

Functional Thinking and Self-Regulated Learning: What Errors do Students Make?

Suci Yuniati^a 
Muhammad Ikram^a 

^aUniversitas Islam Negeri Sultan Syarif Kasim Riau, Faculty of Tarbiyah and Teacher Training,
Departement of Mathematics Education, Riau, Indonesia

^bUniversitas Negeri Makasar, Faculty of Mathematics and Natural Sciences, Department of Mathematics
Education, Makasar, Indonesia

ABSTRACT

Background: Functional thinking is essential for developing students' algebraic reasoning, yet many students still experience difficulties in solving functional thinking tasks. Previous studies have examined newman error analysis and functional thinking separately; however, limited research has explored their relationship within the context of self-regulated learning. **Objectives:** This study aims to analyze students' Newman error patterns in functional thinking tasks based on their levels of self-regulated learning. **Design:** This study employed a descriptive qualitative design. **Setting and Participants:** The participants consisted of 21 eleventh-grade students from a senior high school in Riau, Indonesia, selected through purposive sampling. **Data collection and analysis:** Data were collected through functional thinking tasks, self-regulated learning questionnaires, and semi-structured interviews. The data were analyzed through data reduction, data display, and conclusion drawing. **Results:** The findings showed that students with high and moderate levels of self-regulated learning experienced comprehension, transformation, process skills, and encoding errors. Meanwhile, students with low levels self-regulated learning demonstrated all Newman error categories, including reading errors. Lower levels self-regulated learning was associated with more comprehensive difficulties in solving functional thinking tasks. **Conclusions:** Self-regulated learning influences students' Newman error patterns in functional thinking tasks. The findings provide pedagogical implications for supporting students' cognitive and metacognitive development in mathematics learning.

Keywords: Newman error analysis, functional thinking, self-regulated learning, mathematical problem solving.

Pensamento funcional e aprendizagem autorregulada: que erros cometem os alunos?

RESUMO

Enquadramento: O pensamento funcional desempenha um papel importante no desenvolvimento do raciocínio algebrico dos estudantes, mas muitos ainda apresentam dificuldades na resolução de tarefas de pensamento funcional. Estudos

Corresponding author: Suci Yuniati. Email: suci.yuniati@uin-suska.ac.id

anteriores investigaram separadamente a análise de erros de Newman e o pensamento funcional; entretanto, ainda são limitadas as pesquisas que exploram a relação entre ambos no contexto da aprendizagem autorregulada. **Objetivos:** Este estudo tem como objetivo analisar os padrões de erros de Newman em tarefas de pensamento funcional com base nos níveis de aprendizagem autorregulada dos estudantes. **Desenho:** Este estudo utilizou uma abordagem qualitativa descritiva. **Local e participantes:** os participantes consistiram em 21 estudantes do 11.º ano de uma escola secundária em Riau, Indonésia, selecionados por amostragem intencional. **Recolha e análise de dados:** os dados foram coletados por meio de tarefas de pensamento funcional, questionários de aprendizagem autorregulada e entrevistas semiestruturadas. Os dados foram analisados por meio de redução de dados, apresentação dos dados e elaboração de conclusões. **Resultados:** Os resultados mostraram que estudantes com níveis altos e moderados de aprendizagem autorregulada apresentaram erros de compreensão, transformação, habilidades de processo e codificação. Enquanto isso, estudantes com baixos níveis de aprendizagem autorregulada demonstraram todas as categorias de erros de Newman, incluindo erros de leitura. Níveis mais baixos de aprendizagem autorregulada estiveram associados a dificuldades mais complexas na resolução de tarefas de pensamento funcional. **Conclusões:** A aprendizagem autorregulada influencia os padrões de erros de Newman dos estudantes em tarefas de pensamento funcional. Os resultados oferecem implicações pedagógicas para apoiar o desenvolvimento cognitivo e metacognitivo dos estudantes na aprendizagem da matemática.

Palavras-chave: Análise de erros de Newman, pensamento funcional, aprendizagem autorregulada, resolução de problemas matemáticos

INTRODUCTION

Errors in mathematical problem-solving remain a major issue in mathematics education because they reflect students' difficulties in reasoning, representation, and decision-making processes. Therefore, identifying the types and causes of students' errors is essential for designing effective instructional interventions. One widely used framework for diagnosing students' errors is Newman Error Analysis (NEA), which enables researchers and teachers to identify difficulties that occur during mathematical problem-solving processes (White, 2009). Previous studies have applied NEA to analyze students' errors in mathematical literacy, Higher-Order Thinking Skills (HOTS), and general mathematical problem-solving tasks (Mansor et al., 2021; Kusmayadi et al., 2022; Lubis et al., 2021; Nugraheni et al., 2021; Noutsara et al., 2021). However, most studies have focused primarily on procedural and conceptual errors without examining how students' mathematical thinking processes contribute to these errors.

One important aspect closely related to students' reasoning processes is functional thinking. Research on functional thinking has attracted increasing attention because of its important role in supporting students' algebraic reasoning and advanced mathematical understanding. Previous studies have indicated that students were able to identify patterns, generalize relationships, and use various mathematical representations when solving functional problems (Oliveira et al., 2021; Stephens et al., 2017; Yuniati et al., 2019; Yuniati et al., 2022). Other studies also reported that students employed different strategies in constructing recursive, covariational, and correspondence relationships when solving functional tasks Tanisli (2011). Studies on the development and design of mathematics instruction and tasks have focused on enhancing students' functional thinking abilities (Pang et al., 2022; Pang & Sunwoo, 2022; Deng et al., 2025). These efforts represent a comprehensive approach that integrates curriculum, teaching, assessment, and the analysis of student learning outcomes. Moreover, these studies provide empirical evidence that elementary school students can engage in more sophisticated mathematical thinking and reasoning when they receive consistent, well-structured, and sustained mathematics instruction. Nevertheless, studies investigating students' functional thinking errors using the NEA framework remain limited, particularly at the senior high school level.

In addition to cognitive aspects, affective and metacognitive factors also influence students' mathematical problem-solving performance. One important factor is self-regulated learning, which has been reported to influence students' learning strategies, persistence, and problem-solving performance in mathematics (Amiruddin et al., 2022; Biber, 2022; Cetin, 2022). Previous studies mainly examined self-regulated learning in relation to mathematical achievement and learning outcomes, whereas limited research has explored its relationship with students' functional thinking errors. As a result, how students with different levels of self-regulated learning experience and produce errors during functional thinking processes remains unclear.

Therefore, this study aims to analyze students' Newman error patterns in functional thinking tasks based on their levels of self-regulated learning. This study is expected to contribute to mathematics education research by integrating error analysis, functional thinking, and self-regulated learning within a single framework, as well as offering suggestions for developing more effective mathematics instruction. Specifically, this article addresses the following research question: What types of newman errors do students make in functional thinking tasks based on their levels of self-regulated learning?

THEORETICAL ELEMENTS

Functional Thinking

Functional thinking refers to a representational reasoning process that emphasizes the relationship between two or more varying quantities. According to Martha Blanton and colleagues, functional thinking involves generalizing relationships between covarying quantities and expressing these relationships through different representations, such as natural language, algebraic symbols, tables, and graphs (Blanton et al., 2016; Blanton et al., 2015). Similar perspectives have also been reported in previous studies (Stephens et al., 2017; Yuniati et al., 2022). Furthermore, functional thinking supports the development of students' reasoning abilities, which serve as essential foundations for learning calculus, advanced mathematics, and science. The framework of functional thinking comprises three primary components: recursive patterning, covariational relationships, and correspondence (Morales & Parra, 2022). For instance, in a problem involving the ages of Álvaro and Carmen, where Carmen is five years older than Álvaro, the relationship can be represented by the function $f(x) = x + 5$ (Morales et al., 2018). According to Tanisli (2011), functional thinking provides several benefits for mathematics learning. First, it helps students understand algebraic concepts and the concept of functions more effectively. Second, it offers an alternative way of thinking that enables students to generalize relationships between varying quantities. Third, functional thinking contributes to the development of students' reasoning abilities. Finally, it equips students with fundamental competencies needed to support success in learning calculus, advanced mathematics, and science.

Newman Error Analysis (NEA)

The Newman Error Analysis (NEA) framework can be used by teachers to identify students' difficulties in solving mathematical problems that may result in incorrect answers (Dewantara et al., 2024; Sari et al., 2023). NEA categorizes students' errors into five stages. First, the reading stage refers to students' ability to correctly read mathematical questions, symbols, and notations. Second, the comprehension stage involves students' understanding of a problem, including identifying known information and determining what is being asked. Third, the transformation stage focuses on students' ability to select and formulate appropriate strategies, formulas, or mathematical models to solve the problem. Fourth, the process skills stage concerns the accurate execution of mathematical procedures and calculations. Finally, the encoding stage relates to students' ability to present the final answer correctly (Sari et al.,

2023). The application of NEA offers several benefits for teachers, such as classifying students' errors into distinct categories, identifying factors that contribute to problem-solving difficulties, developing effective instructional interventions, designing learning materials and assessment tasks that align with students' abilities, and serving as an evaluation tool for measuring classroom learning outcomes (Sundayana et al., 2023; Astuik & Purwasih, 2023).

Self-Regulated Learning (SRL)

Self-regulated learning refers to students' ability to manage and control their own learning processes, including cognitive, motivational, and behavioral aspects in achieving learning goals (El-adl & Alkharusi, 2020). In mathematics learning, self-regulated learning plays an important role because it helps students plan learning strategies, monitor their understanding, and evaluate learning outcomes. According to (Khayat, 2022), students with high levels of self-regulated learning tend to be more disciplined, capable of managing study time effectively, and highly motivated in complete mathematical tasks. In contrast, students with low SRL often experience difficulties in maintaining focus and solving mathematical problems effectively. Self-regulated learning is influenced by several factors, including individual, behavioral, and environmental factors (Cetin, 2022; Ma & Guo, 2023). Several studies have shown that self-regulated learning has a positive relationship with various learning approaches, such as inquiry-based learning, problem-based learning, and project-based learning (Amiruddin et al., 2022; Ortega-Ruiperez, 2023). In mathematics education, self-regulated learning supports the development of students' thinking skills, problem-solving abilities, and learning independence. Therefore, self-regulated learning is considered an important factor in supporting students' success in understanding mathematical concepts and completing mathematical tasks effectively.

Integrating Functional Thinking, NEA, Self-regulated Learning

Functional thinking, Newman Error Analysis (NEA), and self-regulated learning are interrelated concepts that support students' mathematical understanding and problem-solving abilities. Functional thinking emphasizes students' ability to identify patterns, recognize relationships between varying quantities, and represent mathematical ideas through symbols, tables, graphs, or algebraic expressions (Blanton et al., 2016; Stephens et al., 2017; Blanton et al., 2015). According to Tanisli (2011), functional thinking helps students understand algebraic concepts, generalize relationships between quantities, develop reasoning abilities, and build foundational competencies needed for advanced mathematics and science learning. In solving functional thinking

tasks, students often experience difficulties during the problem-solving process. Therefore, Newman Error Analysis (NEA) can be used to systematically identify students' errors through five stages: reading, comprehension, transformation, process skills, and encoding (Sari et al., 2023). NEA assists teachers in classifying students' errors, identifying the causes of difficulties, and designing appropriate instructional interventions (Sundayana et al., 2023; Astuik & Purwasih, 2023). Through NEA, teachers are able to analyze how students process mathematical information and determine the stages at which errors frequently occur in functional thinking tasks.

The occurrence of errors in functional thinking tasks is also closely related to students' self-regulated learning. Self-regulated learning refers to students' ability to monitor, regulate, and control their cognition, motivation, and behavior during the learning process (El-adl & Alkharusi, 2020; Khiat, 2022; Mahuda et al., 2022). Students with high levels of self-regulated learning tend to be more capable of planning learning strategies, maintaining focus, evaluating solutions, and managing difficulties when solving mathematical problems. Conversely, students with low levels of self-regulated learning often experience challenges in understanding problems, selecting appropriate strategies, and accurately completing mathematical procedures. In addition, self-regulated learning is influenced by individual, behavioral, and environmental factors (Cetin, 2022; Ma & Guo, 2023), and has a positive relationship with several student-centered learning approaches, such as inquiry-based learning, project-based learning, and problem-based learning (Amiruddin et al., 2022; Ortega-Ruiperez, 2023). Therefore, the integration of functional thinking, NEA, and self-regulated learning provides a comprehensive framework for analyzing students' mathematical problem-solving processes. Functional thinking focuses on students' reasoning and generalization abilities, NEA identifies the types of errors made during problem solving, and self-regulated learning explains how students regulate their learning processes when solving mathematical tasks. Through this integration, teachers can better understand students' difficulties and develop more effective instructional strategies to improve students' mathematical reasoning and problem-solving abilities.

METHODOLOGY

This study employed a descriptive qualitative research design. The participants consisted of 21 eleventh-grade students from a senior high school in Riau, Indonesia. Data were collected through questionnaires, tests, and interviews. The questionnaires were administered at the beginning of the study

to categorize students’ levels of self-regulated learning (SRL) into high, moderate, and low categories. Furthermore, tests were conducted to identify errors made in solving functional thinking tasks. Semi-structured interviews were also conducted with students who provided incorrect answers to obtain more in-depth information regarding the Newman errors they made while solving the tasks.

Research Participants

The 21 students were classified into three categories based on the results of the questionnaires, tests, and interviews, as presented in Table 1.

Table 1

Subject Selection Process

Questionnaires and Tests	SRL Category	SRL and Test Results	Interview	Research Subject
21 Students	3 in the low category 14 in the moderate category 4 in the high category	Low SRL Correct: - Wrong: 3 Moderate SRL Correct: 7 Wrong: 7 High SRL Correct: - Wrong: 4	14 answered incorrectly	1 low SRL student 1 moderate SRL student 1 high SRL student

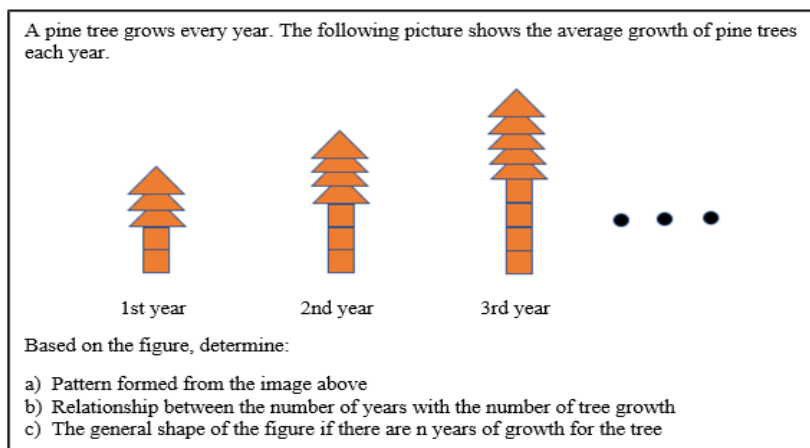
Data Collection Method

Qualitative data were collected using the instruments employed in this study and were subsequently analyzed inductively to understand meanings and unique characteristics, construct phenomena, and test hypotheses (Creswell, 2012). The questionnaire consisted of a set of written questions and statements to be answered by the respondents and was administered to students at the beginning of the study to determine their levels of self-regulated learning, which were categorized as high, moderate, or low. Meanwhile, the test was used to identify errors made by students in solving functional thinking tasks using the answer sheet format presented in Figure 1. Interviews were also conducted through a series of questions asked directly to the respondents. All students who provided incorrect answers were interviewed individually to obtain more in-depth information regarding the Newman errors they made while solving the

tasks. Furthermore, the instruments were validated by three experts: a mathematics expert, a mathematics education expert, and a mathematics teacher.

Figure 1

Question form



Data Analysis

Data were analyzed using the model proposed by Miles and Huberman (1994), which involves data reduction, data display, and conclusion drawing. The data reduction process focused on summarizing and selecting the main and important points from the raw data. The first stage involved reviewing the responses to the 30 items in the self-regulated learning (SRL) questionnaire completed by the 21 students. Scores were assigned to each item according to the assessment guidelines, and the students were subsequently classified into the three categories: high, moderate, and low SRL. The second stage involved examining students' answer sheets for the functional thinking task, which consisted of one open-ended question, with particular attention given to incorrect responses. The third stage involved conducting interviews with students who provided incorrect answers. Subsequently, one student from each SRL category (high, moderate, and low) was selected based on the criteria of having the highest number of Newman errors and demonstrating the ability to communicate and cooperate effectively during the interview process. Thus, three research participants were selected to represent the overall data. Following data reduction, the data were organized and presented to facilitate the drawing of appropriate conclusions. The data consisted of the results of the

SRL questionnaire, students' answer sheets, and interview transcripts. The final stage involved drawing conclusions based on the overall findings of the analysis using a descriptive approach. Data obtained from qualitative research need to be validated to ensure the trustworthiness of the findings (Creswell, 2012). In this study, validation was conducted through triangulation by collecting data using multiple instruments. Specifically, the data obtained from the participants were further confirmed through interviews to identify the causes of the errors made while solving the functional thinking task.

RESULTS AND ANALYSES

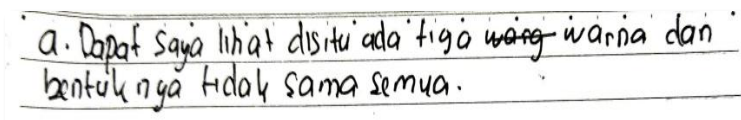
The analyzed data were used to classify students according to the types of Newman errors they exhibited. The results showed that three students were categorized as having low levels of self-regulated learning (SRL), seven students as having moderate levels, and four students as having high levels. However, students within each category demonstrated similar answer patterns. Therefore, one representative student was selected from each category, namely ODA, DS, and CRA, respectively. The data obtained from these students are presented as follows.

ODA WITH LOW SRL

ODA was unable to solve the problem correctly and made errors in identifying the recursive pattern. This was evident from the answer sheet presented in Figure 2 and was further supported by the interview excerpt below.

Figure 2

ODA's Responses to the Recursive Pattern Task



R : Please tell me again, what is the problem presented in the question?

ODA : It is about the difference in the number of trees, right?

R : State the information that is given in the question.

ODA : The patterns are formed in different shapes.

R : Explain how you determined the pattern presented in the question.

ODA : Because there are different colors and shapes. I noticed that there are three colors and different shapes.

- R : Tell me again, what is the problem in the question?
 ODA : The difference in the number of trees, right?
 R : State what information you know in the question!
 ODA : The patterns formed are in different shapes.
 R : Explain how you determine the pattern formed from the questions!
 ODA : Because there are different colors and shapes too, I see there are three colors and different shapes.

ODA exhibited a reading error because, although the student was able to read the questions clearly, ODA was unable to identify the problem and the information presented in the questions. In addition, ODA also exhibited a comprehension error because the student did not understand the instructions provided in the questions, resulting in an incorrect solution.

In determining the covariational relationship, ODA also made errors in solving the given problem. This was evidenced by ODA's worksheet, as presented in Figure 3, and further supported by the following interview excerpt.

Figure 3

ODA's Responses to the Covariational Relationship Task

B. Dapat saya lihat disitu tahun ke 1 (ada 3 pohon)
tahun ke 2 (ada 4 pohon) tahun ke 3 (ada 5 pohon)
dan disitu ada perbedaan setiap tahunnya.

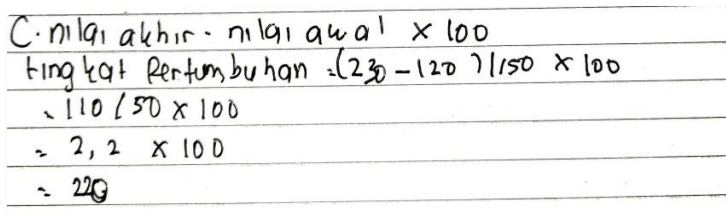
- R : Please explain and show the changes in the relationship between the number of years and the amount of tree growth.
 ODA : I can see that there were 3 trees in the first year, 4 trees in the second year, and 5 trees in the third year, which shows that the number of trees increased each year.

In this case, ODA exhibited a reading error because, although the student was able to read the questions clearly, ODA was unable to identify the problem and the information presented in the questions. ODA also exhibited a comprehension error because the student did not understand the instructions provided in the questions, resulting in an incorrect solution.

ODA also made errors in determining correspondence relationships. This was evidenced by the student's worksheet, as presented in Figure 4, and was further supported by the following interview excerpt.

Figure 4

ODA's Responses to the Correspondence Relationship Task



Handwritten student work showing calculations for a growth rate problem. The work is written on lined paper and includes the following steps:

$$\begin{aligned} & \text{C. nilai akhir} - \text{nilai awal} \times 100 \\ & \text{tingkat pertumbuhan} = (230 - 120) / 50 \times 100 \\ & = 110 / 50 \times 100 \\ & = 2,2 \times 100 \\ & = 220 \end{aligned}$$

R : Please explain and show how you obtained that answer.

ODA : For example, the value was 100 in the first year. Then, I used the initial value, the value in the second year, and the final value to determine the growth rate using the formula $(230 - 120) / 50 \times 100$. Therefore, the final answer is 200.

R : What is the general formula used in this problem?

ODA : The final value minus the initial value, multiplied by 100, Ma'am.

R : So, what is the conclusion of the problem above?

ODA : By using the formula, the final result is 220.

R : Tell and show how you got that answer!

ODA : For example, the value is 100 in the first year, then the initial value, the second year value, the year of work, and the final value were used to determine the growth rate as $= (230 - 120) / 50 \times 100$. Therefore, the final answer is 200.

R : What is the general form of the question?

ODA : Final value-initial value multiplied by 100, Ma'am.

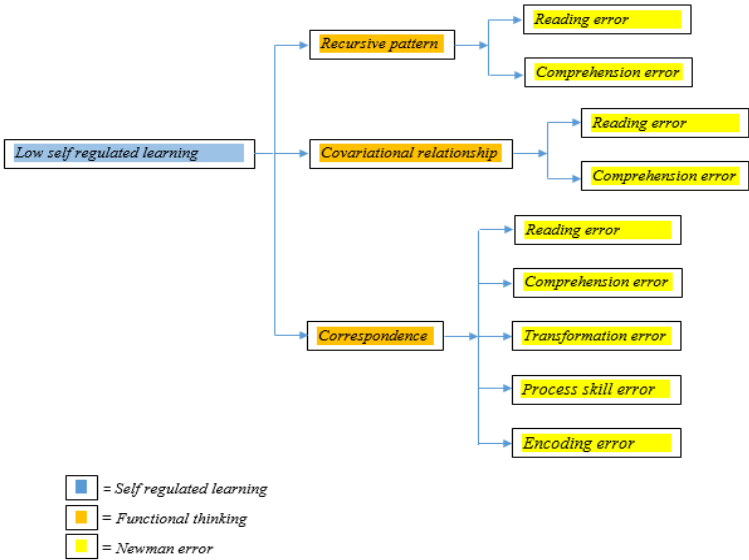
R : So, what is the conclusion from the problem above?

ODA : By using the formula, the final result is 220.

Thus, ODA made reading errors by failing to read the question carefully. Comprehension errors were also identified, as the student did not fully understand the instructions given in the problem. In addition, transformation errors occurred when the student designed an inappropriate solution strategy. Processing skill errors were evident during the execution of the transformation

process, which resulted in incorrect answers. Furthermore, encoding errors were observed because the solution method applied was incorrect from the outset and no final conclusion was provided. Therefore, the errors made by the student in solving functional thinking problems at a low level of self-regulated learning (SRL) are summarized in Diagram 1.

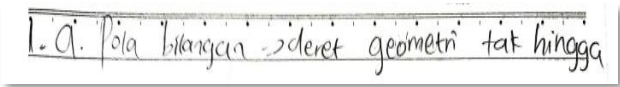
Diagram 1
Errors Made by Students with Low Self-Regulated Learning (SRL) in Solving Functional Thinking Problems.



DS with Moderate SRL

DS was unable to solve the problem correctly and exhibited errors in identifying the recursive pattern. Evidence of these errors can be observed in the student's response shown in Figure 5 and in the interview excerpts.

Figure 5
DS's Responses to the Recursive Pattern Task



R : Tell me again, what is being asked in the problem?

DS : The problem asks us to determine the changes that occur from year to year based on the tree data.

R : What information is provided in the problem?

DS : The information provided is related to the number of pine trees.

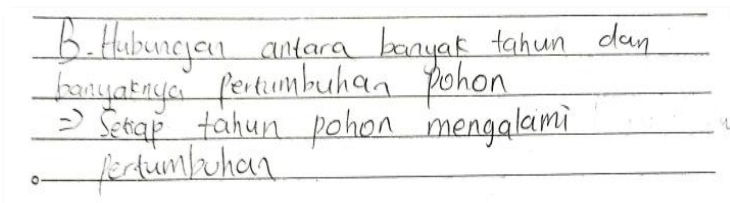
R : Explain how you determined the pattern presented in the problem.

DS : I concluded that the pattern is an infinite geometric series because the number appears to increase every year.

The student's written responses and interview data indicated that DS made comprehension errors due to a failure to fully understand the instructions provided in the problem. Furthermore, DS was unable to identify the covariational relationship, as evidenced by the responses presented in Figure 6 and supported by the following interview excerpt.

Figure 6

DS's Responses to the Covariational Relationship Task



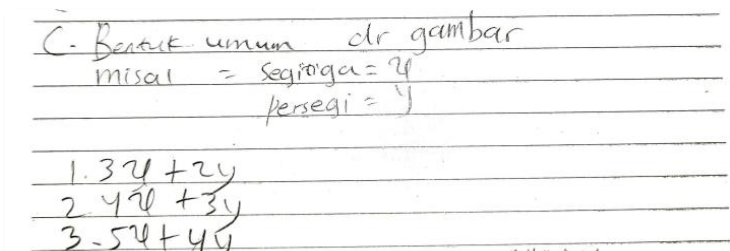
R : Explain and demonstrate the changes in the relationship between the number of years and the amount of tree growth.

DS : Based on the number of years shown, the tree grows over time. At first, the tree has three leaves and two branches, and then the number of leaves and branches appears to increase each year.

DS exhibited comprehension errors, as evidenced by an incomplete understanding of the problem instructions, which led to an incorrect solution. Furthermore, DS made errors in determining the correspondence relationship, as demonstrated in the written responses presented in Figure 7 and corroborated by the following interview excerpt.

Figure 7

DS's Responses to the Correspondence Relationship Task



R : Explain and demonstrate how you obtained your answer.

DS : There are two components in the figure: stems and leaves. I represented the stems by (y) and the leaves by (x). From the first-year figure, I obtained ($3x + 2y$), from the second-year figure ($4x + 3y$), and from the third-year figure ($5x + 4y$).

R : What is the general form of the pattern?

DS : I represented the leaves (triangles) as (x) and the stems (squares) as (y).

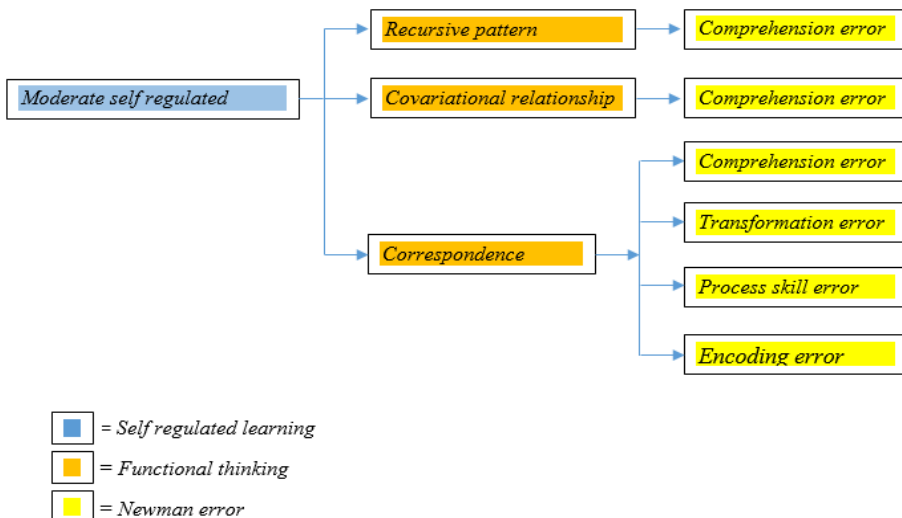
R : What conclusion can you draw from the problem?

DS : The conclusion is that the value of (n) can be determined using (x) and (y).

DS was unable to solve the problems correctly and exhibited comprehension errors due to a failure to fully understand the instructions provided in the questions. The student also demonstrated transformation errors by employing inappropriate solution strategies. In addition, processing skill errors were identified, as DS lacked the procedural knowledge necessary to carry out the solution process correctly. Furthermore, encoding errors were evident because the solution method was incorrect from the outset and no valid conclusion was provided. Overall, the errors made by students with moderate levels of self-regulated learning (SRL) in solving functional thinking problems are summarized in Diagram 2.

Diagram. 2

Errors Made by Students with Moderate Self-Regulated Learning (SRL) in Solving Functional Thinking Problems.



CRA with High SRL

CRA was unable to solve the problem correctly and demonstrated errors in identifying the recursive pattern. These errors were evident in the student's written responses presented in Figure 8 and were further supported by the following interview excerpt.

Figure 8

CRA's Responses to the Recursive Pattern Task



R : Tell me again, what is being asked in the problem?

CRA : The problem is about the growth of pine trees from year to year.

R : What information is provided in the problem?

CRA : The information provided is shown in the figure, where the number of pine tree components increases each year.

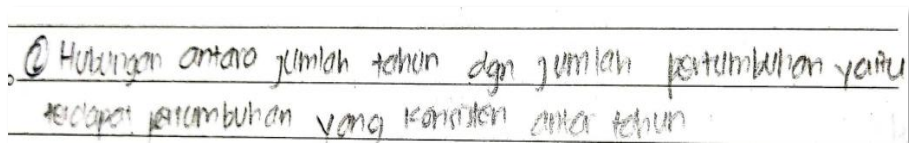
R : Explain how you determined the pattern presented in the problem.

CRA : I identified the pattern by observing the number of geometric shapes in each year. There are five shapes in the first year, seven in the second year, and nine in the third year. Therefore, I concluded that the pattern formed in the figure is an odd-number pattern.

CRA exhibited comprehension errors, as evidenced by an inadequate understanding of the instructions provided in the problem. In addition, the student experienced difficulties in identifying the covariational relationship. These difficulties were evident in the written responses presented in Figure 9 and were further supported by the following interview excerpt.

Figure 9

CRA's Responses to the Covariational Relationship Task



R : Explain and demonstrate the changes in the relationship between the number of years and the amount of tree growth.

CRA : The growth appears to be consistent from year to year. For example, the number of triangles increases from three in the first year to four in the second year.

CRA exhibited comprehension errors, as evidenced by an incomplete understanding of the problem requirements, which resulted in an incorrect solution. Furthermore, CRA experienced difficulties in determining the correspondence relationship. These difficulties were evident in the written responses presented in Figure 10 and were further supported by the following interview excerpt.

Figure 10

CRA's Responses to the Correspondence Relationship Task

Handwritten work on lined paper:

3. $n = x + y$ ($\textcircled{1} n = 3x + 2y$ $\textcircled{2} n = 5x + 4y$)
 $x \rightarrow \text{segitiga}$ ($\textcircled{3} n = 4x + 3y$)
 $y \rightarrow \text{persegi}$

R : Explain and demonstrate how you obtained your answer.

CRA : I used symbols to represent the shapes in the figure, where (x) represents a triangle and (y) represents a square. Based on this representation, the first-year figure consists of three triangles and two squares, which can be expressed as $(3x + 2y)$. Similarly, the second-year figure can be expressed as $(4x + 3y)$, and the third-year figure as $(5x + 4y)$.

R : What is the general form of the pattern?

CRA : Based on the values of (x) and (y), I concluded that the general form is $(n = x + y)$.

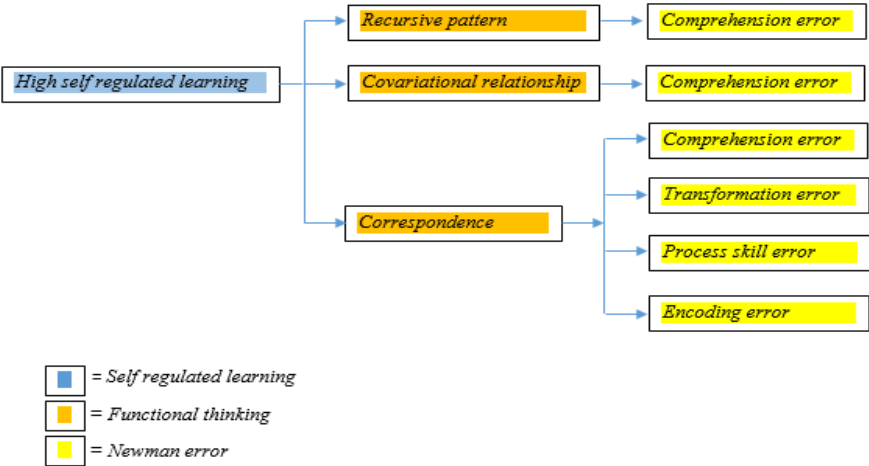
R : What conclusion can you draw from the problem?

CRA : I concluded that the general form $(n = x + y)$ can be used to determine the value for each year. Thus, the first year is represented by $(3x + 2y)$, the second year by $(4x + 3y)$, and the third year by $(5x + 4y)$.

In conclusion, CRA exhibited comprehension errors due to an inadequate understanding of the instructions provided in the problem. The student also demonstrated transformation errors by employing inappropriate strategies to solve the problem. Furthermore, processing skill errors were identified during the execution of the solution process, which resulted in incorrect answers. In addition, encoding errors were evident because the solution method was inappropriate from the outset and no valid conclusion was provided. Overall, the types of errors exhibited by students with high levels of self-regulated learning (SRL) in solving functional thinking problems are summarized in Diagram 3.

Diagram 3

Errors Made by Students with High Self-Regulated Learning (SRL) in Solving Functional Thinking Problems.



The results showed that students exhibited different types of Newman errors when solving functional thinking problems within the three frameworks of recursive patterns, covariational relationships, and correspondence correspondence (Yuniati et al., 2022; Yuniati et al., 2020). The recursive pattern framework focuses on analyzing variations in a sequence of values to identify the underlying pattern (Pinto & Canadas, 2012; Blanton et al., 2016). In this study, recursive patterns were identified inductively using the formula $b = U_n - U_{(n-1)}$, where (b) represents the difference between consecutive terms, U_n is the nth term, and $U_{(n-1)}$ is the preceding term. The findings indicated that students with high and moderate levels of self-regulated learning (SRL) did not exhibit reading errors, as they were able to read the questions accurately and identify the information provided. However, these students demonstrated comprehension errors because they were unable to fully understand the problem requirements despite correctly reading the questions (Agustiani, 2021; Lubis et al., 2021; Nuryati et al., 2022). In contrast, students with low SRL exhibited both reading and comprehension errors due to difficulties in understanding the problems, identifying relevant information, and interpreting the instructions. Reading errors were associated with an inability to recognize symbols, words, or essential information required to solve the problems appropriately. This finding is consistent with those of (Mahuda et al., 2022;

Sutama & Indriyani, 2021; Trapsilasiwi et al., 2021), who reported that reading errors occur when students fail to recognize written words or symbols, resulting in incorrect solutions. Comprehension errors were also attributed to students' misconceptions regarding the meaning of the problems, which prevented them from proceeding to the correct solution stage. This finding supports the results of (Noutsara et al., 2021), who found that comprehension errors occur when students are unable to understand a problem or translate it into appropriate mathematical representations.

A covariational relationship refers to the simultaneous change between two quantities within a pattern or sequence (Wilkie, 2014). (Blanton et al., 2016) further explained that covariational reasoning involves coordinating changes in two or more quantities, particularly independent and dependent variables. The results showed that students with high and moderate SRL did not exhibit reading errors when identifying covariational relationships because they were able to read and understand the information presented in the problems. Nevertheless, comprehension errors were observed due to difficulties in interpreting the instructions. This finding is consistent with previous studies (Agustiani, 2021; Lubis et al., 2021; Nuryati et al., 2022), which reported that comprehension errors occur when students struggle to understand what is being asked, despite being able to read the problem correctly. Meanwhile, students with low SRL exhibited reading errors because they were unable to read the questions carefully and identify the relevant information. They also demonstrated comprehension errors due to difficulties in understanding the instructions. According to (Noutsara et al., 2021; Sutama & Indriyani, 2021), reading errors are commonly associated with students' inability to interpret the meaning of words or symbols, understand complete information, and determine what is required by the problem.

Correspondence refers to the generalization of relationships between quantities and was primarily represented by students through algebraic and symbolic forms. This finding is consistent with (Yuniati et al., 2019), who reported that students often generalize quantitative relationships using algebraic formulas introduced during classroom instruction. Similarly, (Blanton et al., 2015) found that symbolic representations are commonly used to express generalized relationships. Other studies (Blanton et al., 2015; MacGregor & Stacey, 1995) have also reported the use of verbal representations for the same purpose. The findings revealed that students with high and moderate SRL demonstrated comprehension errors due to difficulties in understanding the instructions provided in the problems. Furthermore, transformation errors occurred when students applied inappropriate strategies to develop solutions.

Processing skill errors were also identified because students were unable to carry out the solution procedures correctly. In addition, encoding errors were evident when students used inappropriate methods from the outset and failed to provide valid conclusions. These findings are consistent with those reported by (Wardhani & Argaswari, 2022), who identified comprehension, transformation, processing skill, and encoding errors in students' solutions to trigonometric word problems.

Meanwhile, students with low SRL exhibited reading errors due to difficulties in reading the questions accurately and identifying the relevant information. They also demonstrated comprehension errors because they were unable to understand the problem instructions. Furthermore, transformation errors occurred when students selected inappropriate methods for solving the problems. Processing skill errors emerged during the execution of the solution procedures, while encoding errors were reflected in the use of inappropriate methods and the failure to provide conclusions. These findings are consistent with those reported by of (Agustiani, 2021; Utama & Indriyani, 2021), who found that students commonly experience reading, comprehension, transformation, processing skill, and encoding errors when solving mathematical problems. Such errors are often caused by inadequate conceptual understanding, flawed reasoning processes, forgetfulness, carelessness, insufficient knowledge of formulas and solution procedures, and the cumulative effects of errors made in previous stages. A summary of the findings is presented in Table 2.

Table 2

Research Results

SRL Category	Functional Thinking Framework	Newman Error	Information
High and Moderate SRL	Define a recursive pattern	Comprehension errors	Students were unable to understand the instructions provided in the problem, employed inappropriate solution strategies, which resulted in incorrect solutions,
	Determine covariational relationships	2.1 Comprehension errors	
	Determine correspondence	3.1 Comprehension errors	
		3.2 Transformation errors	

		3.3 Processing skill errors	and failed to provide valid conclusions.
		3.4 Encoding errors	
Low SRL	Define a recursive pattern	1.1 Reading errors	Students were unable to read the questions accurately, identify the relevant information and problem requirements, understand the instructions provided, develop appropriate solution strategies, solve the problems correctly, and formulate valid conclusions.
	Determine covariational relationships	Comprehension errors	
	Determine correspondence	2.1 Reading errors	
		2.2 Comprehension errors	
		3.1 Reading errors	
		3.2 Comprehension errors	
		Transformation errors	
		3.4 Processing skill errors	
		3.5 Encoding errors	

CONCLUSION

This study identified three patterns of Newman errors in students' functional thinking based on their levels of self-regulated learning (SRL). Students with high and moderate SRL exhibited similar types of Newman errors, although the characteristics of their responses differed. These errors included comprehension errors, transformation errors, processing skill errors, and encoding errors, while no reading errors were observed. In contrast, students with low SRL demonstrated all five types of Newman errors, namely reading errors, comprehension errors, transformation errors, processing skill errors, and encoding errors. These findings suggest that students' levels of self-regulated learning may influence the types of errors they make when solving functional thinking problems. Future studies are recommended to examine the relationship between functional thinking and self-regulated learning using broader samples and different research designs.

AUTHORS' CONTRIBUTION STATEMENTS

SY formulate research ideas, seek research theories, develop research instruments, process and analyze data. MI as data collectors, providing research equipment and preparing research permits for schools. All authors play an active role in discussing research results, overcoming problems and agreeing on research conclusions.

DATA AVAILABILITY STATEMENT

All research data available to the first author, SY

FUNDING

This research received no external funding.

INFORMED CONSENT STATEMENT

Informed consent was obtained from all subjects involved in the study.

ACKNOWLEDGMENTS

The researcher would like to thank all parties who have assisted in this research, especially to one of the schools in Riau who has agreed to give permission to conduct the research.

REFERENCES

- Agustiani, N. (2021). Analyzing Students' Errors in Solving Sequence and Series Application Problems using Newman Procedure. *International Journal on Emerging Mathematics Education*, 5(1), 23. <https://doi.org/10.12928/ijeme.v5i1.17377>
- Amiruddin, A., Muhammadiyah, U., Utara, S., Kapten, J., & Basri, M. (2022). The Role of Inductive Learning Models on the Students Self-regulated Learning in Math Journaling. *Cypriot Journal of Educational Sciences*, 17(7), 2428–2445. <https://doi.org/10.18844/cjes.v17i7.7680>
- Astuik & Purwasih, E. P., & Purwasih, S. M. (2023). Field Dependent Student Errors in Solving Linear Algebra Problems Based on Newman's Procedure. *Mosharafa: Jurnal Pendidikan Matematika*, 12(1), 169–180. <https://doi.org/10.31980/mosharafa.v12i1.1684>
- Biber, B.T. (2022). Self-Regulated Learning Strategies Used by Students to Prepare Mathematics Exams. *International Journal of Progressive Education*, 18(4), 0–2. <https://doi.org/10.29329/ijpe.2022.459.16>
- Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., Kim, J., Blanton, M., Knuth, E., Gardiner, A. M., & Kim, J. (2015). The Development of Children's Algebraic Thinking: The Impact of a Comprehensive Early Algebra Intervention in Third Grade. *Journal for Research in Mathematics Education*, 46(1), 39–87. <https://doi.org/10.5951/jresmetheduc.46.1.0039>
- Blanton, M., Brizuela, B. M., Gardiner, A. M., Sawrey, K., Newman-owens, A.,

- Blanton, M., Gardiner, A. M., & Newman-owens, A. (2016). *A Learning Trajectory in 6-Year-Olds' Thinking About Generalizing Functional Relationships*. *Journal for Research in Mathematics Education*, 46(5), 511–558. <https://doi.org/10.5951/jresmetheduc.46.5.051>
- Cetin, B. (2022). Investigation of Fourth Grade Primary School Students' Self-Regulated Learning Strategies and Successes in Mathematics Course in Terms of Socio-Economic Level. *International Journal of Curriculum and Instruction*, 15(1), 219-239.
- Creswell, J. W. (2012). *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research* (4th ed.). Pearson.
- Deng, X., Ding, R., & Huang, R. (2025). Investigating a Learning Progression of Functional Thinking for Elementary Students. *Educational Studies in Mathematics*, 120(1), 81–107.
- Dewantara, A. H, Istiyono, E, Retnawati, H., & Suyanto, S. (2024). Investigating Pre-Service Primary School Teachers' Difficulties in Solving Context-Based Mathematics Problems: An Error Analysis. *Mathematics Teaching Research Journal*, 16(2), 28–47.
- El-adl, A., & Alkharusi, H. (2020). Relationships between Self-regulated Learning Strategies, Learning Motivation and Mathematics Achievement. *Cypriot Journal of Educational Sciences*, 15(1), 104–111. <https://doi.org/10.18844/cjes.v15i1.4461>
- Khiat, H. (2022). A Self-regulated Learning Management System: Enhancing Performance, Motivation and Reflection in Learning. *Journal of University Teaching & Learning Practice*, 19(2), 43–59. <https://doi.org/10.53761/1.19.2.4>
- Kusmayadi, T. A., Sahara, S., & Fitriana, L. (2022). Application of Newman Errors Analysis Theory Related to Mathematical Literacy Problems: A Case Study of Secondary Students in Class 11. *AIP Conference Proceedings*, 2566. <https://doi.org/10.1063/5.0117183>
- Lubis, R. S., Pramudya, I., & Subanti, S. (2021). Mathematics Literacy: Newman's Error Analysis (NEA) Review from Habits of Mind. *Proceedings of the International Conference of Mathematics and Mathematics Education (I-CMME 2021)*, 597, 237–245. <https://doi.org/10.2991/assehr.k.211122.033>
- Ma, Y., & Guo, W. Y. (2023). Exploring the Effect of Learning Motivation and

- Self-Regulated Learning Climate on Undergraduates' Self-Regulated Learning in Higher Education. *International Journal on Social and Education Sciences (IJonSES)*, 5(2), 354-366. <https://doi.org/10.46328/ijonSES.512>
- MacGregor, M., & Stacey, K. (1995). The Effect of Different Approaches to Algebra on Students' Perceptions of Functional Relationships. *Mathematics Education Research Journal*, 7(1), 69–85. <https://doi.org/10.1007/BF03217276>
- Mahuda, I., Nasrullah, A., Mubarika, M. P., Meilisa, R., Evasufi, L., & Fajari, W. (2022). Android-Based Mathematics Learning Media Assisted by Smart Apps Creator on Self-Regulated Learning. *International Journal of Asian Education*, 3(3), 160–165. <https://doi.org/10.46966/ijae.v3i3.292>
- Mansor, A., S., N., Samsudin, N., & Ahmad Shukri, F. N. (2021). Newman's Error Analysis of Common Errors in Solving Convergence of Power Series Using Ratio Test. *International Journal of Academic Research in Business and Social Sciences*, 11(9), 888–898. <https://doi.org/10.6007/ijarbss/v11-i9/11077>
- Miles, M., B., & Huberman, A., M. (1994). *Qualitative Data Analysis: A Methods Sourcebook (Third Edit)*. <https://uk.sagepub.com/engb/asi/qualitative-data-analysis/book246128>
- Morales, R., & Parra, J. (2022). Strategies Employing Prospective Primary Education Teachers in a Functional Relationship Task. *Acta Scientiae*, 24(3), 32–62. <https://doi.org/10.17648/acta.scientiae.6959>
- Morales, R., Cañadas, M. C., Brizuela, B. y Gómez, P. (2018). Relaciones Funcionales y Estrategias de Alumnos de Primero de Educación Primaria en un Contexto Funcional. *Enseñanza de las Ciencias*, 36(3), 59-78.
- Noutsara, S., Neunjhem, T., & Chemrutsame, W. (2021). Mistakes in Mathematical Problems Solving Based on Newman's Error Analysis on Set Materials. *Journal La Edusci*, 2(1), 20–27. <https://doi.org/10.37899/journallaedusci.v2i1.367>
- Nugraheni, N., Sukestiyarno, YL., Wardono, W., Masrukan, M. (2021). Newman Error Analysis to Identify Student Errors in Solving High-Order Thinking Skills Problems Based on Self-Confidence. *International Conference on Science, Education and Technology*, 7 (1), 23–34. <https://proceeding.unnes.ac.id/index.php/iset>

- Nuryati, N., Purwaningsih, S. S., & Habinuddin, E. (2022). Analysis of Errors in Solving Mathematical Literacy Analysis Problems using Newman. *International Journal of Trends in Mathematics Education Research*, 5(3), 299–305. <https://doi.org/10.33122/ijtmer.v5i3.164>
- Oliveira, H., Polo-Blanco, I., & Henriques, A. (2021). Exploring Prospective Elementary Mathematics Teachers' Knowledge: A Focus on Functional Thinking. *Journal on Mathematics Education*, 12(2), 257–278. <https://doi.org/10.22342/jme.12.2.13745.257-278>
- Ortega-Ruiperez, B. (2023). Guidelines for Instructional Design of Courses for the Development of Self-Regulated Learning for Teachers. *South African Journal of Education*, 43(3), 1–13.
- Pang, J., & Sunwoo, J. (2022). An Analysis of Teacher Knowledge for Teaching Functional Thinking to Elementary School Students. *Asian Journal for Mathematics Education*, 1(3), 306–322. <https://doi.org/10.1177/27527263221125112>
- Pang, J., Leena, L., & Sunwoo, J. (2022). Task Development to Measure Functional Thinking: Focusing on Third Graders' Understanding. *The Korean Society of Educational Studies in Mathematics-Journal of Educational Research in Mathematics*, 32(3), 351–372. <https://doi.org/10.29275/jerm.2022.32.3.351>
- Pinto, E., & Cañadas, M. C. (2012). Functional Thinking and Generalisation in Third Year of Primary School. *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education, At Institute of Education, Dublin City University*. (1–8).
- Sari, D. P., Darhim, & Rosjanuardi, R. (2018). Errors of Students Learning with React Strategy in Solving the Problems of Mathematical Representation Ability. *Journal on Mathematics Education*, 9(1), 121–128. <https://doi.org/10.22342/jme.9.1.4378.121-128>
- Stephens, A. C., Fonger, N., Strachota, S., Isler, I., Blanton, M., Knuth, E., & Murphy Gardiner, A. (2017). A Learning Progression for Elementary Students' Functional Thinking. *Mathematical Thinking and Learning*, 19(3), 143–166. <https://doi.org/10.1080/10986065.2017.1328636>
- Sundayana, R., Parani, C., E. (2023). Analyzing Students' Errors in Solving Trigonometric Problems Using Newman's Procedure Based on Students' Cognitive Style. *Mosharafa: Jurnal Pendidikan Matematika*, 12(1), 135–142. <https://doi.org/10.31980/mosharafa.v12i1.762>

- Sutama, S., & Indriyani, Y. P. (2021). Newman Error Analysis (NEA): Detection of Student Learning Barriers in PPKM in Mathematics Subjects. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 10(4), 2901. <https://doi.org/10.24127/ajpm.v10i4.4221>
- Tanisli, D. (2011). Functional Thinking Ways in Relation to Linear Function Tables of Elementary School Students. *Journal of Mathematical Behavior*, 30(3), 206–223. <https://doi.org/10.1016/j.jmathb.2011.08.001>
- Trapsilasiwi, D., Murtikusuma, R. P., Hobri, Oktavianingtyas, E., Wiliandani, I., & Widodo, D. M. (2021). Analysis of Introverted Students' Error Based on Newman in Solving Arithmetic Sequences and Series Problems. *Proceedings of the 1st International Conference on Mathematics and Education*, 371–375. <https://doi.org/10.2991/assehr.k.210508.090>
- Wardhani, T. A. W., & Argaswari, D. P. A. D. (2022). High School Students' Error in Solving Word Problem of Trigonometry Based on Newman Error Hierarchical Model. *Infinity Journal*, 11(1), 87–102. <https://doi.org/10.22460/infinity.v11i1.p87-102>
- White, A. L. (2009). A Revaluation of Newman's Error Analysis. *MAV Annual Conference*, 3(Year 7), 249–257.
- Wilkie, K. J. (2014). Learning to Like Algebra Through Looking. *Australian Primary Mathematics Classroom (APMC)*, 19(4), 24–33. <https://eric.ed.gov/?id=EJ1093277>
- Yuniati, S., Nusantara, T., Subanji, & Sulandra, I., M. (2019). The Use of Multiple Representation in Functional Thinking. *International Journal of Recent Technology and Engineering*, 8(1C2), 672–678.
- Yuniati, S., Nusantara, T., Subanji, & Sulandra, I., M. (2020). Generalisation of Relations Between Quantity Variations Through Arithmetic Sequences in Functional Thinking. *International Journal of Innovation, Creativity and Change*, 13(2), 580–593.
- Yuniati, S., Suparjono, S., & Kurniati, A. (2022). Analysis of Student Ability in Functional Thinking Through the Function Table: The Case of Square Function. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 11(2), 1582. <https://doi.org/10.24127/ajpm.v11i2.4471>
- Yuniati, S., Nusantara, T., Subanji, & Sulandra, I., M. (2022). Investigating Functional Thinking Processes that Impact on Function Composition

Problems in Indonesia. *Acta Scientiae*, 4, 84–118.
<https://doi.org/10.17648/acta.scientiae.6969>