ACTA SCIENTIAE
ISSN: 2178-7727
DOI: 10.64856/acta.scientiae.8374

FO(RM)AP: A theoretical model for teacher development focused on promoting mathematical reasoning

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ABSTRACT

Background: High-quality mathematics teaching is intrinsically linked to teacher education, their professional knowledge, and their capacity for reflection grounded in classroom practice—elements that are essential to their professional development. In recent years, we have been particularly interested in mathematical reasoning (MR), in ways to foster it during mathematics lessons, and in the contributions it offers to students' mathematical learning. **Objectives**: To propose a theoretical model of a formative process that supports teachers' understanding of MR, its underlying processes, and how to promote it in the classroom; and to discuss the key elements that constitute this model. **Design**: Cycles of design-based research (DBR), combined with the theoretical and methodological studies we have developed on this topic, have revealed the need to systematically reflect on the elements that make up such formative processes. **Setting and Participants**: Five formative experiences involving mathematics teachers in different school contexts. **Data Collection and Analysis**: We considered data generated and collected during the five formative processes. These data were organised, transcribed, and analysed to identify evidence of the professional learning enabled by the formative processes for the participating teachers. **Results**: The FO(RM)AP model was structured around three dimensions (introduction, deepening, and consolidation), each providing opportunities for professional development through tasks, documentation, and reflective cycles. **Conclusions**: The model contributes to teacher education by structuring practices that promote MR in students.

Keywords: Mathematical Reasoning; Mathematics Teaching; Professional Development; Theoretical Model; Design-Based Research; Professional Learning Opportunities.

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FO(RM)AP: Um modelo de processo formativo para o raciocínio matemático visando o desenvolvimento profissional de professores que ensinam matemática

RESUMO

Contexto: Um ensino de matemática de qualidade está intrinsicamente relacionado com a formação de professores, com seus conhecimentos profissionais e com a reflexão que fazem a partir da prática, elementos essenciais ao seu desenvolvimento profissional. Um tema que tem nos interessado sobremaneira, nos últimos anos, é o raciocínio matemático (RM), as formas de promovê-lo nas aulas de matemática e a contribuição que ele tem para a aprendizagem matemática. **Objetivos**: propor um modelo teórico de processo formativo que contribua para a compreensão dos professores sobre o RM, seus processos e as formas de como promovê-lo em sala de aula e discutir os elementos que integram. **Design:** ciclos de investigação baseada em design, aliadas aos estudos teóricos e metodológicos que realizamos sobre essa temática, nos evidenciou a necessidade de pensar, de forma sistemática, nos elementos que integram esses processos formativos. **Cenário e Participantes**: Cinco experiências formativas com professores de matemática em diferentes contextos escolares. **Coleta e Análise de Dados**: consideramos dados gerados e recolhidos no decorrer de cinco processos formativos, que foram organizados, transcritos e analisados buscando evidências das aprendizagens profissionais que os processos formativos possibilitaram aos professores participantes. **Resultados**: Estruturou-se o modelo FO(RM)AP em três dimensões (introdução, aprofundamento e consolidação), cada uma promovendo oportunidades de desenvolvimento profissional por meio de tarefas, registros e ciclos reflexivos. **Conclusões**: O modelo contribui para a formação de professores ao estruturar práticas que favorecem a promoção do RM dos alunos.

Palavras-chave: Raciocínio Matemático; Ensino de Matemática. Desenvolvimento Profissional; Modelo teórico; Pesquisa Baseada em Design; Oportunidades de Aprendizagem Profissional.

INTRODUCTION

Within the field of mathematics education, many studies have explored pedagogical alternatives to position students as active subjects in their learning. In this area, the research group “Mathematical Reasoning and Teacher Education” [Raciocínio Matemático e Formação de Professores], led by the first two authors of this article, has been dedicated to investigating the ways of promoting mathematical reasoning (MR) in the classroom, at all levels of schooling (e.g., Araman & Serrazina, 2020; Trevisan & Araman, 2021; Morais et al., 2022; Oliveira et al., 2022; Trevisan et al., 2023).

The relevance of the development of the students’ MR is pointed out by several international studies (Lannin et al., 2011; Jeannote & Kieran, 2017; Mata-Pereira & Ponte, 2018), considering that “without mathematical reasoning there is no mathematics”, that “mathematical reasoning is fundamental for a deeper understanding of mathematics” (Lannin et al., 2011, p. 7), and that the development of reasoning is one of the great objectives of mathematics teaching (Mata-Pereira & Ponte, 2017).

Taking these elements into account, the research group initially focused its efforts on the conceptual understanding of MR, its processes and teacher actions for its promotion in the classroom, which resulted in a set of research results (in the form of master’s and doctoral theses, articles in periodicals and proceedings of events). In parallel, the group focused on the creation, proposal, and development of formative processes that contributed to expanding teachers’ professional knowledge of MR, its potential for mathematical learning, and ways to promote it in the classroom. We assume that teachers’ knowledge of MR directly influences their actions in class (Araman et al., 2019), so that it is “necessary for teachers to know the reasoning processes of their students and reflect on them” (Ponte et al., 2012, p. 375), preferably through reflection on situations lived in the classroom context.

Thus, over the last five years, we have studied, structured, and developed five formative processes involving teachers working at different levels of education, both in initial and continuing education. Such formative processes were structured according to the PLOT (professional learning opportunities for teachers) model (Ribeiro & Ponte, 2020), a theoretical framework for organising and understanding teachers’ learning opportunities in mathematics. This structuring adopts assumptions from the methodology of design-based research (DBR) (Cobb et al., 2016; Ponte et al., 2016), predicting the realisation of several design cycles. This allows each new cycle (formative process) to present improvements over the previous one, based on the experience it generates, and to reformulate the principles that underlie the formative process through collective and collaborative reflection among researchers participating in the research group.

Thus, this article, through the systematisation of research results based on data produced in these formative processes, aims to *propose a theoretical model of the formative process that contributes to teachers’ understanding of MR, its processes, and ways to promote it in the classroom, and to discuss the elements that integrate it*. This amalgamation of the theoretical, methodological, and experiential elements from the design cycles led us to systematically think, structure, and discuss a theoretical model of formative processes in MR that contributes to the development of teachers’ knowledge and, as a result, to their professional development.

To this end, this article is structured as follows: in addition to this introduction, there is a theoretical foundation with principles that support the model; next, we discuss methodological research paths that gave rise to the model; we present the Teacher Education for Mathematical Reasoning model [Formação de Professores para o Raciocínio Matemático]—FO(RM)AP, discussing each of its dimensions. Finally, we discussed, in the final considerations, how this model can be used in the education of teachers who teach mathematics, highlighting its potential.

THEORETICAL FRAMEWORK

To expand teachers' learning possibilities, formative processes can deepen their knowledge (Ball et al., 2008; Carrillo-Yañes et al., 2018) and, consequently, contribute to their professional development (Richit, 2021). We hope that this has repercussions for students' learning, as formative processes and moments of teacher discussion allow them to re-signify their knowledge (Ribeiro & Ponte, 2020) and are related to how students learn. This is different from the concept of teacher education, which, for many decades, was limited to offering "training" to "fill in" possible deficits in their knowledge, without articulation with their practice or with their relationship to student learning (Ponte et al., 2022).

Recently, research has pointed out that teacher education must understand the re-signification of different domains of specialised knowledge for teaching (Carrillo-Yañez et al., 2018), in articulation with practice through formative tasks or professional learning tasks (PLTs) (Ribeiro & Ponte, 2020). In this sense, thinking about teachers' professional learning implies generating opportunities for them to discuss, interact, and socialise their doubts, share suggestions and concerns about classroom practices, enabling them to learn in interaction with other professionals and in the sharing of results and concerns that emerge from their work (Gross et al., 2024). The concept of professional learning opportunity (PLO) adopted in this study has teacher educators as one of its fundamental elements (Ribeiro & Ponte, 2020). Teacher educators establish a complex articulation linking aspects of teaching to teachers, the classroom, students, and content (Prediger et al., 2019).

When considering these aspects, it is essential to understand how to organise formative processes that can enhance PLO for teachers, especially when discussing mathematical reasoning (MR), our focus of interest. However, there is no consensus in the literature on what MR is and what it means to reason mathematically. According to Mata-Pereira and Ponte (2018, p. 38), mathematical reasoning "includes a diversity of reasoning processes, namely, the formulation of questions, the formulation and testing of conjectures, the definition and application of resolution strategies, and justification". In the same vein, Jeannotte and Kieran (2017) define mathematical reasoning as a process of communication with others or with oneself that allows one to infer mathematical statements from other mathematical statements, a definition we adopt in this article. These authors analyse the reasoning in terms of its structural aspect (deductive, inductive, and abductive) and its processes, which involve the search for similarities and differences (generalise, conjecture, identify a pattern, compare, and classify), validation (justification and proof/demonstration) and exemplification (Jeannotte & Kieran, 2017).

The organisation of mathematics teaching and learning environments involving the resolution of exploratory tasks (Ponte, 2005), collaborative work (Brodie, 2010; Ellis, 2011) and the promotion of mathematical discussions (Wood, 1999; Ponte, 2017; Rodrigues et al., 2018) are essential for students to expand their ability to think and to mobilise different MR processes. It is interesting that teachers carefully choose and plan challenging tasks that encourage critical thinking and the search for justifications. Monitoring students' progress and intervening when necessary (Canavarro, 2011) are important aspects of the teacher's performance that contribute to students' MR development.

We assume that the teaching and learning processes connect and complement each other inseparably and that, although the teacher cannot directly direct the students' learning process, they can provide them with learning environments (Steinbring, 1998) with potential for the development of MR, through different actions (Araman et al., 2019). However, studies that address teacher professional development in MR-related formative contexts are still incipient (Davidson et al., 2019; Henriques & Martins, 2022).

In particular, in recent years, relevant investigations in the academic arena have been accentuated by studies seeking to identify, analyse, and recognise how knowledge about MR is articulated with teacher education. Vieira et al. (2020) investigated how pre-service teachers who will teach mathematics in the early years understand MR processes before and after a formative experience, finding greater clarity in their explanations of what they understand these processes to be after this experience. In this same direction, the study by Rodrigues et al. (2021) found that teachers' participation in a formative process contributed to greater knowledge of actions that promote students' reasoning processes and to a better understanding of teaching actions that challenge them to use the processes of generalising and justifying.

Santos et al. (2022) argue that teachers participating in formative processes progressively focus their attention on MR processes, with opportunities to identify and characterise them. The authors state that the design of these processes seems to have contributed to teachers deepening their understanding of core MR processes (generalising and justifying), as well as to their ability to foster and identify such processes when working with their students.

Rodrigues et al. (2021) propose a theoretical framework organised into six levels of knowledge within each of the reasoning processes (generalise, justify, compare, classify, and exemplify), and analyse the evolution of this type of knowledge in a formative context. Based on this framework, Correa et al. (2024) discussed the levels at which essential understandings of MR and its processes are held, highlighting the relevance of developing formative meetings to increase participating teachers' understanding. Similarly, Hebert et al. (2015) propose categories that outline the different perceptions of MR held by teachers working in the early years, presenting a theoretical framework that facilitates monitoring the growth of primary school teachers' awareness of aspects of MR.

Within the scope of the research group in which this study is developed, we highlight three formative experiences described and analysed in the dissertations by Anjos (2023), Oliveira (2024), and Trevisolli (2024). In the first, the results showed that a face-to-face formative process aimed at teachers in the early years of elementary school provided participants with PLOs to understand MR processes, especially generalisation, through mathematical research and reflections on their teaching practices.

Trevisolli (2024) and Oliveira (2024) analysed another formative process structured from discussions about MR and its processes, followed by stages of planning, carrying out, and reflecting on a class. The first study revealed PLOs related to rethinking pedagogical practices, focusing on the search for strategies to promote the processes of conjecturing, generalising, and justifying in mathematics. The second study analysed discussions about the classes these teachers planned and delivered, showing different PLOs that, according to Ball, Thames, and Phelps (2008), were categorised into two groups: specialised knowledge of MR and pedagogical knowledge of MR.

Despite the promising results of these works, to study whether and how a formative process can support the teacher's professional learning about MR, it is necessary to understand more systematically what professional knowledge (Ball et al., 2008) is necessary for teachers to promote the development of students’ MR (Martins et al., 2023). In turn, to study whether and how a formative process can generate PLOs about RM, it is necessary to organise the design of this process for this purpose (Davis & Krajcik, 2005; Fuentes & Ma, 2018) through a theoretical model that guides and supports education programs for teachers who teach mathematics, with a view to promoting students’ mathematical reasoning, as proposed in this article.

We adopt an interactive and interconnected perspective that considers different domains that, together, can contribute to generating PSO for the teacher (Ribeiro & Ponte, 2020), and a structure that aims to break with a linear and compartmentalised logic of conceiving the formative processes that aim to provide learning to the teacher (Goldsmith et al., 2014). Thus, the relevance of outlining parameters, supported by theoretical studies that inform the structuring of actions that promote students’ MR, is justified by the principles and foundations underpinning the model presented in the remainder of this text.

METHODOLOGICAL PATHWAYS

As already mentioned, over the last few years, the research group “Mathematical Reasoning and Teacher Education” (Raciocínio Matemático e Formação de Professores) has been dedicated to studying, structuring, and developing formative processes that aim to support the teacher’s professional learning about MR, its processes, and the ways to promote it in the classroom. The research conducted within the scope of such processes was designed, structured, and carried out using design-based research (DBR) (Cobb et al., 2016; Ponte et al., 2016). This methodological option is due to the cyclical nature of RBD, which comprises phases of preparation, intervention, and retrospective analysis (Cobb et al., 2016), together constituting a design cycle. In turn, the realisation of several cycles allows the development of local theories and the projection of educational interventions (Gravemeijer, 2016). According to Gravemeijer (2016), the theories developed have a limited scope because they pertain to a specific domain. They are studied iteratively and cumulatively, which allows learning about the specific domain being studied to develop.

In fact, the research program we carried out, supported by RBD, enabled us, through a retrospective analysis of the previous cycle, to propose a new cycle, analyse the data obtained, and elaborate a local theory. We highlight that five formative processes have been carried out so far, as shown in Figure 1. The first two resulted in the dissertations of Anjos (2023), Oliveira (2024), and Trevisolli (2024), in addition to a thesis still being prepared, as well as in the publication of related articles (Anjos et al., 2025; Anjos et al., 2024;

Trevisolli et al., 2024). The third and fourth formative processes produced data that are still being analysed by master's students, and the last by a doctoral student.

Figure 1

Formative processes carried out (Authors, 2026

Formative Process 1 - 2022	Formative Process 2 - 2023	Formative Process 3 - 2024	Formative Process 4 - 2024	Formative Process 5 - 2025
Eight teachers who teach mathematics in the early years of elementary school (continuing education)	Ten teachers who teach mathematics at different levels of education (continuing education)	14 students from a mathematics teaching degree course (initial education)	28 students from a teaching degree in pedagogy (initial education)	40 students from a teaching degree in pedagogy (initial education)
Three PLTs were performed and analysed.	Four PLTs were performed (three of which were the same as the formative process 1).	Five PLTs were performed.	Two PLTs were performed.	Five PLTs were performed.

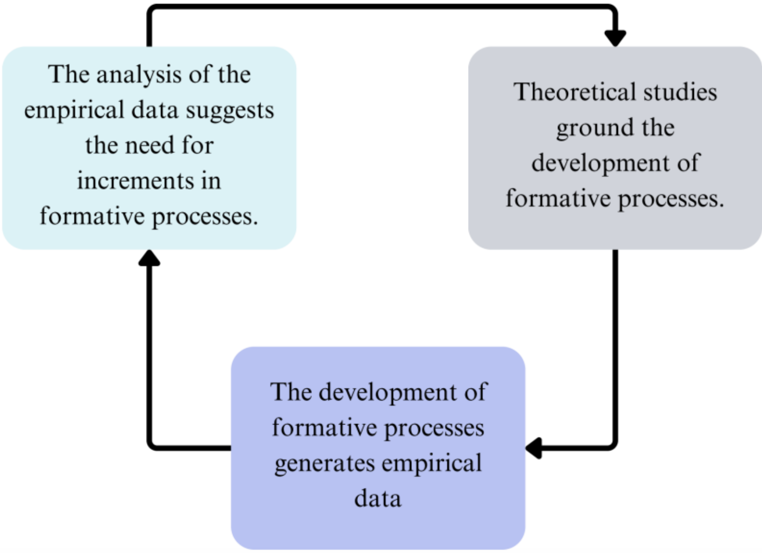
Given the iterative and cyclical nature of the RBD, the analysis of data from each formative process contributed to the development of the subsequent formative process. Each evolution in the theory located in each of the processes assisted in the projection of the next formative process, that is, in a new design cycle (Cobb et al., 2016; Ponte et al., 2016), as shown in Figure 2.

The data generated and collected during the formative processes (Figure 1) comprise: (1) written protocols (the written records of the PLT carried out by the teachers participating in the formative processes); (2) audio and video recordings of the discussions that took place in small groups between the teachers and also in plenary between teachers and the educators during the formative processes; (3) the written records, through field diaries and memories of meetings, of researchers who are part of the research group, and who met during and at the end of each formative process.

These data were collected, organised, transcribed and analysed seeking evidence of PLO (Ribeiro & Ponte, 2020) that the formative processes allowed the participating teachers, especially those related to the development of knowledge necessary for teaching (Ball et al., 2008; Carrillo-Yañes et al., 2018), to the processes of mathematical reasoning (Jeannotte & Kieran, 2017) and to the actions that promote MR in the classroom (Rodrigues et al., 2021).

Figure 2

Iterative process for the development of the formative processes (Araman et al., 2025, p. 11)



Therefore, we present and discuss below a theoretical model, based on our long-term theoretical and empirical results, that has the potential to support the development of new formative processes that promote RM. It comprises three dimensions (introduction, deepening, and consolidation), each of which builds on the previous level, forming a theoretical model that can develop professional knowledge of MR and strategies to promote it in the classroom.

The empirical data employed in this article were generated within the scope of research projects submitted for ethical review, with the voluntary participation of the subjects involved and upon consent for the use of written records, audio and video recordings, and other materials produced throughout the formative processes. In the case of the project approved by the Research Ethics Committee for Human Subjects of the Federal University of Technology

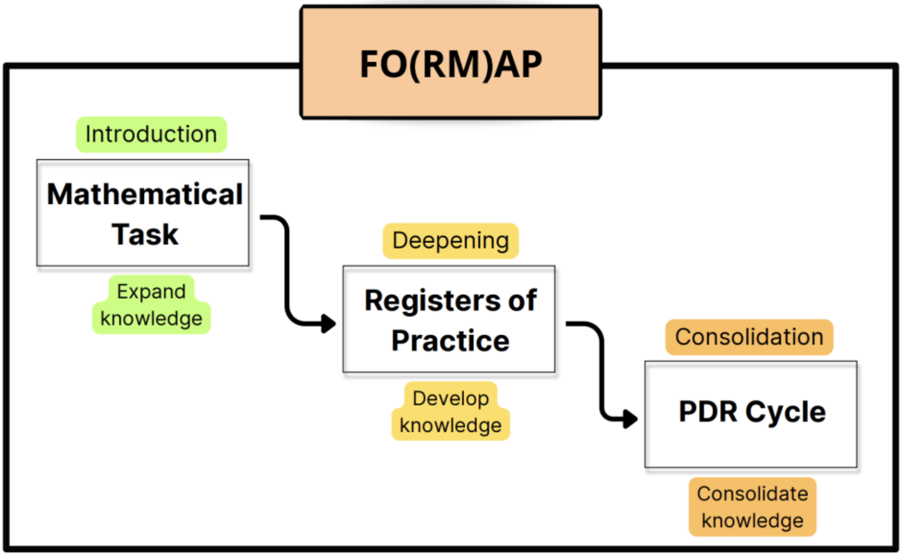
– Paraná (CEP/UTFPR), nº 5,161,835 and CAAE nº 49773821.1.0000.5547 are duly registered. In all cases, privacy, confidentiality, and the anonymity of participants were ensured.

THEORETICAL MODEL

We called the theoretical model we proposed *Teacher Education for Mathematical Reasoning* (Formação de Professores para o Raciocínio Matemático – FO(RM)AP), presenting three articulated and complementary dimensions, with a focus on generating PLOs related to the different dimensions of knowledge of the teachers involved. The dimension of the introduction, based on mathematical task(s), a stage in which the expansion (acquisition) of various knowledge related to RM is expected; the dimension of deepening, supported by the analysis of records from teaching practice, which aims to develop the knowledge acquired in the previous stage; and the dimension of consolidation, in which teachers have the opportunity to consolidate the knowledge developed during the formative process by carrying out a PDR cycle (of planning, developing, and reflecting) (Trevisan, Ribeiro & Ponte, 2020).

Figure 3

Theoretical model FO(RM)AP (Authors, 2026)



The arrows indicate a continuous and interconnected movement between the three dimensions, in which each one integrates knowledge developed in the others, deepening them as they are thought, discussed, and systematised through the experience of prospective teachers in different contexts that enable different PLOs (Ribeiro & Ponte, 2020), i.e.: those generated from the work with the mathematical task, in which prospective teachers place themselves "in the students' shoes," anticipating resolution strategies; those generated through the analysis and reflection of practice records, including written protocols and audio and video records of students' resolution of the mathematical task; and those generated in the experience with the PDR cycle, at which prospective teachers have the opportunity to plan, develop, and reflect on a real teaching experience, carried out in a classroom.

Below, we discuss each dimension:

Introduction This is the first dimension of the FO(RM)AP model, and it is when, in many cases, prospective teachers have their first contact with a systematic study of MR, supported by specific theoretical and methodological contributions from research on this theme. Some research findings indicate that, although there is consensus among teachers who teach mathematics on the importance of promoting students' MR development, understanding what MR is, its processes, and how to develop it remains difficult for most of them.

Thus, the first dimension occupies a unique place in the formative process. In this dimension, aspects related to the definition of MR (Araman & Serrazina, 2020), understanding of its structural and procedural aspects (Jeannotte & Kieran, 2017), essential understandings of MR (Lannin et al., 2011) and the role that mathematical tasks play in its promotion (Ponte et al., 2020) will be addressed.

However, the experience we gained through the various formative processes we carried out (Chart 1) showed that such themes are not easy for teachers to understand immediately without a context in which they make sense (Martins et al., 2023). Thus, the FO(RM)AP model suggests the use of mathematical tasks as a starting point for the development of the formative process, that is, all discussion starts in the mathematical task: a work proposal for students, composed of one or more situations aimed at the development of a specific mathematical idea (Stein & Smith, 1998).

Regarding the selection of tasks that have potential for MR, Ponte et al. (2020) highlight the role of these tasks in promoting students' mathematical reasoning, and that "designing or identifying tasks that promote reasoning and implementing them in the classroom are fundamental aspects of teaching that can influence students' learning opportunities" (Martins et al., 2023, p. 1127).

That said, in this dimension, the proposed mathematical task plays a remarkable role and therefore requires reflection, formulation, and execution to raise questions involving MR and its processes. This is a point to highlight, as the mathematical task to be worked on in that dimension not only presents potential for MR and raises relevant discussions, it can also serve as an example for teachers of how a mathematical task can contribute to MR in the classroom, since this same mathematical task selected for the dimension of the *Introduction* of the FO(RM)AP model will be used in the next dimension, from another perspective.

Nevertheless, what can we consider a task with potential for MR promotion? We know that good mathematical tasks contribute to thinking and arguing, however, more specifically, we find in the Reason project (Ponte, 2022) some principles to consider in the task to engage students in situations that promote mathematical reasoning, such as: (i) "include questions that allow a variety of resolution strategies" (p. 100); (ii) "include questions that involve a variety of representations" (p. 100); and (iii) "include questions that encourage and favor reflection on the reasoning processes used" (p. 101). The Reason Project also brings principles aimed at promoting the MR processes of generalisation and justification (Ponte, 2022).

Having an appropriate mathematical task to promote MR is a factor to consider, but it is not the only one: the way the task is approached in the classroom is also worth mentioning. In the FO(RM)AP model, we propose presenting the mathematical task to prospective teachers and having them solve it following the exploratory teaching approach (Ponte, 2005). Exploratory teaching has as its "main characteristic that the teacher does not seek to explain everything, but leaves an important part of the work of discovery and construction of knowledge for students to carry out" (Ponte, 2005, p. 13). This characteristic is decisive if we think that the development of the ability to reason mathematically is an experience of its own, not something others can achieve, but one that can be fostered through interaction with other students and the teacher in the proposed task.

That said, the option for the exploratory teaching approach is timely, as it encourages collaborative work (Brodie, 2010; Ellis, 2011) and promotes mathematical discussions (Ponte, 2017; Rodrigues et al., 2018; Wood, 1999), essential aspects for the development of MR. Exploratory teaching comprises four phases: "Introduction of the task; students' autonomous resolution, often organised in small groups or in pairs; collective discussion of resolutions with the whole class; and systematisation of the learning carried out" (Serrazina, 2021, p. 3). All these phases provide moments of interaction among the students and between the students and the teacher during the discussion of the task and therefore constitute important components in promoting MR in the classroom.

Given the above, the FO(RM)AP model proposes the following referral to the *Introduction* dimension:

1)The delivery of the mathematical task to teachers and the proposal that it be solved by them, autonomously or in pairs, according to the first and second phases of exploratory teaching (Serrazina, 2021). Some elements need to be considered by the educator at this time, such as selecting a task that, in addition to the indicated characteristics, aligns with the curriculum requirements for the level of education in which the teachers work. At this moment, during the resolution, teachers have the opportunity to experience a type of task that is often not used in the classroom, and to realise the importance of their interaction in the mathematical discussion of the task.

2) Then, there is the phase of collective discussion of resolutions, the second phase of exploratory teaching (Serrazine, 2021). At this moment, pre-service teachers have the opportunity to know how others solved the task, their thinking processes, their resolution strategies, how they produced their written registers, what type of representation they used, among other aspects that may arise. This is an enriching time for teachers and is closely related to the potential of exploratory tasks and exploratory teaching, which allow them to engage with various ways of thinking about and solving a mathematical task (provided that it presents such potential, of course). It is important to consider that, in formative processes involving more participants, one must select and sequence the resolutions to be discussed in plenary sessions (Elliott et al., 2009).

3) The next phase is the systematisation of the topics learned (Serrazina, 2021), conducted by the educator from the discussions that took place in the previous stages. It is at this moment that the MR definitions are presented and discussed, the MR processes mobilised by teachers during the performance of the mathematical task are highlighted, defined and discussed, the characteristics of the proposed mathematical task and the way to promote it (explanatory teaching) are evidenced and treated as primordial elements for the promotion of MR. The teacher's actions should also be highlighted, especially those related to conducting a mathematical discussion, asking good questions, and not providing immediate answers to students. At this moment, the issues inherent in mathematical concepts, properties, definitions, relationships, and,

finally, conceptual elements of mathematics addressed in the proposed task can also be systematised. At this stage, the educator can use materials that address MR definitions, processes, task characteristics, and exploratory teaching to complement the systematisation. Thus, the conceptualisation of MR and its processes, as well as the way to promote it in the classroom, gains meaning from teachers' experience and is not provided a priori.

According to the FO(RM)AP model, in this first dimension, prospective teachers must expand knowledge, and, therefore, we consider that the referral proposed in the *Introduction* dimension offers professional learning opportunities (Ribeiro & Ponte, 2020) that allow teachers to expand their knowledge, specifically in relation to:

— Mathematical tasks that promote MR: exploratory task (Ponte et al., 2020) with the characteristics listed in the Reason Project (Ponte, 2022).

— Exploratory teaching, its phases and the contributions it presents to MR and mathematical learning (Serrazina, 2021).

— MR and its processes (Lannin et al., 2011; Jeannotte & Kieran, 2017; Vieira et al., 2020)

— Teacher actions that promote MR (Araman et al., 2019; Araman et al., 2025), ranging from selecting the mathematical task, approaching it according to exploratory teaching, asking appropriate questions, not responding promptly to students' questions, promoting quality mathematical discussions, and systematising learning.

Finally, we systematised the main elements of the dimension *Introduction* of the FO(RM)AP model in Figure 4.

Figure 4

Systematisation of the first dimension of the FO(RM)AP model – Introduction (Authors, 2026)

First dimension of the FO(RM)AP model – *Introduction*
Math task
Starting point: the mathematical task approached according to the phases of exploratory teaching.
Expand knowledge
Opportunities to expand knowledge, from the context of the classroom, related to: - MR-promoting mathematical task - exploratory teaching - MR and its processes

- teacher's actions - mathematics present in the task

Deepening: This dimension of the FO(RM)AP model consists of deepening the knowledge expanded in the previous dimension, in order to develop them in more depth, as it will address the same mathematical task, but from a new perspective: from records of practice of the resolution made by students — which may include written protocols and/or audio records of the discussion (Ball et al., 2014). When working with this material from teaching practice, teachers can reflect on and build knowledge from real classroom experiences (Aguiar et al., 2021).

Practice records are students' written task resolutions and transcripts of audio recordings made while they solved the task, in the phases described for exploratory teaching (Serrazina, 2021). The practice records are obtained by the research group as follows: a mathematical task (in this case, the task used in the previous dimension) is proposed to a real class of students, and their teacher proposes to approach it through the phases of exploratory teaching. Some participants in the group monitor the application of the task and carry out data collection, always following the established precepts and the Ethics Committee's approval of the research under opinion N. 5.161.835. The collected data are organised, transcribed, and analysed to identify the MR processes mobilised by the students during task resolution. Based on the analysis conducted by the research group, PLTs are developed (Ribeiro & Ponte, 2020) to guide participants in the formative process of reflecting on and (re)signifying mathematical knowledge about content, teaching, students, and curriculum (Gross et al., 2023).

In the case of the FO(RM)AP model, as our objective is for teachers to develop, in greater depth, the knowledge expanded in the previous dimension (Figure 4), the practice records should enable this. Thus, we propose that in this *Deepening* dimension, the following referral be made:

1st) Prospective teachers receive the PLT (Ribeiro & Ponte, 2020), which must consist of practice records from the application of the same mathematical task proposed to prospective teachers in the *Introduction* dimension. This is because, if teachers have already solved such a task, have already experienced its resolution following the phases of exploratory teaching, have already experienced several discussions about MR, its processes and the teacher's actions in the development of the task with the students, they are well familiar with all these issues and we can direct the formative program to what is different in this dimension: students' resolutions and dialogues. Because when the prospective teacher accesses the PLT, they have already followed the same path to solve the task that the student whose practice records they have access to has taken, which makes their analysis more profound and fruitful.

This finding comes from our experience in previous formative processes (see Figure 1), in which, when we approached practice records from a different mathematical task, the prospective teachers spent considerable time and energy understanding and solving the task. Discussing the practice records, which was the objective of the PLT, occupied a secondary role, and we do not want it to happen.

The proposed PLT should contain questions that lead prospective teachers to analyse and discuss aspects similar to those mentioned in the 3rd proposed referral to the *Introduction* dimension, but now focusing on students: observing students' strategies for solving the task; the MR processes they mobilised (their conjectures; whether they tried to validate or refute the conjectures elaborated; whether they justified their answers; whether they could generalise; what mathematical knowledge they used in these processes); what new knowledge was developed; whether the task provided an opportunity to develop MR; whether it is in accordance with the curriculum guidelines; whether it promotes connections between the mathematical contents. In short, a variety of professional development issues a PLT can raise.

2) Prospective teachers must solve the PLT according to the stages of exploratory teaching (Serrazina, 2021). From our point of view, there is no better way for the teacher to appropriate a teaching approach than through their experience. How can one tell a teacher to follow a specific approach without allowing them to experience it and understand, as a learner, its contributions? Thus, we consider that formative sessions must follow an exploratory approach and privilege "in-service and prospective teachers' autonomous exploration of formative tasks" (Ponte, 2022, p. 112), that is, of PLTs.

From this perspective, we consider that formative sessions should also follow an exploratory approach, enabling prospective teachers to experience exploratory teaching while mobilising their professional knowledge when discussing a mathematical task in the context of a PLT (Aguiar et al., 2021; Ribeiro et al., 2020).

In the autonomous resolution phase of PLT, teachers organised in pairs (or trios) have the opportunity to rescue and mobilise the knowledge developed in the previous dimension. In the collective discussion phase of their resolutions, they have the opportunity to share what they have done and listen to the other teachers, which deepens their knowledge. In turn, the systematisation phase, still the responsibility of the teacher educator, is better supported by the teachers' knowledge than in the previous dimension, since much of the knowledge has already been expanded and is now being developed in depth. Thus, the discussion conducted at this stage by the educator must emerge from issues that may still afflict teachers and that, in one way or another, need to be better clarified.

To conclude, we present Figure 5, with the systematisation of the main elements of the *Deepening* dimension of the FO(RM)AP model:

Figure 5

Systematisation of the second dimension of the FO(RM)AP model – Deepening (Authors, 2026)

Second dimension of the FO(RM)AP model - *Deepening*
Practice records
Starting point: PLT elaborated from the practice records from the mathematical task approached in a real teaching context, following the phases of exploratory teaching.
Deepen knowledge
Opportunities to deepen knowledge from practice records, related to: - elements of classroom management (mathematical task, exploratory teaching, and teacher actions) - students' MR and the processes they mobilise, and how they do it - mathematics present in the task, students' difficulties and learnings

Consolidation: This dimension of the FO(RM)AP model concerns the consolidation of knowledge that has been expanded and developed in depth in the previous dimensions. In our understanding, we create Consolidation opportunities when prospective teachers can apply their learning in real-world teaching contexts. Therefore, our choice of the PDR cycle is justified, as it provides for the planning, development, and reflection on a mathematics class in a real-world context, i.e., the classroom.

Trevisan et al. (2020) propose that a formative process aimed at teachers' professional development should be structured as a cycle with three main stages: planning, development, and reflection (PDR Cycle). In the planning stage, teachers collaborate to develop lesson plans focused on specific mathematical concepts, informed by their practical classroom experience. In development, lesson plans are implemented in real teaching contexts, allowing the practical application of planned strategies. Furthermore, the reflection stage enables collective analysis of the classes taught, using practice records (such as video recordings and notes) to promote critical reflection and improve pedagogical practices (Trevisan et al., 2020).

From this perspective, the PDR cycle offers prospective teachers professional learning opportunities (PLOs) to develop and (re)signify knowledge in real teaching contexts, an essential condition for their professional development (Silver et al., 2007).

So, for the *Consolidation* dimension, we suggest the following referral:

1st) Prospective teachers, organised in pairs, trios, or small groups, discuss the selection of the year of schooling and the class (and the teacher) for which the planning will be carried out (stage P of the cycle). Such choices are always conditioned by the number of participants in the formative process, the level of education they work at, and other practical considerations by educators. Finally, with these definitions in place, we move on to planning the class. As MR is a transversal capacity (that is, it is not necessarily linked to specific mathematical content and can be developed through mathematical tasks that address various content), teachers have a lot of freedom to choose the content, since the focus is on the mathematical task.

So, at this moment, teachers need to mobilise several of the knowledge developed and deepened in the previous dimensions in order to carry out class planning: selecting or creating a mathematical task that promotes MR, organising how it will be approached with students (exploratory teaching), analysing mathematical concepts addressed in the task, anticipating the different strategies that students can use to solve the task, anticipating the possible MR processes that students can mobilise during task solving and anticipating questions that the teacher can asked, avoiding giving students ready-made answers.

The developed plans should be shared with the other teachers and the educator to make any necessary adjustments and refinements. In addition, sharing constitutes a moment of professional learning, as teachers learn from one another when discussing the planning they have carried out.

2) In the development stage, the teacher (or teachers) chosen in the planning stage go to the classrooms to develop the planned class. They can be accompanied by the other teachers participating in the formative process, provided that bureaucratic issues are resolved beforehand. We also suggest that the teachers who follow assist in producing practice records and note the aspects they deem relevant for use in the reflection stage. This is a unique

opportunity for professional development, since the teacher (and those who observe them) can carry out the planned class while facing difficulties arising from practice that cannot be fully predicted. So, this moment can foster valuable professional learning, as it develops knowledge that cannot be gained otherwise, helping them make decisions. The quality of the decisions made may be closely tied to the knowledge developed in the previous dimensions, thereby promoting their consolidation.

3) The experience lived in the development stage will foster reflection. The reflection occurs supported by teachers' reports and practice records collected, organised, and prepared by the educator. Teachers share the experience, collectively reflect on it, become aware of their actions, evaluate the planning, and finally make a detailed analysis of the class's development and discuss it in a final session with the whole group. This moment is fundamental to the consolidation of professional knowledge, since it is through reflection on practice that teachers critically evaluate their actions; in doing so, they develop a new understanding of their practice, enriching their repertoires and improving their ability to solve problems and make informed decisions (Serrazina, 1999).

Figure 6, below, systematises the main elements of the *Consolidation* dimension of the FO(RM)AP model:

Figure 6

Systematisation of the third dimension of the FO(RM)AP model – Consolidation (Authors, 2026)

Third dimension of the FO(RM)AP model - *Consolidation*
PDR Cycle Starting point: PDR cycle: planning a mathematics class grounded in previously developed and deepened knowledge; development of the planned class in a real class; joint reflection on the class developed.
Consolidate knowledge
Opportunities to consolidate knowledge, from the planning and development of a class in a real-world context of teaching and subsequent reflection on it, related to: - articulation of the elements of classroom management (the mathematical task, exploratory teaching, and teacher actions) for the purpose of planning and developing a class. - students' MR and the processes they mobilise, and how they do it - mathematics present in the task, students' difficulties and learnings

FINAL CONSIDERATIONS

In this article, we aimed to propose a theoretical model of the formative process that contributes to teachers' understanding of MR, its processes, and ways to promote it in the classroom, and to discuss the elements that integrate it. From the systematisation of empirical evidence from different formative cycles and consolidated theoretical references, organised from the perspective of design-based research (Cobb et al., 2016; Ponte et al., 2016), we built the FO(RM)AP model (Figure 2), structured in three dimensions: introduction, deepening, and consolidation.

We intend that articulating these three axes enables us to organise formative proposals that mobilise and deepen the teacher's professional knowledge, especially in the management of mathematical tasks, classroom interactions, and the MR processes students mobilise. Each dimension of the model is not watertight, but interrelated, constituting a continuous cycle of professional development that starts from real practices, promotes collective reflection and theoretical systematisation, and returns to practice with new elements.

The results from several formative processes (Figure 1), combined with the theoretical frameworks underpinning these studies, indicate that integrating practice records, mathematical tasks with potential to develop MR, and the educator's mediation is central to promoting PLO. Such opportunities favour the re-signification of teaching knowledge and enable the teacher to plan, carry out, and reflect on classes that more effectively support the development of their students' MR.

The FO(RM)AP model, as already indicated, draws on our experience and reflection in structuring and developing various formative processes, involving both in-service teachers in continuing education and prospective teachers (in initial education). However, we understand that, although the model can serve as a guiding element for formative processes in both initial and continuing education, some aspects must be observed when it comes to initial education: the lack of knowledge from experience and, in some cases, the lack of a teaching context (that is, the classroom), in which the PDR cycle can occur completely (Trevisan et al., 2020). Also, we call attention to the teacher educator as a component that warrants highlighting. It is evident that in all formative processes, whether structured according to the FO(RM)AP model or another, the teacher educator's preparation is crucial. Therefore, in addition to discussing each model dimension, we propose a referral for each dimension.

Therefore, to improve the theoretical and methodological foundations of the FO(RM)AP model, in future investigations, we aim to develop formative processes following the proposed model at different levels of schooling and across different formative contexts. In addition, we intend to explore in greater depth questions about the education of teacher educators themselves, considering their actions, knowings, and challenges when operationalising the model. Finally, we understand the need to develop instruments to monitor and assess the PLOs generated by applying the FO(RM)AP model, a promising area for further investigation.

In short, we consider that the FO(RM)AP model represents a relevant contribution to the field of the education of teachers who teach mathematics, by offering theoretical and methodological grounds for the organisation of formative proposals that dialogue with teaching practice and with the challenges of the classroom, especially related to the promotion of students' MR. We hope that its application in different contexts can foster future investigations and expand the possibilities of an education committed to students' mathematical learning.

ACKNOWLEDGEMENTS

The authors thank the National Council for Scientific and Technological Development (CNPq) and the Araucária Foundation (FA) for their support in the development of the research.

AUTHOR CONTRIBUTIONS STATEMENT

Eliane Maria de Oliveira Araman contributed to the conception of the study, the theoretical framework, the systematization of the FO(RM)AP model, the discussion of the results, and the critical revision of the manuscript. André Luis Trevisan contributed to the conception of the study, the organization of the analyzed formative processes, the methodological systematization, the discussion of the results, and the critical revision of the manuscript. Maria de Lurdes Serrazina contributed to the theoretical framework, the analysis and discussion of the proposed model, and the critical revision of the manuscript. João Pedro da Ponte contributed to the theoretical framework, the analysis and discussion of the proposed model, and the critical revision of the manuscript. All authors approved the final version of the article.

DATA AVAILABILITY STATEMENT

The empirical data analyzed in this article are not publicly available due to ethical commitments established with the research participants, particularly regarding confidentiality, privacy, and the preservation of their identities. The data consist of written records, transcripts, and audio and video recordings produced in the context of formative processes and may contain sensitive or potentially identifiable information. Requests for access to specific data may be considered by the authors, subject to the conditions established in the informed consent forms and the corresponding ethical approvals.

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