

Exploring a prospective mathematics teacher's noticing in the teaching of the line of best fit in a microteaching context

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ABSTRACT

Context: Prospective mathematics teachers need to strengthen their professional noticing, particularly in statistics, where attending to and interpreting students' statistical thinking does not develop spontaneously. Statistical content affords rich opportunities to recognise such thinking and respond to it productively. **Objectives:** This study explored that competence in the teaching of the line of best fit during a microteaching session led by a prospective mathematics and statistics teacher. **Design:** An instrumental case study, in which the participant designed and implemented a lesson framed within the topic of association between variables in the Chilean school curriculum. **Data collection and analysis:** Data came from non-participant observation of the session, which was video-recorded and transcribed, complemented by a stimulated recall interview. **Results:** The prospective teacher identified key classroom events and recognised her students' reasoning about parameter estimation, sample and population. Deciding how to respond proved less developed: her replies remained superficial and drew on a narrow range of instructional strategies. The study advances understanding of how noticing operates in statistics teaching and of the part microteaching plays in developing it.

Keywords: noticing; line of best fit; prospective mathematics teachers; microteaching; qualitative.

Uma aproximação ao *noticing* de uma futura professora de matemática no ensino da reta de melhor ajuste em contexto de microensino

RESUMO

Contexto: O desenvolvimento profissional de futuros professores de matemática requer o fortalecimento da competência de noticing, especialmente no campo da estatística, no qual a identificação e interpretação do pensamento estatístico dos alunos não ocorrem de maneira espontânea. O ensino de conteúdos estatísticos, em particular, oferece oportunidades-chave para reconhecer e responder a esse

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pensamento. **Objetivos:** Este estudo investigou essa competência no ensino da reta de melhor ajuste durante uma sessão de microensino conduzida por uma futura professora de matemática e estatística. **Desenho:** Por meio de um estudo de caso instrumental, uma futura professora foi convidada a elaborar e implementar uma aula de microensino sobre a reta de melhor ajuste, enquadrada no tema da associação entre variáveis presente no currículo escolar chileno. O objetivo foi explorar suas competências profissionais em situações de ensino. **Coleta e análise de dados:** Os dados foram coletados por meio de observação não participante, incluindo gravações e transcrições da sessão de microensino. A análise foi complementada por uma entrevista de recordação estimulada. **Resultados:** Os resultados evidenciam a capacidade da professora para identificar momentos-chave em sala de aula e reconhecer o pensamento estatístico dos alunos relacionado à estimativa de parâmetros, amostra e população. No entanto, sua capacidade de responder didaticamente às ideias dos alunos mostrou-se menos desenvolvida, apresentando reações superficiais ao abordar as ideias dos alunos por meio de estratégias instrucionais. Esses achados contribuem para a compreensão do noticing no ensino da estatística e seu papel no desenvolvimento do pensamento estatístico em contextos de microensino.

Palavras-chave: noticing; reta de melhor ajuste; futuros professores de Matemática; microensino; qualitativo.

INTRODUCTION

Advances in science and technology have placed vast amounts of data within easy reach of citizens and of teacher education institutions across every field of knowledge. Availability alone, however, is not enough: making sense of such data calls for statistical knowledge in both its scientific and its educational dimensions (Ferreiro & Fernández, 1988). In the latter case, higher education institutions, and especially those that prepare teachers, must ensure curricular alignment, since prospective mathematics teachers who teach statistics (hereafter PMTs) need to be able to systematise, analyse and interpret data drawn from real contexts (Rodríguez, 2011) as a basis for their teaching.

Teachers' knowledge, and their skill in making sense of students' mathematical and statistical thinking, matters not only at the disciplinary level but also when that knowledge is transposed into classroom practice (Juárez & Inzunza, 2014; Reyes et al., 2025). It opens up different ways of conceiving statistics and its teaching (Wild & Pfannkuch, 1999), grounded in fundamental statistical ideas (Burrill & Biehler, 2011), and allows teachers to observe key aspects of students' learning through a professional lens (Sherin & van Es, 2009; Star & Strickland, 2008).

That professional lens has been conceptualised in several ways (Sherin & van Es, 2009; Star & Strickland, 2008), each addressing the relationship between students' thinking and teaching practice (Jacobs et al., 2010). Some approaches (e.g., Star & Strickland, 2008) focus on what teachers catch or overlook during a lesson; others (Sherin & van Es, 2009) treat it as the capacity to attend to and interpret classroom events by drawing on one's own professional knowledge.

We align ourselves here with Jacobs et al. (2010), for whom professional noticing is a broader process: it takes in not only attending and interpreting but also deciding how to respond, whether the response is planned or improvised. These three components are interrelated both conceptually and temporally, which yields a more integrated account of teaching practice.

Recent research (Nagle et al., 2018; Shin, 2021; Türker Biber et al., 2023; Yilmaz & Yetkin Ozdemir, 2023) has examined PMTs' noticing in the teaching of statistics, identifying the skills they mobilise and what they focus on when interpreting students' statistical thinking. This body of work shows that many PMTs struggle to steer such thinking towards the goals of the task at hand, which points to the need for situated environments in which they can build the statistical knowledge that contextualised teaching demands. Casey and Wasserman (2015), working with in-service teachers, found along these lines that most regarded a line drawn through two points as sufficient to represent sample data or to model the relationship between variables.

Correlation in the Curricular Guidelines for Prospective Mathematics Teacher Education in Chile

In Chile, teacher education institutions organise their programmes according to the Standards for the Teaching Profession [Estándares de la Profesión Docente, EPD] (Ministerio de Educación [MINEDUC], 2021), which comprise pedagogical and disciplinary standards. The former are transversal and define the teaching practices that secure quality learning (MINEDUC, 2021, p. 18); the latter, for prospective mathematics teachers, divide into disciplinary standards and pedagogical content standards. Together these cover both mathematical and statistical knowledge and its classroom enactment, across six broad areas: Numbers and Algebra; Geometry; Probability and Statistics; Calculus; Computational Thinking; and Mathematical Skills and Attitudes.

Standard C addresses the association between variables, covering the analysis of univariate and bivariate data through graphical and tabular representations and using technology (MINEDUC, 2021, p. 88). Standard F incorporates the line of best fit, emphasising the ability to model statistical phenomena in meaningful contexts while taking account of the variability of the data and interpreting them considering the research question (MINEDUC, 2021, p. 99).

Teaching this content in school settings calls for solid statistical knowledge for teaching (Groth, 2007), which spans the formulation of questions, the collection and analysis of data, and the interpretation of results (Franklin et al., 2007). PMTs' professional skills are therefore crucial: a keen eye for classroom events, many of which ordinarily pass unnoticed, allows teachers to grasp the complexity of teaching (Kilpatrick et al., 2001) and to take decisions that foster statistical thinking (Wild & Pfannkuch, 1999).

Microteaching in mathematics teacher education

Teacher education, particularly in mathematics, has undergone successive curricular reforms (Franklin et al., 2007; MINEDUC, 2021; NCTM, 2000) that call for rethinking how teachers are prepared for real classrooms. Kanyongo (2021) argues for moving beyond practices centred on memorisation and towards teaching for understanding. This requires prospective teachers to live through experiences unlike those they had as school students, since reproducing those models would constrain their professional development (Chapman, 2012). As Zhang et al. (2024) put it, the purpose of education is to prepare learners for the kind of thinking that will be asked of them later.

Against this backdrop, teacher education institutions must create spaces in which prospective teachers can practise under conditions resembling the real classroom. Microteaching is an effective strategy for strengthening pedagogical and instructional skills (Alex & Thomas, 2023; Mukuka & Alex, 2024). Bozkurt and Koyunkaya (2022) stress that it affords meaningful formative experiences and brings strengths and weaknesses into view, while Altuk et al. (2012) highlights its value for analysing teaching performance among peers. It also promotes reflection on teaching and builds confidence and pedagogical intentionality (Basturk, 2016; Bilger, 2017).

The present study therefore sits at the intersection of two lines of inquiry, noticing among prospective mathematics teachers, and microteaching as a site of professional preparation, neither of which has paid sustained

attention to the teaching of statistics. We examine how one prospective mathematics teacher notices, *with particular emphasis on the skill of attending*, as she teaches the line of best fit in a microteaching context, and in doing so we contribute to understanding professional development in school statistics.

THEORETICAL FRAMEWORK

In this section, we present the theoretical framework of the study, corresponding to the noticing competence (Jacobs et al., 2010), centered specifically on the skills of *attending*, *interpreting*, and *deciding*.

The NOTICING competence

The learning and development of professional competencies extend beyond what teacher education institutions offer: they also permeate teachers' instructional and pedagogical work, which involves planning teaching situations, anticipating how students will tackle the activities proposed, and afterwards reflecting on what was designed and implemented (Groenwald & Llinares, 2022). Where statistics is concerned, this learning demands a shift in outlook, because a discipline grounded in data requires teachers to think and teach in terms of phenomena marked by uncertainty (Moore, 1997), unlike mathematical thinking.

The teaching of mathematics [statistics], understood as a practice to be learnt across the whole of teacher preparation, thus requires deep knowledge of the discipline, of how students think, and of which elements of design, implementation and classroom management are necessary for learning. This is in keeping with curricular guidelines that have incorporated such ways of teaching and learning as preparation for participation in society (Franklin et al., 2007; MINEDUC, 2021; NCTM, 2000).

For Groenwald and Llinares (2022), noticing is a professional competence understood as a set of skills that draw on pedagogical and disciplinary knowledge to carry out the work of teaching and learning mathematics [statistics]. That work includes setting objectives for the understanding of statistical concepts, designing lessons that mobilise statistical thinking, implementing activities that take context into account and involve posing statistical questions (Arnold & Franklin, 2021), and selecting and adapting situations for statistical problem solving (Wild & Pfannkuch, 1999).

Articulating these processes makes it possible to establish how prospective mathematics teachers' reason about and make sense of teaching situations in statistics (Fernández et al., 2018; Nagle et al., 2018). It brings to the surface how they construe students' statistical thinking, and the levels of cognitive demand that a lesson can sustain, based on how they interpret students' actions and decide how to carry instruction forward.

Noticing has been given various formulations (Jacobs et al., 2010; Mason, 2002; van Es & Sherin, 2008). Van Es and Sherin (2008) treat it as comprising two integrated subprocesses: (i) selective attention and (ii) knowledge-based reasoning. The first concerns prospective teachers' capacity to fix their attention on a specific classroom situation that matters for teaching; the second, their capacity to draw on available knowledge about a particular topic to make sense of it. Llinares (2013) locates this competence at the junction of theory and practice: noticing professionally involves identifying the relevant features of a teaching situation, using that knowledge to interpret them, and connecting them to more general principles about teaching and learning. It is precisely this theoretical grounding that warrants the *professional label*.

In the particular context of mathematical [statistical] thinking, Jacobs et al. (2010) frame noticing as the acquisition and development of three skills: attending to students' strategies by identifying the statistically relevant details within them; interpreting what those details reveal about students' understanding; and deciding how to carry the teaching of statistics forward, drawing on that interpretation to propose actions that help students progress.

METHODOLOGY

To address the stated objective, we adopted a qualitative, interpretive approach (Merriam & Tisdell, 2016) and, within it, an instrumental case study design (Stake, 1995). The aim was to explore in depth the noticing of a prospective mathematics teacher as she taught the line of best fit, part of the topic of simple linear regression, in a microteaching setting. The sections that follow describe the context in which the PMT worked, how the data were collected, and how they were analysed.

The prospective mathematics teacher and her microteaching context

The prospective mathematics teacher, whom we call Belén (a pseudonym), was taking the Statistical Models course in her ninth semester of

ten when the data were collected. She was chosen as an informant for her sustained motivation to learn and to deepen her grasp of statistical concepts, particularly those bound up with modelling. Her microteaching sessions also showed careful planning, with objectives, resources and instructional strategies directed at the key content of the course.

Two criteria proposed by Silverman (2008) informed this choice: (i) the researcher's interest in certain features of the case, and (ii) its temporal and contextual suitability, since she was taking that course and engaging in microteaching during the same period of her preparation.

Statistical Models belongs to the statistics strand of the teacher education programme in which Belén was enrolled. It is organised into three units: simple linear regression, multiple linear regression, and extensions towards generalised models. Beforehand, PMTs take descriptive statistics and its teaching (sixth semester), probability (seventh) and statistical inference (eighth). Alongside their disciplinary content, all these courses are oriented towards teaching it, in line with the objectives of the Chilean school curriculum (MINEDUC, 2015, 2019).

Assessment in the course includes statistical reports, lesson plans, problem solving with software and microteaching. Two microteaching sessions took place during the second half of 2024: the first, early on, addressed simple linear correlation; the second, towards the end, dealt with the line of best fit. In both, disciplinary content was articulated with the objectives of the school curriculum. Figure 1 shows the sequence of work carried out by Belén and her classmates.

Figura 1

Sequence of the activity carried out in class. (Merriam & Caffarella, 1991)



Of that full sequence, this study reports on Belén's work in stage 4, in which she delivered the lesson to her group of classmates (PMTs), who acted as school students, and then opened a discussion for joint reflection. Our purpose is to explore the professional noticing skills that emerge when a prospective teacher implements and reflects on a microteaching session, and specifically on the teaching of the line of best fit.

Data collection

The main data sources were a video recording of the session (Fitzgerald et al., 2013) and a semi-structured interview (Kvale, 1996). Three segments were used to define the units of meaning associated with *noticing*: the opening, the development and the closure. Table 1 presents each segment, its duration and the aspect of the content addressed.

Table 1

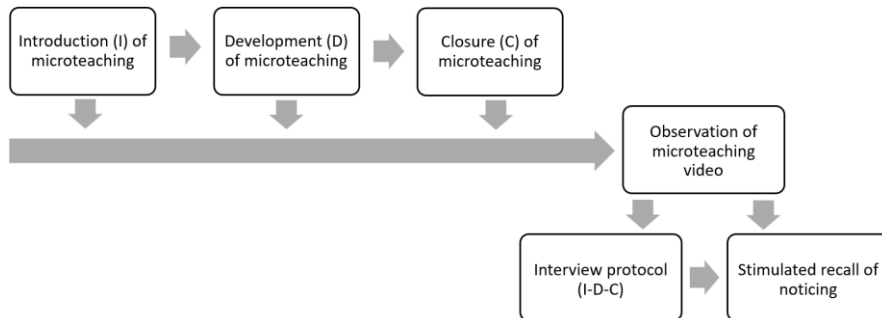
Organization of the microteaching session on the line of best fit. (Own elaboration)

Microteaching segment	Aspects addressed	Duration
Opening	Statement of the lesson objective and prior concepts of dependent and independent variables	06 min
Development	Modeling of the relationship between two quantitative variables and the line of best fit	21 min
Closure	Deeper exploration of conceptual aspects of population and sample related to the line of best fit	03 min

At a time and place agreed with her, Belén then took part in a video-stimulated recall interview (VSRI; Nguyen et al., 2013), so that she could revisit and identify relevant aspects of her teaching and thereby allow us to explore her noticing, that is, her attending, interpreting and deciding how to respond (Figure 2). The technique was chosen because it lets participants see their own teaching in action and helps them recover what they made of classroom events (Lyle, 2003), which here means the understanding students displayed about the line of best fit.

Figure 2

Sequence for the design of the microteaching session on the line of best fit.
(Own elaboration)



The interview protocol was organised around each of the three moments of the implementation (I-D-C). Every question carried the time interval Belén was to watch (see Figure 3), so that she could retrieve the events relevant to the development of her students' statistical thinking and, in doing so, have her noticing stimulated.

Figure 3

Outline of the structure of the stimulated recall interview protocol. (Own elaboration)

Protocol question	Purpose
Q3) 01:06 to 02:50 - Based on the students' responses, what do you consider continuing with the lesson?	This question sought to explore the skill of <i>deciding how to respond</i> , drawing on <i>attending to</i> and <i>interpreting</i> students' thinking about the idea of dependent and independent variables

Figure 3 gives an example of this structure. The first column contains the question number, the interval to be watched and the question itself; the second states the purpose the researcher pursued in exploring noticing in the teaching of the line of best fit.

Data analysis

We began by transcribing both the recording of the session and the interview held afterwards with Belén. The lesson segments (I-D-C) were then divided into units focused on noticing. These episodes corresponded to relevant moments in the teaching of the line of best fit and allowed us to identify the noticing skills (Jacobs et al., 2010) Belén displayed.

Those skills served to explore how the PMT construed her students' thinking. Each unit derived from the I-D-C segments corresponded to Belén's interventions while teaching the statistical topic, and here we concentrated on attending and interpreting. The units drawn from the VSRI supplied the evidence for deciding how to respond, without excluding cases in which the other two skills also surfaced.

To classify the three skills, we used the levels of evidence proposed by Yılmaz (2023): no evidence, weak evidence, partial evidence and robust evidence. This categorisation enabled deductive coding (Patton, 2002) grounded in the professional skills described by Jacobs et al. (2010). Table 2 presents the noticing indicators associated with each level.

Table 2
Analytical framework for the prospective mathematics teacher's noticing.
(Adapted from Yılmaz, 2023)

Levels of evidence	Characteristics of the level
No evidence	The prospective mathematics teacher provides no evidence of her noticing skills during the microteaching session.
Weak evidence	The prospective mathematics teacher's interventions (teaching) or responses (interview) are general and do not take students' statistical thinking into account. She focuses on perspectives centered on procedural and/or mechanical understanding related to students' thinking about the best fit.
Partial evidence	The prospective mathematics teacher attempts to describe students' statistical thinking in detail or consider alternative proposals in response to students' actions. Perspectives grounded in students' understanding are considered, going beyond the procedural or mechanical aspects of the line of best fit.
Robust evidence	The prospective mathematics teacher broadens her understanding of students' thinking, providing support to deepen that understanding through alternative perspectives. She focuses on a conceptual understanding of students' thinking about the line of best fit.

Finally, the analysis was validated through expert consensus triangulation (Flick, 2007), drawing on specialists in the noticing framework

and in statistics education. This consisted of contrasting excerpts from the microteaching interventions with the interview responses, as a multimethod approach to data collection instruments (Baxter & Lederman, 2001).

RESULTS

In this section we present the results, organised around the three moments of the microteaching session (the opening, the development and the closure) and analysed through the three components of the noticing framework: attending, interpreting and deciding how to respond. The analysis draws on a range of classroom episodes and on Belén's responses in the video-stimulated recall interview (VSRI).

Noticing at the Opening (I) of the Microteaching Session

At the opening of the microteaching session, Belén began by reviewing prior knowledge about dependent and independent variables before introducing the content of the line of best fit. She started by setting out the objective of the lesson: to explain the linear regression model (the line of best fit) both by hand and in Excel, working from the relationship between two variables. With that objective in view, Belén was asked the following in the VSRI:

Researcher: Why does your regression lesson begin by revisiting the notion of types of variables? How do these differ from mathematical variables? [Q2) 00:13]

Belén: I wanted to open with this review of independent and dependent variables before the concept of predictor and predicted variables. For that, I first needed the students to recall these concepts and then move on, because they might ask why and how, and why the name changes. The point of revisiting this was to introduce the idea of a relationship between two variables, so that they could understand that these types of variables had these other names in the context of regression. Besides, within mathematics, variables, as in equations, are sometimes rather abstract, or the meaning of the variables is very general; but in this topic of regression, of the regression model, the context, what this variable means, is very important, because it is not just a matter of one being related to the other or one depending on the other, but rather that one is explaining what happens and another is going to be explained on the basis of it, taking decisions and drawing

conclusions, and that allows the students greater understanding.

Belén's initial planning gives evidence of her skill in attending to students' thinking, in that she chose to begin with a review of dependent and independent variables to introduce the terminology of predictor and predicted variables in linear regression. She recognises that, unlike other and more abstract mathematical contexts, in regression it is the context that gives the variables their meaning, which supports conceptual understanding. She also shows partial evidence of interpreting how students understand the line of best fit.

As the session got under way, Belén entered an exchange with the students to clarify and recall the concepts of dependent and independent variable. In response to what she heard, she had the following exchange with one of them:

Belén: What is the dependent and the independent variable?

Student: Height together with the femur.

Belén: Right, and which one would be the independent one?

Student: The independent one, well, I think that one could be like bi

[meaning that it could be either dependent or independent], that one could be

dependent on the other, or the other way round.

Belén: It depends on the context ...

The exchange with the student shows that Belén attends only weakly to his response about the thinking behind types of variables. That response opened an opportunity to interpret and justify the dual nature of variables, that is, that one and the same variable may take on the role of dependent (predicted) or independent (predictor), depending on the research context and on whether those variables make sense within the analysis. Belén, however, attends only to the fact that this duality depends on the context, without pursuing the interpretation any further.

Building on the students' responses, Belén was asked the following in the VSRI: *What of this do you take into account in order to carry on with the lesson?* [Q3] 01:06 to 02:50]. Her answer was as follows:

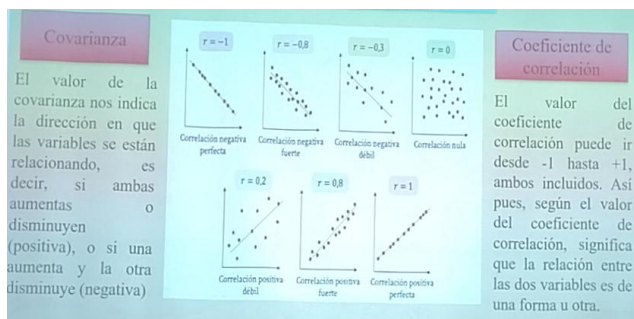
Belén: I consider their understanding of the concepts of dependent and independent variables. Knowing that they had understood gives me the green light to move on to what

comes next about the line in regression. Working from the examples, for instance, the distance they run in a certain time. There, which one is the independent variable, the time or the distance in that context? [example given by Belén]. As I was watching the students, they gave good examples showing that they understood that one variable depended on the other, and with the examples they gave me: one of them was the overall grade average, which would depend on other variables such as the marks, which would be the independent ones; another was marks as the dependent variable and attendance as the independent one.

Belén's answer provides partial evidence of the skill of deciding how to respond, since she identifies and describes the concepts the students show they have understood about the types of examples of variables. She also notes that knowing the students had grasped these concepts tells her they will be able to connect them with the construction of the regression model in the line of best fit. Although she identifies the ideas about types of variables and justifies them in relation to the concepts that follow, she does not go into how that connection would be made.

To close the opening segment of the session, Belén displays a number of symbolic and graphical representations of covariance and linear correlation between two numerical variables, to recall and establish the relationships between the ranges of the correlation and covariance measures and the corresponding scatterplots (Figure 4).

Figure 4
Relationships between graphical representations and measures of association between variables (Taken from the prospective teacher's microteaching session)



She presents and explains these topics with the aim of bringing the concepts back to mind and thereby introducing the students to the notion of the line of best fit in linear regression. She goes over aspects of the relationship between variables, discussing the idea of covariance, $\text{cov}(x,y)$, which indicates above all the direction of the relationship, and correlation, $\text{corr}(x,y)$, which indicates the strength of the relationship between the variables and serves to establish whether or not a relationship exists. Belén complements this with graphical representations. At this point in the interview she was asked about some of the aspects presented in this part of the session, with reference to the components of noticing:

Researcher: What role do the graphs play? Had your students not remembered some of these three topics, how would you have carried the teaching forward and how would you have explained these revision concepts? [Q6) 05:10 to 05:40]

Belén: From how I see the students, the graphs work as visual support; that is, students will not necessarily understand when you explain the correlation ranges that appear above the graphs, but they do understand the concepts of covariance and correlation better when these are complemented with the graphs. When they see the direction of the behaviour, and how the variables are related and how strong that relation is, then with the little dots [ordered pairs] they will be able to see how near or far they are from this theoretical line. With that linear behaviour they say, ..., the relationship is stronger; and if they see it more scattered, they say, there it is weaker, and that visual aid helps them understand.

Researcher: How would you have carried the teaching forward and how would you have explained these revision concepts? [Q6) 05:10 to 05:40]

Belén: I would have used examples, for instance with covariance, how it indicates the direction, or whether both increase simultaneously or not. I would have given examples... let us suppose the temperature rises; when it is very hot, the quantity of ice cream runs out more quickly, or if it is cold, there is a larger quantity of ice cream, because not much is sold. One goes up and the other goes down, as a more everyday example, so that they would understand it through contexts they could relate to.

At the close of the microteaching session, Belén shows partial evidence of interpretation in relation to the skill of attending to students' thinking, specifically in the use of graphical representations as visual support. She grounds this instructional choice in the understanding that graphs allow students to identify visually the direction and strength of the relationship between variables, thereby aiding the interpretation of the correlation ranges $\text{corr}(x,y)$ and their association with covariance $\text{cov}(x,y)$. In observing that “if they see it more scattered, they say, there it is weaker”, Belén shows how a graphical representation can act as a bridge between mathematical abstraction and the student's visual experience.

Partial evidence is likewise observed in relation to the skill of deciding how to respond, when Belén considers the difficulties students face in understanding the concepts of correlation, covariance and association. As a strategy she proposes the use of everyday examples, such as the relationship between temperature and ice cream sales, to reinforce conceptual understanding of these terms.

Noticing in the Development (D) of the Microteaching Session

At the start of the development segment of the session, Belén sets the students the following warm-up activity (Figure 5):

Figure 5

Warm-up activity to introduce the line of best fit (Taken from the prospective teacher's microteaching session).

Contexto: En México se desea predecir la nota de un alumno en un examen de Matemática, en función a la cantidad de horas que estudia en la semana en preparación para este examen. Para saber esto, se tomó a 6 estudiantes como muestra y sus resultados fueron los siguientes:

HORAS DE ESTUDIO (X)	NOTA OBTENIDA (Y)
3	8
6	10
8	15
2	8
1	5
6	12

- Considerando variables del contexto:
 x = Horas de estudio
 y = Nota obtenida
- Y la tabla con los valores entregados

- Grafica los puntos de forma manual en el plano disponible.
- Traza los puntos para formar la línea de regresión.
- Luego haz lo mismo en Excel y compara

Belén uses language belonging to the context of the line of best fit, explaining that the hours of study will be the predictor variable and the marks obtained will be the predicted mark, or that the hours of study will be explaining the mark obtained. She no longer refers to dependent and independent variables. As she explains how the activity is to be carried out,

she stresses that the values, which are ordered pairs (x_i, y_i) , correspond to marks from another country, a point she has planned deliberately.

At the point where Belén makes explicit the intention behind the design of the activity and the relevance of the context for the students' work, the following exchange with the PMT took place in the VSRI:

Researcher: What strategies or mathematical details would your students bring into play? Why? Can you describe some of them? [Q7) 06:45]

Belén: When they plot by hand before using Excel, many of them do not know how to count properly along the axes, for example in tens, and they get confused. They also tend to swap the X and Y axes round, which changes everything. In part c) with Excel, they compare lines and say they are the same because they have the same shape, but they do not check whether they pass through the same points. They do not make that comparison properly. [Q7) 06:45]

Researcher: Based on these mathematical details, how do you interpret them in the context of these students' mathematical and statistical thinking? [Q7) 06:45]

Belén: Yes, there is mathematical and statistical thinking, especially when they compare the lines drawn by hand with those from Excel. They locate points, create graphs and then analyse them. They ask themselves, for example: what is different about the other lines? Does Excel do it better? Why is my line different? The idea is for them to notice that variation. [Q7) 06:45]

Researcher: And based on this understanding, what would you decide to do to carry the instruction forward, whether there is sound mathematical and statistical thinking or difficulties? [Q7) 06:45]

Belén: I would like them to share in class what they got, to compare whether it was the same as or different from Excel, and to conclude why. If the lines are the same, why? If they are not, what changed? I want them to understand why there is a best line, to see that it is not just a matter of drawing it, but of understanding that this line explains better what is going on. [Q7) 06:45]

Analysis of Belén's answers about students' thinking in relation to the line of best fit yielded robust evidence of the skill of attending and partial evidence of interpreting and of deciding how to respond. Belén demonstrates clear attention to the students' work as they estimate the line of best fit, recognising such difficulties as errors in the scaling of the axes or in reversing the order of the Cartesian axes in relation to the variables under study. She also points out that students find it hard to see the estimated line as an approximation rather than as a single exact line, which becomes apparent when they compare their hand-drawn construction with the one produced in Excel without noticing any difference between them. These observations amount to a partial tracking of learning, though one that is sensitive to the students' conceptual difficulties.

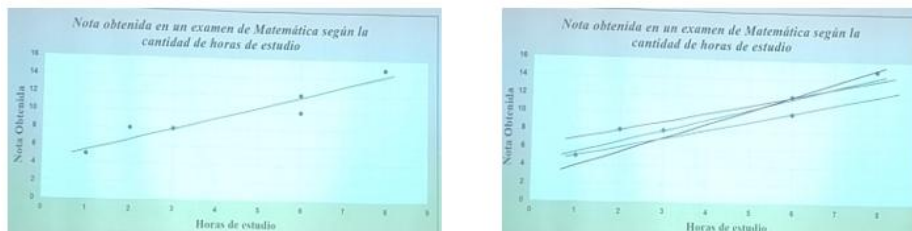
Based on the attending identified, partial evidence is observed of the skill of interpreting students' statistical thinking as they construct the line of best fit. This is because Belén argues, drawing on the students' work on paper and in Excel, that their difficulties centre on a more mathematical form of thinking, joining up ordered pairs without considering that what they are fitting is an estimate. She further notes that, in this process of estimation, the students do not question the differences between the lines constructed with the two resources, or whether the differences between the fitted lines are due to variation between samples.

About the skill of deciding how to respond, Belén supplies partial evidence about how to address students' difficulties in estimating the line of best fit. Although she mentions that one way of helping them is to put questions to them so that they interrogate the differences between the fitted lines, to represent the data jointly, and to conceptualise that any line at all may pass through the points, the aim being to find the best one, she does not go so far as to translate this into specific actions for developing students' thinking and ensuring that they grasp the idea of an optimal estimated line.

After working through the task with the students, where she noticed and interpreted their difficulties in understanding the line of best fit, Belén showed them the scatterplot (Figure 6) with different possible estimated lines drawn from the ordered pairs in the activity.

Figure 6

Scatterplot with different estimated lines (Taken from the prospective teacher's microteaching session).



In presenting the scatterplot, Belén explains the following to the students: *“This line that Excel gave you is a good one [Figure 6, left], but that does not mean that the ones you produced, which did not look alike, were wrong. That is, I could also have drawn a line like this, or like this, or even like this [Figure 6, right]. But why do I keep the one Excel produces? Because Excel gives you the one said to have the best fit, and that is what is useful to us, since, as we said, this is taken from a sample, so I do not know whether it is fitting the data or not. I therefore have to consider that line, the line I am drawing, that comes closest to the data”* [min. 15:13 to 15:56]. This explanation gives some indication of Belén’s intention to deepen students’ statistical thinking around the idea that the line found is an estimate when one is working with a sample, and that this estimate seeks to be the best possible. On this teaching situation, the following exchange with Belén took place in the VSRI:

Researcher: When you show the scatterplot with different fitted lines, what difficulties might students have in learning this concept? How would you decide to carry the teaching forward given the errors or forms of understanding you observe? You also mention the concept of sample when speaking about different fitted lines: why do you do so, and what part does it play in statistics? [Q9] 15:07 to 16:00]

Belén: The lines are one on top of another, and my intention was for the students to notice that each line passes through different points. From what I observed, they understood that well and had no difficulty with the idea that there are several possible lines. One difficulty I did notice is that they cannot grasp why the final line, the one Excel shows, is the best one, and why the others are not, even when it is explained. To

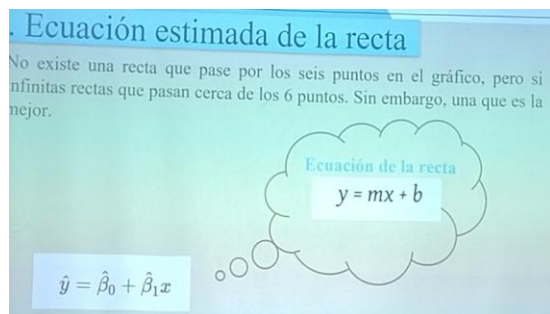
carry the teaching forward, I would show more examples with lines of best fit, each one constructed from different samples. That way they could compare visually, seeing that some points are closer to one line than to another, and so answer why that is the best line and not the others. [Q9) 15:07 to 16:00]

Belén's answer does not supply robust evidence of the skill of attending, although she recognises that the strategy designed to illustrate different lines of best fit was intelligible to the students. Her knowledge of the content and of the students allowed her to notice and to account for the fact that they were developing the concepts of fitted line and of estimation from ordered pairs. This led her to display the correct estimated line to compare it with others, though not necessarily with the best ones. While she identifies relevant aspects of student thinking, no in-depth interpretation of that thinking is in evidence. As for the skill of deciding how to respond, robust evidence was found, since she proposes that, faced with difficulties, she would show further applied situations to illustrate estimated lines, which would allow students to visualise differences and move closer to the best line, thereby supporting their understanding.

On completing the activity on the estimation of fitted lines, Belén formally introduces the equation of the estimated line to the students, relating it to the way Excel works to obtain this best line. She says: "I have to consider the line, or the straight line, that best fits the data, and that is what we are going to learn with this regression model. Since we are dealing with topics in linear regression, we need the equation of the line, and this is the equation of the line in the cloud (Figure 7), but I need to estimate these parameters that the equation of the line has, in the form shown below".

Figure 7

Diagram of the equation of the line together with the line of best fit (Taken from the prospective teacher's microteaching session).



On these aspects of the line of best fit that Belén teaches, she was asked the following: *Given the way your students think, what are you seeking to bring out or to do with these two lines in your teaching? With the $\hat{y}$ line, what is meant by estimating? Why does one estimate on the basis of what the students see?* [Q10] 17:42]

Belén: one of them, which is the equation of the line $[y = mx + b]$, is when I deal with the topic of the population, because they already know this, and in this case, I deliberately put in a sample. In this case that line is of no use to me, so I take hold of this in order to be able to explain to them the estimated equation of that line $[\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x]$, building on their understanding of these concepts; that is, that I need to estimate this line, that I need this line to be transformed in such a way that it becomes the approximation of the line used for the population. I draw these analogies so that they connect the models of the lines at the population and the sample level. What it is going to give me is an approximation, and they know that. With $\hat{y}$ it is like making that approximation, making it as close as possible to the population one. This estimation allows me to have this answer closer to what the population is, and not with the sample; and when working with the sample, I need to fit this line, to estimate it.

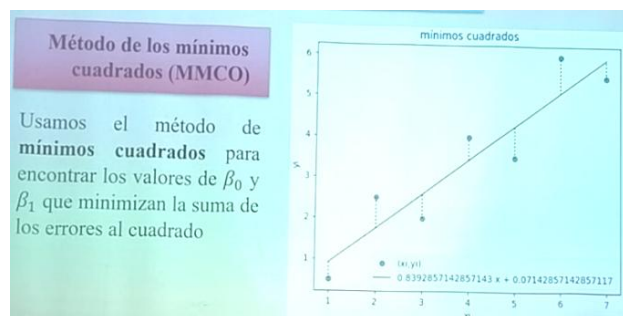
Given that Belén's session gave evidence of the skill of attending to students' statistical thinking about the line of best fit, question Q10 sought to gather evidence about deciding how to respond. Robust evidence is identified

for both interpreting and deciding how to respond. Belén interprets the students' thinking around the best fitted line, addressing concepts such as estimation, the difference between mean and sample, and the fitted line, and putting forward elements for deepening these conceptually in relation to the algebraic models under discussion. She also suggests that one way of advancing the teaching would be to transform the population line $y = mx + b$ into the estimated line obtained from the sample, $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$, articulating it with the students' prior knowledge.

In the explanation she gives the students, Belén presents the population model of the line, $y = \beta_0 + \beta_1 x + \epsilon$, indicating that what is sought is the line that fits the data (x_i, y_i) of a sample. She specifies that the parameters determine the position and the slope of the fitted line, that they minimise the sum of the errors, and that the general model is adapted to one specific to the data. She complements this with the graphical representation in Figure 8 to elaborate the explanation for the students and to deepen their statistical thinking about the line of best fit.

Figure 8

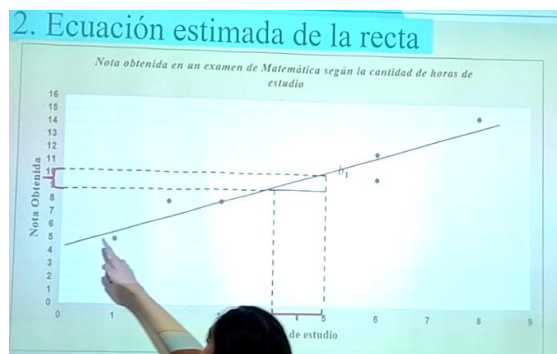
Graphical representation of the line of best fit (Taken from the prospective teacher's microteaching session).



She rounds out the use of the graphical representation by saying: “if I want the data to fit, that is the same as saying that those points are closer to the line” [min. 18:25 to 18:34]. From here, and to close the development and the explanation of the method for estimating the line of best fit, Belén demonstrates the procedure for estimating the parameters, that of $\hat{\beta}_1$ by way of the slope, given two points as an approximation of the parameter (Figure 9):

Figure 9

Graphical diagram for estimating the parameter β_1 in the line of best fit



From this point we asked Belén a question aimed specifically at exploring the skill of deciding how to respond: “If, for example, your students had used different values to carry out the same procedure, how would you have decided to carry the lesson forward on the basis of your students’ thinking?” [Q13] 22:50]. The following excerpt shows the prospective teacher’s answer:

Belén: Given that the students already know something about estimation, I would have remarked that the value of the intercept with the y-axis is 4.4, but that it could be another value, stressing that it is an estimate. For that I would draw on Excel, since it allows several of the concepts already worked on to be taken up again, such as the values of ordered pairs, the estimation of parameters, random samples and the general model. Although the students already know these topics, Excel makes it possible to go deeper without focusing so much on the formulas, which in this case are not very user-friendly.

Belén’s answer provides partial evidence in relation to deciding how to respond, considering the difficulties students might have had if she had worked with different sample ordered pairs. She mentions that, on the basis of the concepts and procedures she identifies the students as knowing, she would use Excel to show how the estimator of the intercept changes; but she does not go deeper into, for instance, the concept of variability, which is a central issue when different samples are taken. She refers only to the students’ use and knowledge of previously explained concepts.

Noticing at the Closure (C) of the Microteaching Session

For the closure of the lesson, following the interpretation of the estimation of the parameters of the line of best fit, Belén opens a dialogue with the students about when it is necessary to estimate those parameters:

Belén: If I am working with data from a population, is it necessary to estimate the parameters of the line?

Student 1: No.

Belén: Why not?

Student 1: Because we are working with all the values, so it is not necessary to do the estimation.

Belén: Good. [she asks another student]. For example, if I did not bring you a sample, and let us suppose the population is 45 students and I have their 45 marks, is it necessary, if I want to apply a regression model to find the line, that I must estimate the parameters?

Student 2: No, because those would already be the parameters of the line.

Belén: Right, what do you mean by that?

Student 2: I mean that that is the equation of the line, which is not like one from a sample [she asks another student].

Student 3: Yes, the same, it is no longer necessary to estimate, because we can already take decisions if we are working with the population.

Belén: As you say, it is not necessary to estimate [...], but as to whether it is necessary, it is not, because we do this when we have the uncertainty as to whether the data fit, whether that is fitting the data of a population, which is precisely what happens when we obtain a sample.

Belén's dialogue with the students (1, 2 and 3) made it possible to evidence the skills of attending and interpreting with regard to statistical thinking around the construction of the line of best fit. Belén is seen to notice and understand that the students distinguish the estimation of the line according to whether the data come from a population or from a sample. In response to an initial question, one student denies that the line has to be estimated and justifies his answer, which is then supported by students 1 and 2. Belén takes up the concepts of population, sample and parameter estimation mentioned by the students, and puts forward a contextualised example (the

marks of a class) that allows these concepts to be linked to a situation close to them.

Finally, faced with the students expressed thinking about estimating a line with population data, Belén concludes that in that case no estimation is carried out, acknowledging that uncertainty applies only when one works with samples. From her attention to the concepts emerging and her interpretation of student thinking, she establishes a link with the notion of uncertainty in the context of probability models.

The findings on Belén’s noticing competence are set out in Table 3 in terms of attending, interpreting and deciding how to respond, following the levels proposed by Yılmaz (2023), as these bear on students’ statistical thinking. Each piece of evidence of Belén’s noticing corresponds, in most cases, to her answers in the video-stimulated recall interview, considering the distinctive events of the microteaching session.

Table 3
Summary of noticing on the line of best fit.

Microteaching segment	Levels of noticing											
	No evidence			Weak evidence			Partial evidence			Robust evidence		
	A	I	D	A	I	D	A	I	D	A	I	D
Opening												
Development												
Closure												

CONCLUSIONS

In line with the aim of the research, which was to explore the noticing of a prospective mathematics teacher in the teaching of the line of best fit in a microteaching context, the results yield relevant evidence, taking into account both the knowledge the PMT displayed and the design of the lesson within the time constraints set. Robust and partial evidence appeared most frequently across the three segments of the lesson, although the skill of deciding how to respond was the least evident during both the lesson and the interview.

It was possible to establish that the attending displayed by the PMT in teaching the line of best fit is robust across the three segments into which the session is divided; yet this does not entail that the level of attending accounts for the skill of interpreting students’ statistical thinking. Where attending is

robust, interpreting remains partial, since, as Fernández et al. (2018) and Jacobs et al. (2010) argue, interpreting students' thinking does not follow automatically from identifying it, but involves going beyond identification, examining in detail how students think and describing their form of reasoning in the face of the learning content.

Notwithstanding the above, in a few passages of the session, specifically in the opening and development segments, attending and interpreting showed no evidence or weak evidence regarding the thinking the PMT identifies in her students. Unlike other studies (Ivars & Fernández, 2018; Zapatera, 2019), however, the skill of deciding how to respond yielded robust evidence from the PMT when she attended to and interpreted students' statistical thinking about the line of best fit.

On this basis, microteaching contexts, supported methodologically by the video-stimulated recall interview, make it possible to bring out different levels of the professional noticing competence, particularly with respect to the line of best fit. This allowed the PMT to become aware of identifying and interpreting statistical ideas that mattered in her students' work, such as estimation, parameter, sample, population and approximation, ideas that give meaning to students' statistical thinking and allow it to be mobilised in the classroom, rather than the more superficial, procedural aspects reported in some of the findings of Yılmaz (2023) and Yılmaz and Yetkin Ozdemir (2023).

These results suggest that microteaching contexts offer a setting in which the prospective mathematics teacher's knowledge for teaching statistics can be explored and deepened, since they open spaces where teachers can strengthen their teaching abilities and skills (Mukuka & Alex, 2024) in relation to students' learning. Complemented methodologically by stimulated recall interviews, these contexts afford the prospective mathematics teacher the opportunity to call to mind relevant teaching events, generating a form of awareness that goes beyond craft knowledge (Kennedy, 2002) about students' statistical thinking. The relationship between microteaching contexts and stimulated recall interviews may be complemented by processes of discussion and reflection among peers, which would generate new knowledge and learning strategies in the development of professional skills (Girit-Yıldız et al., 2023; Santagata et al., 2007).

The study thus contributes to this line of research: although its results are not generalisable, they offer indications as to how the development of professional skills in prospective mathematics teachers might be guided in microteaching contexts. It also provides concrete elements for the design of

formative experiences that allow prospective mathematics teachers to develop an attentive and reflective eye for students' statistical thinking.

ACKNOWLEDGEMENTS

This study was funded under internal research project CIP2023021, “La competencia docente NOTICING [mirada profesional sobre las situaciones de enseñanza] para movilizar el razonamiento estadístico en futuros profesores de Matemática y Estadística de la Universidad Central de Chile” [The NOTICING teaching competence (a professional lens on teaching situations) for mobilising statistical reasoning in prospective mathematics and statistics teachers at Universidad Central de Chile], funded by Universidad Central de Chile, Santiago, Chile.

AUTHORSHIP CONTRIBUTION STATEMENT

NSA collected the data, carried out the analyses and drafted the initial versions of the methodology, the results, the introduction and the theoretical framework. He also conducted the discussion of the results and reviewed and approved the final version of the manuscript. SRS carried out the style revision, the data analysis and the correction of the conclusions.

DATA AVAILABILITY STATEMENT

The data associated with this study will be made available upon request.

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