes de acesso de estudantes ribeirinhos à escola do campo. *Revista Brasileira de Educação Do Campo*, 5, e6897.
<https://doi.org/10.20873/uft.rbec.e6897>
- D'Ambrosio, U. (1990). *Etnomatemática: arte ou técnica de explicar e conhecer*. Editora Ática.
- D'Ambrosio, U. (2008). O Programa Etnomatemática : uma síntese. *Acta Scientiae*, 10(1), 7–16.
<http://www.periodicos.ulbra.br/index.php/acta/article/view/74>
- D'Ambrósio, U. (2001). *Etnomatemática: elo entre as tradições e a modernidade*. Autêntica Editora.
- D'Ambrósio, U. (2004). Etnomatemática e educação. In G. Knijnik, F. Wanderer, & J. C. de Oliveira (Eds.), *Etnomatemática, currículo e formação de professores* (pp. 39–52). Edunisc.
- Fernández-Álvarez, Á. J. (2006). El papel de la Geometría como herramienta de diseño arquitectónico. *EGE-Expresión Gráfica En La Edificación*, 4, 51–61. <https://doi.org/10.4995/ege.2006.12555>
- Ferreira, P. (1955). Nuevos Fundamentos de la Espaciología Social. *Revista Mexicana de Sociología*, 17(2/3), 345–361.
<https://doi.org/10.2307/3537873>
- Formigosa, M. M., & Giongo, I. M. (2022). Jogos de linguagem expressos por alunos ribeirinhos na pesca artesanal do rio Xingu. *Revista de Educação Pública*, 31, 1–21.
<https://doi.org/10.29286/rep.v31ijan/dez.13695>
- Franco, V. D. S., De Souza, E., Pinheiro, A., Dias, T. D. S., de Azevedo, F. de, & Santos, J. D. C. (2015). Evolução mensal da cota fluviométrica do Rio Xingu em Altamira-PA associada aos eventos El Niño e La Niña. *Ciência e Natura*, 37, 104–109.
<https://doi.org/10.5902/2179460x16223>
- Gavarrete, M. E., & Albanese, V. (2018). Abordar la ubicación espacial y el plano cartesiano desde la interculturalidad [Addressing spatial location and the Cartesian plane from an intercultural perspective]. *Uno: Revista de Didáctica de Las Matemáticas*, 82, 23–30.

<http://hdl.handle.net/10481/54483>

- Hage, S. A. M. (2014). Transgressão do paradigma da (multi)seriação como referência para a construção da escola pública do campo. *Educação & Sociedade*, 35(129), 1165–1182.
<https://doi.org/https://doi.org/10.1590/ES0101-73302014144531>
- Le Bras, H. (2000). *Ensaio de geometria social*. Editions Odile Jacob.
- Lopes, R., & Parente, F. de A. (2017). Recomendações para a educação escolar dos ribeirinhos: entre o rio e a rua. In S. B. Magalhães & M. C. da Cunha (Eds.), *A expulsão de ribeirinhos em Belo Monte* (1st ed., pp. 409–433). SBPC - Sociedade Brasileira para o Progresso da Ciência. <https://portal.sbpcnet.org.br/livro/belomonte.pdf>
- Novaes, A. C., Sousa, D. M. de, & Formigosa, M. M. (2021). Educação do campo: uma análise do processo de fechamento de uma escola do campo em Abaetetuba, estado do Pará. *Periferia*, 13(1), 177–195.
<https://doi.org/10.12957/periferia.2021.54613>
- Parra-Sánchez, A. (2018). *Curupira's Walk: Prowling Ethnomathematics Theory Through Decoloniality*. Aalborg University.
- Silva, J. D. C., & Souza Filho, T. A. de. (2002). O viver ribeirinho. In J. D. C. Silva & M. P. De Souza (Eds.), *Nos Banheiros do rio: ação interdisciplinar em busca da sustentabilidade em comunidades ribeirinhas da Amazônia* (pp. 21–35). EDUFRO.
- Thomas, J. (1993). *Doing critical Ethnography*. Sage Publication.
- Wittgenstein, L. (1999). *Investigações filosóficas*. Editora Nova Cultural.