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Kuhn's Epistemology and its Contributions to Understanding the Process of Knowledge Construction and Mathematics Teaching

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ABSTRACT

Context: Scientific progress does not occur linearly but through revolutions that replace paradigms that no longer meet the demands of a given field of knowledge with others that address these requirements and expand the problem-solving horizon. **Objective:** to analyze how Kuhn's epistemology can be applied to understand the paradigm shifts that have occurred in the process of knowledge construction and mathematics teaching. **Design:** this is a bibliographic investigation of an analytical and interpretative nature. **Scope:** the study analyzed how paradigm shifts occurred in the history of mathematics and illustrated these transitions with examples such as the adoption of infinitesimal calculus and the development of non-Euclidean geometry. **Data Collection:** the study was based on a bibliographic review, critically describing the main epistemological themes and concepts developed by Kuhn and his respective commentators. Additionally, it systematized the implications of these paradigm shifts for mathematics teaching, with particular emphasis on the urgency for teachers to remain open to new theoretical-methodological approaches and the use of emerging technologies, such as educational software, which can provide a deeper, more dynamic, and interactive understanding of concepts and facilitate mathematics teaching. **Results:** the use of technology enables students to explore problems, visually verify results, and understand concepts, fostering more meaningful, innovative, and critical learning. It also helps reimagine mathematics education and build a promising future for future generations. Resistance to paradigm shifts in mathematics teaching often stems from adherence to traditional methods and a lack of proper training. **Conclusions:** addressing these challenges, given the need to prepare students for a constantly changing world, requires continuous teacher training, curriculum adaptation, the adoption of active methodologies, and the use of new technologies.

Keywords: Scientific Revolution; Paradigm Shifts; Scientific Crisis; Technologies; Mathematics Teaching; Meaningful Learning.

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A Epistemologia de Kuhn e as suas Contribuições para a Compreensão do Processo de Construção do Conhecimento e do Ensino da Matemática

RESUMO

Contexto: O progresso científico não ocorre de maneira linear, mas por meio de revoluções que substituem paradigmas que não respondem mais às demandas da área do conhecimento, por outros que dão conta destas exigências e ampliam o horizonte de resolução de problemas. **Objetivo:** analisar como a epistemologia de Kuhn, pode ser aplicada para compreender as mudanças paradigmáticas ocorridas no processo de construção do conhecimento e do ensino da matemática. **Design:** trata-se de uma investigação de natureza bibliográfica, de caráter analítica e interpretativa. **Ambiente:** analisou-se como as mudanças paradigmáticas ocorreram na história da matemática e exemplificou-se essas transições, como a adoção do cálculo infinitesimal e o desenvolvimento da geometria não euclidiana. **Coleta de dados:** partiu-se de uma pesquisa bibliográfica, em que se descreveu de forma crítica os principiais temas e conceitos epistemológicos desenvolvidos por Kuhn e de seus respectivos comentadores. Além disto, sistematizou-se as implicações dessas mudanças para o ensino da matemática, com especial destaque para a urgência dos professores estarem abertos a novas abordagens teórico-metodológicas e ao emprego de novas tecnologias, como o uso de *softwares* educacionais, que podem proporcionar uma compreensão mais profunda, dinâmica e interativa dos conceitos e facilitar o ensino da matemática. **Resultados:** O uso das tecnológicas permite aos estudantes, explorar problemas, verificar visualmente os resultados, compreender os conceitos, promovendo um aprendizado mais significativo, inovador e crítico, e reimaginar o ensino da matemática e construir um futuro promissor para as próximas gerações. A resistência a mudanças de paradigmas no ensino da matemática, resulta do apego a métodos tradicionais e da falta de formação adequada. **Conclusões:** O enfrentamento destes desafios, em face da necessidade de preparar os alunos para um mundo em constante mudança, passa pela formação continuada dos professores, adaptação dos currículos, adoção de metodologias ativas e emprego de novas tecnologias.

Palavras-chave: Revolução Científica; Mudanças de Paradigma; Crise Científica; Tecnologias; Ensino de Matemática; Aprendizado significativo.

INTRODUCTION

The approach to teaching mathematics inspired by Thomas Kuhn's (2013) paradigm theory suggests a significant shift in how mathematics is taught and understood. In his work *The Structure of Scientific Revolutions*, Kuhn introduced the concept of paradigms and scientific revolutions, which can be applied to enrich mathematics education, fostering a broader understanding of this subject.

According to Alves and Valente (2021), Kuhn argues that the scientific process does not occur linearly but through periodic revolutions that replace old paradigms with new ones. In the context of mathematics teaching, this perspective can transform how concepts are presented to students. Instead of adhering to a traditional and fixed method, teachers can adopt a more flexible approach, recognizing that different mathematical paradigms can coexist and complement each other.

Applying Kuhn's (2013) theory to mathematics teaching involves encouraging students to question the foundations and underlying assumptions of mathematical concepts. This can lead to a more critical and in-depth understanding of the subject, allowing students to see mathematics not just as a set of fixed rules but as a constantly evolving curricular component of great importance to life.

From a methodological perspective, a bibliographic research approach was chosen, aiming at the analysis, interpretation, and critical discussion of the main themes and epistemological concepts developed by Kuhn in his work *The Structure of Scientific Revolutions* and, consequently, of his respective commentators. The objective is to initiate a reflective and argumentative process regarding the dynamics of science, which, from this perspective, does not occur linearly but through crises and periodic revolutions that replace old paradigms.

In this sense, this paper aims to shed light on how Kuhn's (2013) innovative ideas, especially his concepts of paradigm and scientific revolution, can be used to radically transform the teaching and learning of mathematics. Instead of following a traditional and outdated model, we propose a compensatory approach inspired by Kuhn’s work, which seeks to: deconstruct the false notion of mathematics as a set of absolute and immutable truths; understand mathematics as a dynamic and ever-evolving subject, shaped by different paradigms throughout history; empower students to become active agents in the construction of mathematical knowledge, encouraging them to question, investigate, and seek creative solutions from different perspectives.

In this context, it is necessary to establish some guiding questions that will be addressed throughout this text: How can Kuhn’s concepts of paradigm and scientific revolution be applied to the evolution of mathematics? What are some examples of "paradigm shifts" in the history of mathematics? What are the main implications of Kuhn's theory for mathematics teaching? How can teachers use this perspective to make their classes more dynamic and meaningful? Why is there often resistance to paradigm shifts in mathematics teaching? What are the challenges of implementing new approaches? Thus, this paper invites the reader to embark on an exciting journey through the history of mathematics and the revolutionary ideas of Thomas Kuhn, with the aim of reimagining mathematics teaching and building a promising future for future generations.

THE NOTIONS OF PARADIGM AND SCIENTIFIC REVOLUTION IN KUHN'S WORK

Thomas Kuhn was a physicist, historian, and philosopher of science. He is known for his work, *The Structure of Scientific Revolutions*, published in 1962. In this book, he introduced the concept of the scientific paradigm, arguing that science does not progress linearly and cumulatively, as was often thought, but rather through scientific revolutions. He proposed that fundamental changes in science occur when an established paradigm is replaced by a new one, in a process that is not merely an accumulation of knowledge but a transformation of fundamental assumptions and methods (Kuhn, 2013).

According to Alves and Valente (2021), although Kuhn focused on the history and philosophy of science, he presents concepts and ideas that are highly relevant to the teaching of mathematics. This is because he proposes a model of how scientific knowledge develops through paradigms, which are defined as a set of research commitments shared by a scientific community at a given time, including beliefs, values, and techniques. In other words, a paradigm is like the pieces of a giant puzzle, shaping how scientists interpret the world and conduct their research in a specific historical period (Kuhn, 2013).

For example, the Newtonian paradigm dominated classical physics until it was replaced by Einstein's relativity paradigm, which introduced a new way of understanding space, time, and gravity. This shift occurred due to the influence of Newtonian physics over several centuries, during which Newton's laws accurately and effectively described the motion of objects, the force of gravity, and the behavior of light[1]. This paradigm provided a solid mental model for scientists and engineers, enabling remarkable advances in various fields such as astronomy, mechanics, and civil engineering.

However, over time, some anomalies began to emerge. Phenomena such as Mercury's orbital motion and the deflection of light in strong gravitational fields did not perfectly align with the predictions of Newtonian physics. These anomalies started to raise doubts and questions among scientists, paving the way for a new perspective with the emergence of Albert Einstein's relativity paradigm (Kuhn, 2013).

In 1905 and 1915, Einstein proposed the theory of relativity, which revolutionized our understanding of space, time, and gravity. This theory explained the anomalies that Newtonian physics could not resolve and provided a more comprehensive and accurate view of the universe. The acceptance of the theory of relativity represented a paradigm shift in physics, where the old Newtonian physics, which had dominated science for so long, was replaced by a new perspective, thus paving the way for further groundbreaking scientific discoveries and advancements.

Kuhn argues that the existence of a paradigm does not necessarily imply a complete set of explicit rules. Instead, normal science is guided by concrete examples of problem-solving that the paradigm provides. The direct inspection of these paradigms and paradigmatic examples can partially help determine the activity of normal science through scientific revolutions. According to Kuhn, normal science operates based on research that is firmly grounded in one or more past scientific achievements, which are "recognized for some time by a specific scientific community as providing the foundation for its subsequent practice" (2013, p. 54).

This concept reflects the idea that normal science is not focused on questioning or revolutionizing established foundations but on solving

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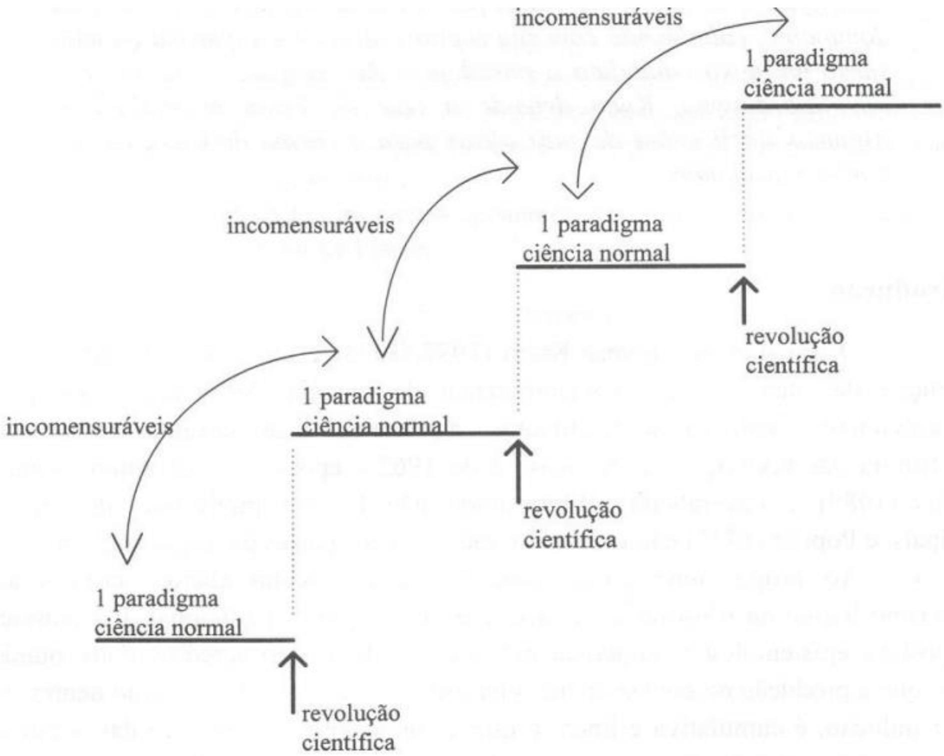
1 https://edisciplinas.usp.br/pluginfile.php/8073043/mod_resource/content/3/Aluno%20-%20Contextos%20sobre%20a%20Meca%CC%82nica%20Newtoniana.pdf

problems and refining details within the theoretical framework already accepted by the scientific community. It is a period of meticulous and cumulative scientific work that may eventually lead to anomalies and, ultimately, to a scientific revolution when a new paradigm emerges to replace the old one. Therefore, normal science can be partially determined through the direct inspection of paradigms, and this process is “aided by the formulation of rules and assumptions but does not depend on them. In fact, the existence of a paradigm does not even necessarily imply the existence of any complete set of rules” (Kuhn, 2013, p. 74).

Normal science is characterized by a period of stability and accumulation of knowledge within the prevailing paradigm. Only when this paradigm enters a crisis and can no longer resolve anomalies does a scientific revolution occur, leading to the adoption of a new paradigm. According to Ostermann (1996), Kuhn’s model of science offers a revolutionary perspective on scientific development, proposing a cyclical dynamic marked by periods of normality and rupture, rather than a linear and cumulative progression, as illustrated in the diagram below.

Figura 1

Diagrama do desenvolvimento científico (Osterman, 1996).



Each step in this diagram represents a paradigm, a shared set of assumptions, theories, and methods that guide scientific research during a given period. The arrows connecting the steps represent scientific revolutions, radical breaking points where a dominant paradigm is replaced by another. The diagram above concisely and clearly illustrates the scientific development model proposed by Kuhn (2013), and each stage can be explained as follows: a) Paradigm (Normal Science): this is the initial stage, where a dominant paradigm governs scientific practice. For example, Newtonian mechanics was widely accepted until Einstein's theory of relativity emerged. b) Incommensurable: when problems or anomalies arise that the current paradigm cannot solve, a period of crisis begins. This is what happened with Newtonian mechanics, which failed to fully explain the precession of Mercury’s perihelion, leading to the need for a new theory. c) Scientific Revolution: as anomalies accumulate and a new theory emerges that solves these problems more effectively, a scientific revolution occurs. This results in the replacement of the old paradigm with the new one, as seen in the transition from Newtonian mechanics to the theory of relativity. d) Paradigm (Normal Science): The new paradigm becomes the new standard of normal science. Subsequent research is conducted within this new paradigm, solving problems and refining theories based on the new principles, as happened after the acceptance of Einstein’s theory of relativity. Thus, this cycle demonstrates how the process of scientific development occurs and can repeat itself as new problems arise that the current paradigm cannot solve, leading to a new crisis, another scientific revolution, and so on.

According to Bartelmebs (2012), Kuhn's work remains relevant in terms of epistemological and structural discussions about the constitution of the sciences, debunking the myth that has formed around science and scientists with the advent of the scientific and technological era. He demonstrates that, in addition to being human constructs, the sciences are also, and consequently, social and historical constructions. This leads to a new understanding of scientific processes, and, why not say, scientific literacy. From this same perspective, Kuhn's work continues to be important as it demystifies the idealized view of science and scientists, showing that science is a human, social, and historical construct. It offers a new perspective on scientific processes and scientific literacy. This more critical and realistic view helps to understand science as a dynamic field influenced by paradigm shifts rather than merely by the accumulation of knowledge.

From the perspective of normal science and paradigms, Kuhn (2013) argues that science does not advance linearly but through revolutions that modify existing paradigms. Kuhn’s view applies to mathematics, where students must first learn to solve problems within a given paradigm or normal science before they are ready to question and revolutionize these paradigms. This is because conceptual change – meaning the shift in how students understand a mathematical concept –

occurs through a process like scientific revolutions. This perspective can help teachers develop teaching strategies that facilitate students' conceptual learning.

Kuhn (2013) emphasizes the importance of the history of science in understanding its development. Similarly, the history of mathematics is crucial for understanding how mathematical concepts and methods have evolved over time and how mathematical problem-solving has transformed this subject. Kuhn's concept of incommensurability, in which rival scientific theories cannot be directly compared, also applies to mathematics. In other words, when students learn new mathematical approaches, they need to understand that these may not be directly compatible with what they previously learned, requiring a shift in perspective. In this regard, Kuhn (2013) highlights the importance of puzzle-solving in normal science, an approach that aligns with problem-solving-based methods in mathematics teaching, where students learn to apply concepts and techniques within an established paradigm.

Problem-solving constitutes a cornerstone of all mathematical activity and a fundamental pathway for the development of mathematical knowledge, being defined as "the search for a means to achieve a certain goal that is not immediately attainable" (NCTM, 2007, p. 134). This highlights the central importance of problem-solving in the development and refinement of mathematical thinking. Thus, problem-solving is essential for the advancement of mathematical knowledge, as it involves the search for methods and solutions to objectives that are not easily attainable.

The teacher, by presenting problem situations, enables students to "mobilize their knowledge to find a solution, and also, the presented situations can be related to different contexts" (Carvalho, 2018, p. 184). The use of problem situations in mathematics teaching is crucial to encouraging the practical application of students' knowledge, providing a contextualized and meaningful learning experience.

Methodologically incorporating challenging problems into teaching, which are related to various contexts such as economics, health, the environment, and technology, helps students develop problem-solving skills, critical thinking, and knowledge integration. This approach not only engages and motivates students but also demonstrates the relevance of mathematics in their daily lives, preparing them to face real-world challenges effectively and innovatively.

However, just as scientific revolutions challenge and transform existing paradigms, presenting students with complex and contextualized problems fosters a revolution in their mathematical understanding, leading them to restructure their ideas and develop new ways of solving problems. This process not only engages and motivates students but also prepares them to face real-world challenges effectively and innovatively, demonstrating the relevance and applicability of mathematics in various contexts. Therefore, Kuhn's (2013) ideas on the nature of science and its historical development provide valuable insights for enhancing mathematics teaching and learning, emphasizing the importance of understanding paradigms, the history of mathematics, and problem-solving.

Normal science takes place within an established paradigm, where scientists solve puzzles and expand knowledge within these predefined boundaries. According to Kuhn, a puzzle refers, in the common sense in which we use the term, to "that particular category of problems that serve to test our ingenuity or problem-solving skills" (2013, p. 66).

Puzzles, both in normal science and in education, play a crucial role in skill development and knowledge advancement. They challenge individuals to apply their knowledge in an ingenious way, fostering a deeper understanding and practical problem-solving skills. By integrating puzzle-solving into teaching, the teacher prepares students to face future challenges with creativity, reflecting an investigative and progressive spirit.

This process occurs because conventional science is characterized by problem-solving within an established paradigm. The scientist is dedicated to solving problems that arise from accepted theories and methods, seeking to understand and apply features without fundamentally modifying the theoretical foundations. This approach emphasizes the continuity of knowledge, where solving puzzles is an essential part of scientific progress.

Thus, puzzles hold great significance for the development of skills and a deeper understanding of science, as scientists require individuals to apply knowledge critically and creatively. That is, through problem-solving, scientists not only apply theories but also deepen their understanding of underlying concepts, resulting in more meaningful learning.

From Kuhn's (2013) perspective, normal science is interrupted by scientific revolutions, which arise from crises and anomalies within the dominant paradigm, leading to the emergence of a new one. This transition involves the reconstruction of scientific problems and techniques. During periods of normal science, scientists work within an accepted paradigm, solving specific (puzzle-like) problems that fit within the established rules and methods. Therefore, the period of normal science is characterized by stability and incremental progress, where the scientific community expands and refines existing knowledge.

However, not all problems can be solved within the prevailing paradigm. Over time, anomalies – observations or problems that do not fit within accepted theories – begin to emerge. When these anomalies accumulate and can no longer be ignored, a crisis arises within the scientific community, creating the need for a scientific revolution. This is because the crisis caused by anomalies can lead to new decisions regarding the direction of research and, therefore, may trigger a scientific revolution, as the dominant paradigm is questioned and eventually replaced by a new one.

This paradigm not only offers new theories but also redefines technological problems, where the transition between paradigms represents a profound shift in how scientists understand and investigate the world. In the educational context, Kuhn’s (2013) concepts can be extremely valuable. Teaching about the nature and dynamics of science, including periods of stability (normal science) and radical changes (scientific revolutions), can help students understand that scientific knowledge is constantly evolving. This encourages critical thinking and cognitive flexibility, preparing students to adapt to new paradigms and technological advances in the future.

THE CRISIS OF CONFIDENCE IN NORMAL SCIENCE AND THE NEED FOR SCIENTIFIC REVOLUTION

The 1960s were marked by a crisis of confidence in science and scientific authority. The Cold War and the arms race between the U.S. and the Soviet Union created an atmosphere of distrust and disagreement about science and its application. This scenario began to change in the late 1980s with the end of the Cold War (Laudan, Donovan, et al., 1993).

The development and use of nuclear weapons during World War II and the constant threat of nuclear conflict during the Cold War generated deep skepticism toward science. The ability of science to create weapons of mass destruction called into question the idea that scientific knowledge always leads to progress. Nevertheless, the use of chemical agents and the escalation of violence in Vietnam revealed that science can be used for political and military purposes, challenging the notion that scientists are neutral and impartial (Andrade, 2019).

In addition to this discredit regarding the use of technology produced by science, the 1960s were marked by a series of complex social problems, such as poverty, racial discrimination, and social inequality. The inability of science to provide simple and quick solutions to these issues undermined confidence in its ability to solve humanity's problems, leading to a scientific crisis (Monteiro, 2020).

Therefore, Kuhn's (2013) theory of scientific revolutions offers, in the 60's, an interesting lens for analyzing the crisis of confidence in science. The accumulation of anomalies, such as the side effects of nuclear technology and the abuses of science during the war, challenged the dominant scientific paradigms and created a climate of uncertainty and dissatisfaction.

Kuhn (2013), through his theory, helps us understand three fundamental aspects of science: a) Science is not a linear process, as its evolution is marked by periods of stability (normal science) and disruptions (scientific revolutions); b) Science is influenced by social and historical factors, as the crisis of confidence in the 1960s was a result not only of scientific issues but also of social, political, and cultural factors; c) The choice of a paradigm is not neutral, meaning that the adoption of a new paradigm involves multiple factors, including its ability to explain anomalies, social and political influences, and the beliefs and values of scientists.

In this sense, it becomes clear that science evolves through revolutions and that scientific paradigms are influenced by social and historical factors. Thus, the complexity of the relationship between science and society can be better understood. Kuhn (2013) presents a new perspective on scientific progress, challenging the traditional idea that science advances linearly and cumulatively. Instead, science evolves through cycles of stability and disruption, characterized by periods of normal science, followed by crises and scientific revolutions. The crisis, in particular, plays a central role in this process, marking the point at which an existing paradigm becomes unsustainable.

During the phase of normal science, scientists work within a paradigm with the aim of solving problems and refining theories. However, over time, anomalies emerge – phenomena that the prevailing paradigm cannot resolve or explain. When these anomalies accumulate and resist attempts at resolution, confidence in the existing paradigm begins to waver, triggering an unprecedented crisis. This period of crisis is marked by instability and uncertainty, where the inadequacy of the current paradigm becomes evident as it fails to address fundamental problems.

According to Alves and Valente,

> […] a crisis can end in three ways. The first possibility is the revelation that the challenged paradigm itself can resolve the causes of the crisis. The second option arises from the persistence of the problem. The third option occurs with the emergence of a new candidate for the dominant paradigm and the confrontation between it and the existing one (2021, p. 48).

The present analysis agrees with this perspective, as it captures the complexity and dynamics inherent in scientific progress, namely: a) The first possibility highlights that the existing paradigm resolves the problems causing the crisis, demonstrating the resilience and adaptability of scientific paradigms. Often, a deep and rigorous examination within the current paradigm's context can lead to innovative solutions that reaffirm its validity and usefulness; b) The second possibility marks the persistence of the problem and serves as an indicator of the limitations of the current paradigm. The persistence of the problem can be crucial, as it exposes weaknesses and signals the need for a new approach or a significant modification to the established paradigm. Recognizing these limitations is fundamental for advancing scientific knowledge; c) The third and most transformative possibility is the emergence of a new paradigm that challenges and eventually replaces the existing one. This process of confrontation and eventual paradigm shift is at the core of the scientific revolutions described by Kuhn. Ultimately, the emergence of a new paradigm that addresses problems more effectively and comprehensively represents a significant advancement and an evolution in the field of knowledge.

These three possibilities are essential for understanding the dynamic and evolving nature of science, highlighting that a crisis can be resolved through internal resolution within the current paradigm, through the persistence of problems, or through the emergence of a new paradigm. In other words, these possibilities demonstrate that scientific progress is a continuous process of adaptation, questioning, and transformation, where each crisis represents an opportunity to deepen and expand our understanding of the world.

The scientific crisis, as described by Kuhn (2013), is a moment of great intellectual tension, as during this phase, scientists become more open to new ideas. Specifically, the crisis sets the stage for a scientific resolution and the emergence and proposition of a new paradigm, eventually challenging and replacing the existing one. This new paradigm not only resolves the anomalies that caused the crisis but also reshapes the way scientists investigate and understand the world.

Therefore, for Kuhn (2013), the crisis is the fundamental element in the cycle of scientific progress. It does not represent a failure of science but rather an essential driving force for innovation and scientific advancement. The crisis triggers the breakdown and collapse of obsolete paradigms, paves the way for new theories, and pushes science toward new standards of understanding and discovery. Thus, by understanding the structure and dynamics of the scientific process and applying them to mathematics teaching, we can create a more dynamic, challenging, and meaningful learning environment where students are encouraged to think critically, question, and construct their own knowledge.

SCIENTIFIC REVOLUTIONS IN MATHEMATICS TEACHING

Kuhn's (2013) epistemological approach can be useful for understanding mathematics teaching. According to Kuhn's perspective, normal science in mathematics would be the process of problem-solving within an established paradigm. This means teaching students to apply existing mathematical concepts to solve standardized problems.

Consequently, as Kuhn argues, normal science is not static. Anomalies – problems that cannot be solved within the current paradigm – lead to crises and, eventually, to scientific revolutions. In mathematics education, anomalies could be represented by non-conventional problems that challenge traditional teaching and problem-solving methods.

Mathematics, with its roots in the early days of humanity, developed in response to demands and necessities, intertwining knowledge from various fields. Its goal is to prepare individuals to deal with everyday problems, fostering minds capable of thinking critically and logically. Although mathematics is a subject with its own laws and characteristics, its applicability transcends boundaries, reaching multiple areas of knowledge. Fundamental concepts such as geometry, calculus, and arithmetic have been carefully developed by philosophers and scientists over time, playing a crucial role in problem-solving, from the simplest to the most complex challenges.

Advancements in studies on the history of science have led to a better understanding of the evolution of scientific and mathematical knowledge. Since the 1960s, the history of science has developed significantly, allowing for a more sophisticated and in-depth discussion about the identity of mathematics. Mathematical knowledge has often been regarded as a set of absolute and immutable truths, concealing a history rich in ruptures, transformations, and revolutions. According to Roque (2012), the past centuries have witnessed moments of profound change, in which old concepts and methods were questioned and replaced by new theoretical perspectives and methodological approaches that have influenced mathematical research and education.

Kuhn (2013), in 1962, proposed a revolutionary perspective on the nature of science, challenging the idea of linear and cumulative progress. That is, he argued that science advances through paradigm shifts – moments of profound transformation in which the foundations of a field of study are questioned and redefined. Kuhn had a significant influence on the history of science, leading to a reassessment of traditional narratives and a more dynamic and context-based approach to understanding scientific development. However, this also sparked greater interest in investigating previously overlooked historical periods that had a major impact on the development of science, particularly in the knowledge and teaching of mathematics.

DEVELOPMENT AND USE OF GEOGEBRA SOFTWARE IN MATHEMATICS TEACHING

The development and use of GeoGebra Software[2] illustrate an inspiring example of the application of Kuhn's notions of paradigm and scientific revolution in the field of mathematics. GeoGebra is a free dynamic mathematics software that allows for the "construction of various geometric objects, such as points, vectors, segments, lines, conic sections, function graphs, and parametrized curves, which can be dynamically modified" (Friske et al., 2016, p. 5). This educational software provides a set of commands related to mathematical analysis, algebra, linear algebra, analytic geometry, statistics, etc.

According to Estevam and Goldoni, GeoGebra (a blend of the words Geometry and Algebra) is a "free, multiplatform dynamic mathematics software that combines geometry, algebra, tables, graphs, statistics, and calculus into a single GUI[3]" (2014, p. 13). In the same perspective, according to Van-Dúnem, GeoGebra is defined as a

[2] GeoGebra - the world's favorite, free math tools used by over 100 million students and teachers

[3] Graphical User Interface

“dynamic mathematics software for use in a classroom environment, integrating geometry, algebra, and calculus” (2016, p. 19).

As explained by Costa and Santos, GeoGebra is presented as a platform that offers numerous possibilities and resources, ranging from the “traditional features of Dynamic Geometry software (points, lines, segments, rays, etc.) to the direct input of equations and coordinates” (2016, p. 34). In this regard, Silva and Fernandes also state that GeoGebra enables the “creation of materials that work more easily and quickly than other softwares, as it allows interactive constructions that facilitate the teaching of certain calculus concepts through dynamic visualization” (2017, p. 68).

The concepts presented by the authors converge significantly in describing GeoGebra as an essential tool for teaching dynamic mathematics. In line with these definitions, it is observed that GeoGebra is widely recognized as a free, multi-platform, and accessible software designed to facilitate the interactive construction and manipulation of mathematical objects.

Additionally, the authors highlight GeoGebra's ability to enable the construction and modification of various geometric objects, such as points, vectors, segments, lines, conic sections, function graphs, and parametric curves. This dynamic feature is essential for learning, allowing students to explore and visualize mathematical concepts in real time, fostering a deeper and more intuitive understanding.

The GeoGebra Software offers a robust combination of features that meet the needs of various branches of mathematics. Its ability to integrate geometry, algebra, calculus, and statistics into a single dynamic platform makes it an invaluable tool for educators and students. The interactivity and capacity for dynamic modifications foster a more engaging learning experience, while its accessibility and intuitive interface ensure that the software is widely adopted in diverse educational contexts.

Therefore, the aforementioned authors converge in their assessment and understanding that GeoGebra is an effective and versatile solution for teaching mathematics, highlighting its relevance and positive impact on modern education. This consensus reflects the importance of GeoGebra as a tool that not only facilitates the understanding of mathematical concepts but also enriches the learning experience, promoting a more interactive and integrated approach to mathematics teaching.

According to João (2021), the idea of using the GeoGebra application in the mathematics classroom has been gaining increasing prominence and strength. Numerous studies confirm this reality, consolidating its importance and the necessity of its inclusion in the daily didactic-pedagogical activities of both teachers and students, both inside and outside mathematics teaching.

As highlighted by Santos (2019), the project initially started with a desktop application. Currently, GeoGebra has expanded to mobile devices, with versions available on the Apple Store, Google Play, and Windows Store. The application continues to be improved to offer the best dynamic mathematics software and services for students and teachers worldwide.

According to Wolff and Silva (2013), the GeoGebra Software was created and developed by Professor Markus Hohenwarter and his team to be used in classroom environments at all levels of education. The first version was released in 2001 as part of a project for his master's thesis. Thanks to numerous research studies, Professor Hohenwarter received awards and financial support from international science academies and institutions. He even won the German Educational Software Award. Due to its significant positive impact on education worldwide, the software's creator later presented the same project in his doctoral thesis at the University of Salzburg, Austria.

Before the GeoGebra Software, the teaching and learning of mathematics were dominated by traditional methods, such as solving problems on paper, using blackboards, expository teaching, pencils, and rulers. From Kuhn's (2013) perspective, this traditional teaching and learning paradigm represents normal science, where teachers follow established practices to teach mathematics. However, the dominant paradigm had limitations, such as difficulties in visualizing abstract concepts, a lack of interactivity, and challenges in adapting teaching to the individual needs of students.

As mathematical concepts became more complex, especially at advanced levels such as calculus and geometry, students faced difficulties in visualizing and fully understanding them. Additionally, the lack of interactivity and personalization in traditional teaching led to an engagement crisis, with many students losing interest in mathematics.

Professor Markus Hohenwarter, in developing the GeoGebra Software, can be said to have introduced a new paradigm in mathematics education by providing an interactive digital environment where students can: a) visualize abstract mathematical concepts – create dynamic graphical representations of functions, geometry, and other mathematical topics; b) manipulate mathematical objects – explore relationships and mathematical properties intuitively by dragging and modifying objects on the screen; c) discover patterns and connections – construct mathematical knowledge through experimentation and independent investigation.

The GeoGebra Software has shifted the focus of mathematics education from memorizing rules and formulas to a more active, constructive, and investigative learning approach. As a new paradigm, GeoGebra: a) encourages students to develop a deeper conceptual understanding of mathematics, moving beyond mere memorization; b) stimulates creativity and curiosity, allowing students to explore different mathematical representations and relationships freely and critically; c) facilitates collaboration and teamwork, creating an environment conducive to student interaction, where they can share ideas and solve problems together.

The paradigm shift for the teaching of Geometry in the teaching of mathematics occurred with the Dynamic Geometry software. An example is GeoGebra, which currently encompasses more than Geometry. It is understood that this change improves the visualization and engagement of students and also allows them to be active participants in the construction of their own mathematical knowledge. The development and application of GeoGebra facilitate a theoretical and practical understanding of how Kuhn's theory of paradigms and scientific revolutions applies not only to scientific progress but also to advancements in education and technology.

An example of the use of GeoGebra, as a theoretical-practical application of Kuhn's theory (2013), is the Pythagorean theorem in the teaching of mathematics with traditional and technological tools. The Pythagorean theorem is a fundamental concept of geometry and has practical applications in various areas, such as in Architecture, Physics, etc.

Let's suppose that an electrician needs to change a light bulb on a pole and, to do so, lean a 6-meter-long ladder against the structure. The base of the ladder is positioned 2 meters away from the base of the post. How high does the ladder lean against the pole? (Consider that the pole and the floor form a 90° angle to each other and use the Pythagorean Theorem to solve the problem).

To solve this problem, the Pythagorean theorem is used, which states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

The resolution of the same problem, now using the GeoGebra Software, which allows visualization and verification.

Steps for solving the problem:

Having a right triangle formed by the wall, the floor and the stairs. The ladder represents the hypotenuse (6 meters), the distance from the wall to the base of the ladder is one leg (2 meters) and the height we want to find is the other leg.

a) Open GeoGebra[4]: access the GeoGebra website or open the application on your computer.
b) Create the points:
    - Click on the "New Point" icon (the symbol of a point).
    - Click on three different points in the workspace to create points A, B, and C. Point A will be the base of the wall, B the

[4] https://www.geogebra.org/?lang=pt_BR

base of the ladder, and C the point where the ladder touches the wall.

c) Create the segments:
   - Click on the "Segment" icon (a line segment).
   - Connect points A and B to represent the ground.
   - Connect points B and C to represent the ladder.
   - Connect points A and C to represent the wall.

d) Define the measurements:
   - Right-click on AB and select "Properties."
   - In the properties window, change the length to 3.
   - Right-click on BC (the ladder) and change the length to 5.

e) Calculate the height:
   - Click on the "Distance or Length" icon (a ruler).
   - Click on points A and C. GeoGebra will automatically calculate the distance between the two points, which is the height you are looking for.

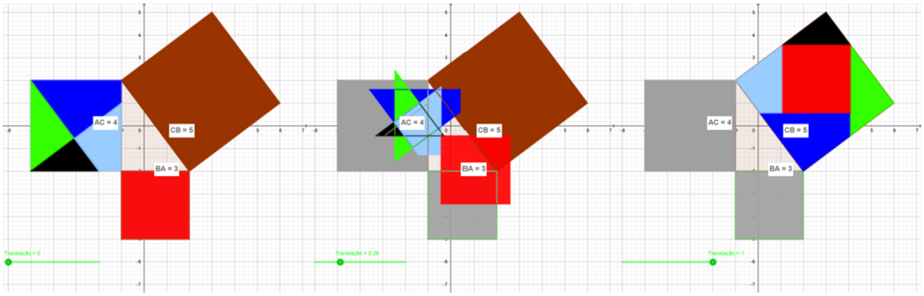
Using GeoGebra, students can construct right triangles and check the relationship ,c-2.=,a-2.+,b-2.. They can manipulate the lengths of the legs and hypotenuse and observe how the equation holds true. GeoGebra shows a right triangle with the correct measurements, and you can view the calculated height. Therefore, with GeoGebra, one can explore different scenarios and visualize the results clearly and intuitively.

The traditional method of solving the problem using the Pythagorean theorem, based on manual calculations and algorithmic procedures, according to Kuhn (2013), reflects normal science, where scientists (teachers and students) work within an established paradigm, using techniques and methodologies widely accepted by the scientific or mathematical education community.

With the construction of a learning object in GeoGebra it is possible for the student to build and visualize, in different sizes of right triangles, visualizing and calculating the Pythagorean theorem, as seen in Figure 3.

Figure 3

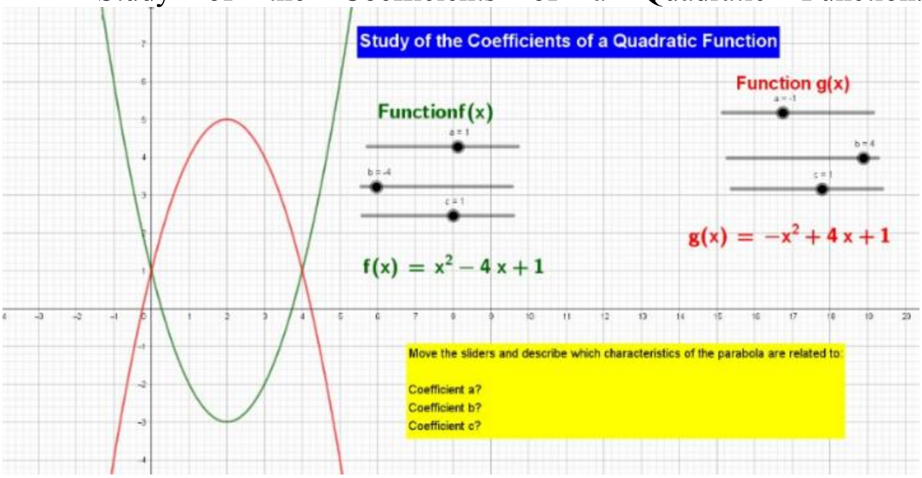
Pythagorean theorem built on GeoGebra software



According to Kuhn (2013), for a new paradigm to be accepted by the scientific or educational community, it must be capable of addressing the old demands of science – or, in this case, issues related to mathematics teaching – while also taking a step forward (solving new problems). In this context, students not only resolve persistent problems but can also use GeoGebra to tackle emerging questions, such as plotting graphs of linear and quadratic functions, exploring how coefficients affect the slope and shape of curves. They can solve quadratic equations visually by identifying roots through their intersection with the x-axis.

Figure 2

Study of the Coefficients of a Quadratic Function.



Using the GeoGebra Software represents the adoption of a new paradigm for problem-solving, where technological tools allow for a more dynamic and interactive visualization and understanding of mathematical concepts. This transition can be seen as a scientific revolution in the educational context, where new tools and methods are adopted to overcome the limitations of the previous paradigm.

The use of technological tools such as the GeoGebra Software empowers students, allowing them to explore mathematical problems in ways that were not possible under the old paradigm. They can visually verify results and gain a better understanding of concepts through interaction with the tool, promoting teaching and, consequently, a deeper and more meaningful learning experience.

Relating Kuhn's (2013) ideas to mathematics education provides an innovative approach to the process of knowledge construction, especially regarding the use of new technologies, such as the GeoGebra Software, to transform learning. This approach not only enriches students' educational experience but also reflects the dynamic nature and evolution of mathematics itself. Thus, by integrating and using new technologies, a scientific revolution in mathematics education is promoted, aligning with Kuhnian theory.

This epistemological model advocated by Kuhn (2013) challenges the traditional view of linear progress in science, highlighting the importance of scientific revolutions and paradigm shifts in the evolution of scientific knowledge. It emphasizes that the development and structuring of scientific and mathematical knowledge is not merely cumulative but rather marked by fundamental ruptures that radically alter the understanding of the nature of knowledge.

FINAL CONSIDERATIONS

Kuhn's theory (2013), with its concepts of paradigm and scientific revolution, offers a valuable perspective for understanding the evolution of mathematical knowledge and enriching the teaching of this subject. Applying these concepts to the history of mathematics reveals how new theories emerge and replace old ones, marking paradigm shifts, such as the transition from the geocentric to the heliocentric system and the introduction of infinitesimal calculus.

The implications for mathematics teaching are profound. Teachers can use Kuhn's epistemology to make their lessons more dynamic and meaningful, that is, adopting new methodologies that enhance the learning experience. Innovations such as GeoGebra, a tool for dynamically visualizing mathematical concepts, exemplify the type of technology that can foster greater interactivity and meaning in classroom environments. This pedagogical practice, in turn, will encourage students to question and explore different approaches and theories, which not only enriches learning but also promotes critical thinking and the ability to innovate in mathematics teaching.

Resistance to paradigm shifts in mathematics education often stems from an attachment to traditional methods and a lack of proper training. Implementing new approaches presents challenges, including the need for continuous teacher training, curriculum adaptation, and overcoming institutional resistance. However, by addressing these challenges, mathematics education can become more relevant, engaging, and effective in preparing students for a constantly changing world.

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STATEMENTS OF AUTHOR CONTRIBUTIONS

N.J.A. and M.A.A. jointly developed the research, organized the theoretical part, the methodological design, performed the data analysis, discussed the results and systematized the final version of the article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study are under the responsibility of T.F.L. and M.A.A. and may be made available upon reasonable request by others, by signing a liability waiver.

REFERÊNCIAS

Alves, M. A. & Tatsch, K. J. S.(2017). Epistemologia, história e ensino da matemática: reflexões sobre formação e aprendizagem significativa. *Revista de Ensino de Ciências e Matemática (REnCiMa)*, v. 8, p. 78-93. https://doi.org/10.26843/rencima.v8i3.1258

A lves, M. A. & Valente, A. R. (2021). A estrutura das revoluções científicas. *In: O estatuto científico da ciência cognitiva em sua fase inicial:* uma análise a partir da Estrutura das revoluções científicas de Thomas Kuhn [online]. Marília: Oficina Universitária; São Paulo: Cultura Acadêmica, 2021, pp. 25-52

Ausani, P. C. & Alves, M. A. (2020) Gamificação e ensino: o jogo dialógico como estratégia didática ativa e inovadora. *Research, Society and Development*, v. 9, p. e139962736-e139962756. https://doi.org/10.33448/rsd-v9i6.2736

Andrade, R. d. (2019). Resistência à ciência: Crise de confiança suscita debate mundial sobre como enfrentar ataques ao conhecimento científico. *https://revistapesquisa.fapesp.br/resistencia-a-ciencia/*

Bartelmebs, R. C. (2012). Resenhando as Estruturas das Revoluções Científicas de Thomas Kuhn. *Ensaio***,** *14,* 351-358.

Carvalho, M. (2018). Literatura e Resolução de Problemas Matemáticos no Curso de Pedagogia. In: R. F. Carneiro, A. C. Souza, & L. F. Bertini, *A Matemática nos anos Iniciaias do Ensino Fundamental: Práticas de Sala de Aulas e de Formação de Professores* (p. 180-187).

Costa, A. P., & Santos, M. C. (2016). Estudo dos quadriláteros notáveis por meio do GeoGebra: um olhar para as estratégias dos estudantes do 6º ano do ensino fundamental. *Revista do Instituto GeoGebra de São Paulo, v.5 n.2*, 3-17.

Estevam, M. I., & Goldoni, E. J. (2014). *O geogebra e a Matemática da educação básica.* Curitiba: íthala.

Friske et al., A. l. (2016). *Minicurso de GeoGebra.* Santa Maria: UFSM.

João, K. S. (2021). Formação de formadores em GeoGebra, uma oportunidade de formação contínua dos professores de Matemática do ensino secundário angolano: Estudo de caso na fase da familiarização do aplicativo. *SENSOS-e,* 50.

Kuhn, t. s. (2013). *A Estrutura das Revoluções Cientificas* (12ª ed.). (B. V. Boeira, & N. Boeira, Trads.) São Paulo: Editora Perspectiva.

Laudan, L., Donovan, A., Laudan, R., Barker, P., Brown, H., Leplin, J., Wykstra, S. (1993). *Mudança científica: modelos filosóficos e pesquisa histórica.*

Monteiro, R. (2020). É verdade que estamos vivendo uma Crise da Verdade? *Revista Brasileira de História da Ciência, 13*(2), 308-319.

Moro, L. & Alves, M. A. (2021). Pesquisa e reflexão sobre a formação e a prática docente: uma percepção dos egressos do PIBID/Matemática. *Revista de Ensino de Ciências e Matemática (REnCiMa),* v. 12, p. 1-22. https://doi.org/10.26843/rencima.v12n4a17

NCTM. (2007). *Princípios e Normas para a Matemática Escolar.*

Ostermann, F. (1996). A Epistemologia de Kuhn. *Cad.Cat.Ens.Fis, v.13,n3*, 184-196.

Roque, T. (2012). *História da matemática. Uma visão crítica, desfazendo mitos e lendas.* Rio de Janeiro. Zahar.

Santos, E. B. (2019). *Cálculo Diferencial e Integral: Uma abordagem prática mediante o uso do Software Geogebra.* Recife.

Silva, N. M., & Fernandes, D. M. (2017). O GeoGebra na aprendizagem das isometrias do plano com alunos do 6º ano.

Revista do Instituto GeoGebra de São Paulo,9657, v.6 n.2, pp 65-80.

Van-Dúnem, I. A. (2016). *A matemática e as tic no processo de ensino e aprendizagem. o geogebra no ensino de funções e gráficos de uma função* (Dissertação do Mestrado). Lisboa.

Wolff, M. E. & Silva, D. P. (2013). O software geogebra no ensino da matemática. *os desafios da escola pública paranaense na perspectiva do professor PDE,* 15.

Saito, F. História da matemática: uma visão crítica, desfazendo mitos e lendas (resenha crítica). *Revista Brasileira de História da Matemática*, São Paulo, v. 13, n. 26, p. 85–94, 2020. doi: 10.47976/rbhm2013v13n2685-94