ISSN: 2178-7727
DOI: 10.64856/acta.scientiae.8268

Contemporary Transversal Themes: contextualization for the teaching of functions in High School aiming at Critical Mathematics Education

Rosângela Ferreira Domingues [a]
Clarissa de Assis Olgin [b]

[a] Secretaria de Estado de Educação de Mato Grosso (SEDUC/MT), Rondonópolis, MT, Brasil
[b] Universidade Luterana do Brasil (ULBRA), Programa de Pós-Graduação em Ensino de Ciências e Matemática (PPGECIM), Canoas, RS, Brasil

ABSTRACT

Background: The content of functions permeates the entire High School mathematics curriculum, and its applicability is perceived in several everyday situations. Due to its importance, there is a need to seek strategies to improve the teaching and learning process of students. **Objectives**: To analyze the application of a didactic sequence with the study of functions developed in a contextualized way, using Contemporary Transversal Themes (TCT), exploring the different representations, aiming to enhance the teaching process, seeking critical formation for High School students. **Design**: The methodology adopted was qualitative, with interpretative analysis of the data. **Environment and participants**: The research site was a state school in Rondonópolis/MT. There were 20 students in the 3rd year of High School. **Data collection and analysis:** Data collection and analysis were performed based on the application of the didactic sequence. **Results**: The didactic sequence allowed the exploration of learning environments and other aspects of Critical Mathematics Education (CME), as well as the use of Register of Semiotic Representation (RSR). **Conclusions**: The didactic sequence presented positive results, as it was possible to verify that there were significant advances regarding the function content, mainly from the use of RSR. In addition, the work with TCT and with the CME assumptions indicates that democratic competence was developed, providing knowledge and skills so that the participating students can act as critical and responsible citizens in their actions in the various sectors of society.

Keywords: High School; Critical Mathematical Education; Contemporary Transversal Themes; Register of Semiotic Representation; teaching and learning of functions.

Corresponding author: Rosângela Ferreira Domingues.
Email: rosangela.domingues@rede.ulbra.br

Temas Contemporâneos Transversais: contextualização para o ensino de funções no Ensino Médio visando uma Educação Matemática Crítica

RESUMO

Contexto: O conteúdo de funções perpassa todo o currículo de Matemática do Ensino Médio e sua aplicabilidade é percebida em várias situações do cotidiano. Por sua importância, há a necessidade de se buscar estratégias de como melhorar o processo de ensino e aprendizagem dos estudantes. **Objetivos**: Analisar a aplicação de uma sequência didática com o estudo de funções desenvolvido de forma contextualizada, utilizando os Temas Contemporâneos Transversais (TCT), explorando as diferentes representações, visando potencializar o processo de ensino e aprendizagem, buscando uma formação crítica aos estudantes do Ensino Médio. **Desenho**: A metodologia adotada foi qualitativa, com análise interpretativa dos dados. **Ambiente e participantes**: O local da pesquisa foi uma escola da rede estadual de Rondonópolis/MT. Os participantes foram 20 estudantes do 3º ano do Ensino Médio. **Coleta e análise de dados**: A coleta e análise de dados foram realizadas a partir da aplicação da sequência didática. **Resultados**: A sequência didática permitiu a exploração dos ambientes de aprendizagem e outros aspectos da Educação Matemática Crítica (EMC), bem como a utilização dos Registros de Representação Semiótica (RRS). **Conclusões**: A sequência didática apresentou resultados positivos, pois foi possível verificar que houve avanços significativos referentes ao conteúdo de função, principalmente a partir do uso dos RRS. Além disso, o trabalho com os TCT e com os pressupostos da EMC, indica que foi desenvolvida a competência democrática, proporcionando conhecimentos e habilidades para que os estudantes participantes possam vir a agir como cidadãos críticos e responsáveis na atuação deles nos diversos setores da sociedade.

Palavras-chave: Ensino Médio; Educação Matemática Crítica; Temas Contemporâneos Transversais; Registro de Representação Semiótica; ensino e aprendizagem de funções.

INTRODUCTION

The National Common Curricular Base (BNCC) presents the Contemporary Transversal Themes (TCT) as an extension of the already well-known Transversal Themes found in the National Curriculum Parameters (PCN). Thus, the BNCC advocates that these TCT should be addressed in a way that contextualizes the teaching of content across different areas of knowledge.

The teaching of Mathematics requires a continuous pursuit of improvement in the implementation of educational practices that promote a more flexible and contextualized curriculum, providing students with opportunities for active and participatory learning. For Basic Education teachers, when developing this curriculum, it is crucial not only to master the subject content but also to understand the various ways in which students assimilate and construct knowledge.

In the pursuit of a contextualized teaching approach, during the development of the first author's doctoral research, a study was designed that integrates Contemporary Transversal Themes (TCT) with the mathematical concept of function. This integration aims to foster the critical competence of High School students through the principles of Critical Mathematics Education (CME) from the perspective of Skovsmose (2001, 2010, 2015). Thus, an investigative study was conducted through the implementation of a didactic sequence, with the objective of examining how the study of functions, taught through activities contextualized by the TCT, contributes to students' development, thereby promoting a CME approach.

CONTEMPORARY TRANSVERSAL THEMES

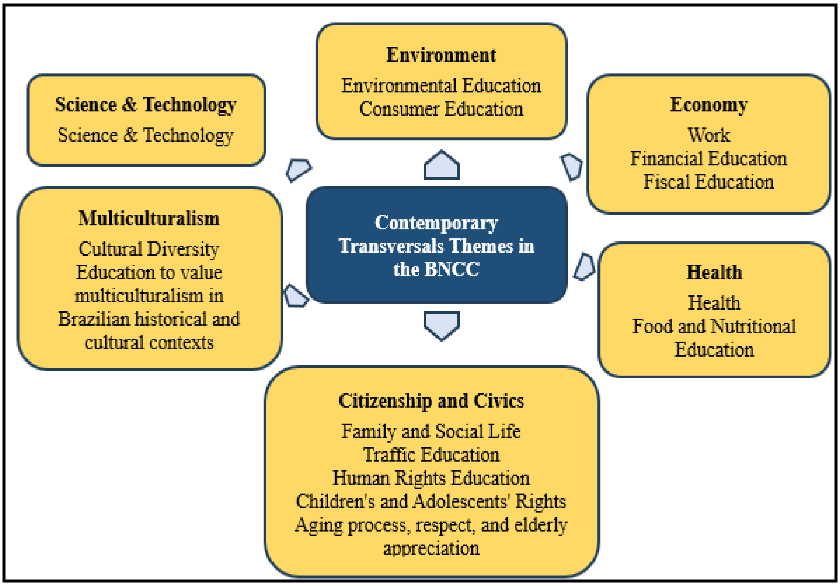
The Contemporary Transversal Themes (TCT) are present in the BNCC (Brazilian National Common Core Curriculum) as mandatory and guiding themes for teaching in Brazilian Basic Education. They should be used to contextualize what is taught, as they bring topics that can be interesting and relevant to the students' development as citizens, highlighting that: "the main goal is for students to complete their formal education not having only encountered abstract and decontextualized content, but also recognizing and learning about topics that are relevant to their role in society" (Brasil, 2019a, p.7).

Thus, these themes are called contemporary because they directly engage with the reality experienced by Brazilian students. They are considered transverse because they span across various areas of knowledge, making them relevant for understanding and addressing the challenges of today's society. In the school context, they refer to issues that respond to contemporary demands – themes present in the daily lives of communities, families, students, and educators – and, therefore, influence the educational process (Brasil, 2019a).

In this sense, the BNCC organizes these themes into six macro-thematic areas, which in turn are divided into fifteen themes, as shown in Figure 1.

Figure 1

The Contemporary Transversal Themes. (Translated from Brasil*, 2019a, p.13)*



Educational practices involving the TCT should be present throughout the entire school curriculum. According to the BNCC, these proposals can be developed within a single curricular component or by integrating multiple components through intradisciplinary, interdisciplinary or transdisciplinary approaches, but always in a transverse manner across the areas of knowledge (Brasil, 2019b). The approach to the TCT should be connected to the social dynamics of daily life, making their study relevant. This connection allows the integration of their content with the topics addressed in the classroom and with the objectives of the general competencies outlined by the BNCC, which aim at fostering citizenship and the development of attitudes and values (Brasil, 2019a).

In this way, working with the TCT is meaningful and adds relevance to school content. Using them in the teaching of functions can be a distinguishing factor in promoting a contextualized learning experience for High School students. Furthermore, addressing the issues covered by these themes is essential for building a fairer, more inclusive, and sustainable future for our youth, which aligns with the principles of Critical Mathematics Education (CME), to be discussed in the next section.

CRITICAL MATHEMATICS EDUCATION

Critical Mathematics Education (CME) involves the relationship that Mathematics Education (ME) has with the democratic competence every citizen needs to act in society. To address this issue of democracy, Skovsmose (2001) emphasizes the importance of ME being engaged in building this democratic competence within society, where mathematical content should be developed in a way that serves as a tool for democratization.

An essential aspect of developing democratic competence is reflection, which can be considered one of the main pillars of CME. Through reflection, individuals can make decisions about actions that should or should not be taken in specific situations, whether they relate to everyday issues, political matters, or social concerns, including those in Mathematics Education. This understanding is based on Skovsmose's (2001) finding that the concept of critique must be developed from the notion of uncertainty. He argues that critique cannot be predefined and, even within a critical approach to CME, it must be built through an investigative process. Thus, approaches to CME should not start from something fixed or predetermined but should remain open to exploring new educational possibilities through investigation and reflection. Moreover, reflection should not only address the learning content but also consider the relevance of applying this content to the student's life. For this, students must have the autonomy to discuss what they are learning and how they are learning it.

Skovsmose (2010) broadens the discussion about actions as objects of reflection, as he argues that actions can be seen as both enchanting and timely or the complete opposite. Therefore, there is a need for reflection on any action, particularly those materialized through mathematics. In this context, the author states that actions must be evaluated through reflection, which may involve deep ethical considerations concerning both the individual reflecting and the action being examined. Thus, reflections are directly tied to the judgment of one's own actions or those of others. However, this is not a simple concept, as there are many perspectives to consider, especially within the school context in Mathematics Education (Skovsmose, 2015).

Thus, mathematical reflection can be utilized in various learning situations, whether in a project-based class or a lesson focused solely on exercises from a textbook. It is possible to create scenarios that encourage students to reflect. Students can question data, analyze mathematical properties, discuss results, propose changes to data, among other possibilities. Reflections require dialogue, functioning more as a process of interaction rather than an individual activity, although individual reflections can also occur.

All the reflection explored leads to the development of reflective knowledge, which Skovsmose (2001) asserts is necessary for evaluating the actions and decisions of government leaders. With the increasing influence of technology on contemporary societies, its impact on democracy also grows. To advance the work with reflective knowledge through Mathematics Education, it is necessary to also question mathematical calculations: whether they are correct, truly necessary, if there are alternative methods of solving, and whether the results are sufficient or reliable. Skovsmose (2001) emphasizes the importance of contextualization in conducting these inquiries, considering a broad educational setting that fosters reflection within the framework of Critical Mathematics Education (CME). Thus, Skovsmose (2015) highlights that CME is more than just a subdivision of Mathematics Education; it is regarded as a form of Mathematics Education that focuses on reflecting on social reality and fostering a critical perspective among students.

In this context, Skovsmose (2015) introduces the concept of *mathemacy*, defined as the ability to deal with various mathematical techniques. The author explains that this competence can be understood as the skill to comprehend and manipulate mathematical ideas, algorithms, and procedures; to apply them in different situations; and to critically reflect on these applications.

Thus, Skovsmose (2015) emphasizes that Mathematics Education should play a role in preparing young people to acquire these skills, integrating students into perspectives, discourses, and techniques that will be essential for their future careers and for the development of current economic and technological frameworks. In this sense, *mathemacy* is crucial, but it must also be acknowledged that groups of students in different contexts (whether economic, social, or cultural) experience mathematics in diverse ways. Consequently, *mathemacy* will also develop differently. Therefore, Mathematics Education must address this diversity, with the challenge of starting from the students' realities without limiting their knowledge but instead expanding it so they can develop *mathemacy* to act in contexts that may differ from their own.

In this process, to develop lessons aimed at promoting Critical Mathematics Education (CME) for the students involved, Skovsmose (2010) introduces six learning environments, classified based on three types of reference: pure mathematics, semi-reality, and reality. These environments relate to classroom practice and are explored through exercises or investigation scenarios[1]. The semi-reality, as described by the author, represents a type of learning situation inspired by reality but does not require precise data. The Figure 2 presents the environment:

Figure 2

Learning Environments. (Adapted and translate from de Skovsmose, 2010)

References	Class Practice	Environments
Pure Mathematics	Exercises	1-Dominated by pure Mathematics, for example, expressions or equations presented without any relation to reality.
	Investigation Scenario	2-Characterized as one that involves numbers and geometric figures, for example, an exercise with translation of figures to a numerical table.
Semi-reality	Exercises	3-Consists of exercises that involve semi-reality, for example, an exercise or problem involving a situation based on a real shopping scenario.
	Investigation Scenario	4-There is a reference to the production of exercises, but with explorations and explanations by students, including simulations of real situations with space for students to analyze data and make decisions.
Reality	Exercises	5-Exercises based on reality are worked on, for example, real-life situation diagrams can be used, and from them, questions about the content being taught by the teacher can be explored, allowing for dialogue between the teacher and students and enabling students to question or add information to the exercise.
	Investigation Scenario	6- Reality is considered as an investigation scenario, where mathematical investigations can be conducted through projects.

Skovsmose (2010) draws a parallel between teaching practices that rely solely on exercises and those utilizing investigation scenarios within the learning environments. He concludes that the choice of which path to follow

---

[1] Skovsmose (2010) differentiates classroom practice based on scenarios from that centered solely on exercises. According to the author, this distinction occurs based on references — such as actions, motives or contexts — that guide students in constructing meanings related to mathematical concepts.

should be made collaboratively by the teacher and the students, potentially encompassing more than one environment within a single lesson. It is also important to emphasize that in environments involving an investigation scenario, students can explore, question, propose solutions, and engage in other actions that lead to reflection.

Based on these principles, a didactic sequence was implemented with the aim of fostering the critical competence of the students participating in the research.

THE REGISTER OF SEMIOTIC REPRESENTATION

The study of Register of Semiotic Representation (RSR) was significant for working with the concept of functions, a mathematical object analyzed in the doctoral thesis. Since this concept is highly abstract, it requires different registers for its comprehension. Duval's studies on mathematical learning emphasize the need to develop a cognitive system that goes beyond natural language and/or images. This system must rely on an entire semiotic system of representation and expression, utilizing symbols, natural language, algebraic formulas, figures, and more, all of which can facilitate the understanding and learning of mathematical objects.

Thus, for understanding learning, it is essential to explore the various ways students acquire knowledge. Focusing specifically on the understanding of mathematics learning, this section presents the theoretical contributions of Duval (2009, 2011a, 2011b, 2012, 2016a, 2016b, 2018) to advance the understanding of learning in this context.

According to Duval (2009), learning in Mathematics is characterized as a field of study that encompasses the analysis of essential cognitive activities such as conceptualization, text interpretation, reasoning, and problem-solving. For this learning to occur in a way that enables students to gain an effective understanding of Mathematics, contributing to their overall education, it is necessary to develop a cognitive system different from that practiced in other disciplines (Duval, 2011b, 2016b). Thus, in the development of these cognitive activities, representations beyond natural language or images are required. The author characterizes these as semiotic systems of representation and expression, which utilize graphic symbols, natural language, algebraic formulas, figures, graphical representations, and others (Duval, 2009).

The semiotic representations are those formed using signs specific to a system of representations, each with its own operational characteristics and meanings (Duval, 2012). The term "sign" refers to what is used for communication, such as hieroglyphs, alphabets, and other elements that form a system of signs, enabling the execution of the "Semiotic Operation," which is classified as distinct from physical, mental, or conceptual operations (Duval, 2016a, p.10). This operation consists of substituting one or more symbols in a mathematical calculation, as seen in computational procedures.

For the development of mathematical thinking, Duval (2016b) highlights two fundamental characteristics of cognitive activity: the importance of semiotic representations and the diversity of ways in which these representations are used in Mathematics. According to the author (2009, 2011b, 2012), they are essential to thought, as they directly contribute to the formation of mental representations, the execution of cognitive functions, and the construction of knowledge. The author emphasizes that understanding Mathematics requires distinguishing the mathematical object from its representation, as the same concept can be expressed in different ways. Thus, learning occurs when students grasp both the concept and its multiple representations, enabling them to establish connections between them.

Regarding the wide variety of semiotic representations used in Mathematics, Duval (2016b) presents natural language, numeral systems (numerical notation), geometric figures (geometric notation), algebraic writings (algebraic notation), and graphical representations (graphical notation). The articulation between these notations is what enables the understanding of mathematical knowledge objects (Duval, 2009, 2011a, 2011b, 2012, 2016a, 2016b, 2018). This articulation occurs through transformations (operations) that can be performed either from one notation to another (conversion between notations) or within the same notation (treatment).

According to Duval (2016b), treatment refers to the transformation of representations within the same semiotic notation, while conversion involves changing from one notation to another while maintaining the same mathematical object. In this context, the author highlights that some transformations are specific to each notation.

The choice between conversion or treatment depends on the purpose of the transformation. While treatment is more significant for mathematical demonstrations, conversion is crucial for the cognitive development of mathematical learning. Thus, according to Duval (2018), mathematical learning occurs when there is an immediate recognition of the same object in different representations. He emphasizes that when a student achieves this recognition, they can perform conversions between notations, replacing a given representation with a completely different one. Moreover, for this recognition to truly occur, working with the mathematical object must involve changing the representation notation in both directions of conversion.

Duval (2018) states that mathematical objects require their semiotic representations to be accessed. In this sense, a mathematical function can be visualized through its representations. According to the author, semiotic representations should be described according to the notation in which they were produced, as this determines their content. He highlights that this content has two inseparable characteristics: first, the presentation of certain properties of the object while concealing others; and second, the complete dependence on the semiotic system used to produce the representation of that object. Thus, the choice of the type of representation depends on the operations intended to generate other representations, whose content will reveal new data or information.

In studying the concept of function, it is essential to articulate different registers of semiotic representation. The algebraic register is used to define the function rule; the numerical register helps analyze roots and other key points; the figural register is used to construct tables; and the graphical register represents the function in the Cartesian plane. Since semiotic representation registers (RSR) can also serve as tools for observing and analyzing students' understanding or misunderstanding in learning Mathematics, the chosen method should allow for the observation of students' work in a way that distinguishes between treatment and conversion of registers (Duval, 2016b). This distinction is important because different types of register transformations correspond to different cognitive domains. When a student correctly performs a treatment, it indicates mastery of the rules within the chosen register. However, when they recognize and successfully convert between registers, Duval argues that a representational transformation occurs, signaling a deeper understanding of the studied concept. In the specific case of functions, the method should aim to assess whether the teaching approach enables students to develop a meaningful understanding of this mathematical object—one that allows them to make connections with real-world situations and problems beyond the classroom.

METHODOLOGY

The methodology used was qualitative research, aiming to investigate the teaching and learning process through the application of a didactic sequence developed for teaching functions to High School students, using TCT to contextualize the work with this knowledge object. Qualitative research is employed to address highly specific questions, as this type of research focuses not only on facts but also on a broader universe that encompasses the meanings, values, aspirations, beliefs, and attitudes of the individuals involved (Minayo, 2016).

For the development of the didactic sequence, textbooks from the 2021 National Textbook and Teaching Material Program (PNLD) for High School were used. Contextualized activities related to TCT (financial education, consumer education, and science and technology) were selected and adapted based on the topics chosen by the participating students through the initial research questionnaire. In addition to contextualization, the activities aimed to explore Semiotic Representation Registers (RSR) in learning environments referencing students' semi-reality or reality (environments 3, 4, 5, and 6) according to Skovsmose's (2010) classification. It is also worth noting that environments with investigative scenarios were used; however, they were considered in terms of activities involving some form of investigation rather than as the focus of the proposed approach.

The research, approved by the ethics committee (approval n. 5.069.909), was conducted with 20 students from the 3rd year of High School in 2023 at the Escola Estatal Professor Domingos Aparecido dos Santos, located in Rondonópolis/MT, where the first author works and resides. The following resources were used to develop the activities: Google Sites, GeoGebra Classroom, a data projector, mobile devices (Chromebooks[2]), paper, pens, among others.

APPLICATION AND ANALYSIS OF THE DIDACTIC SEQUENCE

The didactic sequence was designed to cover the introductory aspects of the topic of functions, addressing the following topics: the idea of a function, the concept and representation of a function (natural language, algebraic representation, figural representation, graphical representation); function graphs (using GeoGebra for constructions and problem-solving); function characteristics (growth and decay, injectivity, surjectivity, bijectivity, inverse function; study of the sign of a function to solve inequalities).

[2] Chromebooks are mobile devices provided by the State Secretariat of Mato Grosso to all schools in the state network for educational use by students (State Decree N. 1,497, of October 10, 2022).

For the implementation of the didactic sequence, the lessons were conducted based on activities presented to the students by the researcher-teacher. The scenario was introduced through the website[3] *"The Study of Functions and Contemporary Transversal Themes"*, created on Google Sites, which students access using Chromebooks or their own smartphones. The website included the results of the initial research questionnaire, the Contemporary Transversal Themes (TCT), theoretical content on the topics covered, developed activities, and proposed activities with links for solving them in GeoGebra Classroom.

After reading the situation presented in the activity, students were encouraged to propose solutions while the researcher-teacher guided them toward the formal conceptualization of the topic in question. At the end of each topic, a list of activities was provided for students to solve in groups or individually. Figure 3 illustrates one of the moments of activity development.

Figure 3

Students developing activities. (The research)

The didactic sequence consisted of 60 activities, organized into two categories: Learning Situations (LS) and Proposed Activities (PA). The lessons for implementing the didactic sequence took place during the second semester of 2023, totaling 23 classes.

---

[3] The site is not publicly accessible; students were added as readers and accessed, it using their institutional email accounts, to the link: https://sites.google.com/edu.mt.gov.br/sequenciadidatica/in%C3%ADcio.

The LS included examples, problem situations, and activities aimed at constructing the addressed concepts, as well as problem situations where the developed concepts were applied to solve a problem. On the other hand, the PA was carried out with the purpose of assessing learning.

Analysis of activities with TCT Consumer Education

The TCT Consumer Education was implemented with the objective of raising students' awareness about conscious consumption, addressing situations related to sustainability. According to the BNCC (Brasil, 2022b, p. 27), this theme aims to "educate individuals so that they are aware of their rights as consumers of goods and services, as well as the impact of their consumption habits on natural resources". This awareness should be reflected in their actions as active participants in their community or society, as well as in their relationship with the environment. Thus, the activities contextualized within this theme cover topics such as electricity consumption, water tariff calculations, fossil fuel and biofuel consumption, among others.

In the "concept of functions" topic, two learning situations and one proposed activity were used (Table 1).

Table 1

Activities involving TCT consumer education with the idea of function. (The research)

Activities	Aspects da CME	RSR
LS 1	Reflective knowledge; learning environments 4 and 6; development of *mathemacy*.	Natural language, registers: figural, numerical, and graphical.
PA 2	Reflective knowledge; learning environments 3.	Natural language, registers: algebraic and numerical.
PA 3	Reflective knowledge; learning environments 5 and 6; development of *mathemacy*.	Natural language, registers: figural, algebraic, and numerical.

In the topic "concept and representation of function", four LS and two PA were used (Table 2).

Table 2

Activities involving TCT consumer education with concept and representation of function. (The research)

Activities	Aspects da CME	RSR
LS 6	Learning environments 3 and 4.	Natural language, registers: algebraic, figural, and numerical. Conversion from the figural register to the numerical and natural language.
LS 9	Learning environments 5 and 6	Natural language, registers: algebraic and numerical.
LS 10	Reflective knowledge; learning environments 3 and 4.; development of *mathemacy*.	Natural language, registers: algebraic, numerical, and graphical. Conversion from natural language to graphical and algebraic.
LS 12	Reflective knowledge; learning environments 5 and 6; development of *mathemacy*.	Natural language, registers: algebraic and numerical. Conversion from the numerical register (table) to the algebraic.
PA 14, PA 15, PA 16	Reflective knowledge; learning environments 3, 4, 5 and 6; development of *mathemacy*.	Natural language, registers: algebraic, graphical, and numerical. Conversion between all registers, including from graphical to algebraic.

In the "function graph" topic, one learning situation (LS) and four proposed activities (PA) were used (Table 2).

Table 3

Activities involving TCT consumer education with function graph. (The research)

Activities	Aspects da CME	RSR
LS 22	Learning environments 3 and 4.	Natural language, registers: algebraic, numerical, and graphical.
PA 19, PA 20, PA 21, PA 23	Learning environments 3, 4, 5 and 6; reflective knowledge; development of *mathemacy*.	Natural language, registers: numerical, algebraic, and graphical. Conversion between all registers, including from the graphical register to the others.

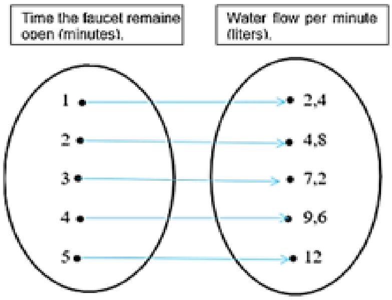
The analysis of activity PA 16 (

Figure 4) is presented, which addresses TCT Consumer Education and was proposed to explore the concept and different representations of functions.

Figure 4

Proposed activity in the concept and representation of functions topic (PA 16). (Adapted from Cevada et al., 2020)

PA 16 (Adapted from Cevada et al., 2020). The principal of Domingos School issued a notification requesting all members of the school community to find ways to reduce water consumption at school, as the neighborhood was facing a shortage of this essential resource.
To raise awareness among students about this issue, the third-year Mathematics teacher assigned a group of students to record the amount of water flowing from a faucet in the schoolyard, which was often left open by students. The students recorded some values in the arrow diagram next to them.
Then, the teacher asked the group to:

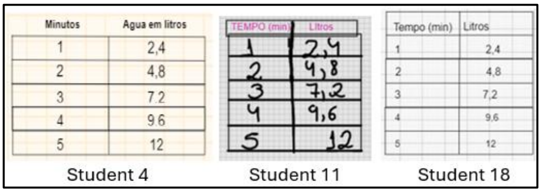


a) Record these data in a table.
b) Analyze whether this relationship is a function and justify your answer.
c) If so, determine the domain and range.
d) Construct a graph representing the table's values in the Cartesian plane using the GeoGebra application.
e) Observe the points in the Cartesian plane. How are these points arranged? Is it possible to describe them using a function rule? If so, what is this rule?
f) Find the values of f(10) and f(15). What do these results represent?
g) Is it possible to determine the value of x for which f(x) = 43.2? If so, what would this value be?
h) Is it possible to determine x belonging to the domain such that f(x) = –4.8?
i) Considering the identified function, if the faucet is left open for 1 hour every day for a week, how many liters of water will be wasted during this period?

This activity (PA 16) was adapted to incorporate the students' reality by using the name of their school and a situation that could occur. In addition to exploring the different representations of functions, it also introduced students to working with the GeoGebra software. For this purpose, GeoGebra Classroom was used, accessed via Chromebooks. Item "a" of this activity focuses on converting the figural register to the numerical register. It is worth noting that the students did not face difficulties with this task. Figure 1 presents images of the solutions provided by three students.

Figure 1

Resolution of 3 students in item "a" of PA 16. (Production of research participants)



Student 4

Minutos	Agua em litros
1	2,4
2	4,8
3	7,2
4	9,6
5	12

Student 11

TEMPO (min)	Litros
1	2,4
2	4,8
3	7,2
4	9,6
5	12

Student 18

Tempo (min)	Litros
1	2,4
2	4,8
3	7,2
4	9,6
5	12

It is noteworthy that the students used the Chromebook resources and GeoGebra Classroom with great ease in developing their solutions.

In item "b", students had to determine whether the given relationship was a function by applying reflective knowledge (Skovsmose, 2001). It was observed that students conducted online searches and used the didactic sequence website, providing a variety of responses (Figure 2).

Figure 2

Answers presented for item "b" of PA 16. (Production of research participants)

Student 3: Yes, because yes.

Student 4: "It is a function because it will have a final value."

Students 7, 14, 17, and 18: "The set X is the set of input values. The application's function operates in some way with x and produces an output: a y value from set Y. The program associates a single y response for each x input."

Students 3 and 11: "Yes. We understand a function as the relationship between sets A and B, in which every element of set A has a unique corresponding element in set B. When this relationship exists, it is described as f: A → B (a function from A to B)."

Student 12: "Set X is the set of values that, in some way, produce a y value in set Y."

Student 16: "Yes, because a function is a rule that relates each element of one set (represented by variable x) to a single element of another set (represented by variable y). For each x value, we can determine a y value, so we say that y is a function of x."

Student 19: "Yes, because it follows the rules: set X is the set of input values, and each time value generates a different water value (the value does not repeat)."

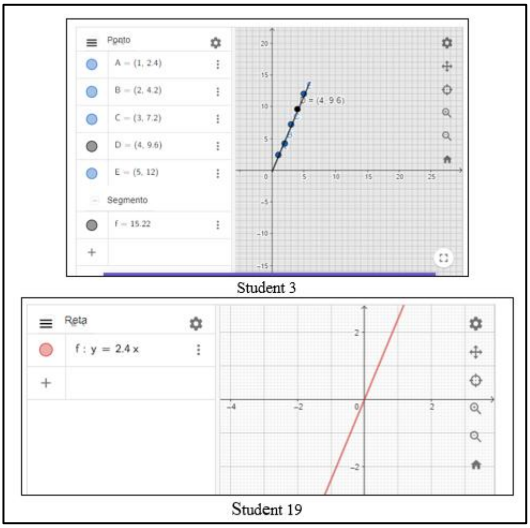
It was observed that, despite having access to research tools, only 10 students responded, and their answers were not entirely appropriate for the question, as they did not correctly relate the data from the activity. Eight participants simply copied the definitions they found in their research, and one student did not provide any justification. Notably, Student 19 was able to align their response with the quantities presented in the exercise. Thus, it can be concluded that students still face difficulties in formalizing the concept of a function.

Regarding item "c" of PA 16, which required identifying the domain and range of the function: seven students responded that the domain was time, and the range was liters; three students stated that the domain consisted of the values presented in the first diagram, and the range consisted of the values from the second diagram. This indicates that students grasped the concept of domain and range, but they did not fully understand that the values in the diagram represent only a fraction of these sets meaning they encompass all non-negative real values.

For the graph construction requested in item “d” (PA 16): seven students first converted the data into the algebraic register, writing the function rule y = 2.4x, before converting it into the graphical register. Three students used the ordered pairs obtained from the figural register to construct their graphs. Figure 3 presents two examples of these graph constructions.

Figure 3

Response of 2 students to item “d” (PA 16). (Production of research participants)



In Student 19’s response (item "d") and those of the other six students who followed the same approach, the domain was not restricted to the set of non-negative real numbers. As a result, the function line did not accurately represent the problem's data. This mistake can likely be attributed to a lack of understanding of this concept in the previous item.

Regarding item "e", all 10 students responded correctly. For item "f", all 10 students correctly performed the calculation, but only two provided a justification for their answers. In item "g", students were given a range value and asked to determine the corresponding x-value, five students solved the equation, five students used a trial-and-error approach, selecting values and calculating their outputs to find the answer, despite using different methods, all 10 students answered correctly. For item "h", the correct response should have been that no value of x satisfies the condition because the range must be positive. However, only Student 4 recognized this fact, while the others answered incorrectly. The final item ("i") required converting between different representations and performing numeric register transformations, such as converting time units from minutes to hours, days, and months. Six students answered correctly, and four students provided only partial calculations. This final item also encouraged the exploration of consumer education, prompting students to reflect on water waste and linking mathematical concepts to the TCT used (Brasil, 2019a).

The PA 16 activity reinforced conversion between registers and within-register processing, following the rules of each register (Duval, 2018). The TCT Consumer Education aspect stood out particularly in item "i", where students calculated water waste over a month, assuming the faucet was left open for one hour per day. This exercise sparked reflections on how simple actions, such as closing a faucet, can lead to significant savings in water consumption. According to Skovsmose (2010), mathematical reflection serves as a tool for decision-making in economic, political, and social contexts.

Analysis of activities with the TCT Financial Education

The TCT Financial Education was implemented to provide students with the necessary knowledge to develop judgment skills, make decisions, and act critically and reflectively regarding economic issues and possible solutions in the society they live in (Brasil, 2022a).

In the topic “idea of function”, two learning situations and one proposed activity were used (Table 4).

Table 4

Activities involving TCT financial education and the idea of function. (The research)

Activities	Aspects da CME	RSR
LS 1	Reflective knowledge; learning environment 4; development of *mathemacy*.	Natural language, registers: figural, numerical, and graphical.
LS 3	Reflective knowledge; learning environment 3; development of *mathemacy*.	Natural language, registers: algebraic and numerical.
PA 5	Reflective knowledge; learning environment 3; development of *mathemacy*.	Natural language, registers: algebraic and numerical.

In the topic "Concept and Representation of Function", three Learning Situations (LS) and seven Proposed Activities (PA) were used (Table 5).

Table 5

Activities involving TCT financial education and concept and representation of function. The research)

Activities	Aspects da CME	RSR
LS 7	Reflective knowledge; development of *mathemacy*; learning environment 4.	Natural language, registers: algebraic and numerical. Conversion of the natural language register to the algebraic register.
LS 13	Reflective knowledge; development of *mathemacy*; learning environment 4.	Natural language, registers: algebraic and numerical. Conversion of the natural language register to the algebraic register.
LS 14	Learning environment 4.	Natural language and graphic register. Conversion of graphic register to natural language.
PA 6, PA 7, PA 8, PA 9, PA 10, PA 12, PA 14	Reflective knowledge; learning environments 3, 4, 5 and 6; development of *mathemacy*.	Natural language, registers: algebraic, graphic, and numeric. Conversion from graphics to algebraic and algebraic to natural language.

In the topic "function graph," four LS and four PA were used (Table 6).

Table 6

Activities involving TCT financial education and function graph. (The research)

Activities	Aspects da CME	RSR
LS 19	Learning environments 3 and 4.	Natural language, registers: numerical and graphical. Works on converting the algebraic meaning to the graph.
LS 20	Reflective knowledge; development of *mathemacy*; learning environment 4.	Natural language, registers: algebraic, numeric and graphical. Conversion of the natural language register to other registers.
LS 21	Learning environment 3.	Natural language, registers: algebraic and graphic. Conversion of the register from the graphic register to the algebraic register.
LS 23	Learning environment 4.	Natural language, registers: numeric and graphic. Conversion of graphic register to numeric and to natural language.
PA 17, PA 18, PA 19, PA 23	Learning environments 3 and 4. Reflective knowledge; development of *mathemacy*.	Natural language, registers: numeric, graphic and algebraic. Conversion between all registers, including from graphics to algebraic and natural language.

In the topic “function characteristics”, six LS and four PA were used (Table 7).

Table 7

Activities involving TCT financial education and function characteristics. (*The research)*

Activities	Aspects da CME	RSR
LS 24	Learning environment 3.	Natural language and algebraic register.
LS 25	Learning environments 3 and 4.	Natural language, registers: algebraic and numerical.
LS 26	Learning environment 3.	Natural language and numerical register.
LS 29, LS 30	Reflective knowledge; development of *mathemacy*; learning environments 3 and 4.	Natural language, registers: numeric, algebraic, graphic and figural. Conversion in both directions between graphic, algebraic and natural language registers.

LS 31	Reflective knowledge; development of *mathemacy*; learning environments 3 and 4.	Natural language, registers: numeric, algebraic and graphic. Conversion from graphic register to algebraic and natural language.
PA 26, PA 27, PA 28, PA 29	Learning environment 3. Reflective knowledge.	Natural language, registers: numeric, algebraic and figural. Conversion of the figural register (sketch) to natural language.

Analysis of activity PA 18 (Figure 4) is presented, which was proposed with the aim of working on the TCT financial education in a learning environment (types 3 and 4). It involves a semi-realistic situation in which students had to use the RSR (natural language, algebraic, numerical, and graphical registers), making the necessary conversions and treatments for problem-solving and presenting the results.

Figure 4

Proposed activity on affine function graph (PA 18). (Adapted from Cevada et al., 2020)

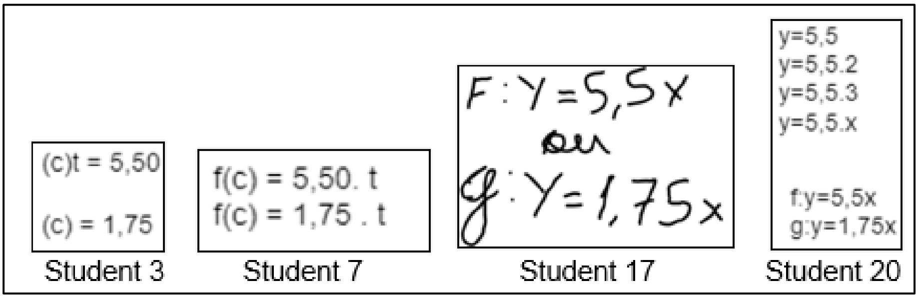
PA 18 (Adapted from Cevada et al., 2020) - If João has breakfast at the bakery, he spends R$ 5.50; if he has it at home, he spends R$ 1.75.

a) For each of these situations, determine a function rule that relates the cost *c* (in R$) of João's breakfast with time *t* (in days).
b) Sketch the graph of both functions in the same Cartesian system using GeoGebra.
c) Calculate the annual expense for each option. How much would João save if he had breakfast only at home?
d) How many people could he share breakfast with if he were willing to use these savings for a social cause?

For the resolution of item "a" of the activity in question, it was necessary to convert natural language into the algebraic register. It is noteworthy that 13 students answered this item. Figure 5 presents the solutions of four students.

Figure 5

Resolution of four students to item "a" of PA 18. (Production of research participants)

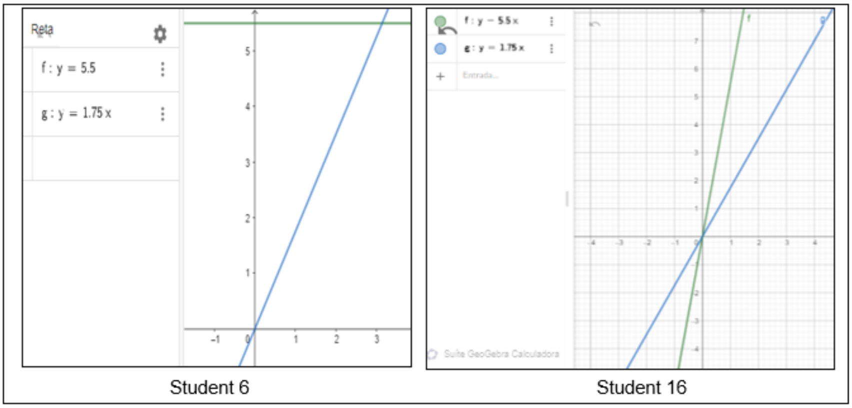


Observing the presented answers, it is noted that Student 3 was unable to perform the conversion correctly, as they did not identify the time variable in the function rule. Student 7 managed to make the conversion correctly but failed to name the functions differently and used the same letter to represent the variable. Notably, two other students answered in a similar manner. Student 17 expressed the function rule correctly, easily making the conversion; in addition to this student, one more provided the same response. Student 20 initially worked with some image values before writing function "f" in the algebraic register. However, for function "g", they immediately identified the function rule. Similarly, six other students gave similar responses, though two of them only expressed the rule for the first function.

Regarding item "b," PA 18 required the conversion from the algebraic register to the graphical representation by constructing the graph of both functions using GeoGebra. Figure 6 presents the solutions of two students.

Figure 6

Resolution of Students 6 and 16 for item "b" of PA 18. (Production of research participants)

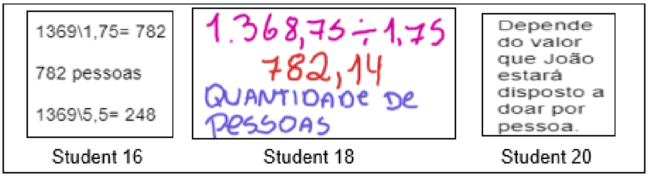


Student 6 did not correctly apply the rule of the first function, resulting in an incorrect graphical representation relative to the exercise. Another student made the same mistake. In contrast, Student 16 accurately represented both functions, but the graph lacked a domain restriction to only positive numbers. Nine other students made similar constructions. It is also worth noting that Student 3, who made an error in the previous item, did not provide a response for this item.

In item "c" of PA 18, students were asked to calculate the annual breakfast expenses for both options, having breakfast at the bakery or at home, as well as determine the savings from the cheaper option. To solve this, they needed to calculate the image of each function for 365 days, which could have been done using the graphical register in GeoGebra. However, the students chose to convert the function rules into the numerical register instead. Only nine students submitted their solutions for this item, and all of them arrived at the correct answer. However, only two students converted their response into natural language. For item "d", students were asked to determine how many people could share breakfast using the savings from the cheaper option. Figure 7 presents the solutions of three students.

Figure 7

Responses of three students to item "d" of PA 18. (Production of research participants)



Item "d" (PA 18) does not clearly specify how breakfast should be shared, whether at home or at the bakery. Therefore, to make this decision, the students had to reflect on Mathematics (Skovsmose, 2010), meaning that, based on the calculations performed in the previous item, they chose a prize and applied it. Among the eight students who answered this item (Figure 7), it was found that: Student 16 did not make this decision and presented calculations for both situations; six students used the data from the cheapest breakfast, like Student 18; and Student 20 did not present any calculations and gave a somewhat random response using natural language, which, however, cannot be considered wrong since the question was somewhat open-ended.

This activity (PA 18) enabled the visualization of two functions in multiple representations (natural language, numerical, algebraic, and graphical). It is noteworthy that, out of the thirteen students who started the activity, only eight completed all the items, with seven of them experiencing no difficulties regarding the treatments and conversions necessary for solving the problem. Observing the students' development, it can be concluded that they are not accustomed to presenting their answers in natural language. This suggests the predominance of a Type 1 learning environment, where only pure Mathematics exercises (calculate, determine, solve) are used, leaving no room for reflection and leading students to provide responses solely in mathematical language (Skovsmose, 2010). It is also worth noting that the Type 4 environment initially piqued the interest of most students (13 participants), but as the calculations became more demanding, some of them gradually dropped out of the process. Given this, the use of various learning environments is essential to gradually encourage students to change their attitudes toward problem-solving, whether these problems are academic, real-life challenges they will encounter in society, in the workplace, or elsewhere (Skovsmose, 2015).

Analysis of activities with TCT Science and Technology

The activities selected to be worked on with TCT Science and Technology include aspects related to the sciences (physics, chemistry, biology, and computing) as well as issues of technological evolution (internet, robotics, and others).

In the "idea of function" topic, one LS and four PA were used (Table 8).

Table 8

Activities involving TCT Science and Technology and the idea of function. (The research)

Activities	Aspects da CME	RSR
LS 4	Learning environment 3.	Natural language and numerical register.
PA 1, PA 2, PA 3, PA 4	Reflective knowledge; learning environments 3, 5 and 6; development of *mathemacy*.	Natural language, registers: figural, algebraic and numerical.

In the topic "concept and representation of function", one LS and three PA were used (Table 9).

Table 9

Activities involving TCT Science and Technology and concept and representation of function. (The research)

Activities	Aspects da CME	RSR
LS 8	Learning environment 3.	Natural language, algebraic and numerical register.
PA 11, PA 12, PA 13	Reflective knowledge; development of *mathemacy;* learning environments 3 and 4.	Natural language, registers: algebraic and numeric. Conversion of natural language to algebraic register.

In the topic "function graph" two LS and three PA were used (

Table **10**).

Table 10

Activities involving TCT Science and Technology and function graph. (*The research*)

Activities	Aspects da CME	RSR
LS 16	Learning environment 5.	Natural language, registers: graphic, figural and numerical. Conversion from numerical register to graphic and from graphic to natural language.
LS 18	Learning environment 3.	Natural language, registers: numeric, algebraic and graphic. Conversion between algebraic and graphic registers.
PA 20, PA 22, PA 23	Learning environments 3, 4 and 5; reflective knowledge; development of *mathemacy.*	Natural language, registers: numeric, algebraic and graphic. Conversion of graphic register to numeric and to natural language.

In the topic "function characteristics" three LS and two PA were used (Table 11).

Table 11

Activities involving TCT Science and Technology and function characteristics. (The research)

Activities	Aspects da CME	RSR
LS 31	Learning environments 3 and 4.	Natural language, registers: algebraic, graphic, figural and numeric. Conversion from numeric to graphic register and from graphic to natural language.
LS 27	Learning environment 4.	Natural language, registers: numerical, graphic and figural. Conversion of figural register to numerical and to natural language.
LS 28	Learning environment 5.	Natural language, registers: numeric and graphic. Conversion of graphic register to numeric and to natural language.
PA 24, PA 25	Learning environments 3 and 5. reflective knowledge; development of *mathemacy.*	Natural language, registers: numeric, algebraic and figural. Conversion of the figural register to the other registers.

Activity PA 25 (Figure 8) addresses TCT Science and Technology by incorporating physics concepts (speed, distance, and time) and presenting a situation involving the use of an electric scooter, which can be considered a technological resource for personal transportation..

Figure 8

Activity proposed in the topic function characteristics (PA 25). (Adapted from Bonjorno, Giovanni Junior & Souza, 2020)

PA 25 (Adapted from Bonjorno, Giovanni Junior & Souza, 2020)

Marcelo lives in a city where it is possible to rent electric scooters for transportation. The maximum allowed speed for these scooters is 20 km/h, but it is recommended that inexperienced users do not exceed 12 km/h.

To estimate the time, it would take to travel from a subway station to his workplace using an electric scooter, Marcelo assumed he would maintain a constant speed of 3 meters per second and created a table to relate the distance traveled (in meters) to time (in seconds).

Distance d (in meters)	3	6	9	12	15	...
Time t (in seconds)	1	2	3	4	5	...

Based on this information, the answer:

a) What is the function rule that relates the distance d (in meters) Marcelo will travel to the time t (in seconds)?

b) Is this function classified as increasing or decreasing? Justify your answer.

c) Marcelo took 10 minutes to complete his planned trip under the given conditions. What distance did he travel?

d) Considering the traffic situation in Rondonópolis/MT, do you think this electric scooter rental service would be beneficial for the population?

The situation presented in PA 25 represents a Type 3 learning environment, as it involves an exercise explored in semi-reality (items “a”, “b”, and “c”). However, it is also classified as a Type 5 environment when the scenario is transferred to the real context of the city where the participants live (item “d”). Throughout the activity, various Semiotic Representation Registers (RSR) were worked on and explored, including numerical, algebraic, figural, and natural language representations (through treatment and conversion processes).

The lesson in which this activity was conducted involved 18 students, but only 14 submitted their responses. In item “a”, students were asked to provide the function rule representing the scenario, requiring them to convert from natural language and figural representations to algebraic form. Eight students answered correctly. In item “b”, which required students to determine whether the function was increasing or decreasing and provide justification: two students did not respond; four students answered incorrectly; eight students answered correctly, providing diverse justifications such as: “increasing, because the numbers are being multiplied by three in ascending order.” –

Student 2; "the function is increasing because x and y are growing at the same time" – Student 10; "increasing because as time passes, the distance increases" – Student 15.

For item "c", students had to convert the given time unit (minutes) into seconds before calculating the required function value. The eight students who answered item "a" correctly also solved this item correctly, successfully handling the necessary treatments and conversions from algebraic to numerical representation. The remaining students managed to convert the time unit but did not present the function's output value.

Regarding item "d", students were required to reflect on the feasibility of electric scooter rentals in their city, considering factors such as traffic conditions, distances, infrastructure, and street topography. Responses varied: four students found the service viable; six students considered it unfeasible; one student gave a mixed answer, stating both yes and no. The responses of the 11 students who answered this item are presented in Figure 9.

Figure 9

Students' resolution to item "d" of PA 25. (Production of research participants)

Student 3: "Yes, because there would be less pollution."

Student 5: "I believe both yes and no, because where I live, it is not very suitable for riding an electric scooter, but those with more skill might manage."

Student 8: "Yes, because there would be less pollution, and it would be fun for most people."

Student 11: "Yes, because it would be profitable."

Student 12: "No, because it is not something that would be very appealing."

Student 13: "No, because everything here is far apart, and traffic is very busy."

Student 14: "Not really, as the roads and streets are not suitable for scooter use."

Student 15: "Not in my neighborhood because of the potholes, but in other parts of the city, it would be beneficial."

Student 17: "No, because there are many potholes, and depending on the time of day, it would take too long."

Student 18: "Yes. Some people cannot afford a vehicle, so I believe having an electric scooter for transportation would be interesting."

It is noteworthy that this item of PA 25 provided an opportunity for reflection beyond TCT Science and Technology, as the responses also addressed issues related to TCT Financial Education and Consumer Education, as well as other topics that were not the focus of this research but could also be explored, such as Traffic Education. The use of a Type 5 learning environment fostered the necessary reflection for students to understand how the function concept can be applied to real situations, thus promoting the development of mathemacy (Skovsmose, 2015).

The development of PA 25, through the work with Semiotic Representation Registers (RSR), facilitated the mathematical understanding of the study object (Duval, 2011b). However, five students who participated in this activity were unable to perform all the necessary conversions. This suggests that further exploration is needed, especially regarding the conversion from natural language to algebraic representation, which was the main difficulty perceived here.

CONSIDERATIONS

Through the analyses carried out, it is believed that the didactic sequence developed in the research, exploring the content of functions in more than one learning environment, along with the use of reflective knowledge, contributed to the enhancement of mathematical literacy. In addition, working with TCT while contextualizing all activities, which also involved the exploration of RSR, demonstrated progress in the teaching and learning process of functions, as students' oral participation and recorded responses indicate a better understanding of the studied concepts.

Based on the results presented by the participating students, it is evident that there were advancements in their understanding of the function-related concepts. However, it was observed that students still face difficulties in converting between different representations, especially from the graphical representation to others. Another challenge was the conversion into natural language, as most students (18 students) generally did not present their answers in natural language. This suggests a lingering effect of predominantly using only numerical and/or algebraic representations in problem-solving activities.

About the work with TCT, it was observed that the contextualization of activities sparked students' interest, as the topics were always related to something they knew or wanted to learn about. The TCT on consumer education encouraged reflections on themes such as sustainability and the importance of better utilizing natural resources in students' daily lives. The TCT on financial education prompted discussions on how to manage financial resources to ensure sufficient income for covering essential living expenses. The TCT on science and technology encompassed activities related to physical and chemical quantities, as well as contemporary technological issues that greatly interest students. Moreover, it became evident that this approach could be extended to other TCT, not only due to their significance but also to comprehensively cover the content of functions.

Regarding the teaching of functions, the students, who were in their third year of high school, had not yet fully consolidated all concepts related to this mathematical topic. During the activities, it was observed that 16 students faced greater difficulties in solving problems. However, by the end of the didactic sequence, significant progress was noted among these students, who demonstrated signs of understanding the concepts, particularly in recognizing a function and representing it in algebraic form. This suggests that if a more intensive approach had been applied from the first year of High School, considering the proposed contextualization with TCT and the use of RSR, better results could have been achieved in the teaching and learning process of functions.

The weaknesses identified in the development of the work were related to the time required to carry out the activities, as well as the fact that many students chose not to complete them since they were not directly linked to the Mathematics assessment process for their academic progression. It is believed that the methodological resources used, as well as the theoretical foundations supporting the proposal, were valuable and can be explored in future research to achieve similar or even better results with other groups of students. Considering the possibility of expansion, this approach could incorporate additional TCT and/or cover more topics within the functions content. An interdisciplinary approach could also be developed in future research, as TCT can be explored in this context as well.

Thus, it is concluded that the didactic sequence yielded positive results regarding the proposed objectives, as significant progress was observed in topics related to functions, mainly from the use of RSR. Additionally, the integration of TCTs and the principle of CME suggests that democratic competence was fostered, equipping students with knowledge and skills to act as critical and responsible citizens in various sectors of society.

AUTHORS' CONTRIBUTION STATEMENTS

All authors contributed to the research and writing of the article. RFD conducted the research, data analysis, and writing of the text, while CAO supervised the field research and contributed to data analysis, writing, and revision of the article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author (RFD) upon reasonable request.

REFERENCES

BONJORNO, J. R., GIOVANNI JÚNIOR, J. R., & SOUSA, P. R. C. (2020). *Prisma Matemática: conjuntos e funções*. (Manual do professor). São Paulo: Editora FTD.

BRASIL (2019a). Ministério da Educação. *Temas Contemporâneos Transversais na BNCC: contexto histórico e pressupostos pedagógicos*. Brasília: MEC.

BRASIL (2019b). Ministério da Educação. *Temas Contemporâneos Transversais na BNCC: propostas de práticas de implementação*. Brasília: MEC.

CEVADA, J. *et al*. (2020). *Matemática nos dias de hoje: funções*. (Manual do professor). São Paulo: Editora SEI.

DUVAL, R. (2009). *Semiósis e pensamento humano: registro semiótico e aprendizagens intelectuais*. Trad: Lênio Fernandes Levy e Marisa Rosâni Abreu da Silveira. São Paulo: Livraria da Física.

DUVAL, R. (2011a). Gráficos e equações: a articulação de dois registros. Trad: Méricles Thadeu Moretti. *REVEMAT: Revista Eletrônica de Matemática*, 6 (2), 96-112. https://doi.org/10.5007/1981-1322.2011v6n2p96

DUVAL, R. (2011b). *Ver e ensinar a Matemática de outra forma: entrar no mundo matemático de pensar: os Registros de Representação Semiótica*. Org. Tânia M. M. Campos. (1.ed.). São Paulo: PROEM.

DUVAL, R. (2012). Registros de Representação Semiótica e funcionamento cognitivo do pensamento. Trad: Méricles Thadeu Moretti. *REVEMAT: Revista Eletrônica de Matemática*, 7 (2), 266-297. https://doi.org/10.5007/1981-1322.2011v6n2p96

DUVAL, R. (2016a). Questões epistemológicas e cognitivas para pensar antes de começar uma aula de Matemática. Trad: Méricles Thadeu Moretti. *REVEMAT: Revista Eletrônica de Matemática*, 11 (2), 1-78. https://doi.org/10.5007/1981-1322.2016v11n2p1

DUVAL, R. (2016b). Registros de Representação Semiótica e funcionamento cognitivo da compreensão em Matemática. In: MACHADO, S. D. A. (org.). *Aprendizagem em Matemática: Registros de Representação Semiótica*. [livro eletrônico]. Campinas, SP: Papirus.

DUVAL, R. (2018). Como analisar a questão crucial da compreensão em Matemática? Trad: Méricles Thadeu Moretti. *REVEMAT: Revista Eletrônica de Matemática*, 13 (2), 1-27. https://doi.org/10.5007/1981-1322.2018v13n2p1

MINAYO, M. C. de S. (2016). O desafio da pesquisa social. In: MINAYO, M. C. de S. (org.), DESLANDES, S. F., & GOMES, R. P*esquisa social: teoria, método e criatividade.* Petrópolis, RJ: Vozes.

SKOVSMOSE, O. (2001). *Educação Matemática Crítica: a questão da democracia*. Campinas: Papirus.

SKOVSMOSE, O. (2010). *Desafios da reflexão em Educação Matemática Crítica.* (2ª ed.). Campinas: Papirus.

SKOVSMOSE, O. (2015). *Um convite à Educação Matemática Crítica*. Campinas: Papirus.