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Mathematical Connections in High School: gaps and perspectives revealed in a systematic review (2015-2024)

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ABSTRACT

Context: Several authors point out that mathematics teaching relies heavily on the use of formulas, neglecting investigative and creative processes, which are valuable for understanding the subject. This problem highlights gaps in the literature and points to the importance of students consciously making mathematical connections among different topics. **Objective:** We aim to gather and synthesize research on mathematical connections that contribute to the teaching and learning of mathematics in high school. **Design**: This research is characterized as a qualitative study of bibliographic nature. **Environment and participants**: 21 articles published in journals indexed in the Scopus database, from 2015 to 2024. **Data collection and analysis**: The selection of articles used the descriptors *mathematical connections* and *mathematical connection* in the title, abstract, and keywords, as well as *high school* and *mathematics education* throughout the text. We read the abstracts and the parts in which these descriptors were used and selected only those studies which contributed to high school education. For the analysis, we identified and categorized the main topics addressed in the theme. **Results**: We found that there are a small number of studies on the production of mathematical connections for high school, most of which focus on student connections in relation to the topics: functions, equations, geometry, and calculus. **Conclusions**: Studies on mathematical connections are recent, which explains the lack of research in high school. Exploring the varied content of this segment from the perspective of mathematical connections promotes a deeper and more lasting understanding, develops creativity, and stimulates mathematical thinking.

Keywords: mathematical connections; mathematics teaching; High School; mathematics education.

Conexões Matemáticas no Ensino Médio: lacunas e perspectivas reveladas em uma revisão sistemática (2015-2024)

RESUMO

Contexto: Diversos autores apontam que o ensino da Matemática recorre ao uso exaustivo de fórmulas, renunciando aos processos investigativo e criativo, valiosos

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para sua compreensão. Essa problemática evidencia lacunas na literatura e aponta para a relevância da produção consciente, pelos estudantes, das conexões matemáticas entre os conteúdos. **Objetivo:** Pretendemos reunir e sintetizar as pesquisas, no âmbito das conexões matemáticas, que colaboram com o ensino e a aprendizagem da matemática para o Ensino Médio. **Design**: Esta pesquisa caracteriza-se como um estudo qualitativo de natureza bibliográfica. **Ambiente e participantes**: 21 artigos publicados em periódicos indexados na base Scopus, de 2015 a 2024. **Coleta e análise de dados**: A seleção dos artigos recorreu aos descritores *mathematical connections* e *mathematical connection* no título, resumo e palavras-chave, além de *high school* e *mathematics education* no decorrer do texto. Realizamos a leitura dos resumos e das partes em que esses descritores foram empregados, e selecionamos exclusivamente pesquisas que apresentaram contribuições para o Ensino Médio. Para a análise, identificamos e categorizamos os principais tópicos abordados no tema. **Resultados**: Verificamos que há um número reduzido de pesquisas sobre a produção de conexões matemáticas para o Ensino Médio, sendo essas majoritariamente centradas nas conexões de estudantes, no tange aos temas: funções, equações, geometria e cálculo. **Conclusões**: Os estudos sobre as conexões matemáticas são recentes, o que justifica as poucas pesquisas no Ensino Médio. Explorar os variados conteúdos deste segmento sob a perspectiva das conexões matemáticas favorece uma compreensão mais densa e duradoura, desenvolve a criatividade e estimula o pensamento matemático.

Palavras-chave: conexões matemáticas; ensino de Matemática; Ensino Médio; Educação Matemática.

PRELIMINARY CIRCUMSTANCES

Teaching mathematics in schools has been a challenge for teachers throughout time. According to D'Ambrosio (2011), this problem happens due to mathematics being, since ancient times, the "Science of numbers, shapes, relations, measurements and inferences" (p. 74), in addition to its characteristics pointing to the requirement for precision, rigor, and accuracy. The author discusses curriculum organizations and the traditional sequence of content, suggesting that they may be limiting mathematics education, which has negative consequences for learning: "You learn something in 3rd grade so you can learn something else in 4th grade so you can learn more in 5th grade; and so, the content is linearly linked together. In this model, if you don't do well in 3rd grade, you can't keep up in 4th grade, and so on" (D'Ambrosio, 2016, p. 3).

According to Assemany (2020), the sequencing of content to be taught, stimulated by the fragmentation of topics included in the school curriculum, encourages teachers to perpetuate a teaching practice that lacks mathematical connections. Carreira's (2010) work supports this idea, as it suggests that opportunities to use connections in teaching should not be wasted, in order to integrate and promote the coherence of concepts, emphasizing the importance of their use so that students in basic education realize mathematics is not a collection of isolated ideas.

The production of mathematical connections in teaching is highlighted in several studies in the field, such as the study by Vanegas and Giménez (2018), which recognizes the (extra)mathematical connections used by future teachers in planning teaching sequences. For the authors, this type of connection relates mathematics to real life and highlights the potential of interdisciplinary teaching sequences, making mathematics more meaningful for students. We also highlight the theoretical-bibliographic research conducted by Allevato and Onuchic (2019), which sought to identify different ways of establishing mathematical connections based on problem-solving scenarios, confirming it as a fertile and relevant field for teaching, as well as being a catalyst for mathematical connections.

The National Common Curricular Base – BNCC (Brasil, 2018), as a guiding document for the minimum curriculum that all Brazilian students should have access to, provides explicit guidance for elementary school mathematics regarding the use of connections as a means of producing knowledge: "The meanings of these [mathematical] objects result from the connections that students establish between them and other components, between them and their daily lives, and between different mathematical topics." (Brasil, 2018, p. 274). However, the reference does not extend to the high school curriculum, leaving gaps in the didactic-pedagogical guidelines on the use of mathematical connections for this segment.

Rezende's studies (2003) corroborate the need to use mathematical connections in teaching. The author highlights that the fragmentation of the curriculum leaves high school students unprepared for mathematical thinking. The questions we ask is: what are the contributions to teaching and learning in high school that emerge from the use of mathematical connections?

Given this problem, we set the following research objective: to gather and synthesize research within the scope of mathematical connections, that contributes to the teaching and learning of mathematics in high school. To this end, we conducted a qualitative bibliographic study (Lakatos & Marconi, 2003), based on a systematic review of the literature with articles published in journals indexed in the Scopus database from 2015 to 2024, which presents research on the use of mathematical connections in high school. Data processing and analysis employed content analysis (Bardin, 2011) as a way to categorize the aspects highlighted in the studies found and emphasize possible contributions to the teaching and learning of mathematics.

This article is divided into six sections. Next, we present the theoretical framework regarding mathematical connections. Subsequently, we describe the methodology, present the results, and analyze the data. Finally, we highlight our considerations, pointing out gaps in the literature and some perspectives for future research.

THEORETICAL OVERVIEW OF MATHEMATICAL CONNECTIONS

The introduction of the idea of connections in education was highlighted as a central element in the field of mathematics education by the National Council of Teachers of Mathematics (NCTM) in 1991. Above all, the concept of connections gained significant prominence when this topic was identified by the NCTM (2000) as an essential mathematical process to be developed by students from early childhood education through high school. In this document, the authors emphasized the use of mathematical connections in teaching in order to foster the development of students' creativity, autonomy in seeking solutions to problems, and curiosity in the interaction between mathematical topics and in contexts that relate mathematics to their area of knowledge.

The first work on the topic of mathematical connections available in the literature is Evitts' doctoral thesis (2004), which presents research on the mathematical connections made by future mathematics teachers when solving problems. The author highlights the need for teachers to have a flexible and interconnected understanding of mathematics in order to promote the construction, emphasis, and use of mathematical connections: "It is in this flexibility and interconnectedness where conceptual understanding may reside" (Evitts, 2004, p. 6). From the results, the author highlights that each research subject demonstrated a unique approach to problem solving, using different mathematical connections.

The types of connections that emerged in Evitts' (2004) study were categorized as follows: i) Modelling connections*,* which emerged in the modelling of real situations based on an aspect of mathematics; ii) Structural connections, understood through the use of comparable situations, analogous problems, remembered stories, and isomorphic connections to seek similarities between the current problem and another situation; iii) Representational connections*,* expressed when there is more than one mathematical representation of the same situation in solving the problem, for example, graphs, numbers, and symbols; iv) Procedure-concept connections*,* conceived through the use of formulas or the chosen mathematical procedure; e v) Connections between strands of mathematics, highlighted by associations with different aspects of the content.

As a result of this research, Evitts (2004) identified four groups that comprise characteristics of connections: Views of Mathematics, Connection-Seeking Indicators, Problem-Solving Attributes, and Cognitive Characteristics, which suggest that the production of mathematical connections must be accompanied by a predisposition toward habits and attitudes regarding their use.

Based on this study, we found subsequent research that sought to contribute to discussions about the potential of mathematical connections in teaching and learning (e.g. Businskas, 2008; Cai & Ding, 2017; Eli et al., 2011; Marshall, 1995). Before expressing the contributions of this topic to mathematics education, we will present the concepts and definitions for mathematical connections.

After all, what are mathematical connections?

According to NCTM (1991), mathematics is a network of closely connected ideas, which highlights a clear position on the nature of mathematics. This document emphasizes that mathematical ideas are connected by specific relationships, and these connections can be identified a priori and independently of whether the student is aware of them. Alongside, Coxford (1995) conceptualized connections as broad ideas or processes that can be used to connect different topics in mathematics, pointing out three characteristics: unifying themes, mathematical processes, and mathematical connectors.

Evitts' studies (2004) present a metaphorical structure of connectivity and are based on the concept described by Hiebert and Carpenter (1992) that understanding something is related to what is already known, that is, that an idea or procedure is understood when it is linked to existing networks formed by strong and numerous connections. The author does not present a formal definition for mathematical connections but proposes that connectivity is not only a way of thinking about the discipline of mathematics, but also a way of interpreting a learner's mathematical knowledge.

Subsequently, Businskas (2008) analyzed how high school teachers define and understand mathematical connections. In her work, the author highlights several concepts used in understanding what mathematical connections are, and concludes that most of them are based on the common sense idea of connecting, using some of the results in the classification of the connections found, described by the author as true relationships between two mathematical ideas, which allows the construction of a network of interconnected concepts.

Gamboa and Figueiras (2014) point out that mathematical connections occur through a coherent logical relationship between elements, which extends to mathematical ideas. For the authors, mathematical connections constitute a network of links and bonds that enable the construction of meanings and that will successively produce new connections with other mathematical ideas. Concurrently, the studies by Eli et al. (2011) highlight mathematical connections as components of a mental schema, constructed from the memory developed by the individual. The authors emphasize the use of prior knowledge to establish or strengthen the understanding of the relationship between two or more mathematical elements (ideas, concepts, and representations) in a mental network.

> This model suggests that learning mathematics for understanding involves assimilating or connecting new information into mental networks, forming new connection(s) between existing knowledge components, and accommodating or reorganizing schemata to address perturbations in knowledge structures and to correct misconceptions. Thus, the building and refining of such mental structures through the establishment and strengthening of connections plays an important role in the development of students' learning of mathematics. (Eli et al., p. 299)

Based on these and other definitions, Assemany (2020) conducted a study on the appropriation of these connections by mathematics teachers based on the concept of vectors and conceived a meaning for this notion in the context of mathematics education, through a constructivist bias (Vygotsky, 1991). The author proposes that mathematical connections supplant the simple amendment of mathematical topics or content, or even the conception of using only prior knowledge to establish relationships as the only ways to make connections. Carreira (2010) supports this idea when she proposes, based on Fischer's (1993) definition, that mathematical activity in teaching is characterized as a means and as a system: "When we think of mathematics as a means, we emphasize above all its character as a tool; when we take mathematics as a system, we consider mainly the way in which mathematics organizes and constructs ways of thinking and acting" (Carreira, 2010, p. 14). This requires the urgency of (inter)connecting mathematical concepts in a way that makes sense to those who relate them and, consequently, "(...) mathematical activity as a territory of connections implies looking at mathematics simultaneously as a system and as a means" (p. 18).

Based on the premise that all knowledge is constructed through experiences that relate prior knowledge to new knowledge, Assemany (2020) argues that connections are intrinsically related to mathematical learning, consistent with Hiebert and Carpenter (1992), who propose mathematical understanding as a web of representations of mathematical ideas, procedures, and facts. From this perspective, the stronger and more numerous the connections are, the deeper the mathematical understanding will be. In addition, Cai and Ding (2017) argue that this network should be characterized as a dynamic and continuous process, just as Assemany (2020) considers understanding to be a result of the production of connections.

Based on Vygotsky's (1991) sociocultural constructivist theory, also known as schema theory, Marshall (1995) developed a foundation for conceptualizing mental schemas in the field of educational psychology. For the author, the schema is a means by which similar experiences are assimilated and added together to be remembered quickly and easily, that is, it is an essential component for learning. The author's studies indicate that a schema is a non-rigid structure, whose main characteristic is the existence of connections, and the strength of the connectivity of the components within the schema itself or the group of schemas is responsible for constituting it in a powerful and cohesive manner.

Given these concepts, Assemany (2020) proposes that (mathematical)[1] connections are represented by a web — a group of schemas — consisting of interconnections between concepts, in a logical and meaningful manner for the individual who relates them. The interwoven relationships develop from the individual (and unique) experience of the subject, allowing the construction of a network of memories driven by connections. The author points out that the stronger the connections are, the deeper the mathematical understanding will be.

The definitions highlighted mathematical connections as intrinsic to learning, emphasizing mathematics as an area consisting of interrelated elements, whether within its own content or with topics external to it. Mathematical

[1] In the studies by Canavarro (2017) and in the guidelines expressed by the NCTM (2000), we find the equivalence of the terms connections and mathematical connections, due to the fact that mathematical activity is considered an exercise in making connections (Hiebert & Carpenter, 1992). Therefore, in this article, we will use both expressions to denote the same idea.

connections help students understand concepts and build processes, interweaving their schemas and forming a system. For Carreira (2010), "developing mathematical connections is, fundamentally, not wanting to stop there and realizing that things are connected, they are not a collection of separate ideas, «they are not like loose peas in a bag», to use Vygotsky's words" (p. 18, emphasis added by the author), promoting more lasting and meaningful learning.

Next, we highlight the types of mathematical connections, emphasizing the categories defined by some authors, presented in the literature up to the time of this investigation.

Types and categories of mathematical connections

The work of García-García and Dolores-Flores (2017) emphasizes that when two or more mathematical ideas are connected, intra(mathematical) connections are established. Canavarro (2017) points out that this type of connection is made intrinsically between mathematical ideas, such as when we recognize a relationship between topics in the mathematical curriculum, allowing them to be addressed as content that unfolds logically, helping students to perceive mathematics as an articulated whole.

On the other hand, when we relate mathematical ideas to other disciplines or real-life knowledge, we establish (extra)mathematical connections (García-García & Dolores-Flores, 2017), considered as relationships with other areas of knowledge, such as Physics, Chemistry, and Medicine, but also with everyday practices that enhance the student's cultural experience.

Some researchers have focused on categorizing mathematical connections, with particular emphasis on research in the categories of (intra)mathematical connections (e.g. Businskas, 2008; Eli et al., 2011; Evitts, 2004; Rodríguez-Nieto et al., 2020).

The study by Businskas (2008), conducted with high school teachers, investigated different ways of establishing (intra)mathematical connections, resulting in the identification of five categories: *multiple representations, part-whole relationships, implication, connections in procedures,* and *teaching-oriented connections*. Subsequently, Eli et al. (2011) conducted an investigation based on task solving with high school teachers, which resulted in the identification of five other categories of (intra)mathematical connections: *categorical, procedural, characteristic, derivation,* and *curriculum*.

In addition, García-García and Dolores-Flores (2017) investigated the connections manifested in high school students when solving calculation tasks and identified two other categories: *connection by reversibility* and *connection in meanings*. Rodríguez-Nieto et al. (2020) identified the category of *metaphorical* connections when conducting a theoretical study with the aim of contributing to research on mathematical connections. Table 1 presents the set of categories of (intra)mathematical connections that emerged in literature by the above-mentioned authors, highlighting a definition and an example.

Table 1

Categories of (intra)mathematical connections. (Elaborated by the authors)

Category	Definition	Example
Multiple representations	Using alternative or equivalent representations for the same concept	Representations by algebraic writing and by a curve in the graph of a given quadratic function
Part-whole relationships	Relating ideas by the hierarchy between concepts, whether by a particularity or generalization	A square is a specific type of rectangle
Procedural	Using procedures, formulas, or algorithms to relate ideas	Using the quadratic formula to determine its roots
Teaching-oriented connections	Using the teacher's prior knowledge to instruct the teaching practice of another concept	Choosing the language to teach a particular subject
Categorical	Recognition of properties that define a mathematical object	A quadrilateral is any polygon that has four sides
Characteristic	Describing the characteristics of a mathematical concept	A rectangle has two pairs of parallel sides and four right angles
Derivation	A certain mathematical knowledge serves to construct or explain another knowledge	Using knowledge of area and circumference to arrive at the formulas for surface area and volume of a cylinder
Curriculum	Relating topics or ideas that impact the curriculum structure	Addressing circle area and circumference length so that students understand the number π

Reversibility	Recognizing bidirectional relationships between concepts	The integral and the derivative are inverse operations, just like addition and subtraction
Connection in meanings	When meaning is assigned to a mathematical object	Saying that the integral is the area under a curve
Metaphorical	Relating mathematical ideas to meaningful references	Attributing the curve of a graph to the notion of distance traveled

Connections in procedures and *connections by implication* were not included in the table because their definitions overlap or are included in others. *Connections in procedures*, according to Businskas (2008), have a similar definition to *procedural connections*, according to Eli et al. (2011). The *connection by implication*, from Businskas (2008), resembles the *connection by derivation*, from Eli et al. (2011). In order to simplify the classification, each pair of similar categories was condensed into one category, maintaining the *procedural* and *derivation* nomenclature. In total, we are considering 11 categories of (intra)mathematical connections.

METHODOLOGICAL DESIGN

To gather and synthesize research in the field of mathematical connections that contribute to the teaching and learning of mathematics in high school, we conducted a qualitative investigation (Denzin & Lincoln, 2006), since this study is an activity that situates us in the world and consists of a set of material and interpretive practices that codify the object to be analyzed through a series of representations, using numerous analysis strategies, such as semiotics, narrative, content, discourse, etc.

This research is exploratory in nature (Bogdan & Biklen, 2013), bibliographic in procedure (Lakatos & Marconi, 2003), and systematic in its study of the literature (Arantes, 2025), as it aims to highlight what exists and reveal gaps based on materials already produced, namely scientific articled on mathematical connections focused on high school, which are indexed in the *Scopus*[2] database. Knowing that the first studies on the topic appeared in 2004, and with the aim of finding the most current articles on mathematical connections, the period selected in the journal search focused on the years 2015 to 2024.

[2] Available on: https://www.elsevier.com/products/scopus

The *Scopus* international database was chosen because it is considered one of the largest multidisciplinary platforms for peer-reviewed literature abstracts and citations, with bibliometric tools to track, analyze, and visualize research, ensuring greater visibility and credibility within the scientific community. This database offers intelligent tools that stand out for their contribution to bibliographic research, especially when searching for international publication trends in languages other than English.

The initial search for scientific articles used the descriptors *mathematical connections* or *mathematical connection* in the title, abstract, and keyword parameters, considering that these are essential terms in this research. In terms of areas of knowledge, we selected articles classified in at least one of the following areas: Social Sciences, Mathematics, and Multidisciplinary, totaling 171, a result that we kept only as a quantitative note of studies that indicate mathematical connections in these areas.

Next, as our study focuses on mathematical connections geared toward high school education, we added the terms *high school* and *mathematics education* to the search for the body of the next, maintaining the previous descriptors. We found 72 articles, and we read the abstracts and excerpts from the texts in which the descriptors were used to understand whether the context was consistent with the objectives of this research. This process resulted in a total of 33 articles, mostly written in English, with two in Spanish and one in Portuguese.

We read these articles in their entirety and conducted a third screening, excluding those that had not researched mathematical connections and those that did not use them for teaching and learning in high school students. In addition, it is important to note that we found terms that were not compatible with what we were looking for; for example, high school appeared in the expression junior high school, a connotation that indicates research conducted in the final years of elementary school. Situations such as these led us to disregard some texts, resulting in 21 articles selected for use in this research, on which we made notes and recorded relevant information, such as the objective, a summary of the methodological procedures, the main results, and considerations.

Based on the records, we highlighted some units of analysis: *teachers, methodological proposals, future teachers, tasks, function, calculation, vectors, content, teaching, learning, high school, students, (intra)mathematical and (extra)mathematical connections*, which guided the thematic axes used for categorization. Next, we created the categories, with caution and vigilance to preserve the principles of mutual exclusion, homogeneity, relevance, productivity, objectivity, and fidelity, characteristic of content analysis (Bardin, 2011). Below, we present the categories found and their respective definitions:

1. *Production of (Intra)mathematical Connections by High School Students:* This category refers to studies on (intra)mathematical connections that are mainly produced by high school students, emphasizing their importance for mathematical learning.
2. *Methodological Teaching Proposals that Promote Connections:* This category refers to studies that propose methods and models for teaching mathematics, with an explicit emphasis on mathematical connections, focusing on methodological teaching proposals.
3. *(Extra)mathematical connections between mathematical and Physics knowledge in High School:* This category refers to studies that investigate the (extra)mathematical connections that link knowledge of mathematics and Physics in the context of high school education.
4. *Production of Mathematical Connection by Mathematics Teachers:* This category refers to studies that emphasize the mathematical connections produced by mathematics teachers in professional or training environments.
5. *Production of Mathematical Connections by Future Mathematics Teachers*: This category refers to studies that emphasize the mathematical connections produced by future mathematics teachers in their initial training.

We would like to note that, although we have been conducting this search since 2015, the oldest scientific article we found relating mathematical connections and high school education was published in 2017, highlighting the embryonic state of studies in this area. In the next topic, we highlight the texts organized in each category.

RESULTS

In this section, we present the 21 scientific articles organized into the five categories mentioned above. Tables 2, 3, 4, 5 and 6 highlight the texts that represent the thematic axes of the same category.

The articles selected for the thematic axis of *(intra)mathematical connections* constituted the category *Production of (Intra)mathematical Connections by High School Students*. Among the five categories identified in this study, this one comprised most of the research: 10 out of a total of 21. Of these articles we observed that six presented, within the scope of this axis, research with results focused on the use or validation of existing categories in the literature, as highlighted by Table 1, or on the suggestion of others. Therefore, we chose to include only one article that represented them, authored by García-García and Dolores-Flores (2017), which was also the oldest article found in the entire systematic study.

The other four studies related to this thematic axis sought to extrapolate the analysis of categories of mathematical connections and contributed to other aspects, such as understanding the processes of producing (intra)mathematical connections. Table 2 lists the articles that represent this category.

Table 2

Data from selected article in the category Production of (Intra)mathematical Connections by High School Students. (Elaborated by the authors)

Article title	Author(s)	Year	Journal	Country
Intra-mathematical connections made by high school students in performing calculus tasks	Javier García-García and Crisólogo Dolores-Flores	2017	International journal of Mathematical Education in Science and Technology	Mexico
High School Students' Difficulties in Making Mathematical Connections when Solving Problems	Jailani, Heri Retnawati, Ezi Apino and Agus Santoso	2020	International Journal of Learning, Teaching and Educational Research	Indonesia
Analysis of students' mathematical connection abilities in solving problem of circle material: transposition study	Nanang Diana, Didi Suryadi and Jarnawi Afgani Dahlan	2020	Journal for the Education of Gifted Young Scientists	Indonesia
Empirical Study of Factors Affecting the Students' Mathematics Learning Achievement	Wawan and Heri Retnawati	2022	International Journal of Instruction	Indonesia
Learners' algebraic and geometric connections when solving Euclidean geometry riders	Hans Bila, Kgaladi Maphutha and Paul Mutodi	2024	Pythagoras	South Africa

Of all articles in this study, the work by García-García and Dolores-Flores (2017) was the first to focus on mathematical connections in the context of high school. The authors identified and classified the (intra)mathematical connections produced by Mexican students when solving calculus tasks. The results indicated 24 mathematical connections, quantified and classified according to the categories listed in Table 1. Following up on this, Bila et al. (2024) drew on the study by García-García and Dolores-Flores (2017) to explore the mathematical connections established by 10^{th}-grade students at a public school in South Africa when solving Euclidian geometry exercises. The results show that mathematical connections form a web of relationships that allow other connections to be constructed. The authors highlight *characteristic* connections subsequently emerge, such as *procedural* ones.

Jailani et al. (2020) conducted a case study to describe the difficulties high school students face in solving mathematical problems within the Indonesian educational system. From the results, the authors identified the students' poor ability to make mathematical connections, which necessarily implied a lack of understanding of the problem and/or success with the solution procedures. *Part-whole* connections stood out as the most complex for these students. Similarly, the work of Wawan and Retnawati (2022) aimed to discuss factors that affect mathematical learning, such as anxiety when learning and the ability to make mathematical connections. The quantitative study, conducted with 10^{th}-grade students in Indonesia, showed that the ability to establish mathematical connections increased student motivation.

The study by Diana et al. (2020) aimed to describe the analytical geometry skills of ten students from Indonesia when creating mathematical connections involving the equation of a circle and the equation of the line tangent to it. According to the authors, the results allowed them to infer that the production of connections was an effective tool for students to remember what they had studied.

With regard to the thematic axis of *methodological proposals*, we found four studies whose purpose was to emphasize or recommend a teaching method that would encourage the production of mathematical connections. However, there were two studies that presented research whose objectives deviated from our analysis proposal. Maphutha et al. (2023) focused on the methodological proposal without pointing out its relationship with the mathematical connections that emerged in the research. The study by Bicer et al. (2023), related to the same thematic axis, defended the use of mathematical connections to promote creative processes in students, suggesting a model to describe the relationship between mathematical creativity, mathematical ability, and mathematical connection.

Thus, the two works that constituted the category *Methodological Teaching Proposals that Produce Connections*, as they describe the production of mathematical connections by high school students based on the context of a teaching proposal presented, are indicated in Table 3.

Table 3

Data from selected articles in the category Methodological Teaching Proposals that Promote Connections. (Elaborated by authors)

Article title	Author(s)	Year	Journal	Country
Core Model on Improving Mathematical Communication and Connection, Analysis of Students' Mathematical Disposition	R. Poppy Yaniawati, Rully Indrawan and Gita Setiawan	2019	International Journal of Instruction	Indonesia
Learning Mathematics Using a Collaborative RME Approach in the Indoor and Outdoor Classrooms to Improve Students' Mathematical Connection Ability	Didik Sugeng Pambudi, Sunardi and Titik Sugiarti	2022	Jurnal Pendidikan Matematika	Indonesia

The research by Yaniawati et al. (2019) aimed to improve mathematical communication, mathematical connections, and the willingness of Indonesian high school students to engage in mathematical activities, based on the CORE learning model: (C) connecting old and new concepts and information, (O) organizing ideas, (R) reflecting, exploring, and investigating, (E) expanding, using, and discovering. From the results, we highlight that students who underwent the CORE model used and improved the mathematical connections produced in the study of trigonometry.

Similarly, Pambudi et al. (2022) conducted a study to assess the effectiveness of the RME (Realistic Mathematics Education) model in promoting mathematical connections regarding the similarity of triangles. The model emphasized collaborative activity among Indonesian high school students, using an active and participatory methodology in which students went to the schoolyard and investigated actions on the flagpole, such as observing the angles between the pole and the wood and measuring the angles and lengths. The results indicated that the RME model promoted the production of (intra)mathematical and (extra)mathematical connections among students.

Regarding the thematic axis of *(extra)mathematical connections*, we found three articles in the systematic study. One of them contributed to the relationship between (intra)mathematical and (extra)mathematical connections, revealing that, although both connections are 'sufficiently good', the indicator of connection between mathematics and everyday life was not sufficient (Rafiepour & Faramarzpour, 2023). However, this study does not suggest research intrinsic to (extra)mathematical connections, as defined in the constitution of the category *(Extra)mathematical connections between mathematical and Physics knowledge in high school.* We can see in Table 4 the articles that represent this category.

Table 4

Data from selected articles in the category (Extra)mathematical connections between mathematical and Physics knowledge in high school. (Elaborated by authors)

Article title	Author(s)	Year	Journal	Country
Exploring mathematical connections of pre-university students through tasks involving rates of change	Crisólogo Dolores-Flores, Martha Iris Rivera-López and Javier García- García	2018	International journal of Mathematical Education in Science and Technology	Mexico
Conexiones matemáticas asociadas al concepto vector em un texto de secundaria de la Nueva Escuela Mexicana	Flor Monserrat Rodríguez-Vásquez, Viana Nallely García-Salmerón and Jesús Romero-Valencia	2024	Avances de Investigación en Educación Matemática	Mexico

The work of Dolores-Flores et al. (2018) refers to the (extra)mathematical connections of pre-university students in Mexico, in tasks on rates of change, which connect mathematical content with the ideas of speed and acceleration in Physics. The authors highlighted the presence of some categories (Table 1) among mathematical concepts, such as *multiple representations* and *procedural representations*, but emphasized the absence of (extra)mathematical connections, pointing to possibilities for future work.

The research by Rodríguez-Vásquez et al. (2024) investigated the mathematical connections involved with the concept of vectors in a textbook used in high schools in Mexico. The results show that the concept of vectors stands out in (extra)mathematical connections with Physics, being used to represent physical phenomena. According to the authors, the vector is perceived as a pictorial representation, as in images and diagrams, in the subject of Balance of Forces. On the other hand, in the topics of Dynamics, the vector is used to represent concepts such as forces, motion, velocity, and acceleration, in which its main elements are addressed: magnitude, direction, and sense.

Within the thematic axes of *teachers and contents*, we found three studies focusing on professional teaching activities, which constitute the category *Production of Mathematical Connections by Mathematics Teachers*, as shown in Table 5.

Table 5

Data from selected articles in the category Production of Mathematical Connections by Mathematics Teachers. (Elaborated by authors)

Article title	Author(s)	Year	Journal	Country
Conexiones matemáticas identificadas en una clase sobre las funciones exponencial y logarítmica	Karen Gisel Campo-Meneses and Javier García-García	2023	Bolema	Mexico
Mathematical connections established in the teaching of functions	Vesife Hatisaru	2023	Teaching Mathematics and its Applications: An international Journal of the IMA	Australia
Conexões Matemáticas Reveladas na Formação de Professores de Matemática	Daniella Assemany	2024	Bolema	Brazil

Campo-Meneses and García-García (2023) conducted research using onto semiotic analysis on the mathematical connections that emerged during high school mathematics classes on exponential and logarithmic functions. The production of mathematical connections by students and, especially by teachers, when teaching the content was investigated. Because logarithmic and exponential functions are inverse, it was expected that teachers would show the production of *reversibility*-type connections, which did not occur. The results indicated that the mathematical connections most used by teachers were *multiple representations, procedural,* and *characteristic.*

Hatisaru (2023) described a study on the mathematical connections established by mathematics teachers in their teaching practices. The author observed the classes of 13 Australian teachers in 9th grade elementary school and freshman year of high school classes on the topic of functions. Two teachers were then selected for the case study, in which Hatisaru (2023) identified different teaching strategies between them and, consequently, different connections. From the results, we highlight that this difference in the connections established is due to different beliefs about the nature of mathematics and the teaching processes.

Assemany's (2024) study highlighted the mathematical connections produced by Portuguese elementary and high school teachers in the context of a continuing education course, in which the topic of vectors was used as prior knowledge to produce connections in exploratory tasks. The categories of (intra)mathematical connections identified by the teachers were *multiple representations, derivation, teaching-oriented,* and *curriculum-oriented.* The author points out the need for the conscious production of mathematical connections and further studies on the topic, also focusing on teachers' perspectives of connections.

The last category, *Production of Mathematical Connections by Future Mathematics Teachers*, relates to the thematic areas of future teachers, teaching, and learning, and includes an article that describes the mathematical connections that emerge from future mathematics teachers in situations involving lesson planning and monitoring in high school, within the scope of the supervised internship process (Table 6).

Table 6

Data from selected articles in the category Production of Mathematical Connections by Future Mathematics Teachers. (Elaborated by authors)

Article title	Author(s)	Year	Journal	Country
Prospective Teachers' Pedagogical Considerations of Mathematical Connections: A Framework to Motivate Attention to and Awareness of Connections	Jonathon K. Foster and Hwa Young Lee	2021	Mathematics Teacher Education and Development	United States

The study by Foster and Lee (2021) was conducted over eight weeks with 16 American students studying for a bachelor's degree in mathematics, who participated in activities such as observing high school classes, participating in laboratory sessions with the teacher in charge, and observing teaching materials and lesson plans.

The future teachers participating in the study attended high school classes on the following topics: i) advanced algebra (operations/factoring with/of polynomials), ii) algebra support (interval notation, characteristics of polynomial functions); and iii) pre-calculus (solving trigonometric equations, law of sines and law of cosines). The laboratory sessions lasted one hour and required prior work by research subjects to solve the exercises proposed by the teacher in charge in an online forum.

After this period, five categories emerged in Foster and Lee's (2021) research, named: *connection by comparison; connection of something specific to a generalization; connection of methods; connection by logical implication;* and *connection to the real world*. The work of these authors contributes to the mathematical connections produced by future mathematics teachers, and, in addition, their results contribute to studies related to the categories of connections found in the literature (Table 1).

The classification of connections into categories was verified by reading all the texts in this systematic study, resulting in occasional collaboration, and we therefore classified them as cross-cutting contributions to the thematic axes being analyzed (Tables 2, 3, 4, 5 and 6).

In order to respect the units of registration (Bardin, 2011), the thematic axes sought to classify the connections produced according to the subjects and their intentions, distinguishing the contexts of study and, in this research, avoiding a detailed look at the mathematical content and objects whose connection was evident. Although the relationship between the content addressed in the production of mathematical connections did not play a role in the creation of the categories of analysis, when we noticed the strong focus on the same themes in the high school mathematics curriculum to the detriment of others, we decided to present this data to share our perceptions and gaps as contributions to this research. Table 7 highlights the mathematical topics that were used in the present studies on mathematical connections in high school.

Table 7

Mathematical topics covered in the selected articles. (Elaborated by authors)

Topic	Connected themes	Author(s)
Rate of change	Speed, acceleration, geometry	Dolores-Flores, Rivera-López and García-García (2018)

Vectors	Plane and spatial geometry, circumference, trigonometry, functions, matrices, integers, and complex numbers	Assemany (2024)
Exponential and logarithmic functions	Potentiation, population growth	Campo-Meneses and García-García (2023)
Linear equations, trigonometry, and circumference	Fraction, quadratic equation, area	Jailani, Retnawati, Apino and Santoso (2020)
Linear equations and functions	Plane geometry, rate of change	García-García (2024)
Calculus (derivatives and integrals)	Linear and quadratic functions	García-García and Dolores-Flores (2017)
Circumference	Angles, Pythagorean theorem, system of equations	Bila, Maphutha and Mutodi (2024) Diana, Suryadi and Dahlan (2020)
Function	Set theory, counting	Hatisaru (2023)

In the next section, we present an analysis of the results obtained, considering the theoretical basis of this study.

DATA ANALYSIS

This research revealed five categories in the results, related to (intra)mathematical connections (among students, teachers, and future teachers), (extra)mathematical connections, and teaching methodologies that promote connections. Through these, two cross-cutting themes emerged: classifications and content expressed about mathematical connections (Tables 1 and 7). Therefore, our analysis sought to respect the various interconnections that emerged in this research, abandoning the idea of a discussion based exclusively on the five categories.

The identification of (intra)mathematical connections produced by high school students (Table 2) was the objective of most of the selected articles, highlighting them as a fundamental element of mathematical thinking. In the study by Bila et al. (2024), the authors highlighted that, for students to be successful in solving Euclidean geometry exercises, it is necessary to produce mathematical connections of the *characteristic* type (Eli et al., 2011), since classifying triangles, for example, based on the characteristics of their sides, is essential for understanding the geometric object triangle. This mathematical connection favors the emergence of other types of mathematical connections, as shown in Table 1. Similarly, in Table 4, Dolores-Flores et al. (2018) emphasized the importance of *characteristic* connections in the study of the rate of change as a driving force for other mathematical connections. These results are supported by Evitts' (2004) conception of mathematics as flexible and consisting of interconnected elements, which promotes the construction of other connections, such as a network, a web, or a schema (Marshall, 1995).

Similarly, in Table 2, Diana et al. (2020) gave the example of a student who, despite recognizing the formulas that could be used in exercises on circumference, demonstrated difficulties in solving a problem on the same topic. Although the problem-solving scenario stimulates and provokes the production of mathematical connections (Allevato & Onuchic, 2019), this production must be continuous and specific to the individual (Assemany, 2020), which explains the importance of increasingly numerous, active, and deep connections for students to form their own network (Hiebert & Carpenter, 1992). Therefore, merely knowing the formulas to solve a problem shows an incipient, weak connection, with few experiences, which requires other intertwined relationships that allow the construction of a network of memories, with an understanding of mathematics as a means and as a system (Carreira, 2010).

Bila et al. (2024) confirm their findings by showing that mathematical connections enable the construction of other connections, in line with Gamboa and Figueiras (2014), when the authors point out that the network of links and ties formed by connections enables the construction of meanings and new connections with other mathematical ideas. Similarly, the work of Diana et al. (2020) highlights connections as effective tools for students to remember what they have studied (Marshall, 1995). Both studies emphasize the potential of mathematical connections for high school students' learning and highlight the concept *interwoven* in/by schemas in the concept of mathematical connections.

In the works of Diana et al. (2020) and Jailani et al. (2020), from Table 2, and Campo-Meneses and García-García (2023), from Table 5, the authors relied on their students' mathematical connections to perceive, monitor, and infer their mathematical understanding. This direct association between connections and mathematical understanding is based on several authors, among whom we highlight Hiebert and Carpenter (1992), who consider mathematical understanding as a web of representations of ideas and connections; followed by Carreira (2010) and Assemany (2020), who defend mathematical connections as the foundation of mathematical understanding.

In Table 2, the research by Wawan and Retnawati (2022) shows us that mathematical connections go beyond the relationship with mathematical learning, also having an impact on students' motivation when solving activities. The authors point out that when students perceive their own production of connections, they show more interest in studying mathematics. According to Evitts (2004), this disposition is fundamental for the production of mathematical connections. This result is also in line with the principles of the NCTM (2000), when its authors highlighted connections, indicating them as strategies for developing students' autonomy and curiosity, directly impacting their predisposition to learn.

In Table 5, Hatisaru (2023) highlighted the difference between the connections produced by two teachers when teaching. Of the total connections identified, 65% refer to the first teacher's classes, and 35% to the second teacher's classes. Although both used connections in the same categories, there was a large difference in the quantities of each. The author highlights, among other justifications, that their initial training is different and there is a divergence in opinions and beliefs about mathematics. This scenario is supported by Evitts (2004), who emphasizes the influence of the subjects' individual experiences as a driver of unique approaches, with the production of different mathematical connections. Furthermore, Hatisaru (2023) points out that this distinction suggests different opportunities for mathematical learning, highlighting the importance of social interaction in the learning process (Vygotsky, 1991).

The emphasis on producing mathematical connections through the task methodology was one of the results of Assemany's (2024) study, shown in Table 5. In the context of a continuing education course, the teachers participating in the research recognized the potential of exploratory tasks for producing mathematical connections, since this methodology allows for open situations with different resolutions, encouraging the emergence of connections of the *multiple representations* type (Businskas, 2008). The research highlights the situation in which teachers apply the exploratory tasks carried out during training, with high school students, and return to the course with their impressions and circumstantial narratives. The author highlighted the conscious production of connections as a driving force for teaching nourished by mathematical connections, complementing the studies by Businskas (2008) and

Vanegas and Giménez (2018) by encouraging teachers to reflect on connections in teaching, awakening them to their importance in the mathematics classroom.

Similarly, the work of Foster and Lee (2021), in Table 6, raised awareness of connections for teaching (NCTM, 2000). The authors emphasized the remarkable production of mathematical connections in discussions about teaching practices experienced by future teachers during the research, highlighting the production of *teaching-oriented* connections (Businskas, 2008) and connections in the *curriculum* (García-García & Dolores-Flores, 2017). These results are supported by D'Ambrosio (2016), who discusses the fragility of the linear and serial system in education. Furthermore, the study highlights the importance of (initial) teacher training to reflect on one's own professional practice in order to provide means to stimulate the production of mathematical connections in teaching and learning (Allevato & Onuchic, 2019).

In table 3, the studies by Yaniawati et al. (2019) and Pambudi et al. (2022) highlight teaching methods that proved successful as high school students produce the mathematical connections assumed by the authors. The two studies propose methodologies – CORE and RME – that converge to encourage active student participation, stimulating autonomy by taking the lead in activities, collaborative work through organization, group resolution and discussion, and curiosity to relate mathematical ideas. According to the NCTM (2020), these characteristics are closely related to the production of connections by students, based on the conscious motivation of the teacher through their own connections, which guide the proposed activities. This also supports the study by Foster and Lee (2021), which emphasizes the importance of collaborative work for students to establish mathematical connections.

The article by Dolores-Flores et al. (2018), which sought to understand students' (extra)mathematical connections with the ideas of speed and acceleration in Physics (Table 4), highlighted the great difficulty students had in establishing (extra)mathematical connections based on the concept of rate of change. Although the objective did not focus on (intra)mathematical connections, the authors pointed to a propensity to produce them rather than relate the areas of mathematics and Physics. The obstacles encountered in connecting the rate of change to situations in Physics or real life highlight the urgency of studies on (extra)mathematical connections (Vanegas & Giménez, 2018), which have been identified to a lesser extent than research on (intra)mathematical connections.

Although it was not the focus of this study, and due to the introductory nature of the research on mathematical connections related to secondary education, we consider it highly relevant to highlight the data on categories (Table 1) and mathematical content (Table 7) that emerged in the research and were present across all the thematic areas.

Classification into categories is a trend that contributes to the understanding and development of research on the subject, since we conceive mathematics as a network of closely related ideas (NCTM, 1991), a schema that enables the construction of meanings and the production of new connections (Gamboa & Figueiras, 2014). Therefore, the stronger and more numerous the connections, the more intense and meaningful mathematical understanding will be (Hiebert & Carpenter, 1992).

The categories of connections that were present in the works that constituted the categories of analysis referred to (intra)mathematics, presented in order of frequency: *procedural* (8), *multiple representations* (7), *derivation* (6), *characteristic* (5), *part-whole* (4), *teaching-oriented* (4), *connections in meanings* (2), *reversibility* (2), *curriculum* (2), and *metaphorical* (1), as shown in Table 1. This result shows that the type of category called *categorical*, by Eli et al. (2011), was not cited as a form of connection in this systematic study. According to García-García and Dolores-Flores (2017), the *characteristic* category is equivalent to the *categorical* category, and therefore they chose to use only the former in their texts, which gives us evidence to consider a sparse dissemination of *categorical* connections.

Regarding Table 7, we note that most of the research conducted with high school students or teachers addresses the topic of functions, with an emphasis on their diversity. We also highlight the topics of vector, linear equations, trigonometry, circumference, and calculus, which emerged as the most prominent. These mathematical topics appeared in studies in connection with each other or with others, mathematical or otherwise. For example, the connection between velocity, acceleration, matrices, systems of equations, and population growth (Dolores-Flores et al., 2018), and vector, which were connected to circumference, trigonometry, functions, integers and complex numbers, matrices, and geometry, highlighting other connections between these topics (Assemany, 2024).

The topics highlighted in Table 7 corroborate the defense of interconnected mathematics (Evitts, 2004), which motivated students and teachers to perceive its concepts and processes as interrelated, as a means and a system (Carreira, 2020), and not just as disconnected subjects that distort the perception of mathematics that refers to schemas (Marshall, 1995). Thus, the interconnections pointed out in the table form a network of ideas or processes that are used to connect different topics in mathematics (Coxford, 1995).

Given this analysis, we move on to the final section of the study with the perception of some contributions and the assessment that there are several gaps in the literature on research in the field of mathematical connections for high school. Even in works of mathematical connections intrinsic to the concept of connections itself, or in various scenarios – elementary school, higher education, initial and continuing teacher training – the results are still primitive and require many paths to be explored in mathematics education (Canavarro, 2017). The teacher's intention in assuming the production of connections in their teaching methodologies must be the significant change to modify this scenario.

SOME CONSIDERATIONS

Curiosity. Hope. Doubt. Disillusion. Concern. Anguish.

The clarity of these feelings leads us to a conscious position that there are insufficient conclusions representing the potential of mathematical connections for teaching and learning mathematics. We place this research in a scenario of preliminary considerations, which highlight the importance of urgently producing research on mathematical connections, whether in elementary school, high school, higher education, initial or continuing teacher training, or other contexts.

According to the Oxford English Dictionary, *liminal* relates to a transitional state of a process[3]. In this case, the considerations presented here cover an indefinite and limited period of time, in which contributions will still be scarce, but have the potential to become objects that form theories; and the gaps will highlight vacuums and absences, which may turn into small hiatuses.

Discovery. Anxiety. Restlessness. Joy.

The studies were unanimous regarding the perception of individuals – whether students, teachers, or future teachers – regarding the relevance of making or perceiving mathematical connections. Teachers and students showed motivation toward mathematical activity as a system and as a means, and appropriated the connections as a framework, demonstrating the possibility of building other connections based on those previously produced. Mathematical

[3] One of the definitions of the word *liminal*, from the Oxford English Dictionary, https://www.oed.com/dictionary/liminal_adj.

understanding was confirmed by the presence of connections, awareness of which was intertwined with encouragement and promotion to produce them.

The development or use of methodological teaching proposals, when designed to produce mathematical connections, align with their purpose and encourage active student participation, autonomy, collaborative work, and curiosity to relate mathematical ideas. Exploratory tasks, CORE, and RME thereby enhance the construction of mathematical connections, driving the use or creation of other teaching methods that contribute to the perspective of mathematical connections.

Reflection. Tension. Discovery. Organization. (Re)Cognition.

The various categories that emerged in the studies denote significant contributions to understanding and raising awareness of mathematical connections, which can – and should – be incorporated and grouped into the 11 categories indicated in Table 1. It is important to highlight that, among these, the following categories show the greatest propensity for connections: *procedural, multiple representations,* and *derivation*. Furthermore, we highlight the small number of studies on (extra)mathematical connections, accounting for 15% of the total articles used, with works on (intra)mathematical connections prevailing.

We highlight, in Table 7, the mathematical content that was addressed and connected in the articles included in the study. The scenario shows us the lack of other topics and fields of knowledge that can be researched in light of mathematical connections in high school. This research points to the study of (intra)mathematical connections between other topics in the curriculum as prospects for future work, such as: probability, combinatorial analysis, financial mathematics, areas, volumes, rational, irrational and real numbers, measures of tendency, and many others.

This investigation aimed to analyze articles indexed in the *Scopus* database from 2015 to 2024 that present research on mathematical connections in high school. We believe that, far beyond analytically and quantitatively expressing data contained in the studies, we reveal the embryonic state of research on the topic, a scenario that we cannot ignore.

Disillusion became Hope. Anguish became Curiosity.

Reflection became (Re)Cognition. Discovery became Joy.

This investigation turned into a Denouncement.

AUTHORS' CONTRIBUTION DECLARATION

DA e MP conceived the idea presented. DA developed the introduction and theoretical framework. MP conducted the literature review and created the data collection and analysis methods. DA and MP analyzed the data together. All authors actively participated in the discussions of the results and the writing of the entire text, reviewed and approved the final version of the work.

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