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A problematising approach to exponentiation and radicals

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ABSTRACT

Background: This investigation emerged from teaching experience and the recurrent observation that, in everyday school life, mathematics is mostly taught as a set of mechanical procedures based on the application of formulas and algorithms, with no space for the development of numerical sense or conceptual understanding of the content. **Objectives**: To analyse the impact of an alternative approach to traditional mathematics teaching on the conceptual understanding of exponentiation and radical operations. **Design**: This is a qualitative, interpretive research study, based on participant observation. **Setting and Participants**: The study was conducted in a public school in the municipality of Cachoeiras de Macacu, in the state of Rio de Janeiro, with a 6th-grade class in elementary education consisting of 14 students. **Data collection and analysis**: Data were collected from written records produced by students during the activities. The application of tasks and the qualitative data analysis followed the principles of problematised mathematics, which proposes the problem as the starting point for the construction of mathematical knowledge. **Results**: The analysed data indicate that the adopted strategy favoured a more solid understanding of the concepts explored, strengthened students' self-esteem, and increased classroom interaction. **Conclusions**: The results highlight the positive impact of pedagogical practices that promote active student participation, making the learning process more meaningful, reflective, and collaborative.

Keywords: Problematised mathematics; Number sense; Elementary education; Mathematical mindset; Mathematical exploration.

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Potenciação e radiciação sob uma lente problematizadora

RESUMO

Contexto: Esta investigação surgiu da experiência docente e da recorrente percepção de que, no cotidiano escolar, a matemática tem sido majoritariamente trabalhada como um conjunto de procedimentos mecânicos, baseados na aplicação de fórmulas e algoritmos, sem espaço para o desenvolvimento do senso numérico ou da compreensão conceitual dos conteúdos. **Objetivos:** Analisar o impacto de uma abordagem alternativa ao ensino tradicional da matemática na compreensão conceitual das operações de potenciação e radiciação. **Design:** Trata-se de uma pesquisa qualitativa, de natureza interpretativa, fundamentada na observação participante. **Ambiente e participantes:** O estudo foi realizado em uma escola pública do município de Cachoeiras de Macacu, no estado do Rio de Janeiro, com uma turma de 6º ano do Ensino Fundamental, composta por 14 alunos. **Coleta e análise de dados:** Os dados foram obtidos por meio de registros escritos produzidos pelos estudantes durante as atividades. A aplicação das tarefas e a análise qualitativa dos dados seguiram os princípios da matemática problematizada, que propõe o problema como ponto de partida para a construção do conhecimento matemático. **Resultados:** Os dados analisados indicam que a estratégia adotada favoreceu uma compreensão mais sólida dos conceitos explorados, além de contribuir para o fortalecimento da autoestima dos alunos e para a ampliação da interação em sala de aula. **Conclusões:** Os resultados evidenciam o impacto positivo de práticas pedagógicas que promovem a participação ativa dos estudantes, tornando o processo de aprendizagem mais significativo, reflexivo e colaborativo.

Palavras-chave: Matemática problematizada; Senso numérico; Ensino fundamental; Mentalidade matemática; Exploração matemática.

INTRODUCTION

The motivation for this research arises from teaching experience and the recurring observation that some mathematics teachers close to me who work in the final years of elementary school report that many students present significant conceptual gaps in basic mathematical knowledge, an example of which can be observed in arithmetic. This deficit may be related to a pedagogical approach that, institutionally, prioritises the execution of procedures over the construction of conceptual understanding. In this way, students end up having contact with mathematics in a mechanised way, through pre-established examples and the repetition of exercises, without due reflection on the meanings of the operations involved (Humphreys & Parker, 2019). This teaching model reinforces the mistaken belief that mathematical learning is limited to memorising rules and algorithms, neglecting the understanding and attribution of meaning to problem situations and the fundamental concepts of the subject (Boaler, 2018).

In contrast to the belief that mathematics should be taught primarily through memorisation and procedural repetition, we propose an approach that prioritises conceptual construction. Our goal is to investigate how 6th-grade students can learn various arithmetic properties and develop their number sense by exploring exponentiation and radicals through meaningful activities. We seek to understand how an alternative methodology to the traditional one, which, instead of following the sequence: definition – example – exercise, prioritises the attribution of meaning to arithmetic procedures, can impact the conceptual understanding of these operations. Thus, we ask: how does this differentiated approach influence the way students build their understanding of exponentiation and radicals, promoting more reflective learning connected to mathematical foundations?

The activities selected for this study are based on a teaching-learning approach that is part of the context of problematised mathematics (Menezes & Quintaneiro, 2023). This approach seeks to question pre-established mathematical structures, promoting a deeper reflection on concepts and their meanings. Furthermore, our work is based on the concept of explorations, as defined by Ponte (2003), who considers this type of task important for the development of students' mathematical thinking. To analyse the data generated from these activities and answer our research question, we developed an analytical lens that articulates different theoretical frameworks. This analysis engages with the idea of problematised mathematics, critically addressing the disconnect, often present in teaching, between formalised mathematics and its construction processes. We also incorporate the contributions of Boaler (2018) and other researchers who advocate for the importance of mathematical growth mindsets and the attribution of meaning to concepts as essential elements for students' learning experience.

This research was conducted at a public school in the municipality of Cachoeiras de Macacu. The study involved a 6th-grade elementary education class composed of 14 students. Eleven of them are of a regular age for this stage of education and are attending the 6th grade for the first time. The choice of this class was not random but rather motivated by the fact that the researcher, who is also the first author of this work, teaches the group, which enabled closer monitoring of the investigative process.

This text is the result of research focused on teaching practice, functioning as a pilot application of the activities planned for the first author's master’s research at PROFMAT[1], called: *Desenvolvendo uma mentalidade matemática por meio de uma abordagem problematizada no ensino de áreas geométricas* [Developing a mathematical mindset through a problem-based approach to teaching geometric areas], under the guidance of the second and third authors.

The decision to transform the teaching work environment into an investigative space is grounded in Cochran-Smith and Lytle’s (1999a, 1999b) perspectives. The authors defend the importance of teachers developing theoretical knowledge based on their own pedagogical practice. This approach allows teaching to be understood not only as a field of application of pre-existing knowledge, but as a dynamic space for the construction of knowledge, in which sociocultural and institutional factors are considered. Teaching research, in this context, assumes a reflective and procedural character, characterised as an “investigative stance” (Cochran-Smith & Lytle, 2009). Continuing this research communication, we first present the theoretical foundation that guides the study; then we describe the methodology adopted; subsequently, we present the applied activities and analyse the students’ contributions; finally, we conclude with our final considerations.

THEORETICAL FRAMEWORK

Boaler (2018) argues that all students can succeed in mathematics, provided they are exposed to an appropriate pedagogical approach and receive encouraging, positive messages throughout the learning process. For the author, the potential to achieve academic performance is not predetermined by innate factors but can be expanded through inclusive and motivating educational practices. However, the traditional teaching model, based predominantly on the expository transmission of content and students’ passivity, has proven to be at odds with this perspective.

Boaler (2018) also affirms that many of the traumas people experience with mathematics originate from a traditional procedural approach that prioritises the memorisation of rules and the mechanical application of methods. She also highlights that this rigid approach, focused on right or wrong answers, tends to generate anxiety and frustration in students. This is because they are

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not encouraged to deeply understand concepts or explore diverse problem-solving strategies, which limits their mathematical understanding and skills.

In this context, the author distinguishes two types of mindsets that influence learning: the growth mindset and the fixed mindset. The first refers to the belief that mathematical skills can be developed through effort, practice, and overcoming challenges, promoting greater engagement and persistence in the face of difficulties. A fixed mindset is characterised by the belief that intelligence or ability for mathematics is innate and immutable, which tends to limit students' confidence and restrict their performance.

In her work, Boaler (2018) argues that mathematics goes beyond the simple memorisation of facts and methods, being, in fact, a conceptual field. For her, developing a mathematical mindset involves a dynamic approach in which students become active protagonists, seeking to understand and attribute meaning to mathematical knowledge. Furthermore, this perspective proposes that mathematical production be seen as a process of exploration, investigation, and creativity, in which students exercise conjecture and justification rather than limit themselves to finding correct answers.

The teaching approach is fundamental to transitioning from a fixed to a growth mindset, as it directly influences how students engage with learning. This is because

> [...] all people have a mindset, an essential belief about their way of learning (Dweck, 2006b). People with a growth mindset believe that intelligence increases with hard work, while those with a fixed mindset believe that you can learn but cannot change their baseline level of intelligence. Mindsets are critically important because research shows that they lead to different learning behaviours, which, in turn, create different learning outcomes for students (Boaler, 2018, p. 12).

According to Boaler's (2018) research, the development of a growth mindset involves using open-ended problems that allow students to explore different methods, approaches, and representations. Furthermore, it is recommended to incorporate opportunities for exploration and investigation, and to work through the problem with students before presenting traditional resolution methods. Introducing visual elements and questioning students' perspectives on mathematics are also essential practices. Other scholars, such as Humphreys and Parker (2019), reinforce this approach, highlighting that it alters students' roles in mathematics classes. Students should be encouraged to test new ideas, understanding that mistakes are part of the learning process. Wrong answers, in this context, are not seen as failures but as learning experiences that value the thinking process and discovery over the search for the correct answer.

We understand that learning occurs both through the activities themselves and through reflection on them. In this sense, the search for mathematical meanings, rather than the passive acceptance of algorithmic procedures, directly affects how students construct their perceptions of the world. This can lead students to understand that questioning and problematisation are essential for knowledge and the advancement of science, rather than simply accepting what is presented to them.

It is precisely within this perspective that we find the concept of problematised mathematics (Menezes & Quintaneiro, 2023). This is an approach that views mathematics and its teaching from the perspective of its production processes, also considering pedagogical approaches and their possible social effects. This approach aims to denaturalise previously established mathematical ideas that were approached in a structured manner, proposing a questioning of this dominant mathematical structure and emphasising the order of invention. Concisely, these orders are understood as follows:

> When it comes to an epistemic discussion about mathematics itself, the perspective of the order of invention is opposed to the perspective of the order of structure. The second aspect has as its backdrop the idea of mathematics as a systematised body of knowledge, with relevance to already organised ideas and structures. Thus, from the perspective of the order of structure, mathematics is a body of knowledge organised from axioms, definitions, and theorems. From the perspective of the order of invention, mathematics resides in the unfinished, not beginning with axioms and ending with theorems; instead, it resides in its production processes (Menezes & Quintaneiro, 2023, p. 65-66).

From the perspective of problematised mathematics, the word "problem" must be understood differently from the usual negative interpretation. Rather than an obstacle, it represents a stimulus to investigation and exploration, being seen as an opportunity to question, create, and develop new solutions within mathematical knowledge.

> [...] the problem exists in itself, dispensing with a solution to gain materiality as a problem. That is, a problem is not a lack that will be overcome by knowledge of the pre-existing solution, but rather an invention, a novelty, a coming-to-be that creates something that never existed. Deleuze draws on Henri Bergson to consider the field of problems as autonomous with respect to the field of solutions. In other words, a problem can have a charge of truth in itself, regardless of whether it receives a solution and whether it is correct. An important consequence of this autonomy of problems is the emergence of a perspective according to which the fact that a problem remains unsolved does not disqualify its existence as a problem. [...] it is the problem that generates its possible solutions (Giraldo & Roque, 2021, pp. 12-13).

The problematised mathematics approach proposed by Menezes and Quintaneiro (2023) offers a broad understanding of the various dimensions of mathematics, encompassing its scientific, pedagogical, and social aspects, with the concept of "problem" as the central element that drives the production of mathematical knowledge. The scientific dimension addresses the epistemological aspect of mathematics, considering the historical processes that shaped its development. The pedagogical dimension focuses on how content is taught, as well as the practices and discussions that involve teaching mathematics. The social dimension investigates the affections, feelings, and perceptions of the world that are generated when teaching and learning mathematics. This study has a particular interest in the pedagogical dimension, recognising, however, that the boundaries between these dimensions are fluid and interdependent, as the authors themselves emphasise in their works.

In the context of teaching, the proposal is to problematise, i.e., to question, for example, why a definition is formulated in a specific way rather than another, or why a procedure follows a specific path rather than another. Furthermore, we seek to understand the meanings associated with these definitions and procedures. It is a critical perspective that recognises teaching approaches as essential in building perceptions of the world. Because we realised that this perspective directly connects with the development of a growth mindset, we decided to integrate it into our analytical lens. Note, for example, the conception of errors.

> [...] in mathematics education, in recent decades, important contributions from theoretical perspectives have shifted the

> role of "error" in teaching from a sign of deficiency to an inherent and constitutive aspect of learning processes (e.g., Cury, 2007). We believe that a perspective on problematised mathematics can contribute to other views on these debates in mathematics education (Giraldo & Roque, 2021, p. 16).

In other words, even errors or gaps in understanding are seen as essential parts of the learning process, as discussed by Boaler (2018) and Humphreys and Parker (2019) in the context of the growth mindset. Problematised mathematics, in turn, aligns with the perspectives of these authors, especially when emphasising the effects of pedagogical approaches in their social dimension, which impact both students and teachers.

The relevance of self-esteem of both students and teachers in the learning process becomes evident to us. Furthermore, we understand that a flexible approach to teaching mathematics—one that simultaneously considers institutional demands and theoretical foundations—can contribute to the construction of solid mathematical knowledge and to the formation of a solid foundation for future learning. Given this perspective, we chose to analyse the task definitions proposed by Ponte (2003, p. 1) also as a methodological path for data production, recognising that:

> Certain tasks are suitable for some classes, while other tasks may be suitable for other classes. Certain tasks may be better suited to specific times, others to others. Typically, the teacher should not propose the same type of task or proceed in the same way in the classroom. Instead, one should choose tasks and act based on events and student responses. A good strategy usually involves different types of tasks, so one of the teacher's problems is finding the "ideal mix" of tasks suitable for their students.

In the same study, the author classifies four types of tasks, highlighting the importance of considering their intentions in the teaching-learning process: exercises, problems, investigations, and explorations. According to him, these categories are defined and situated in a specific way, as described below:

- The exercise is characterised as a low complexity task, the resolution of which requires the direct application of a previously learned procedure, resulting in a single correct answer.
- Problem (which does not refer to the concept of problematised mathematics) is a closed task that requires a greater level of cognitive

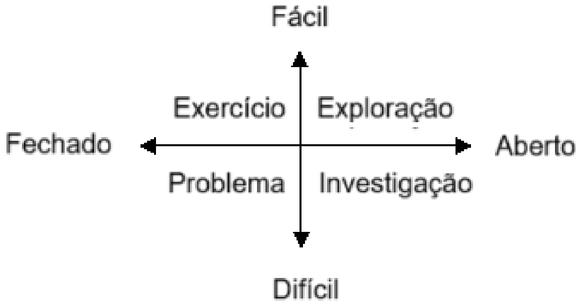
effort to be resolved. Although it has a single correct answer, it is more complex, requiring the student to connect different mathematical concepts and develop strategies to find the solution.

- Investigation is an open task, which does not presuppose a single answer and, in some cases, may even indicate the non-existence of a definitive solution. This type of task requires a high level of cognitive effort, encouraging students to explore different approaches, formulate hypotheses, and justify their reasoning, thereby promoting deeper, more reflective learning.
- Exploration differs from investigation mainly due to the degree of difficulty involved. While investigating requires significant cognitive effort, exploration consists of an open-ended task that students can access and develop more easily. In this way, exploration allows students to experiment with mathematical concepts more intuitively, without necessarily facing complex challenges, promoting an environment conducive to the gradual construction of knowledge.

Figure 1 illustrates these definitions, positioning the different task types according to their levels of openness and cognitive difficulty. In this way, it facilitates understanding of the distinctions between exercise, problem, investigation, and exploration, helping visualise the pedagogical intentions behind each approach.

Figure 1

Relationship between tasks. (Ponte, 2003, p.4)



Based on this theoretical dialogue, we developed our pilot activity, which constitutes the object of this research, and which will be detailed in the following sections, accompanied by our analyses. We intended to adopt a method consistent with the theoretical discussion presented, ensuring its applicability in a regular classroom while accounting for institutional limitations.

METHODOLOGY

We designed the activities to achieve what Ponte (2003, p. 1) defines as an "ideal mix" of tasks, considering the suitability for the profile of the students participating in the research, a 6th-grade class of an elementary school. In this way, we structured an intervention plan[2] that addressed concepts of exponentiation and radicals through a sequence of tasks. The professor-researcher, the first author of this article, conducted this plan across two meetings, each consisting of three 50-minute classes.

We adopted a qualitative research approach, following the perspective of Bogdan and Biklen (2007), as we believe that, in mathematics education research, the methodology must be aligned with the researcher's conceptions of education and knowledge, including their views on mathematics and mathematics education (Araújo & Borba, 2020). This choice is justified by its consistency with our objectives, since the application of quantitative tests tends to emphasise gaps in knowledge without necessarily revealing the learning processes involved. Furthermore, we understand that, in qualitative investigations, the construction of truth is a social and contextualised phenomenon and cannot be dissociated from the environment in which the research takes place.

Data were collected through students' written records and the teacher-researcher's notes, aiming to understand, in a broad and contextualised way, the participants' interactions, behaviours, and reflections.

DATA PRODUCTION AND ANALYSIS

On the first day of the activity, the teacher introduced exponentiation by inviting his students to jointly construct some powers of square numbers. The initial proposal followed a traditional approach, and we sought to gradually

[2] The research described in this article was not submitted to the Research Ethics Committee (Comitê de Ética em Pesquisa - CEP), given that the researcher worked as a class teacher and the activities developed were part of the school routine, compatible with the previously established content and pedagogical practices. We emphasise that there was no type of exposure or risk to the students involved. Finally, we explicitly exempt the *Acta Scientiae* from any consequences arising from failure to submit this study to the Research Ethics Committee.

implement a careful change that would not cause any surprises. Such care proved necessary, as problematisation and questioning of mathematical concepts were not standard practices in the class, making the familiarisation process indispensable.

By revealing that the power reading was "squared," the teacher prompted the students to reflect on the reason for this reading, asking whether anyone knew its origin. In a short time, many students could relate the idea of a square to the construction of the power of 2, highlighting a significant connection between the mathematical concept and geometry. At that moment, the initially proposed task worked as an exercise (Ponte, 2003), which was later enriched by problematising the meaning (Menezes & Quintaneiro, 2023) of the word "square" in the context of exponentiation.

Continuing the activity, the powers from 0^2 to 10^2 were constructed in a table in a column format, followed by other columns containing the powers from 10^2 to 20^2 and from 20^2 to 30^2. During this process, the teacher asked the students to carefully observe the results. After making Table 1, additional time was given for students to analyse the values and record in their notebooks any patterns or curious characteristics that had caught their attention, seeking to identify interesting or intriguing aspects in the data presented.

Table 1

Powers

$0^2 = 0 * 0 = 0$	$10^2 = 10 * 10 = 100$	$20^2 = 20 * 20 = 400$
$1^2 = 1 * 1 = 1$	$11^2 = 11 * 11 = 121$	$21^2 = 21 * 21 = 441$
$2^2 = 2 * 2 = 4$	$12^2 = 12 * 12 = 144$	$22^2 = 22 * 22 = 484$
$3^2 = 3 * 3 = 9$	$13^2 = 13 * 13 = 169$	$23^2 = 23 * 23 = 529$
$4^2 = 4 * 4 = 16$	$14^2 = 14 * 14 = 196$	$24^2 = 24 * 24 = 576$
$5^2 = 5 * 5 = 25$	$15^2 = 15 * 15 = 125$	$25^2 = 25 * 25 = 625$
$6^2 = 6 * 6 = 36$	$16^2 = 16 * 16 = 256$	$26^2 = 26 * 26 = 676$
$7^2 = 7 * 7 = 49$	$17^2 = 17 * 17 = 289$	$27^2 = 27 * 27 = 729$
$8^2 = 8 * 8 = 64$	$18^2 = 18 * 18 = 324$	$28^2 = 28 * 28 = 784$
$9^2 = 9 * 9 = 81$	$19^2 = 19 * 19 = 361$	$29^2 = 29 * 29 = 841$
$10^2 = 10 * 10 = 100$	$20^2 = 20 * 20 = 400$	$30^2 = 30 * 30 = 900$

At this point, the task took on an exploratory character (Ponte, 2003), allowing students' responses to demonstrate how the diversity of perspectives contributes to the construction of mathematical knowledge, reflecting the idea that mathematics is a social production (Menezes & Quintaneiro, 2023). For the analysis, we identified them as E1-E14 and transcribed some of their statements, selecting those relevant to the development of the activity. Below, we present their responses to the curiosities they identified in the table.

> E1: The last numbers repeat. And the last numbers don't change; even though they are different, the calculation is the same.
>
> E2: Some numbers in the table are repeated. Is number 2 [exponent] in all accounts?
>
> E4: What caught my attention was that the numbers are repeating themselves. It's the same number in all tables in the last house.
>
> E6: What caught my attention is that the number doesn't change the order.
>
> E9: All numbers end in 0, 1, 4, 5, 6, and 9.
>
> E11: That they continue in the same order. That even numbers end in an even number. That odd numbers end in an odd number. And look! After 10, the numbers are above 100. That the numbers are always ending in 1, 4, 5, 6, and 9.

Based on the students' speeches, we observed that their analyses complemented each other in their search for patterns within the potencies presented. Some students focused on the last digits of the powers, while others focused on the pattern of ordering the numbers. One student in particular, identified as E11, took advantage of the classroom discussion to explore the issue of parity in more depth. Furthermore, he expressed surprise at the rapid increase in potency, exceeding the order of hundreds. This aspect was particularly interesting, as most students were concerned with finding a single correct answer, reflecting the recurrent view that in mathematics there is always an exact, unique answer. In this context, we began to observe signs of a growth mindset emerging (Boaler, 2018; Humphreys & Parker, 2019), as, although initially hesitant to make mistakes, students began to report their perceptions, their conjectures, and explore alternatives, moving away from a mechanistic approach to a more reflective and investigative one.

During the presentation and discussion of the students' contributions, the opportunity to reflect on and value each of them arose. An example of this is the observation of student E2, who brought a conjecture about how to write the power of degree 2. Although the teacher and colleagues already understood this conjecture, E2 needed to express it explicitly to consolidate the understanding of the writing of the exponentiation itself. This moment led us to a critical reflection on our teaching practice, as we realised that some of the statements had not been fully understood throughout the activity, which prompted the need for that explanation. Furthermore, when we observed that student E2, as well as other students whose statements we have not transcribed here, when reinforcing the association between the exponent and the number of times the base should be multiplied, felt motivated to continue participating in the discussion, we realised the importance of creating an environment that values even seemingly obvious observations, as they contribute significantly to collective learning.

This situation revealed the perception of the patterns of power 2 degree as a problem that, when discussed, led to mathematical production around the concept in question, rather than being seen simply as something to be solved, without relevance to the advancement of mathematical studies (Giraldo & Roque, 2021). This moment also indicated that an approach that involves reflection on teaching practice brings to light the social dimension of problematised mathematics (Menezes & Quintaneiro, 2023), highlighting the importance of recognising students as capable of producing mathematical knowledge instead of merely as recipients of content.

At this point in the field research, our objective was to problematise the emergence of mathematical definitions and structures from discussions like the one presented. It is worth noting that this was this class's first experience with a mathematics activity that intentionally proposed exploration and problematisation as pedagogical strategies for research purposes. The professor-researcher then chose not to pressure the students to deepen their contributions, considering that everyone – including the professor himself – had long been immersed in a traditional teaching culture. Overextending the discussion could lead to frustration and disinterest in the new approach. Furthermore, the students seemed satisfied with the observations made during the activity. This decision not to lengthen the questions was influenced by a reflection on Boaler's (2018) studies, which highlight the importance of preserving students' self-esteem to keep them engaged in the learning process.

The teacher-researcher's decision to value the individual observations of the majority of students aligns with Boaler's (2018) and Humphreys and Parker's (2019) ideas. The authors highlight the importance of maintaining positive, encouraging communication, which is essential for developing a mathematical growth mindset. However, all the mathematical questions raised by the students were discussed and problematised in class during the same meeting, with the aim of preparing the students for the activity that relates to exponentiation and radicals. That students observed patterns in the last digits of powers, and that the existence of an order was essential. If these patterns had not been identified, the teacher-researcher would take steps to lead students to these observations, since such facts are essential to the study of square roots.

In the second meeting of the research-focused activity, students explored the concept of radicals, with a specific focus on square roots. When asked by the professor-researcher why this concept had this name and how it could be defined, the students quickly connected it to the formation of a square and the idea that "root" refers to the origin or beginning of something. This initial understanding was fundamental to the construction of the concept. Then came the idea of carrying out the reverse process of squaring, which was a crucial moment to revisit the table constructed in the previous meeting.

The discussion on radicals deepened, allowing the teacher to emphasise that a square root is considered exact[3] when it results in an integer. If the result is not an integer, the root is called non-exact, but it is still a number. This clarification was important for the students, as it provided a more precise understanding of square roots, deepening their understanding of the relationship between exponentiation and radicals.

Within this context, our questions were presented in a staggered manner: each new question was asked only after the previous one had been answered. Thus, we discussed different alternatives for finding the square roots of some numbers, using a task that encouraged students to explore and reflect on the process.

1- What does the square root of a number mean to you? Explain it in your own words.

2- About $\sqrt{5329}$:

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[3] Personally, we prefer to refer to square root answers as being integer or non-integer, but we are respecting the school's institutional material in this activity and calling them exact and non-exact.

a) Do you think this square root can result in an integer? In other words, can it be accurate? Explain the reasoning you used to reach this conclusion.

b) Which natural numbers are candidates to be the square root of 5329? Explain the reasoning that led you to this conclusion.

c) From the options you determined as possible square roots of 5329, was it necessary to check all of them? Explain what you did and why you made that choice.

d) Write a short text that brings together all the information presented in letters A, B, and C, explaining what the square root of 5329 is. The goal is for the text to contain a complete explanation of how you arrived at your result.

In the second stage of observation, a reduction in the students' fear of participating and answering questions was observed, compared to the first meeting. Responses became more spontaneous, reflecting a greater interest in actively contributing. This behaviour aligns with the ideas of Boaler (2018) and Humphreys and Parker (2019), who note that the first positive contact had a favourable impact on students' self-esteem. As a result, inhibition decreased and student contributions increased. Below, we present the students' responses regarding the second activity. For the first question, we highlight the following arguments:

> E1: It is a number multiplied by itself.
>
> E2: The square root of a number is a number that, when multiplied by itself, results in the original number.
>
> E4: It is a type of mathematical operation, like addition, multiplication, among others. It is the inverse operation of the power of two.
>
> E5: It is a number that, when multiplied by itself, gives a number.
>
> E12: The square root of a number is the number that, when multiplied by itself, gives that first number.

By sharing their conceptions, students are encouraged to reflect and explore ideas that contribute to understanding and attributing meaning within mathematics, as highlighted by Boaler (2018). We noted that, although the responses showed some progress, the idea of a learning that is still very much focused on the procedure remains very present in the explanations, reflecting an operational tendency in their approaches. Some students provided more detailed answers, while others left certain aspects implicit. In our analysis, we would need to observe, in practice, the impact of each explanation on students' actual understanding of the concepts. For example, E4 seems to have realised the relationship between power and root with the exponent and index, respectively. This initial understanding will likely facilitate your understanding of more advanced concepts, such as cubic and higher-order roots. Afterwards, we proposed that students work on issues related to the number $\sqrt{5329}$, to expand their reflections and begin to generalise the concept of square roots.

Following Boaler's (2018) suggestion for developing a growth mindset, we sought to transform procedural exercises into open-ended problems, offering students the opportunity to practice their perceptual, reasoning, and creative skills during this phase of the activity. The proposal thus aimed to make the task an exploration experience, as suggested by Ponte (2003). The questions asked were intended not only to observe how students applied their procedures, but also to direct them to record their answers at the end of the activity, allowing us to generate problematisations. Below, we present the answers we received to questions a) to d), summarising the main points of each.

> E5: Yes, because the 9 has roots. 3 or 7, because they end in 9. I did 73×73 and it worked.
>
> E7: I don't think so, as it's an odd number that I could find. The numbers $71, 72, 73, 74, 75, 76, 77, 78$ and 79 are candidates, since the square root of 5329 is between $70^2 = 4900$ and $80^2 = 6400$. It wasn't necessary to check all the numbers squared; I did $71^2 = 5041,\ 72^2 = 5184,\ 73^2 = 5329$, therefore, the square root of 5329 is 73.
>
> E9: Yes, because the number has a root, which is $73 \times 73 = 5329$. 7 and 3, $7 \times 7 = 49$ and $3 \times 3 = 9$, then, numbers ending in 9. It was not necessary to square all the numbers: $71^2 = 5041,\ 72^2 = 5184,\ 73^2 = 5329$. So the root is 73.
>
> E11: First, it may have a root because the number ends with 9. Then, I investigated the power table and found numbers 77 and 73, one of which was the exact root of 5329. The exact root is 73 and I didn't need to do both; I analysed which one was closer.

> E12: Yes, because it ends with 9. It is number 3. It wasn't necessary to check them all. First, I checked 75×75 and got 5625, then I checked 73×73 and got 5329.

We observed that students converged on the possibility that 5329 has an exact square root, especially since the number ends in 9. However, there was one exception: student E7, who associated the problem with parity, opening up a new possibility for exploring an error as a starting point for mathematical construction (Giraldo & Roque, 2021). The teacher then asked whether the parity of a number (whether it is even or odd) influences whether it has an integer square root, which sparked an interesting discussion about the relationship between these properties.

Despite this, E7 demonstrated good number sense by suggesting that the number should be between 70 and 80, which significantly contributed to collective reflection. Another student, E11, also contributed to the discussions by looking for a possible pattern, making an estimate and choosing the closest answer, although without confirming its accuracy. In our analysis, we were careful to take into account the 6th graders' stage of argumentative development and to understand the meaning behind their answers, even when they were laconic.

Another important point was that, when searching for patterns of numbers with integer square roots, students adopted different strategies to discover information about the number in question. For example, E5, E9, E11, and E12 used multiplication of equal numbers, in which the unit digit is 9, although this is implicit, as in the cases of E5, E11, and E12. Furthermore, we observed that students naturally began to work with notions of intervals, estimating values between two numbers, without having been formally introduced to the concept. This type of approach, where ideas emerge spontaneously and exploratively, is highly desirable in a problematised methodology, as it favours the development of mathematical reasoning in a more organic and meaningful way.

Still in this context, students were asked to analyse the $\sqrt{7043}$. It is worth noting that at this point, they were not given a list of topics to cover; instead, they were given the freedom to decide how to conduct the analysis, which provided greater autonomy in exploring and approaching the problem.

At this point in the activity, students were informed that, if they wished, they could use the calculator freely. This decision aligns with Silva's (1986) statement, which highlights the positive contributions of this technology to problem solving. According to the author, the use of the calculator helps students explore new strategies, not only through trial and error and successive approximations, but also by facilitating data organisation, hypothesis formulation and verification, and faster calculations. This process contributes to the development of mathematical reasoning, encouraging a more dynamic and reflective approach.

Therefore, we will examine the students' analyses of the square root of 7043, noting the approaches and reasoning that emerged during the activity.

> E2: It has no exact root. Numbers that have an exact square root are those that end in $0, 1, 4, 5, 6$ e 9. What comes closest to $\sqrt{7043}$ is 83, because 84 exceeds.
>
> E7: I verified that 7043 is between 6400 and 8100, then its root would have to be between 80 and 90. Performing the multiplications, I saw that $83 \times 83 = 6889$ and $84 \times 84 = 7056$. Therefore, number 7043 does not have an exact square root.
>
> E11: Number 7043 does not have a natural number as its square root, since it ends in 3. Doing the math, the ones that come closest are 83 and 84. So, it gets such as this: $83 < \sqrt{7043} < 84$. Approximating on the calculator, the number that comes closest is $83{,}92$.
>
> E12: I started by making the approximation, knowing that $85 \times 85 = 7225$, so I lowered it to 84×84, which is 7056. Then I did 83×83, and got 6889. And as 7043 is much closer to 7056; then, of 6889, I think that $\sqrt{7043}$ is approximately $83{,}9$.

Observing students' contributions, we noticed that some limited themselves to stating that there was no natural square root for the number, considering this to be sufficient. Others, although they also mentioned the non-existence of the natural root, went further in determining the interval in which the root could be located. Additionally, some students not only performed the previous steps but also used the calculator to obtain a decimal approximation of the square root of 7043, demonstrating a higher level of exploration and precision in the resolution process.

The most interesting aspect in this context is that the vast majority of students did not seek the teacher's approval when completing the proposal. Many did not question whether their answers were adequate or whether they needed to deepen their analyses, which is natural behaviour at this stage of schooling. This fact reflects a significant advance in students' autonomy, as well as in the self-confidence they have developed in interpreting requests and providing their answers (Boaler, 2018).

The calculator was an important addition to the activity. Due to time limitations, it would not be possible to delve too deeply into the non-natural roots, and this technology brought agility to the process (Silva, 1986). In addition to increasing the students' interest, the calculator motivated them to explore further, with some seeking several decimal places for the square root of 7043. They were challenged by the novelty of being able to approach the desired number but never quite reach it. This process helped students explore and become familiar with the idea of approximation, expanding their understanding of the concept.

It was evident during the development of the activity that the strategies adopted by the students to solve the problem were more interesting than the answer itself, as highlighted by Ponte, Brocardo and Oliveira (2020). During the tasks, students explored and applied the properties of multiplication, ordering, exponentiation, and radicals, demonstrating a deeper, more reflective approach than simply seeking a final answer. This process highlighted the richness of learning and promoted familiarity with fundamental mathematical concepts.

This approach, which invites participation, questioning —assumptions of problematised mathematics (Menezes & Quintaneiro, 2023)—, and exploration (Ponte, 2003), seems to have encouraged greater engagement among students in the construction of mathematical knowledge, in addition to contributing to the development of a growth mindset (Boaler, 2018). Once this stage is complete, we will present our considerations.

CONCLUSIONS

In this work, our objective was to observe the construction and appropriation of different arithmetic properties and number sense by 6th-grade elementary education students through activities focused on exponentiation and radicals, adopting an approach that moved away from traditional teaching. We avoided the classic triad of content presentation— definition, example, and exercise— Instead, we adopted a problematising stance, inspired by the ideas of problematised mathematics, and developed tasks that comprised the central activity of this investigation.

Initially, we noticed that the students showed some fear during their first encounter with the activity. However, when comparing their participation between the first and second days, we observed a significant transformation: their contributions became more elaborate, and they were more willing to expose themselves to possible errors, remaining involved in the process. The students understood that, in this type of task, it is not just about presenting the answer to a problem, but about explaining the thought process that leads to that solution, i.e., the process of mathematical production itself. This path involves analysis, hypothesis formulation and refutation, and the construction of conclusions. We understand that this change demonstrates the active involvement of students in the construction of knowledge throughout classes.

During the search for the square root of specific numbers, students used properties of multiplication, such as monotonicity, while acknowledging numerical order. Furthermore, the development of the notion of interval was observed, since, in the absence of a direct procedural method for calculating the square root, students adopted a logical approach, gradually adjusting and reducing the interval until they approached the answer.

This process also demonstrated a growing understanding of the relationships between the concepts of exponentiation and radicals, as observed at some points in this study. Furthermore, students made observations on the arithmetic properties of parity, which opened the possibility of generalising to even, odd, and multiple numbers, offering a fertile field for future investigations. The curiosity about irrational numbers was also remarkable, especially due to the dedication of some students who were not content with approximations but tirelessly sought to reach the square root of 7043.

We believe that this research reveals that a problematised approach offers possibilities for mathematical production that, although unpredictable at first, are highly significant for the development of mathematical knowledge. For example, the proposed activity could be expanded to introduce the concept of function, if we included questions such as: “What if the exponent increases (or decreases) and the base varies in the same way, what happens to the result?”, allowing the answers to guide students in understanding the relationships between variable values.

This unpredictability, far from being an obstacle, should be seen as a fundamental element in pedagogical planning, as we found that this approach valued students’ contributions and had a positive impact on their self-esteem during the learning process. Furthermore, it encouraged them to participate actively, fostering a growth mindset. For this reason, we invite other teachers and researchers to experiment with this approach, so that more data on the social and pedagogical effects of this practice can be collected and analysed.

Motivated by the overall development of the activity, we can affirm, as Menezes and Quintaneiro (2023) do, that involving students in problem solving in mathematics is extremely beneficial. In this approach, students have the opportunity to observe, discover, and explore mathematical properties, as well as construct meanings and contribute to understanding concepts. These elements indicate important social effects, such as the emergence of new forms of participation in the classroom and the possibility of generating new meanings about the world through the practice of problematisation. This is an opportunity for a mathematical experience guided by the order of invention, in which the consolidated structures and definitions are the result of a dynamic, creative process rather than a starting point.

In short, when experimenting with a problematised mathematics approach in practice, we observed that the leading role in the teaching-learning process alternates between teachers and students. The teacher's intentional mediation in this proposal allowed students to make their own observations, choose different paths, and question aspects they considered relevant to problem-solving. With this, we are increasingly convinced that a problematised approach to teaching not only constructs concepts in different ways but also prompts us to reflect on conceptions of mathematics and its structural definitions, thereby influencing the importance and meanings attributed to the different forms of participation in the world.

AUTHORSHIP CONTRIBUTION STATEMENT

The study design, activity implementation, and data collection were conducted by FVGM, who also wrote the final version of the manuscript, under the supervision of FMS and PCP. The three authors collaborated in the data analysis and discussion of the results, and FMS and PCP were responsible for the final review of the text.

DATA AVAILABILITY STATEMENT

The data that support the results of this research may be made available by the corresponding author, FVGM, upon a duly justified request.

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