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How elementary school students explain their understanding of algebraic problems: An analysis in light of mathematical language

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ABSTRACT

Background: The comprehension of mathematical texts, especially algebraic problems, has been discussed in mathematics education, based on Ludwig Wittgenstein's philosophy of language and on studies of mathematical language, with emphasis on the role of language in the production of meaning. **Objectives:** What explanations do students provide about their own understanding of algebra problems? How do they mobilise mathematical language when interpreting those problems? **Design:** This study is a qualitative inquiry, descriptive and exploratory in nature, and was carried out through semi-structured interviews and open-ended questionnaires that included three mathematical problems. **Setting and participants:** 16 students from four public schools in the rural and urban areas of Canaã dos Carajás, in the state of Pará, Brazil. **Data collection and analysis:** data were collected through questionnaires and interviews, and the analysis focused on students' responses to the following question: Give me your thoughts about what this text says. **Results:** The findings indicate that most students show a superficial understanding of algebraic problems, relying mainly on intratextual elements as an explanatory strategy. **Conclusions:** Although students experience difficulties understanding algebraic problems, they can identify relevant information, indicating potential for developing comprehension through language use and explanation.

Keywords: explanation; comprehension; algebraic problems; algebra; mathematical language.

Como os alunos do ensino fundamental explicam sua compreensão sobre os problemas algébricos: Uma análise à luz da linguagem matemática

RESUMO

Contexto: A compreensão de textos matemáticos, especialmente de problemas algébricos, tem sido discutida na educação matemática, com base na filosofia da linguagem de Ludwig Wittgenstein e em estudos sobre linguagem

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matemática, destacando o papel da linguagem na produção de sentido. **Objetivos:** Quais explicações os alunos apresentam sobre sua compreensão de problemas algébricos? Como mobilizam a linguagem matemática ao interpretar esses problemas? **Design:** A pesquisa é qualitativa, de natureza descritiva e exploratória, realizada por meio de entrevistas semiestruturadas e questionários abertos com três problemas matemáticos. **Ambiente e participantes:** Participaram 16 estudantes de quatro escolas públicas das zonas rural e urbana do município de Canaã dos Carajás, Pará, Brasil. **Coleta e análise de dados:** Os dados foram coletados por meio de questionários e entrevistas, sendo analisadas as respostas à questão: "Me conte a sua opinião acerca do que esse texto fala" **Resultados:** Os resultados indicam que a maioria dos alunos apresenta compreensão superficial dos problemas algébricos, utilizando principalmente elementos intratextuais como estratégia de explicação. **Conclusões:** Conclui-se que, embora os alunos apresentem dificuldades na compreensão de problemas algébricos, conseguem identificar informações relevantes, o que indica potencial para o desenvolvimento da compreensão por meio do estímulo à linguagem e à explicação.

Palavras-chave: explicação; compreensão; problemas algébricos; álgebra; linguagem matemática.

INTRODUCTION

Research on how students explain their understanding of algebraic problems arises from both teaching experiences and ample evidence of recurring difficulties in mathematical learning. In the school context, elementary school students often present limitations in reading, interpreting, and understanding algebraic problems, which directly impacts their school performance. This scenario is corroborated by large-scale assessments, such as the Program for International Student Assessment (PISA) (Brasil, 2018) and the National School Performance Assessment (Prova Brasil) (Brasil, 2019), which show low levels of proficiency in mathematics in Brazil, especially in tasks that require interpretation and problem solving.

To situate this study in the field, a mapping of research on the reading, interpretation, and understanding of mathematical and algebraic texts was carried out based on the CAPES and the Digital Library of Theses and Dissertations (Biblioteca Digital de Teses e Dissertações — BDTD). From the mathematics education criteria, 24 studies published over the last 20 years were selected. In general, these studies converge in showing students' recurring difficulties in reading and interpreting mathematical texts across different content areas, such as basic operations, geometry, and algebra. Such investigations, for the most part, point to the centrality of reading and textual understanding as conditions for the development of mathematical thinking.

However, most of these studies adopt a predominantly descriptive approach to difficulties, emphasising factors such as the absence of reading and writing practices in mathematics and the dependence on rote learning. Studies such as Weber's (2012) show that the limitations of working with mathematical texts favour mechanised resolution strategies. However, they make little progress in problematising the meaning processes involved in students' understanding.

On the other hand, a set of research shifts the focus to the production of meaning in mathematical learning, especially in algebra. From this perspective, authors such as Silveira (2005), drawing on philosophical references such as Kant and Wittgenstein, argue that the construction of mathematical concepts involves dynamic processes of meaning, in which students reinterpret and re-signify concepts across different contexts. Although such studies contribute to understanding the nature of mathematical learning, they tend to privilege conceptual and theoretical analyses, paying less attention to the concrete ways students express their understanding.

Similarly, investigations based on Ludwig Wittgenstein's philosophy of language, such as those of Teixeira Júnior (2016), highlight the importance of language use in the learning of algebra, understood as a system of rules and practices. These approaches emphasise that meaning does not reside in essences but in subjects' uses of language across different language games. Although they recognise the role of language, such studies do not always empirically examine how students mobilise it when explaining their understanding of problems.

In addition, some investigations emphasise the relevance of communication and dialogue as mediators of mathematical learning, as observed in Lacerda (2010), who highlights the importance of language games in reading and writing in mathematics. However, even in these studies, communication is often analysed in collective contexts or centred on written production, with less attention to students' individual oral explanations as a space for meaning production.

Given this panorama, we identify an important gap in the literature: although the centrality of language and interpretation in mathematical learning is recognised, the ways in which students explain their understanding of algebraic problems through speech remain little explored. In particular, there is a lack of research on how students explain their interpretations and mobilise linguistic resources by attributing meaning to mathematical statements.

This study, situated in this context, seeks to investigate how students, through their interview responses, explain their understanding of algebraic problems. The data and results presented here are part of the 1st category of the master's thesis. Thus, the research is guided by the following question: How do students explain their understanding of algebraic problems? What textual resources do they use? Is the understanding carried out in a superficial, basic, or proper way?

THEORETICAL FRAMEWORK

The research is based on Ludwig Wittgenstein's (1889–1951) philosophy of language and the authors who study the Austrian philosopher. Wittgenstein composed two opposing reflections about language: in his first reflection, in *Tractatus Logico-Philosophicus* (1921), he argues that knowledge is outside language, and that it only represents the contents and objects of the world and/or the human mind. For this, language should obey the logical conditions to represent the world in a search for the essence of objects in the world. In the second reflection, the philosopher demonstrates, in *Philosophical Investigations* (1951), an understanding that opposes his previous idea: language, its construction and transmission, is what provides knowledge. For the second of Wittgenstein's (2014) thinking, the meaning of a word depends on the use we make of it in different situations and contexts.

A bit about language

Language in mathematics education, according to Danyluk (2015), is the organisational ability to express our actions. The author points out that when we speak, we open a space for creativity to manifest itself, organising and transmitting the imaginary, also present in mathematics. Thus, the line of reasoning we will follow about language, in general, is that it is not only concerned with the physical and empirical representation of the world as a correspondence with reality (Hebeche, 2003), because language is not mere reference.

According to Machado (2011), language is an instrument of communication; therefore, people create codes and representation systems to communicate certain types of messages, and its use is simpler than that of a language. Thus, a language cannot be described as a code, since the units of language do not preexist language, and since language does not consist only in labelling objects of reality. It represents a self-organisation based on experience data.

Thus, this organisation is also portrayed in Silva, Silveira, and Oliveira (2020) and Gottschatk (2008) as, based on Wittgenstein's philosophy, a set of grammatical rules of language. Such rules, therefore, are associated with rules of a game that requires the mastery and perception of the use of language, signs, and symbols. However, mathematics lacks orality; consequently, it needs a mother tongue to express its meanings (Cunha & Silva, 2018), and its meanings must be translated into that mother tongue.

Thus, we will address aspects of language aimed at its foundation in the reading, understanding, and interpretation of mathematical texts, although their concepts and definitions are intertwined and interdependent. Vilela and Mendes (2012) say that language collectively establishes the meaning and understanding of words, signs, and symbols, which are also linked to language, but not as a designation.

Reading, interpreting, and understanding in Wittgenstein

The approach to reading adopted in this research is textual, as the objective is to delimit its role in the understanding and interpretation of mathematical texts. We recognise that there is not only textual reading. The act of reading is comprehensive and is not limited only to reading written words (Danyluk, 2025) because, in the face of different types of expressions, human beings make different types of reading possible. However, we emphasise the reading of mathematical texts and how it has been carried out in mathematics education.

Reading is understood as the act of understanding, interpreting, and transforming. It is perceived as the understanding of language expression, not just the deciphering of traits encoded on paper. There is an association among the three definitions as interconnected concepts, because reading, in the sense of learning, requires a union of understanding and interpretation for transformation to occur in learning mathematics (Danyluk, 2015).

Mathematical writing is an important way to record the mathematical language that the child understands (Danylu, 2015). This fact reinforces mathematical literacy, and mathematical reading and writing. In this sense, mathematics considered as a formal language does not include orality, being characterised as an exclusively written symbolic system (Machado, 2011). An understanding of the mathematical writing system presented by Danyluk (2015) denotes the existence of a developed and communicated understanding:

> The misconception of some books and even some educators that mathematical literacy is "reading, writing, and counting"

> is a fallacy, since for them reading refers to the mother tongue, counting to mathematics, and writing to a simple mechanical copy. The true meaning of writing in mathematical language is far beyond this: "And writing causes existential understanding and interpretation to be developed, fixed, and communicated by the record made (Danyluk, 2015, p.15).

Mathematical writing is the result of a mathematical reading, understanding, and interpretation; it is the externalisation of what students have internalised. The student's difficulty in learning mathematics is similar to learning the mother tongue, but is aggravated by the need to write unfamiliar (mathematical) symbols, not just written words. Thus, for many people, learning mathematics is like learning a foreign language (Pimm, 1987; Danyluk, 2015).

This approach becomes evident when considering that linguistic ability usually involves attributing meaning to what is heard and read, as well as the possibility of expressing understandings through speech and writing. Such processes are also present in mathematical learning, to the extent that the student must interpret statements, construct meanings, and communicate their ideas using a specific language.

> Considering the communicative competence of language, fluidity in speech suggests the ability to handle implicit resources and use them for other purposes. In addition, it would allow students to move from the concrete to the abstract, manipulate formal language, and later control their own thoughts in learning mathematics (Pimm, 1987, p. 31).

The manipulation of formal language is fundamental to learning mathematics because of the dependence between the mother tongue and mathematical language. Machado (2011) argues that, from Saussure's perspective, the functions of the mother tongue show that language is a system of signs that expresses ideas and that it also serves the functions of describing, interpreting, creating meanings, imagining, and understanding the world. Therefore, we observe that one of the functions of language is to interpret and understand, and these functions are only possible with reading.

Regarding interpretation, scholars discuss this concept in the scope of mathematics education. Mathematics is much more than "learning techniques to operate with symbols, it is viscerally related to the development of the ability to interpret, analyse, synthesise, signify, conceive and transcend the sensitive immediate, project and extrapolate" (Machado, 2011, p.101).

We, including students, try to make sense of everything we hear and read. The interpretation here is that the search for meaning can lead us to unusual conclusions and that words in the mother tongue have different meanings. If the listener is not aware of the specific use for that context and attributes the inappropriate meaning in the mathematical environment, it can be assumed that there will be several difficulties in understanding (Pimm, 1987):

> An example is in the question: < What is the difference between 24 and 9? > a nine-year-old boy answers: < one is even and the other is odd> many others answered: < one has two digits and the other has one> these answers indicate a failure to understand the word difference used in the mathematical sense of subtraction. Algebra and arithmetic present important cases of metaphors, and these cases constitute a central aspect of the expression of mathematical meaning (Pimm, 1987, p. 36).

The use of the word difference mentioned in the question: "What is the difference between 24 and 9?" is in the mathematical language game, since it means the result of a subtraction. However, confusion in the meaning of the word can be explained by comparing it with the mother tongue. Gottschatk (2008) notes that, for Wittgenstein, mathematics is just one of the language games that are part of our forms of life, in the context in which they are used, and the modes of communication, in which understanding is a manifest capacity in use.

> Whether the word "number" is necessary in the ostensible definition of the two depends on whether a person conceives it, without that word, in a different way from what I want, and this will certainly depend on the circumstances in which it is given, and on the person to whom I give it. And the way they 'conceive' the explanation is shown in the way they make use of the explained word. (Wittgenstein, 2014, p. 30).

Wittgenstein (2014) reports that understanding the definition of a word, as in the example of the word 'two', does not necessarily imply the use of the word 'number', as expected by those who ask. If other words are used by the person who provides the answer, this will manifest itself in the way the interlocutor conceives the explanation of that word and is evident in the way they use it.

Authors and researchers on Wittgenstein report that the philosopher believes that reading is the act of understanding, interpreting, and transforming. Reading here is not only the decoding of signs and symbols in an interconnection with the definitions of interpret and understand, since reading expresses a language as stated by Danyluk (2015). On the other hand, interpreting mathematical texts can be understood as a search for meaning. Therefore, it may lead to mistaken conclusions due to the use of specific words in the mathematical environment, which have meanings different from those in the mother tongue (Pimm, 1987).

Another understanding established in a Wittgensteinian analysis is the ability to use, react to, and explain words. Thus, as with understanding words, to understand mathematical texts, the student must also have this ability, which is considered a permanent condition. Glock (1998)

Thus, the reading, understanding, and interpretation of mathematical texts, grounded in Wittgenstein's foundations, led us to identify trends and assumptions in the development of school algebra and, consequently, incorporate them into this article.

School algebra

School algebra is described in the National Common Curriculum Base (Base Nacional Comum Curricular—BNCC), which is a normative document that defines an organic, progressive set of essential learning that all students must develop across the stages and modalities of basic education (Brasil, 2018). It guides the development of basic education curricula in Brazil. According to the document, basic education is divided into three stages: early childhood education (ages 0 to 5), elementary school (ages 6 to 14), and high school (ages 15 to 17).

In elementary school (*ensino fundamental*, in Brazilian school organisation), mathematics is one of the areas of knowledge and algebra makes up a thematic unit in which it requires the development in students of competencies and skills aimed at developing algebraic thinking, "which is essential to use mathematical models in the understanding, representation and analysis of quantitative relations of magnitudes and also of mathematical situations and structures, making use of letters and other symbols" (Brasil, 2018, p. 270). Note that some algebraic knowledge is related to other mathematical knowledge, such as geometry, as shown in Figure 1.

Figure 1

Question about algebraic expression (Giovanny Jr. & Castrucci, 2018, p. 102)

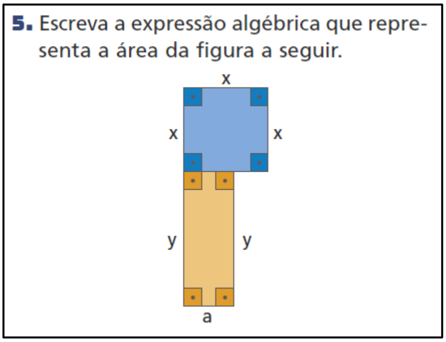


Figure 1 presents a question about an algebraic expression, and we can see that it previously required knowledge of geometry, such as area. This relationship between algebra and geometry reveals that the development of algebraic thinking, as suggested by the BNCC, requires recognising it for the use of mathematical symbols. That is, reading the problem can cause several interpretations, such as the word area, which can lead the student to search in its context for the meaning of the word area and relate it to the balcony, for example, with the meaning of an open space inside a construction, among other meanings, and not to understand that, in this game of mathematical language, the meaning would be of a surface measurement and, to obtain it, it is enough to multiply the length and width.

Thus, the student must first know what an area is in the mathematical context, and then identify the measures in the figure, remember the 1st property of potentiation. For the multiplication of the length and width of the square, multiplication of equal bases, the base and the sum of the exponents are preserved ($A = x.x$ / $A = x^2$). Consequently, the student should remember that when there is no exponent at the top right of the variable, it means the exponent is 1. Thus, they should do the same for the area of the rectangle and then write the algebraic expression that expresses the total area of the figure ($A = x^2 + ay$), where **A** represents the area, **x** represents the side of the square, **a** is the length of the rectangle, and **y** represents the width of the rectangle.

Notably, in the final years of elementary school, algebra is expressed in students' abilities to recognise equality relations, differentiate numerical variables, generalise properties, use algebraic symbology, seek regularity patterns in a recursive and non-recursive sequence, investigate an unknown value and determine the variable between two or more magnitudes, solve and elaborate problems with quantities directly or inversely proportional, understand the factorisation process and other skills. Brasil (2018) also indicates that students establish connections between variables and functions, between unknowns and equations, and learn techniques for solving equations, inequalities, and the Cartesian plane.

However, according to Brasil (2018), the definition of algebra is linked to the fundamental mathematical ideas of the thematic unit (algebra): equivalence, variation, interdependence, and proportionality. It is also linked to what this thematic unit should emphasise: “the development of a language, the establishment of generalisations, the analysis of the interdependence of magnitudes, and the resolution of problems through equations and inequalities” (Brasil, 2018, p. 270). To this end, the algebra thematic unit has different teaching and learning processes for elementary school.

METHODOLOGY

This article presents the results of category 1 of analysis of the master’s thesis research. It is noteworthy that the thesis is a component of the larger research project titled “Linguistic approach to mathematical literacy: pedagogical theory and practice” (Abordagem linguística ao letramento matemático: teoria e prática pedagógica). It was developed by the research group Linguistic Processes in Mathematics Education (Processos Linguísticos em Educação Matemática—Prolem) at Unifesspa. This project complies with the criteria established by the Research Ethics Committee of the Federal University of Pará (UFPA) under opinion No. 4.339.214 and CAAE No. 36527520.9.0000.0018.

The research is descriptive and exploratory and the approach adopted is qualitative because it has the characteristics mentioned by Yin (2016), i.e.: studying the meaning of people’s lives (seeking the meaning that students attribute to the words of the algebraic problem), in real life conditions (in the school where they study); representing people’s opinions (bringing their explanation of the key words of the problem); covering the contextual conditions in which people live (urban and rural schools); contributing with revelations about existing concepts that can help explain human behaviour (understanding of mathematical texts) and striving to use multiple sources of evidence (questionnaire and interviews).

The research reported in this article was conducted in four public schools in the municipality of Canaã do Carajás, south of the State of Pará, including one rural and three urban schools, with 16 students in the final years of elementary school. The three algebraic problems that served as the basis for the interview were selected from the Mathematical Problems Test (Prova de Problemas Matemáticos), one of the research instruments applied by the Linguistic Approach to Mathematical Literacy (Abordagem Linguística ao Letramento Matemático) project, which, in turn, were selected by the members of the Prolem from old tests of the National Examination for Certification of Skills in Youth and Adult Education (Exame Nacional de Certificação de Competências da Educação de Jovens e Adultos—Encceja), as they include comprehensive knowledge from the four years of the final years of elementary school and because they are questions already validated in a large-scale exam.

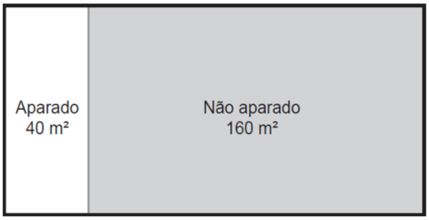
We will present the questions with their numbering in the Mathematical Problems Test. Question 10, the first question of the questionnaire, talks about hiring a gardener to provide a service, as shown below:

Figure 2

Question 10 – Hiring a Gardener (Prova de desempenho de Resolução de Problemas, 2021)

QUESTÃO 10

Uma pessoa contratou um jardineiro para cortar o gramado de sua residência. O tempo gasto por esse jardineiro para cortar a grama de uma região de 40 m² foi de 80 minutos. A figura representa o gramado, indicando as regiões onde o trabalho já foi realizado e onde o gramado ainda deve ser aparado.



Considere que o jardineiro mantenha o mesmo ritmo de trabalho no restante do gramado.

Quanto tempo, em minuto, o jardineiro levará para concluir o restante do corte do gramado?
A) 320
B) 200
C) 120
D) 100

Figure 2 shows question 10, covering the thematic unit of algebra. The object of knowledge is variation of magnitudes: directly proportional, inversely proportional, or non-proportional; the BNCC skill is EF08MA13: solving and elaborating problems involving magnitudes directly or inversely proportional, through varied strategies, and the Encceja H16 matrix skill is: solving problem situations involving the variation of magnitudes directly and inversely proportional.

Question 14, the second question in the questionnaire, concerns the purchase of decorative pieces for resale, as shown in Figure 3.

Figure 3

Pieces of Decoration (Prova de desempenho de resolução de problemas, 2021)

QUESTÃO 14

Um comerciante quer comprar peças de decoração, que custam 3 reais cada uma, para serem revendidas em uma feira de artesanato. Para transportar todas as peças até a feira, o comerciante terá um gasto de 50 reais. Considerando que ele revenda todas as peças, cada uma por 5 reais, o comerciante pretende obter, descontado o frete, um lucro de 250 reais.

Para obter o lucro desejado, a quantidade de peças que o comerciante deverá comprar é
A) 60
B) 100
C) 125
D) 150

Figure 3 shows question 14, which has the thematic unit of algebra; the object of knowledge is "1st degree polynomial equations"; the BNCC skill is EF07MA18: solving and elaborating problems that can be represented by 1st degree polynomial equations reducible to the form ax + b = c, making use of the properties of equality, and the encceja matrix skill is H21: solving problem-situation through first degree equations.

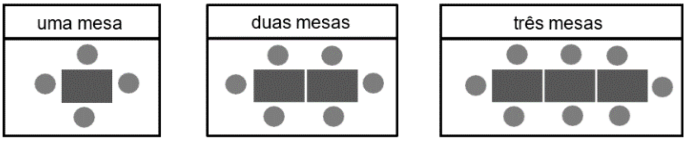
Question 18, the third question in the questionnaire, concerns tables in the pizzeria that should be side by side, as shown in Figure 4.

Figure 4

Tables at the Pizzeria (Prova de desempenho de resolução de problemas, 2021).

QUESTÃO 18

Em uma pizzaria, o gerente organiza as mesas lado a lado, de acordo com a quantidade de pessoas no grupo. Observe algumas composições possíveis com uma, duas ou três mesas.



Para grupos com mais de oito pessoas, é possível acrescentar mais mesas, seguindo esse mesmo padrão. Um grupo de pessoas foi a uma pizzaria numa comemoração de fim de ano. Para acomodá-las foram necessárias 11 mesas dispostas lado a lado,não sobrando nenhum lugar. Sabe-se que, nesse grupo, para cada 4 pessoas foi consumida uma pizza grande que custava R$ 20,00.

O valor, em real, referente ao consumo total de pizzas desse grupo foi de

A) 55
B) 80
C) 120
D) 220

Figure 4 presents question 18, which deals with the thematic unit of algebra; the object of knowledge is recursive and non-recursive sequences; the BNCC skill is EF08MA11: identifying the regularity of a recursive numerical sequence and building an algorithm through a flowchart that allows indicating the next number, and the skill of the Encceja H27 matrix is: identifying regularities present in numerical sequences.

The question*: "Give me your thoughts about what this text says" was used for the three questions, and from the answers* we obtained data for the analysis of category 1: **How students explain their general understanding of the problem.** In analysing the answers, we identified a common pattern in their structure. This standard can be divided into four parts:

I— A **professional** (gardener, merchant, and manager);

II— A **service** or place that provides a service (mowing lawns, buying and reselling, and a pizzeria and/or organisation of tables);

III— A **mathematical situation** [proportionality - time spent due to the trimmed area; equation (profit = resale value - cost value) and algebraic expression (value paid according to the quantity consumed);

IV— They present the **question.**

In the first analysis, we used the four parts of the problem structure as a basis. We systematised the students' understanding at three levels for the first analysis, which would be classified (classification prepared by the authors) into:

1– Superficial: it identifies one-off information from one or at most two of the structures I, II, and/or IV of the problems (summary: it cites something from any of the structures, except the mathematical situation);

2– Basic: it identifies and basically relates information from structures I, II, and/or IV to the III of the problems (summary: it cites, superficially, information from any of the other structures with the mathematical situation);

3– Adequate: it identifies and suitably relates information from structures I, II, III, and IV of the problems (summary: it relates the information from the context to the mathematical situation and the question of the problem).

In the second analysis, we observed how students explain the structures (I– Professional, II– Service, III– Mathematical situations and IV– Questions), and the systematisation was divided into three groups:

I– Intratextual: the explanation given by the students with information present only in the base text of the problem;

II– Contextual: it encompasses explanations based on information from the broader context provided by the problem, such as situations from their daily lives that are not present in the text but do not affect the understanding of the problem.

III– Extrapolated: it brings, in the explanation, information that is not in the text, and that causes damage to the interpretation.

RESULTS AND ANALYSIS

The article includes two analyses carried out in Category 1, "How students explain their general understanding of the problem". Danyluk (2015, p. 198) emphasises that, "when communicating the perceived, the process of existential understanding and intellectual understanding is already underway," for mathematics education, the written and oral expressions must be both interpreted. Although the research in this article focuses on students' oral explanations, it corroborates Danyluk's view that, among the possibilities for organising thinking and communicating what is perceived, the construction of concepts is one of them. Thus, we seek to identify which concepts and resources students use when communicating their understanding of the algebraic problems presented to them.

We investigate, then, how students explain, broadly, their understanding of the mathematical text—superficial, basic, and/or adequate—and what resources they adopt to do so—intratextual, contextual, or extrapolated.

Thus, in the first analysis of Category 1, we aimed to understand whether students identify information about elements of the problem structure. Thus, we check what they focus on in reading the problem and whether the explanation of their general understanding is superficial, basic, or adequate. The level of understanding considered **SUPERFICIAL** indicates that the student presented only one or two problem structures without mentioning any mathematical context. Table 1 shows some of the answers analysed as superficial to the question: Give me your thoughts about what this text says.

We will abbreviate the word question (Question 10 to Q10, Question 14 to Q14, Question 18 to Q18) in the 1st column of Table 1. We will explain the coding used to identify the students: the letter **A** indicates the student; numbers from 1 to 25 indicate the order of the interviewees; and ordinal numbers indicate the year of elementary school (final years) that the students attend, from **6th to 9th grade**.

Table 1

Category 1 answers – Superficial explanation (Authors, 2023).

Student	**Answer**	**Summary**	**Structure**	**Level**
A1.6th Q.10	For him to finish mowing things...the lawns. It says here, right: A person hired a gardener to mow their lawn at their residence.	Professional Service	I and II	Superficial
A13.8th Q.14	About the...hold on...a merchant who wants to buy kind of a product, right? And that’s all I understood from here.	Professional Service	I and II	Superficial

A6.8th Q.18	Um …a pizzeria.	A place that provides a service	I	Superficial

Table 1 shows, in the speech of student A1.6th, that he looks for words in the text and focuses only on two structural elements of the problem—professional and the service provided—in question 10. This same description of the structure is in the speech of student A13.8th for question 14. In the student's answer, A6.8th presents only one structural element—the description of the place that provides the service. Therefore, of the total of 25 responses, 15 are at the superficial level, as they present only one or two structural elements. Thus, we noticed that 60% of students give superficial explanations. This reveals that most do not master the technique of identifying the main elements of the text. Wittgenstein (2014) mentions in aphorism 199 that, "understanding a sentence means understanding a language. Understanding a language means mastering a technique. " We note, therefore, that most students do not master the technique of identifying the main structural elements of the mathematical text.

For the **BASIC** level of understanding, we consider the relationship the student explains between structural elements I, II, or IV and structural element III — the mathematical situation. We look at the answers in Table 2.

Table 2

Chart of answers in Category 1 – Basic explanation (Authors, 2023)

Student	**Answer**	**Summary**	**Structure**	**Level**
A13.8th Q10	About the gardener and how long it takes him to...how do we say it?...weeding, practically, here we say weeding the garden.	Professional Service Mathematical situation	I, II, and III	Basic
A10.8th Q14	She is wondering the profit of...of the pieces he will resell at the fair.	Service Mathematical situation	II and III	Basic
A16.6th Q18	In my opinion, the statement is about a	A place that	II and III	Basic

	pizzeria where a group of people went to eat at the end of the year, and now it is asking us to determine how many pizzas the group will need.	provides a service Mathematical situation

Table 2 shows some of the explanations classified as basic, as it reveals that students identify and thoroughly relate information from structures I, II, and/or IV to structure III of the problems. They superficially cite information from any of the other structures in the mathematical situation. We noticed that 8 of the 25 answers were at a basic level. Student A13.8th's answer indicates the professional, the mathematical situation, and the service provided; student A10.8th mentions the reselling service and the profit, which refers to a mathematical situation; and student A16.6th presents in his answer the place that provides the service and partially informs the mathematical situation.

Therefore, we found that 32% of the students explain what they understand at a basic level; their speeches include elements that indicate a relationship among the words of the mathematical text, indicating algebraic understanding. Here we note that they begin to identify propositions and/or mathematical situations, and, according to Gottschalk (2022, p. 47), "understanding a mathematical proposition is analogous to understanding a rule. Moreover, understanding a rule is being able to follow it." Therefore, the identification of mathematical situations marks the beginning of the development of mathematical language-game skills.

The level considered **ADEQUATE** for the general understanding of the problem refers to the presentation of the four structural elements: (professional, service, mathematical situation, and question), which shows that the student identifies the main information and fully externalises their answer. Table 3 follows.

Table 3

Category 1 – Table of answers Category 1 – Adequate answer (Authors, 2023)

Student	**Answer**	**Summary**	**Structure**	**Level**
A10.8th Q10	He talks about a gardener and his work that he mows lawns in a region, right? And in the text, it is asked what he... how long, in minutes, he would take to mow the remaining lawn; [that] he was mowing a lawn and then he stopped, and it took about 80 minutes to mow a part of the lawn.	Professional Service Mathematical situation Answer	I, II, III, and IV	Adequate
A11.9th Q18	It talks about a group... it talks about a pizzeria and a manager... it's for a group of eight people, no, my bad. It mentions a group that went to a pizzeria, where they set a table for four people, two tables for six people, and three tables for eight people. And it is asking... the question is the real value of the total consumption of pizza in this group was... how much the total number of pizzas they consumed was, and how much it cost.	Professional A place that provides a service Mathematical situation Answer	I, II, III, and IV	Adequate

Table 3 presents only the two answers that adequately explain the structure of the problem. Student A10.8th cites the base text as a whole and mentions the professional, the service, the mathematical situation, and the question; student A11.9th also describes the three structural elements and the place that provides the service. The analysis showed that only 8% of the students suitably explained their understanding of the problem. Gottschalk (2022) notes that the knowledge process is complex: the meanings of the problem or a word do not occur immediately. "A word only acquires meaning when you operate with it, that is, following a rule" (Gottschalk, 2022, p. 48). The author confirms the use of the word in its mathematical context, and this complex process of knowledge is slow; we observe this in the smallest number of students who can present the main elements of the mathematical text.

In the second analysis of the category in the group classified **as** INTRATEXTUAL, we noticed in the student's answer A7.6th to question 10: **Researcher**: Give me your thoughts on what this text says. **A7.6th:** It tells about a gardener hired to mow a lawn.

The answer reveals the focus on the professional and the type of service he provided, i.e., mowing the lawn; thus, we realise that the information they explain is all present in the text, without the addition of contextual information that could harm the interpretation of the text or not, and therefore it received this classification. Therefore, of the 25 answers, we found that most (16) present this characteristic, and we also observed that students mainly look for words from the text to explain the question. In the group of answers classified as **CONTEXTUAL**, we noticed in the student's answer A13.8th to question 14**: Researcher:** Give me your thoughts on what this text says. **A13.8th:** About the... hold on... a **merchant** who wants to buy a kind of **product**, right? And that's all I understood from here.

The speech introduces the word product, which is not in the text but is part of the context conveyed by the base text; it does not impair the understanding of the problem. According to Marcuschi (2008), a text can have numerous interpretations, but not an infinite number of them. Freedom of interpretation is limited, as the text provides contextual clues. With these characteristics, we identified six contextual answers and realised that they introduced other synonymous words from everyday life for use in explaining the question.

In the group of **EXTRAPOLATED** answers, we observed that, in the speech of student A14.7th, he provided the following answer to question 18: **Researcher**: Give me your thoughts on what this text says. **A14.7th:** Soa family went for a snack somewhere at the end of the year, then he had to choose a table with 11 chairs.

He reported that there were 11 chairs; however, the text refers to 11 tables. This misconception can lead to a loss when a faulty procedure is calculated and executed, resulting from the insertion of incorrect data. Thus, we found that only three students extrapolated when explaining the question.

In the first category, as students explain their general understanding of the problem, we divided the analysis into three levels (superficial, basic, and adequate) and three groups (intratextual, contextual, and extrapolated). The analysis summary is shown in Table 4.

Table 4

Category 1 – Percentage of students in each of the comprehension levels (Authors, 2023).

Level	Quantity	Percent
Superficial	15	60%
Basic	8	32%
Adequate	2	8%
Total	25	100%

Table 4 presents the quantitative distribution and the percentage of the answers of the comprehension levels. We noticed that, in general, students explain their comprehension superficially, as 60% of the results presented only two elements (professional, service, and/or question) without the mathematical situation element. While 32% explain in a basic way, using one of the structural elements plus the mathematical situation. Only 8% demonstrated an adequate understanding, as they mentioned all four structural elements. Let us observe the summary of the second analysis in Table 5.

Table 5

Category 1 - Resource groups (Authors, 2023)

Resource	Quantity	Percent
Intratextual	16	64%
Contextual	6	24%
Extrapolation	3	12%
Total	25	100%

In Table 5, we note that 64% of the total search for intratextual resources —information present in the base text— to explain their general understanding, 24% presented contextual resources with information from their context without harming the interpretation, and 12% extrapolated the information, which can harm the interpretation, leading to misunderstandings and inducing errors in the calculations.

CONCLUSION

The research aimed to investigate how students in the final years of elementary school explain their general understanding of algebraic problems. The results of the first analysis show that most students explain their general understanding superficially, citing only 1 or 2 structural elements of the text and mentioning no mathematical situation, suggesting they fail to consider relevant mathematical contexts in their explanations. In the second analysis, the results show that many students use intratextual resources to explain their understanding; they use words from the problem text to address what they understood.

Furthermore, we noticed that the answers from the first analysis, classified as basic and adequate, accounted for 40% of the explanations that cited the mathematical situation as a structural element. Although this understanding does not guarantee the correct resolution of the algebraic problem, it indicates an important identification in the base text that, when continuously developed in the classroom, will help raise the level of understanding.

In addition, we observed in the second analysis that the number of extrapolations was the lowest. That is, among students who seek only explanations in the base text and those who bring contextual information that does not compromise interpretation, which is the majority, suggesting a search for safe textual resources for understanding. However, as already mentioned, this does not guarantee that they solve the problem correctly, and this was not the objective of the research. However, we see that students seek to interpret the text by focusing on key elements of the problem, leading to a better level of understanding.

Notably, further studies should be conducted to verify the limits of understanding for effective resolution. Therefore, it is indisputable that students should be encouraged to identify the main information and to explore their speech as a learning issue. The speech prompts the student to consider what to answer, and even if they are unsure whether the answer is right or wrong, they will reflect on the evolution of their understanding.

AUTHORSHIP CONTRIBUTION STATEMENT

VPTJ was responsible for the conceptualisation, elaboration of the theory, and supervision of the work. NSVA adapted the methodology to the context of the study, developed the models, conducted the activities, and collected the data. NSVA and RBR performed data analysis. All authors actively contributed to the writing of the discussion of the results, reviewed the manuscript, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, NSVA, upon reasonable request.

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