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A look at activities using dynamic geometry software in textbooks: how are they characterized?

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ABSTRACT

Background: The National Textbook and Didactic Material Program (PNLD) 2020 was the first to foresee the development of competences and abilities that are advocated in the National Common Curricular Base, which includes specific guidelines about the use of Digital Technologies, among them the dynamic geometry software (DGS). **Objectives:** In that context, the present research aimed to characterize and understand the role of dynamic geometry software in the textbook proposed activities in the Elementary School, approved by PNLD 2020. **Design:** Four collections were submitted to a document analysis aiming at a methodological procedure which considered the construction and visualization associated with experimentation-with-technologies and mathematical investigation. **Setting and Participants:** Through that analysis, 36 activities with the use of DGS were identified and classified according to the methodological procedure created. **Data production and analysis:** The classification was the following: corroboration of conjecture understanding (16), mathematical discovery (13), domestication of technology (5) and simple observation (2). In the discussion about mathematical investigation, it was observed that only one activity had such approach, because the students were invited to justify the raised conjectures. **Results:** Results show that in three out of the four collectors studied, dynamic geometry software helps in the conjecture understanding, while, in one of them, it plays a more related role to mathematical discovery. **Conclusions:** When teachers become aware of those roles in the textbooks, they can make an adaptation or a restructuration of the activities, as they find it necessary, potentializing pedagogical gains in their classes. Furthermore, the importance of authors/publishers of textbooks developing propositions for the use of DGS, to create a more favorable environment for mathematical learning, is reinforced.

Keywords: Digital Technologies; PNLD; Elementary School; Mathematics Education.

Um olhar para as atividades com o uso do software de geometria dinâmica nos livros didáticos: como elas se caracterizam?

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RESUMO

Contexto: O Programa Nacional do Livro e do Material Didático (PNLD) 2020 foi o primeiro a prever o desenvolvimento das competências e habilidades previstas na Base Nacional Comum Curricular. Essa inclui orientações específicas sobre o uso de Tecnologias Digitais; dentre elas, o software de geometria dinâmica (SGD). **Objetivos:** Nesse contexto, a pesquisa que apresentamos buscou caracterizar e compreender o papel do software de geometria dinâmica nas atividades propostas nos livros didáticos (LD) dos Anos Finais do Ensino Fundamental aprovados no PNLD 2020. **Design**: Foi realizada uma análise documental em quatro coleções, tendo em vista um procedimento metodológico que considerou a construção e visualização associadas à experimentação-com-tecnologias e à investigação matemática. **Ambiente e participantes**: A partir dessa análise, foram identificadas 36 atividades com o uso do SGD e classificadas de acordo com o procedimento metodológico criado. **Produção e análise de dados:** A classificação foi a seguinte: corroboração da compreensão de conjecturas (16), descoberta matemática (13), domesticação da tecnologia (5) e simples constatação (2). No âmbito da discussão sobre a investigação matemática, observou-se que apenas uma atividade possuía tal abordagem, pois havia um convite aos estudantes para justificarem as conjecturas levantadas. **Resultados**: Os resultados mostram que, em três das quatro coleções estudadas, o SGD auxilia na compreensão de conjecturas, enquanto, em uma, ele desempenha um papel mais voltado para a descoberta matemática. **Conclusões:** Tendo ciência desses papéis nos LD, os professores podem realizar uma adaptação ou uma (re)estruturação das atividades, quando julgarem necessário, potencializando ganhos pedagógicos em suas aulas. Além disso, reforça-se a importância de que autores/editores de LD desenvolvam propostas que utilizem o SGD de forma a criar um ambiente mais propício à aprendizagem matemática.

Palavras-chave: Tecnologias Digitais; PNLD; Anos Finais do Ensino Fundamental; Educação Matemática.

TEXTBOOK AND DIGITAL TECHNOLOGIES

Brazil has one of the largest textbook evaluation and distribution programs in the world: The National Textbook and Didactic Material Program (PNLD). That program aims to offer, in a universal and freeway, didactic works – specially textbooks – to all students from Elementary School to High School in Brazilian public schools.

The PNLD also establishes guidelines to authors and publishers about the content those books should contain. Since 2014, those guidelines have specifically approached the inclusion of Digital Technologies (DT), involving the use of softwares, such as the dynamic geometry software (DGS) as part of the material evaluation and approval process.

Furthermore, the PNLD 2020 (the first one addressed to the Elementary School after the enactment of the National Common Curricular Base – BNCC, in 2018), supports that the main goal of the Program is to contribute to the development of BNCC (Brasil, 2018) competences and abilities, and the same happens with the PNLD 2024, again intended to the Elementary School.

As an example, number 4 general competence from the Base emphasizes: “understand, use and create digital technologies of information and communication in a **critical, meaningful**, **reflexive and ethic way** in several social practices (including school ones) [...]” (Brasil, 2018, p. 9, authors’ emphasis).

The BNCC described abilities show ideas that are aligned with digital competence: out of 121 Mathematics abilities for the Elementary School, 11 ones refer to Digital Technologies, from which eight specifically mention the dynamic geometry software. That means present textbooks must include activities that use that kind of software to be approved.

However, just mentioning Digital Technologies in books does not guarantee the authors are fully exploring their potentialities. In some cases, the Technologies may be incorporated just as a formal request to meet the PNLD demands, without necessarily contributing to some learning that promotes deep understanding of contents.

Despite the regulatory guidelines for textbooks established in the PNLD notices since 2014, researches show the materials do not always follow them. Gitirana, Bittar and Ignácio (2014), after having analysed the inclusion of Digital Educational Objects (DEO) in the approved books by PNLD 2014, identified discrepancies in that process. Although five collections have been approved, 200 Digital Educational Objects were submitted to evaluation, out of which only 16 were accepted and incorporated to the books, distributed in three collections – while two of them did not include any of those Objects.

The high number of rejected Digital Educational Objects (184) is mainly due to low conceptual quality, lack of efforts in the integration of Digital Educational Objects to the book pedagogical approach, conceptual mistakes, misleading, reinforcement of stereotypes, and the underuse of technological potentialities.

Many of evaluated resources did not show meaningful differentials compared to the printed book, compromising their role in teaching and learning.

Amaral-Schio (2018), when analysing activities from five collections of High School textbooks approved by PNLD 2015, focusing on the use of technologies for the development of Geometry knowledge objects, highlights the low presence of those tools in the materials. Besides, even when there is such presence, Ribeiro and Amaral (2016, p. 73), when researched three collections for the Elementary School, point out that Digital Educational Objects in the books do not favor interactivity and mathematical content is not always considered a priority (sometimes it is not even presented in a correct mathematical way).

In the light of such presented research results and questionings, we hereby share the results of a masters research (Carvalho, 2022) whose guideline question was: what is the role of dynamic geometry software in the proposed activities by Mathematics textbooks for Elementary School? Its objective was to characterize and understand the role of dynamic geometry software in proposed activities in the textbooks of the Elementary School, approved by PNLD 2020, in the geometry thematic unit chapters.

We understand such research brings contributions to Mathematics Education field, since it analyses a set of textbooks approved by a national program of public policy and they are in many Brazilian public schools. PNLD, in its different phases, evaluates the proposed books by publishers, and the ones approved and chosen by the teachers will be distributed in schools. However, there is not a Program evaluation process (Amaral et al., 2022).

Such researches may instigate the reflection on evaluation important aspects, such as the presence of technologies in the proposed activities. The questions discussed here may still expect teachers to ponder relevant aspects of teaching practice and PNLD authors and evaluation team to rethink the role of digital technologies in Basic Education Mathematics textbooks.

CONSTRUCTION AND VISUALIZATION ASSOCIATED TO EXPERIMENTATION-WITH-TECHNOLOGIES AND MATHEMATICAL INVESTIGATION

From the perspective of Dynamic Geometry, in which the movement is intrinsic to the processes that involve it, there is an important distinction between construction and drawing. According to Laborde (1998), construction refers to creating geometric figures that keep their fundamental properties, even when some of their elements – such as a point or a line – are dragged. In this way, the geometric figure is said to be resistant to dragging test (Zulatto, 2002). If the figure does not resist such movement, it is considered a drawing (Borba, Scucuglia & Gadanidis, 2020), as a photo of a geometric object.

That idea of movement can be explored through visualization because, when manipulating geometric figures dynamically, there is the possibility of visualizing and manipulating the objects. When planning an activity based on DGS, it is essential to explore the visual aspect, since the software offers almost immediate feedback. Such feedback requires interpretation from students, which makes visualization a fundamental aspect in Dynamic Geometry (Laborde, 1998).

Zimmermann and Cunningham (1991) define visualization as the process of forming images with a purpose. It is not just creating or observing images but using them to generate discoveries or promote mathematical understanding. To the authors, that image formation can be done mentally, with paper and pencil or through digital technology. In that context, visualization is directly related to a problem understanding and to mathematical discovery, demanding an active participation of those who are visualizing it.

Therefore, after building a geometric object, it is crucial to have an objective to visualize it, whether it is by raising hypotheses, making discoveries and validating properties or confirming previously formulated ideas. The interaction with the created object is fundamental, and that is possible by the manipulation offered by DGS.

As that exploration takes place, through manipulation, visualization occurs, since the first offers means for the connections to happen, essential process by the adopted visualization definition. It is a perspective in which visualization favors new mathematical understandings and, consequently, mathematical learning. That emphasizes the importance of exploration in mathematical activities.

According to Borba and Villarreal (2005), an experiment is made to verify the truth of a hypothesis in order to accept it or reject it, to find something unknown or to provide examples of a known truth. The experiment idea of discovering something unknown is associated to the idea of mathematical discovery, which is primordial in the production of mathematical senses, as Borba, Scucuglia and Gadanidis (2020) point out when they say that the discovery of patterns or singularities between representations of mathematical objects stimulates the production of mathematical senses. This way, there is an empirical dimension that involves both mathematical thinking and learning.

In the empirical dimension of mathematical learning, Borba, Scucuglia and Gadanidis (2020), through the theory of experimentation-with-technologies, stated that digital technological resources took a pivotal role due to their essentially experimental and visual character. So, the relevance of experimental and visual (already discussed) dimensions in the use of DGS is notorious.

In spite of that, it is possible to use Digital Technologies, and consequently the DGS, in a domesticated way (Borba, Scucuglia & Gadanidis, 2020). It means using the software in a way that the user keeps formerly done practices with conventional technologies intact; for instance, after building the graphic of a first-degree function by using paper and pencil, the user will do the same in a software. The DGS has a great deal of visual possibilities; not exploring that capacity, mainly in a topic the student knows about, results in a tendency to such domestication, not creating possibilities for the students.

Baldini and Cyrino (2012) point out that in learning environments, it is interesting to integrate mathematical investigation to softwares such as Geogebra, since this tool enables the creation, manipulation and exploration of mathematical situations. Besides, it favours the analysis, conjecture formulation, identification of regularities, result discussions and concept generalization.

From the conceptual point of view, investigating may be understood as the search for relations between mathematical objects, either known or unknown, aiming to identify their properties (Ponte, Brocardo & Oliveira, 2019). Those authors highlight that mathematical investigation is not necessarily associated to sophisticated problems, but rather to the formulation of open questions, i.e., questions that do not have only one answer and can be explored in different ways. That process allows new problems to emerge and demands fundamental and accurate answers whenever possible (Borba, Scucuglia & Gadanidis, 2020).

Mathematical investigation involves four main moments: (1) situation recognition, exploration and formulation of questions or initial concerns; (2) conjecture formulation; (3) testing and possible conjecture refinement; and (4) conjecture justification through demonstration and evaluation of the work done (Ponte, Brocardo & Oliveira, 2019). The authors emphasize that those moments, especially the first three ones, may occur simultaneously.

Among those four moments, the justification of conjectures, whose role was previously discussed, is emphasized. Ponte, Brocardo and Oliveira (2019) warn that such stage is – usually – left in a secondary place, or even forgotten, especially in school first grades. However, when structuring the mathematical investigation in stages, the authors make the importance of conjecture justification very clear in order that the investigative process does not become poor. This moment is essential to consolidate mathematical knowledge and to guarantee the proposed activities may promote a deeper and more meaningful investigation. As Zulatto (2002, p. 29) pointed out:

> Moving in that direction, raising hypotheses, making explorations, testing by dragging, may be an alternative for the use of softwares in Dynamic Geometry. However, in order that its use does not become an obstacle to demonstration, it is necessary to no longer emphasize confidence, which is achieved, many times, by handling the software, but rather to instigate students to explain the veracity of their conjectures, thus avoiding the demonstrations to be forgotten or left to a secondary place.

With this perception, it is possible to notice an approximation between experimentation-with-technologies and mathematical investigation. Besides justification, mathematical discovery is also seen associated to both, as Borba, Scucuglia and Gadanidis (2020) observe. They emphasize the use of a DGS may be meaningful for mathematical learning when the didactic-pedagogical scenario that involves the activities using those softwares covers the complexity of mathematical thinking, just like the fact that the search for developing such complexity is fundamental in the (re)elaboration of investigative activities.

The idea of (re)elaboration stems from a simplified categorization that Borba, Scucuglia and Gadanidis (2020) suggest to show a distinction between adaptation and (re)struturaction of an activity, which is described as follows: adaptation of an activity occurs when the activity objective, as well as its nature, are preserved as aspects related to construction that can be modified, not undergoing meaningful changes or reinforcing the experimental character of the activity. In the (re)structuration of the activity, the activity objective is kept, but the nature of the construction is not.

Such ideas support what is considered important in this research, since, with the objective of using the DGS according to its potentialities for mathematical learning, it is required the activities presented by textbooks to be adapted, structured or (re)structured, having in mind the experimentation-with-technologies and mathematical investigation, in accordance with the need.

Therefore, it is extremely important, when elaborating activities with the use of a DGS, that authors and publishers consider integrating construction and visualization through the exploration of their approaches, not suggesting the use of the software in a domesticated way, but rather offering the teacher propositions that can potentialize learning through its use. So, the characterization of such activities is primordial to understand the role DGS has played in activities presented in textbooks.

METHODOLOGY

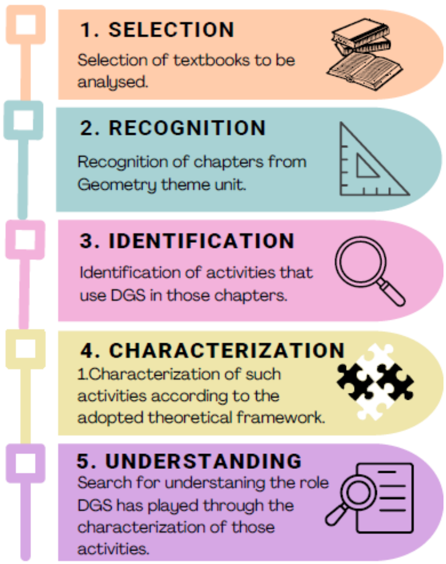
The present research adopts a qualitative approach, in which, according to Alves-Mazzotti (2001), the researcher is considered the main agent of investigation and must keep constant contact with the data source and offer detailed descriptions about the analysed elements.

So, in order to reach the proposed objectives in this qualitative research, considering that textbooks are the object of study, we believe a document analysis would be appropriate. According to Lüdke and André (2011, p. 38), that approach may be a valuable technique for qualitative data treatment, to reveal new aspects of a theme or problem.

According to Cellard (2008), the research based on document analysis must try to get the greatest possible amount of information from the analysed materials in order to obtain relevant elements that can build an adequate *corpus* for subsequent analysis. This way, the detailed exam is part of the researcher's adopted approach, which is related to textbooks, which will be the document source (Cechinel et al., 2016). With that base, the methodological path of this research is shown in Figure 1.

Figure 1

Methodological path of this research.



It was decided to analyse four collections out of the 11 ones approved for the Elementary School in PNLD 2020, which were the most recent ones when the research was started. Each collection includes four books designed for the sixth to ninth grade, totaling 16 analysed books. Of the more than ten million textbooks distributed in 2020 to Brazilian public schools, around 70% belong to those four collections, which highlights the representativity of those data (Brasil, s.d.). The books that were used in the present research were donated by a public school, and all of them were Teacher's Guides. Although the analysis has been developed in printed books, the images presented along this paper are from online books, aiming to offer a better image quality of the pages.

In the 16 analysed textbooks, the corresponding chapters to Geometry theme unit were identified, according to the organization presented by the developers in the initial pages, aligned with BNCC proposed structure. In those sections, activities that were developed with the use of DGS were searched, usually located in specific sections for that purpose. In total, 36 different activities were found. Each of them was named by a code consisting of two letters followed by a number, varying according to the collection.

One of the collections is *Apoema* (Longen, 2018), in which nine activities were found, named AP01, AP02, and so on, up to AP09. In the collection *The Conquest of Mathematics* (Giovanni Júnior & Castrucci, 2018), CM01 up to CM04; and in *Araribá Mais* (Gay & Silva, 2018), AR01 up to AR15; finally, *Teláris* (Dante, 2018), TE01 up to TE08.

In order to characterize them, and based on the theoretical framework, a methodological procedure was created (Carvalho, 2022), which basically consisted of two questions for each one of the 36 found activities: *1. Have the worked concepts in the activity been approached before, either in the book or in previous books from the same collection in the Elementary School? 2. Is there an invitation to exploration, through visualization and movement tests, being promoted by the activity?*

The exploration through visualization and movement occurs from dragging moving elements of a built object in the DGS, demanding to be mathematically interpreted. That process will be named visualization and movement test, since it involves the manipulation of the construction with the aim that the formed images help understand or discover concepts (Zimmermann & Cunningham, 1991).

Based on Borba and Villarreal (2005), it is understood that activities that explore either known or new concepts – as long as they present exploratory characteristics – fit in the experimentation-with-technologies approach. In those activities, interaction with the software provides tests and instantaneous visual feedback, favoring mathematical learning.

Considering the distinction made by the authors about previously approached knowledge for the student, it is necessary to distinguish between two different kinds of activities. When the activity invites exploration through visualization and movement, and it is based on unknown concepts to the student, it promotes *mathematical discovery.* In that case, mathematical results previously unknown are obtained, aligning with one of the exploration objectives: the formulation of new conjectures (Borba & Villarreal, 2005).

In turn, if the activity stimulates exploration, but works on concepts and conjectures already presented in textbooks, its role becomes to *corroborate the understanding of conjectures,* helping the student in the mathematical convincing process.

As for situations in which there is no invitation to exploration – where the experimentation-with-technologies is not present – a distinction between the kinds of activities is also observed. In case the activity deals with concepts that haven't been worked yet, its objective is restricted to *simple observation*, by using the DGS to present something previously unknown by the student, without promoting an exploratory involvement.

When the activity approaches known concepts and does not explore the constructed figures, it may be interpreted under the perspective of *domestication of technology*. The DGS offers a lot of visual possibilities, and not exploring such potential in a familiar context to the student may lead to a limited approach, without creating opportunities for more meaningful learning.

Therefore, a methodological procedure structured in a matrix 2x2 is proposed – as shown in Figure 2 – categorizing the analysed activities. The categories in the left column involve experimentation-with-technologies, while the ones in the right column do not show that approach.

Figure 2

Sistematização 2x2 das características gerais das atividades. (Carvalho & Amaral, 2024)

	WITH INVITATION TO EXPLORATION	WITHOUT INVITATION TO EXPLORATION
PREVIOUSLY PRESENTED CONCEPT	Corroboration to conjecture understanding	Domestication of technology
NOT PREVIOUSLY PRESENTED CONCEPT	Mathematical discovery	Simple observation

The classificatory difference between activities that corroborate conjecture understanding and the ones that have mathematical discovery approach is whether the concept worked in the assignment has already been previously approached in the textbook. However, it is important to point out that differentiation is not just represented by a change in the textbook pages. The position an activity has in a book reveals the authors' ideological or pedagogical intentions (Choppin, 2004). It is more than provision: it is about understanding how the authors see the purpose of an activity in the textbooks for the students, since they have been impregnated with those authors' conceptions of Mathematics teaching and learning, either explicitly or not (Borba & Villarreal, 2005).

Provided the approaches of experimentation-with-technologies and mathematical investigation complement each other, the first one is of importance to culminate in the mathematical demonstration (Borba, Scucuglia & Gadanidis, 2020). That stage is essential in the students' mathematical investigation (Ponte, Brocardo & Oliveira, 2019). As Mazzi (2018) points out, stimulating students to conjecture, test and be convinced of a result before the formal demonstration may arouse the interest for a more accurate justification.

For this reason, the research aims to discuss whether the activities promote mathematical investigation. In this regard, the following question is considered: (1) *Is there an invitation for the justification of raised conjectures?* If such invitation is made to students, since the experimentation-with-technologies has already been provided, then, the activity is said to also favor mathematical investigation.

Therefore, when analyzing the activities of the textbooks based on the fronts of construction and visualization, some adaptations will be presented in some propositions, according to the previously proposed definition. Such propositions aim at the improvement of those activities with the use of a DGS in order that the visual character of experimentation and investigation can be potentialized in them, having in mind that the students' mathematical investigations may considerably contribute to Mathematics learning and motivate them as well.

CHARACTERIZING THE ACTIVITIES

The 36 activities found in the four collections were characterized according to the four categories of analysis presented: corroboration to conjecture understanding, mathematical discovery, domestication of technology and simple observation. Next, there will be a discussion whether such activities have the mathematical investigation approach, from the view of the invitation to conjecture justification.

Experimentation-with-technologies

Table 1 shows the characterization of the 36 activities with the use of DGS. A meaningful variation among the collections is observed: *Araribá Mais* contains 15 activities; *Apoema*, nine; *Teláris*, eight and; *The Conquest of Mathematics*, four. Besides, each collection highlights different software roles. To better understand the role of DGS in the analysed textbooks, one activity of each collection in each one of the four categories will be exemplified – respectively – to show how roles differ depending on the collection.

Table 1

Characterization of the 36 activities found with the use of DGS (Adapted from Carvalho, 2022, p. 103)

	WITH INVITATION TO EXPLORATION	WITHOUT INVITATION TO EXPLORATION
PREVIOUSLY PRESENTED CONCEPT	Corroboration to conjecture understanding	Domestication of technology
	AP01, AP02, AP03, AP07, AP08, AR02, AR07, CM02, CM03, CM04, TE01, TE03, TE04, TE05, TE06, TE08 Total: 16 (44,4%)	AP04, AP05, AP06, AR05, TE02 Total: 5 (13,9%)

NOT PREVIOUSLY PRESENTED CONCEPT	Mathematical discovery	Simple observation
	AR01, AR03, AR04, AR06, AR08, AR09, AR10, AR11, AR12, AR13, AR14, AR15, CM01 Total: 13 (36,1%)	AP09, TE07 Total: 2 (5,6%)

The analysis of table 1 suggests – at first sight – that most activities of the four collections adopt the experimentation-with-technologies approach, given the concentration in the left column. However, when examining each collection individually, it is observed that they present distinct characteristics. Next, these aspects will be detailed, starting from the collection *The Conquest of Mathematics*, shown in Figure 3.

Figure 3

Classification of activities of The Conquest of Mathematics collection

	WITH INVITATION TO EXPLORATION	WITHOUT INVITATION TO EXPLORATION
PREVIOUSLY PRESENTED CONCEPT	CM02, CM03, CM04	-
NOT PREVIOUSLY PRESENTED CONCEPT	CM01	-

As Figure 3 shows, all activities of that collection follow the experimentation-with-technologies approach, which points to a valorization of that kind of interaction in Geometry teaching. However, it should be noticed that such collection contains the smallest number of activities involving DGS, which can restrict a broader exploration of that technological resource by students. Although literature emphasizes the relevance of such approach for students' mathematical learning, its presence should be increased throughout the collection, offering more opportunities for students to develop a continuous familiarity with the software.

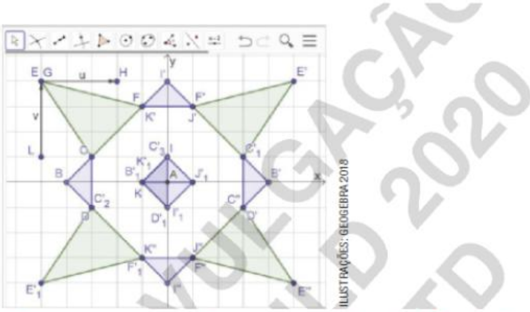
Nowadays, students would deal with Geogebra only once a year, since there is just one activity in each book of the Elementary School, which might be insufficient for the experimentation to be really used in a coherent way in teaching and learning process. So, a larger number of activities might favor a more progressive construction of geometric concepts, promoting more dynamic learning. As an example of the role played by the activities of that collection, Figure 4 shows the activity CM03, which helps in the conjecture understanding, allowing students to validate hypotheses from the interactive manipulation and observation of geometric objects.

Figure 4

Final part of CM03: 8th grade (Giovanni Júnior & Castrucci, 2018, p. 195).

1 On Geogebra, build the previously presented geometric pattern. You can follow step by step what Talita and Fernando used or make the geometric transformations in another order. Personal Answer.

2 After building the geometric pattern, by using the tool [cursor icon] click on a vertex of one of the first built polygons and drag. See the example below.



What have you noticed? Possible answer: the changes made in the first built polygons occur in the other polygons obtained as transformation in the plane of the first ones.

Prior to what is shown in Figure 4, a construction of a geometric pattern made by Talita and Fernando is presented. It is shown that they were given a challenge to make a geometric pattern by using symmetries of reflection, translation and rotation in the DGS, with restriction to use just determined tools and each of them maximum twice. To that end, they initially built two triangles and applied reflections related to x and y axes. Next, they made a translation and a rotation of one of the triangles. Finally, they applied a new translation and reflected the obtained triangles to make the intended geometric pattern.

The authors show that construction step by step and – as it can be seen in Figure 4 – they ask students to do it in a similar way, and they can change the order of geometric transformations. After that, in question 2, they ask them to drag the vertexes to see what happens. It is observed that it is not possible to be done with all vertexes; therefore, students could only drag some of them, depending on the performed construction.

Initially, they ask students to build the geometric pattern that was previously presented, but the focus of the activity in geometric transformations does not necessarily require students to build that exposed geometric pattern. They could create other ones and, similarly, notice that dragging one of the vertexes implies dragging the reflected, rotated and translated geometric drawing through DGS transformation tools, even though they were applied in different orders.

In the question "What have you noticed?", after dragging the vertexes, the activity instigates an active visualization to try to notice what happens when such dragging is done. Still, by the red answer of the textbook, it is observed that students are expected to give a generic and superficial answer. Such activity could be adapted to questions about, for instance, the distance from the dragged vertex to x or y axis, and the distance from the reflected vertex to the same axes.

That activity emerges after the explanation about isometries, at the end of the chapter, right before the review activities that, traditionally, are at the closing of a chapter. So, it is observed that the use of DGS here is less emphasized. This aspect is reinforced by the excerpt immediately presented before that activity in the textbook, where the elaborators state the following:

> The objective of the proposed activities is that students, by using their geometric instruments, will be able to work on geometric transformations on the plane and notice the construction differences in each of them. **The use of dynamic geometry softwares may help in the construction of isometries.** But it is important that stage of construction, by using graph paper, ruler, compass and squares, be worked in the classroom (Giovanni Júnior & Castrucci, 2018, p. 193, authors' emphasis).

That activity could be restricted to a simple visualization, without giving a specific purpose to drag the vertexes, even if such purpose were stimulating creativity. However, it could be adapted to deepen its objectives, by balancing the use of DGS and conventional tools, such as graph paper, ruler, compass and square, emphasizing that each one provides different forms of exploration. So, the activity took on the role of corroborator of conjecture understanding, since the exploration had already been done in the textbook, but the use of DGS allowed to reinforce that verification and potentially facilitate students' conviction.

By focusing on *Apoema* collection, Figure 5 shows it does not contain activities that promote mathematical discovery, just one that starts from a non-previously presented concept (AP09), and there is a balance between activities that have the approach of experimentation-with-technologies (five out of nine) and the ones that do not have it (four out of nine), of which three have the role of domestication of technology (AP04, AP05 and AP06) and one of simple observation (AP09).

Figure 5

Classification of activities of Apoema collection

	WITH INVITATION TO EXPLORATION	WITHOUT INVITATION TO EXPLORATION
PREVIOUSLY PRESENTED CONCEPT	AP01, AP02, AP03, AP07, AP08	AP04, AP05, AP06
NOT PREVIOUSLY PRESENTED CONCEPT	-	AP09

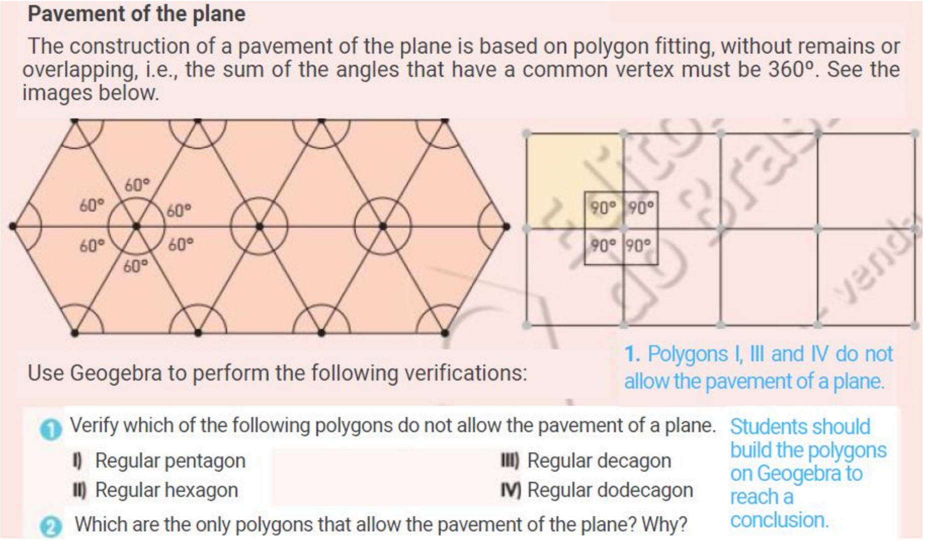
AP09 is based on a pavement of a plane with polygons in a real situation, but from a theoretical point of view, aiming that students build mosaics and see which polygons allow paving a plane, in groups of three or four students. To this end, it was necessary to consider that a pavement of a plane is just possible with those regular polygons that have internal angles divisible by 360.

Initially, the activity named "Exercise – building regular polygons" asked students to use the 'regular polygon' tool from Geogebra, select two defined points in one of the axes with measure 2, and type '3' in the showing dialogue box, i.e., creating an equilateral triangle of side 2. It was required that process to be repeated to create a square and a regular hexagon.

It is important to emphasize that this is really the construction of regular polygons; even when moving one of the vertexes of the presented polygons, the geometric figures remain unchanged (Laborde, 1998). However, the proposed exercise objective is to learn how to use Geogebra 'regular polygon' tool, and not the construction of that polygon independently. Next, the pavement of the plane is required, as it is shown in Figure 6.

Figure 6

Part of AP09: 9th grade (Longen, 2018, p. 197)



The AP09 is described as based on the EF09MA15 ability: "Describe, in writing and through a flow diagram, one algorithm for the construction of a regular polygon whose side measure is known, by using [...] softwares (Brasil, 2018, p. 319). This way, it is possible to notice that the activity follows exactly the BNCC ability, describing, in writing, the algorithm for the construction, in this case, of an equilateral triangle whose side 2 is known. However, it is important to highlight that the Base abilities are focused on the student, i.e., the activity should help students in the elaboration of that description, and not do the job in their place, which ends up not favoring the learning process.

From the options presented in question 1, it is required the simple observation that only the mosaic with the regular hexagon is possible, since the mosaic with the equilateral triangle and the square has already been presented in the figures. That was not previously approached in the textbook, but there is no invitation to exploration, i.e., students are not expected to reach that conclusion themselves when they drag the moving elements. The possibility of creating mosaics just when the sum of internal angles with common vertex is 360º has already been mentioned. When students know, for instance, the value of internal angles of a regular pentagon, decagon or dodecagon, they can already answer that question through calculations, and so, AP09 has the role of simple observation about the question of polygon internal angles summing 360º in common vertexes for students by using Geogebra.

The activity could be restructured by not mentioning the 360º sum before questions 1 and 2, but it could ask students about that characteristic and expect them to reach that conclusion from the developed construction and exploration. It could also leave the measure of those polygon sides open, since they have no influence on that property.

Now, the classification of activities of the collection *Araribá Mais* will be shown in Figure 7. Unlike *Apoema*, it is possible to notice that collection caused the biggest influence in the left column of Table 1. First, because it was the collection that presented the most activities with the use of DGS (15 in comparison to nine, eight and four), but also because, with one exception, all the other activities have the approach of experimentation-with-technologies (14 out of 15), with an interesting concentration of 12 out of 14 with the role of mathematical discovery.

Figure 7

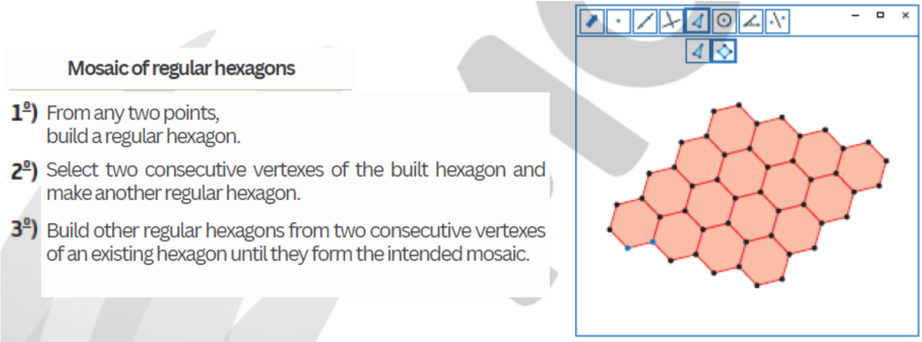
Classification of activities of Araribá Mais collection.

	WITH INVITATION TO EXPLORATION	WITHOUT INVITATION TO EXPLORATION
PREVIOUSLY PRESENTED CONCEPT	AR02, AR07	AR05
NOT PREVIOUSLY PRESENTED CONCEPT	AR01, AR03, AR04, AR06, AR08, AR09, AR10, AR11, AR12, AR13, AR14, AR15	-

Figures 8 and 9 exemplify part of one out of those 12 activities classified as mathematical discovery, AR06. It also proposes the construction of mosaics composed of different polygons by using the pre-defined function ‘Regular polygon’, and from this, the discovery of internal angles of such polygons, without the use of formulas.

Figure 8

Part of AR06: 7th grade. (Gay & Silva, 2018, p. 201)

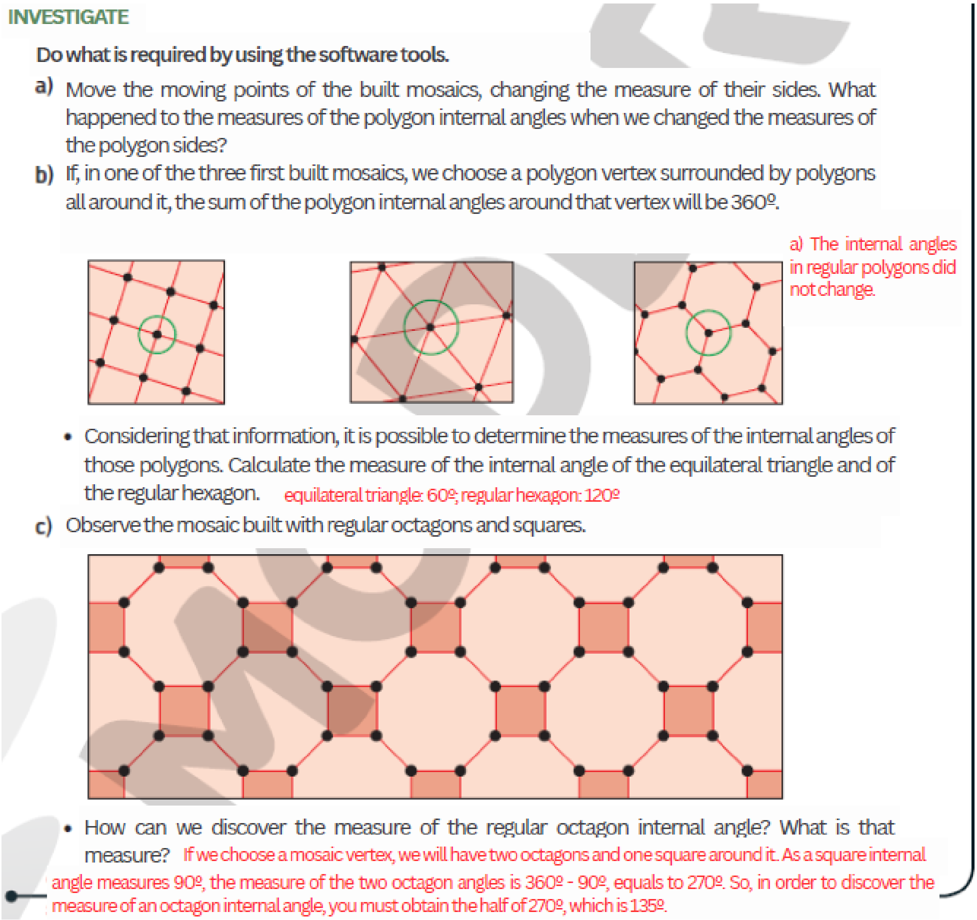


Contrasting to the prior activity – AP09 – it is observed that, when students are asked to build the mosaic with the regular hexagon, neither the polygon side measure nor the way to use the pre-defined function to build that polygon is specified. Besides, there is no clear guideline about the location of the two necessary initial points for the construction; these aspects do not have any influence on the intended final construction. After asking for the construction of mosaics with squares, equilateral triangles, regular hexagons and a compound of regular octagons and squares, the activity leads to the section 'Investigate', shown in figure 9.

In the a) item, students are encouraged to notice that, regardless of the modifications in the polygon sides, the internal angles of regular polygons remain invariable. In the following items, students are invited to calculate the values of those angles, which are determined with basis on the first three mosaics. Item C proposes an additional challenge, asking students to combine the previously calculated value of a square internal angle with the angles of two octagons. That process is directly related to EF07MA27 ability: "Calculate measures of regular polygon internal angles, without the use of formulas, and establish relations between polygon internal and external angles, preferably linked to the construction of mosaics and tiles" (Brasil, 2018, p. 309).

Figure 9

Final part of AR06: 7th grade. (Gay & Silva, 2018, p. 202)



INVESTIGATE

Do what is required by using the software tools.

a) Move the moving points of the built mosaics, changing the measure of their sides. What happened to the measures of the polygon internal angles when we changed the measures of the polygon sides?

b) If, in one of the three first built mosaics, we choose a polygon vertex surrounded by polygons all around it, the sum of the polygon internal angles around that vertex will be 360º.

a) The internal angles in regular polygons did not change.

- Considering that information, it is possible to determine the measures of the internal angles of those polygons. Calculate the measure of the internal angle of the equilateral triangle and of the regular hexagon. equilateral triangle: 60º; regular hexagon: 120º

c) Observe the mosaic built with regular octagons and squares.

- How can we discover the measure of the regular octagon internal angle? What is that measure? If we choose a mosaic vertex, we will have two octagons and one square around it. As a square internal angle measures 90º, the measure of the two octagon angles is 360º - 90º, equals to 270º. So, in order to discover the measure of an octagon internal angle, you must obtain the half of 270º, which is 135º.

It is interesting to notice that, as there is no previous approach of the values of regular polygon internal angles, students are invited to discover such values from a construction they make themselves; even by moving the sides of such polygons, the angles do not change. This way, such discovery may arouse the production of mathematical senses to students. In spite of that, there is room for the activity to be improved, since it is possible to allow students to be more creative, searching for mosaics in their own way and also realizing that not all regular polygons allow such tiling.

When analyzing the referred collection, it is possible to notice that difference in the pedagogical approach the authors propose in contrast to the other collections. By corroborating what Hohenwarter et al. (2008) defend, instead of giving students the answer to a problem whose answer they do not know, first, such explorations allow a more meaningful introduction to an abstract concept. Finally, Figure 10 brings the classification of the last analysed collection, *Teláris*.

Figure 10

Classification of activities of Teláris collection

	WITH INVITATION TO EXPLORATION	WITHOUT INVITATION TO EXPLORATION
PREVIOUSLY PRESENTED CONCEPT	TE01, TE03, TE04, TE05, TE06, TE08	TE02
NOT PREVIOUSLY PRESENTED CONCEPT	-	TE07

It is emphasized that, just like *Apoema*, this collection does not propose activities that promote mathematical discovery, which could enrich students' learning. Focusing on TE02, this is an activity that starts from a previously presented concept, and without an invitation to exploration, i.e., playing the role of domestication of technology.

In that activity, students are invited to use Geogebra software to build quadrilaterals. The first step is to build a pair of parallel lines by using pre-defined functions, like the options "Line" and "Parallel line". The second step is to build two perpendicular lines to those parallel lines. Finally, in the third step, students must use "Polygon" tool to connect the intersection dots of the built lines, forming a polygonal region. The assignment culminates in the reflection about the kind of formed quadrilateral, based on the obtained intersections.

This activity is based on the EF06MA22 ability: "Use instruments such as rulers and squares, or softwares, to represent parallel and perpendicular lines and construction of quadrilaterals, among others" (Brasil, 2018, p. 303), and its continuance, from the fourth step, can be seen in Figure 11.

Figure 11

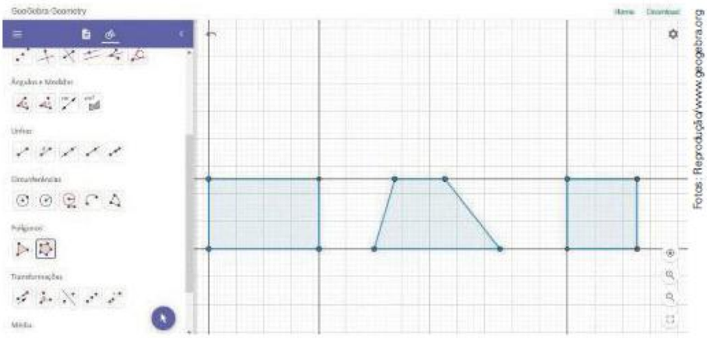
Final part of TE02: 6th grade. (Dante, 2018, p. 155)

4th step: By using the pairs of lines you initially built and the “Polygon” option, build a trapeze.

5th step: By using again the pair of parallel lines you initially built, you can also build a square. To this end, we are going to use another option for polygon building from the tool menu.

At first, make another perpendicular line to those parallel lines. Then, click in “Regular polygon”, select the 2 intersection points of that perpendicular line with the parallel lines and type “4”.

You will obtain quadrilaterals like these ones.



However, there is no interaction with those constructions or even confirmation that they are really the constructions mentioned. There is no request for moving any of the moving points either, in order that students themselves would do the dragging test, i.e., that they would notice that the mentioned polygons would be in the constructions, even if the moving points were moved.

So, this is an example of domestication of technology, in which the DGS potentialities are not explored; besides, there is little or no interaction with the performed constructions, thus avoiding a deeper exploration. Furthermore, it approaches concepts that have already been worked in the textbook former pages. Therefore, the precariousness of activities in this format is reaffirmed, since they use Digital Technologies only as a pretext not to work with conventional technologies, such as paper and pencil. Not incorporating such potentialities may be considered an inefficient use of the DGS.

Mathematical investigation

Having in mind the discussion about the activities inviting students for justification of conjectures, it was observed that only one activity (AP08) makes such invitation. In several instances in the collections there is a demonstration of such conjectures, before or right after the activities, except for that one, always from the textbook itself, not from the students.

The referred activity starts by asking participants to make groups of three or four students and to use ruler, compass, paper and pencil to develop it. So, there are instructions for the first part of the activity, which is not to be initially developed through Geogebra. Students are invited to make a circumference, a quadrilateral inscribed in it, and to measure the four internal angles with the help of a protractor.

That first activity expects students to notice that the sum of the opposed angles of that quadrilateral is 180º, even with possible imprecision mistakes (Longen, 2018). At the end of the first part, students are asked to repeat such procedure by making other circumferences and quadrilaterals to see that the sum of the opposed angles is still 180º. The second part can be seen in Figure 12.

Figure 12

2nd part of AP08 activity: 9th grade. (Longen, 2018, pp. 102-103)

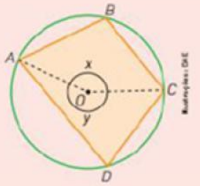
Instructions for the 2nd part

1. Read the following property.

Property

If a quadrilateral is inscribed in a circumference, its opposed angles are supplementary. Reciprocally, if the angles of a convex quadrilateral are supplementary, then the quadrilateral can be inscribed in a circumference.

2. To demonstrate that the opposed angles of a quadrilateral inscribed in a circumference are supplementary, make a drawing like the one in the following example:



To demonstrate that property, you can use the relation between inscribed angle and central angle of a circumference.

3. Try to demonstrate algebraically that B and D angles are supplementary. Exchange ideas with your classmates about how it can be done and present the justifications to all class. See the answer in the Teacher's Guide.

In the 2nd part it is possible to see the students are asked to demonstrate, in groups, the property that was indicated in the 1st part, providing an initial guideline to use the previously seen and demonstrated property in the textbook about the relation of the central angle being the double of the inscribed angle in a circumference.

Finally, the activity presents its 3rd and last part to be performed on Geogebra. First, it is observed that the activity brings this part in order to verify the previously mentioned property. The same construction made in pencil and paper is requested, now with the use of DGS through the angle tool. It was not required to drag the construction points till then. Next, some final questions are made in order that students become convinced of the relation of opposed angles of the inscribed quadrilateral in the circumference summing 180º, requiring some vertexes of the quadrilateral to be dragged, verifying that the property is always valid.

As for that activity, it is observed that the author's objective is to use Geogebra as a way to verify the property, as it is shown in the beginning of the 3rd part: "By using Geogebra software, you will verify the previously mentioned property" (Longen, 2018, p. 103). However, it is possible to question the reason for repeating the same construction with conventional resources and later with Geogebra. In this way, this sequencing reduces the relevance of using the DGS, since the student would be redoing something that may have already convinced them through active visualization.

The students' manual work (to be done in pencil, paper and protractor and repeated a few times) could be reduced if it were done with Geogebra help, as time would be saved, since this is one of the benefits of such tool (Borba; Scucuglia; Gadanidis, 2020). Besides, there would be the possibility of instantaneous visual feedback for students, with fewer imprecision mistakes, naturally found when conventional materials are used.

Since the usual is to demonstrate something that is already convinced to be true (Hanna, 2000), we suggest an improved adaptation of the activity, as pointed out by Borba, Scucuglia e Gadanidis (2020), preserving the mathematical and pedagogical objectives and nature, but in need of changes in the construction of such activity to reinforce the necessary experimental character in activities with the use of DGS. So, according to literature, the 3rd part could be the starting point, not the verification point, since after the property being tested several times that the sum of opposed angles is 180º in the quadrilateral inscribed in the circumference, from active visualization, students could be convinced it was true, and then could try to justify such property algebraically, as it was requested in the 2nd part. Therefore, the approach of mathematical investigation presented in the activities with the use of DGS in the analysed textbooks was discussed, focusing on the invitation to conjecture justification.

UNDERSTANDING THE ROLE OF DGS

From the methodological procedure, it was possible to understand that the role of DGS in three out of the four collections is – mainly – of contributing to understand conjectures; in only one, it is of promoting mathematical discovery – *Araribá Mais* collection. Since BNCC and, consequently, PNLD do not specify the activities with the use of DGS, it is possible to notice different amounts and different approaches of activities when they are compared.

The Conquest of Mathematics collection was the one that presented the fewest number of activities with the use of DGS – four in total, one in each year. As it was the most chosen /distributed collection in 2020, almost half the total, it was expected to offer more activities to give students the opportunity to use DGS more often. It also draws attention to the fact that three of them start from a familiar concept, with corroboration and understanding of conjectures. It is also possible to notice that those ones always meet at the end of the chapters where they are present, before the chapter review exercises, approaching already broadly explored concepts in the textbook previously. CM01 is an exception because it approaches a non-presented concept till then (with the role of mathematical discovery), however, just as a matter of apparent curiosity, since the new topic does not have to be approached in the 6th grade and the book does not mention it anymore. So, it is possible to question the decision of the authors/publishers regarding the approach of activities with the use of DGS, which does not seem to be a priority if compared to conventional resources.

In *Apoema* collection – for example – there was no approach of mathematical discovery in any of its nine activities and we can notice the lack of exploration of the software potentialities (three with domestication of technology and one with the role of simple observation) presenting several questions by using only the DGS pre-defined functions, as well as not inviting for dragging in order that students could explore the geometric objects that were being built. Besides, only one activity started from a not yet approached concept in the textbooks: the collection does not use DGS as a starting point for experimentation by students, bringing previously performed activities with the use of conventional resources (five with the role of corroboration to conjecture understanding).

In the *Araribá Mais* collection, the 15 activities with the use of DGS – except for one – present invitation to exploration; in addition, 12 start from a non-approached concept in the textbook, favoring the mathematical discovery. The collection shows the use of the software in a more potentialized way when it proposes its use as the starting point and creates a more favorable environment to experimentation by students, containing several invitations for conjecture elaboration and not providing previously defined answers.

Last, in *Teláris* collection, two activities that did not have invitation to exploration were presented (one of domestication of technology and another of simple observation). All eight activities had clear objectives, but, what called the most attention in the collection was that there were many questions that did not have an open character, restricting students about where they should put a point, or the specific measure they should make a segment in, or how they should call a constructed object; all of those elements were not essential for the activity mathematical objectives; besides, there was only one activity that started from an unknown concept.

In the discussion about stimulation to mathematical investigation, it was also possible to identify that none of the 36 activities in the four collections had that approach, mainly because there was no invitation to justification of the conjectures that were raised (fourth point of investigation). The only activity that contained that invitation needed some improvement, since Geogebra was being used just as a way to verify that the property was valid, after the demonstration invitation, decreasing the relevance of the DGS.

FINAL CONSIDERATIONS

The results of the research we share here show that it is fundamental that authors/publishers of textbook collections propose activities that use the DGS in a way to create a favorable environment for mathematical learning, preventing it from being used in a limited and domesticated way, just as a simple observation tool. The use of DGS in a non-potentialized way – as a resource that seems to be present in the activities just to guarantee its approval by PNLD – reduces its creative potential. Therefore, we understand that the responsibility for that approach also goes right on the Program.

Going back to the first Program evaluation criterion, it is clear that the objective is to make sure that the materials will contribute to the development of abilities and competences expected by BNCC, which, in turn, offers limited guidelines regarding the use of DGS. In PNLD context, it is possible to see the presence or absence of certain technologies; however, in order to offer a quality textbook in the classroom, not only the presence of digital technologies is necessary, but also their quality. Making digital technologies compulsory in textbooks was a major advance, but stopping here would be a backward step, since digital technologies are in constant evolution and the mere presence of those tools does not ensure the activities will promote a favorable environment for mathematical learning.

In the Mathematics Education area, the use of DGS – in its interactive and visual approach – has potential to increase the way the geometric concepts are taught and learned. By offering tools that allow students to explore those concepts in a dynamic way, the focus of the present research, which is understanding the role of DGS in textbook activities, is essential to identify how those tools have been used and to what extent they need improvement. That process provides valuable information about practices teachers can adopt to potentialize students' learning.

Furthermore, important questions emerge for future researches, such as the analysis of textbooks of other segments of Basic Education, or the investigation on how the role of DGS may vary in other collections of the Elementary School in the PNLD 2020 enactment. And in High School, how are the activities developed with those resources? How may new roles of DGS emerge in the published PNLD 2024 books? Were there modifications in the activities with the use of DGS? And how are the adaptations regarding other digital technologies? Would the roles discussed here be applicable to other technologies presented in the books? Such questions are pertinent and open the way to new researches, expecting the textbook field to continue expanding and evolving in Mathematics Education in the years to come.

AUTHOR STATEMENT AND CONTRIBUTION

AMC designed the study within her master's research and performed the production and initial data analysis. RBA guided the research, participated in discussions about the data, as well as actively contributed to writing and reviewing this manuscript. Both authors approved the final version of the present work.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this paper, because it is research of publicly available bibliography.

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