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Histogram in Primary and Secondary Education Mathematics Textbooks in Chile: An Approach to Literacy and Statistical Thinking

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ABSTRACT

Background: The growing interest in improving initial mathematics teacher training and guiding the teaching practices of in-service teachers in Chile has prompted researchers to analyse textbooks, which play a fundamental role in both the school system and training institutions, through the introduction of early practices in which pre-service teachers interact with them. **Objective**: To analyse histogram-related activities in Primary and Secondary Education textbooks in Chile and how these promote the acquisition of statistical literacy and thinking. **Methodology**: A qualitative approach was adopted through a content analysis of textbooks distributed by the Chilean Ministry of Education between 2016 and 2023. **Participants**: The sample consisted of six textbooks selected through purposive non-probability sampling. **Results**: The main results show that the histogram definitions are similar to those presented in statistics books, without distinguishing between discrete quantitative variables, even when the activities are associated with such variables. Likewise, questions related to histogram situations involve literal readings and comparisons of bar heights, preventing in-depth exploration of the data, even though statistical inference concepts have been introduced at the school level. **Conclusion**: The findings show that histogram-related activities do not promote the development of higher-level skills.

Keywords: Textbooks, Histograms, Statistical Literacy, Statistical Thinking, Activities.

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Histograma en los libros de texto de matemática de Educación Primaria y Secundaria en Chile: un acercamiento a la alfabetización y al pensamiento estadístico

RESUMEN

Antecedentes: El creciente interés por mejorar la formación inicial de profesores de matemática como la orientación de los procesos de enseñanza de los profesores en activo en Chile ha invitado a los investigadores a analizar los libros de texto, los cuales juegan un rol fundamental tanto en el sistema escolar como a nivel de instituciones formadoras con la introducción de prácticas tempranas donde los profesores en formación interactúan con los libros de texto. **Objetivo**: analizar las actividades relacionadas con el histograma en los libros de texto de Educación Primaria y Secundaria en Chile y cómo estas favorecen la adquisición de una alfabetización y un pensamiento estadístico. **Metodología**: Se adopto un enfoque cualitativo, mediante un análisis de contenido de los libros de texto distribuidos por el Ministerio de Educación chileno entre 2016 al 2023. **Participantes**: La muestra estuvo compuesta por seis libros de texto seleccionados por un muestreo no probabilístico del tipo intencionado. **Resultados**: Los principales resultados dan cuenta que las definiciones de histograma son similares a las presentadas en los libros de estadística, sin hacen diferencia entre variables cuantitativas discretas, aun cuando las actividades se asocian a variable de dicha naturaleza. Asimismo, las preguntas relacionadas con situaciones que involucran histogramas se asocian a lecturas literales y la comparación entre las alturas de las barras, no permitiendo profundizar en los datos, a pesar que a nivel del sistema escolar se han introducido conceptos de inferencia estadística. **Conclusión**: Los hallazgos evidencian que las actividades relacionadas con histograma no favorecerían el desarrollo de habilidades superiores.

Palabras clave: Textos escolares, Histogramas, Alfabetización estadística, Pensamiento estadístico, Actividades.

Histograma em livros didáticos de matemática do ensino fundamental e médio no Chile: uma abordagem à alfabetização e ao pensamento estatístico

RESUMO

Contexto: O crescente interesse em melhorar a formação inicial de professores de matemática e orientar os processos de ensino de professores em serviço no Chile convidou pesquisadores a analisar livros didáticos, que desempenham um papel fundamental tanto no sistema escolar quanto no nível de instituições de formação, com a introdução de práticas iniciais onde os futuros professores interagem com os livros didáticos. **Objetivo**: Analisar atividades relacionadas a histogramas em livros didáticos de educação primária e secundária no Chile e como estas promovem a aquisição de alfabetização e pensamento estatístico. **Metodologia**: Foi adotada uma abordagem qualitativa, por meio de uma análise de conteúdo de livros didáticos distribuídos pelo Ministério da Educação do Chile entre 2016 e 2023. **Participantes**: A amostra foi composta por seis livros didáticos selecionados por amostragem não

probabilística do tipo proposital. **Resultados**: Os principais resultados mostram que as definições de histogramas são semelhantes às apresentadas em livros didáticos de estatística, sem diferenciação entre variáveis quantitativas discretas, mesmo quando as atividades são associadas a variáveis dessa natureza. Além disso, questões relacionadas a situações envolvendo histogramas estão associadas a leituras literais e comparações entre alturas de barras, impedindo análises aprofundadas dos dados, apesar de conceitos de inferência estatística terem sido introduzidos no sistema escolar. **Conclusão**: Os achados mostram que atividades relacionadas a histogramas não favorecem o desenvolvimento de habilidades de nível superior.

Palavras-chave: Livros didáticos; Histogramas; Alfabetização Estatística; Pensamento Estatístico; Atividades.

INTRODUCTION

Graphical representations are fundamental to data analysis across society and at different levels of education, as they help describe data distributions (American Statistical Association [ASA], 2016). Those that, as Boels et al. (2019) mention, represent not only data but also statistical concepts such as variability, shape, and central tendency. In this regard, one of the most commonly used statistical graphs to synthesise and analyse the behaviour of univariate data of a quantitative nature is the histogram (Arnold & Pfannkuch, 2014; Kaplan et al., 2014), which allows revealing characteristics of the data such as shape, central tendency, variability, and outliers. The histogram accounts for the distribution of the data (Ben-Zvi & Amir, 2005), where the context gives meaning to reading and critical interpretation, skills related to statistical literacy (Garfield, 2002), and supports evidence-based decision-making.

In this sense, the ASA (2016) provides suggestions to teachers who teach statistics at the school and university system level, among which are the introduction of skills related to literacy and statistical thinking, such as, for example, the interpretation and critical reading of graphic representations (Garfield & Ben-Zvi, 2008). The taxonomy proposed by Curcio (1989) plays an important role in it, as it allows guiding the work with statistical graphs, from a literal reading to a reading that goes beyond the data, as well as deciding on the relevance of the graphic representations according to the nature of the data. This skill is associated with statistical thinking, understood as the mental strategies people use to make decisions and the methods and procedures used to analyse and summarise data on both qualitative and quantitative variables (Triola, 2006).

In this context, one of the graphic representations present in the school curriculum associated with quantitative data is the histogram, which, at the

school system level, is introduced into textbooks from the final years of primary education, both internationally and in the Chilean case (Rojas-Rojas et al., 2023). Textbooks, from the perspective of Gracin (2018), are "artefacts that translate policies into pedagogy and represent a link between the planned and the implemented curriculum [in the classroom]" (p. 1). In addition, as Barcelos and Hollebrands (2017) affirm, the tasks included in textbooks influence the teaching and learning of mathematics and statistics in particular.

In line with the above, this article aims to analyse the activities related to histograms in primary and secondary education textbooks in Chile and how they fosterstudents' literacy and statistical thinking.

Graphic representations in the Chilean school curriculum

Statistics topics have gained greater prominence in school curricula, especially those related to graphical representations, both internationally and in the Chilean case (Díaz-Levicoy et al., 2016), with these topics introduced from primary education (Ministerio de Educación [MINEDUC], 2016). In particular, graphic representations of quantitative data are present in the school curriculum from the seventh grade of primary education in Chile. In this regard, it is noteworthy that there is no explicit reference at the secondary education level, even though they are present in the textbooks of the first grade of secondary education in Chile (Table 1).

Table 1

Learning objectives about graphics in primary education in the Chilean school curriculum. (MINEDUC, 2016, p. 115)

Level	Grade	Learning Objectives
Education Primary	7	Representing data obtained in a sample through tables of absolute and relative frequencies, using appropriate graphs, manually and/or with educational software.
	8	Representing the diagrams manually and/or with educational software. Evaluating the way the data are presented: Comparing information from the same data represented in different types of graphs to determine the strengths and weaknesses of each;

Level	Grade	Learning Objectives
		Justifying the choice of the graph for a given situation and its corresponding data set; Detecting manipulations of graphs to represent data.
Education Secondary	1	Compare populations by making "xy" graphs of two attributes for each sample, in a concrete, pictorial way.

In the learning objectives delivered by the MINEDUC, it is noteworthy that the histogram is not explicitly mentioned, even though it is one of the most widely used representations for showing the behaviour of quantitative data (Kaplan et al., 2014). Likewise, nothing is said about the skills required to read and interpret information in graphic representations, which are associated with statistical literacy. However, the skill associated with statistical thinking is emphasised (Garfield & Ben-Zvi, 2008), which involves justifying the type of graph based on the nature of the data. On the other hand, at the first grade of secondary education, only the introduction of two-dimensional representations is mentioned, namely the dispersion graph, which allows the study of the visual behaviour of two variables.

The histogram and its presence in the books of primary and secondary education

The histogram is one of the representations that makes visible cardinal concepts in statistics, such as variability, whose understanding is closely related to literacy and statistical thinking (Ben-Zvi & Garfield, 2004). However, Bruno and Espinel (2009) indicate that the histogram is a graphical representation that synthesises a frequency distribution by rectangles whose widths represent class intervals and whose areas are proportional to the associated frequencies. In this regard, Triola (2006) mentions that these bars are adjacent, i.e., there is no space between them. In these definitions, there is an implicit reference to the fact that the histogram allows the synthesis of data of a continuous quantitative nature, summarised in a frequency distribution table, a process associated with transnumeration, a skill associated with statistical thinking (Wild & Pfannkuch, 1999).

In this regard, in the school system, histograms are introduced in textbooks in the final years of primary education in some countries; for example, in Spain (García-Alonso, 2021), a bar graph is used to represent the behaviour

of discrete quantitative variables. Nevertheless, Parks et al. (2000) note that although it appears to be a bar graph, it is essentially a histogram. From the authors' perspective, this fact could generate semiotic ambiguities for readers of textbooks, particularly for students in primary and secondary education who are assimilating these statistical concepts, in accordance with what was proposed by delMas et al. (2005). Given the structure of the histograms, this representation allows a transition to density functions as an idealised model (delMas et al., 2005). For example, graphically contrast the empirical behaviour of a quantitative variable with theoretical models such as the normal distribution, similar to the sample distribution of the average, the basis of statistical inference (Madden, 2011).

Literacy and statistical thinking

Among the skills associated with statistical literacy are those related to reading and interpreting graphic representations, for which Curcio's (1989) taxonomy proposes reading levels corresponding to the difficulty of the tasks that can be posed. These levels are: a) *Reading the data*. It is related to literally reading the information in a graph; b) *Reading into the data*. It invites us to obtain information that does not stand out to the naked eye within the graph, for which it is necessary to perform some mathematical calculations; c) *Read beyond the data*. It consists of a more reasoned reading of a graph to infer the information it contains about a population under study, in short, to make an informal inference. Finally, the fourth level, introduced by Shaughnessy (2007), is called reading behind the data, in which the origin of the data, the construction of the graph, and its relevance are questioned, depending on the nature of the data. This last level is closely related to statistical thinking (Garfield & Ben-Zvi, 2008).

Along the same lines, the process of changing the representation of data to frequency tables, graphs, or measures of central tendency is called transnumeration (Chick, 2003), a skill associated with statistical thinking (Wild & Pfannkuch, 1999), in line with the learning objectives of the Chilean school curriculum (MINEDUC, 2016), as well as the perception of variation, the integration of statistics and the context that allows generating, searching, interpreting, criticising, and judging information related to the resolution of statistical problems. Skills that are not foreign to professional statisticians when they synthesise the data in numerical summaries or graphical representations, when they respond to the objectives of some type of quantitative study.

Activities related to the histogram

Activities in statistics play a fundamental role in developing skills related to literacy and thinking. In this context, the interpellations should mobilise knowledge to assess, monitor, or provide feedback on students' learning, among other possibilities, while paying attention to highlight the object of study and acknowledging that stochastic and real contexts are essential.

In this regard, the ASA (2016) proposes a categorisation of activities related to the teaching of statistics: A1. On concepts and/or inquiring about common misconceptions in statistics; A2. Promote the understanding of key statistical concepts, such as statisticians, representativeness, sampling, variability, etc., and not only to procedural activities; A3. It is oriented toward students to draw conclusions based on summarised data through graphic and/or numerical representations; A4. They can argue based on data, and that context provides meaning. It follows that activities should be oriented to the intermediate and upper levels of Anderson and Krathwohl's (2001) taxonomy, in a way that favours the development of critical thinking on statistical issues. In short, it is about giving meaning and understanding to this statistical object, which is useful for modelling quantitative variables.

METHOD

To address the research objective, we used a qualitative methodology and content analysis (Krippendorff, 1997) in the sections where the histogram appears. The sample, selected through a non-probabilistic sampling of the intended type, consisted of six textbooks distributed by the MINEDUC in 2016, 2020, and 2023 for use in teaching and learning by primary and secondary education students aged 10 to 14. Some of the textbooks have been reissued from the original version, so, for analytical purposes, only the most recent edition will be considered (see Table 2).

Table 2

Selected and analysed textbooks

Code	Authors/Year /Title/Publisher
TEP5	Alfaro, S., Espinoza, Y., Riquelme, M., Ainardi, V., Aldunate, V., Falconi, P., & Olivares, C. (2016). *Texto del estudiante, Matemática 5° Básico.* Houghton Mifflin Harcout.
TEP7	Merino, R., Muñoz, V., Pérez, B., & Rupin, P. (2016). *Texto del estudiante, Matemática 7° Básico*. SM.
TEP8	Catalán, D., Pérez, B., Prieto, C., & Rupin, P. (2016). *Texto del estudiante, Matemática 8° Básico*. SM.
TES1	Muñoz, G., Santis, M., & del Valle, J. (2016). *Texto del estudiante, Matemática 1 medio*. SM.
TEP5	Ho, Soon, G., & Ramakrishnan, C. (2020). *Texto del estudiante, Matemática 5° Básico*. Santillana.
TEP7	Iturra, F., Manosalva, C., Ramírez, M., & Romero, D. (2023). *Texto del estudiante, Matemática 7° Básico*. SM.
TEP8	Torres, C., & Caroca, M. (2023). *Texto del estudiante, Matemática 8° Básico*. Santillana.

The content analysis of the textbooks was based on Mayring's (2000) methodological proposal. In this sense, we selected the textbooks to be analysed; then, we chose the chapters that involved histograms; and, finally, we analysed the definitions, concepts, activities, questions, and their scope checking against the skills and knowledge related to literacy and statistical thinking (Garfield & Ben-Zvi, 2008). In addition, they were contrasted with the guidelines given by the ASA (2016) on the integration of real and contextualised data and on understanding statistical concepts, such as variability.

For data collection, we prepared an Excel matrix in which the levels of texts (primary or secondary), year of publication, activities and/or definitions, and page number were indicated. The code used to present the data was: [elementary or secondary school text-year] and the page number in the school text. For example, [TEP8-2016, p. 300] refers to the eighth-grade school text published in 2016, and the activity is on page 300 of that book.

RESULTS

Definition of the histogram in textbooks

Interpretation is one of the skills related to statistical literacy (Chance, 2002; Garfield, 2002), as well as statistical concepts (Ben-Zvi & Garfield, 2004). In this context, only the textbooks of the eighth grade of primary education and the first grade of secondary education present different definitions of a histogram:

> A histogram is a type of bar chart that represents the frequency of the categories in which the data is ordered. It is used to represent continuous variables, such as time or mass, or discrete variables when their values are grouped into intervals [TEP8-2016, p. 293]
>
> A histogram is a graphic representation in the form of continuous bars, in which their heights are proportional to the absolute frequency of the intervals represented [TES1-2016, p. 252]
>
> The histogram is a graph formed by contiguous bars, where each one represents a range of values. It serves to express information about data that are grouped [TEP8-2023, p. 178]

These definitions emphasise that, regardless of the year of publication of the textbooks, histograms are continuous bar graphs; this reinforces the continuity of the data, i.e., they allow us to represent quantitative variables when the data are grouped into intervals (frequency distribution tables). In the definitions, it is implicitly stated that the intervals have constant amplitude; likewise, the case of discrete quantitative data is not explicitly stated. On the other hand, regarding its interpretation, the textbook for the first grade of secondary education explains that "We can interpret a histogram from the form of data distribution or concentration" (TES1-2016, p. 252).

This description focuses the interpretation of the histograms on the form (symmetry or asymmetry) of the data distribution, bearing in mind the location of the measures of central tendency and, implicitly, the variability of the data, making the concepts concentration and variability synonymous. A similar situation occurs in specialised statistical books such as Devore's (2008) or Triola's (2006), even though the latter author notes the difference between the associated histogram and the continuous and discrete quantitative data. This

would eventually prevent the generation of semiotic ambiguities associated with this type of representation (Kim & Fukawa-Connelly, 2019).

Questions or activities related to the histogram in the textbooks

In coherence with the interrogative cycle associated with statistical thinking, Table 3 shows the articulation of the questions or instructions related to the histogram according to the recommendations of the ASA (2016), Shaughnessy (2007), as well as Curcio's taxonomy (1989). Here, we note that in these books there are no interpellations related to cardinal concepts, nor do students encounter situations in which they inquire about erroneous conceptions of histogram construction, such as the absence of labels in the representations or the inappropriate use of this type of representation. Likewise, there are no activities in which readers must provide arguments about contextualised problematic situations.

Table 3

Orientation of the questions associated with the problem activities according to the ASA proposal and Curcio's taxonomy related to the histogram

Categories about statistics teaching (ASA, 2016)	Texts 2016				Text 2020	Texts 2023	
	TEP5	TEP7	TEP8	TES1	TEP5	TEP7	TEP8
A1: On concepts and/or questions about common misconceptions in statistics							
A2: Understanding of key statistical concepts	2	4	11	6		4	6
Reading the data	2	1	2				
Reading into data		2	6	6		4	6
Reading beyond the data		1					
Reading behind the data			3				
A3: Drawing conclusions based on data			1	1			
Reading the data							
Reading into data				1			
Reading beyond the data			1				
Reading behind the data							
A4: Arguing based on data							
Total	2	4	11	6		4	6

We observe that, in general, the activities are classified in category A2, i.e., these questions promote the understanding of statistical concepts rather than only procedural activities. Those that are essentially classified in the first

two levels of Curcio's (1989) taxonomy, i.e., to make a literal reading of the information that summarises the histogram, as well as to make comparisons between the heights of the bars that make up this representation.

> In which shoe number do most students fall? (TEP5-2016, p. 288)
>
> What was the number of trips taken most often? (TEP7-2016, p. 327)
>
> What information do your bars represent? (TEP8-2016, p. 293)
>
> How many people were surveyed? (TEP7-2023, p. 200)
>
> Looking at the graph, we can see that the highest bar corresponds to 40, indicating that the shoe size with the highest frequency is 40 (TEP8-2023, p. 177).

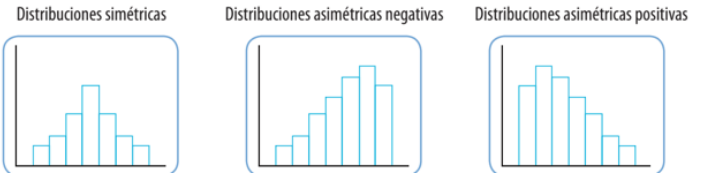
Other questions, associated with the category of reading within the data (Curcio, 1989), require students to understand concepts such as the asymmetry and/or symmetry of a data set's distribution and their relationships with the measures of central tendency. An example of this is the following questions:

> How is the distribution of people's hours in each workshop? (TEP7-2016, p. 325)
>
> What is the relationship between the distribution of the sample data and the measures of central tendency? Discuss your response with your peers. (TEP7-2016, p. 327)

Note that the first grade of secondary education, the textbook provides graphical representations of histograms that allow visualisation of the shape of the distributions (Figure 1).

Figure 1

Graphic representations to visualise the asymmetry of the distribution of a data set. (TES1, 2016, p. 252)



In the representations, the case of unimodal distributions is implicitly considered, as well as the location of the measures of central tendency relative to distribution bias. On the other hand, summarised data are decoded into a histogram to calculate position measures such as quartiles or percentiles.

> Calculate the quartiles Q1, Q2, and Q3 of the test scores. (TEP8-2023, p. 190)
>
> Which is the 35th percentile? (TES1-2016, p. 267)

In these activities, students must relate the values of the quantitative variable (on the horizontal axis) to the observed frequencies (on the vertical axis).

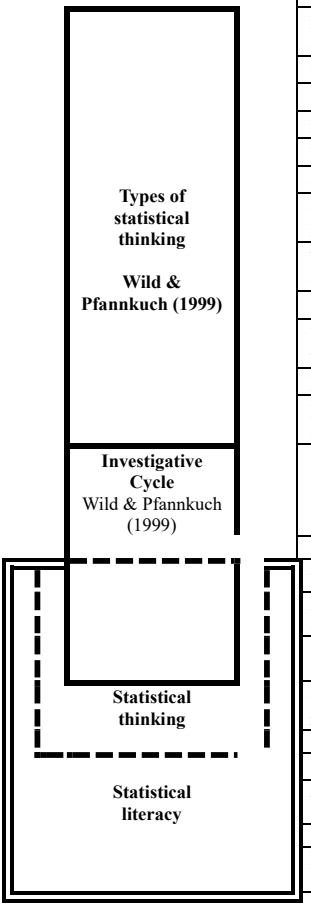
In summary, the questions and/or activities in the textbooks related to histograms of quantitative and/or qualitative variables promote understanding of concepts and, less frequently, aim to draw conclusions—the latter, related to Level 3 of the ASA proposal (2016).

Activities present in textbooks and their relationship with literacy and statistical thinking

Figure 2 shows the relationship between the questions and/or activities related to the histograms present in the textbooks analysed and literacy and statistical thinking skills (Garfield & Ben-Zvi, 2008; Wild & Pfannkuch, 1999). In particular, the type of thinking related to transnumeration, according to Chick's (2003) classification.

Figure 2

Skills and knowledge about literacy and statistical thinking in the questions and/or activities on histograms in the analysed textbooks. (Wild y Pfannkuch, 1999, Garfield y Ben-Zvi, 2008, Chick, 2003)



‘	Year 2016				Year 2020	Year 2023	
	TEP5	TEP7	TEP8	TES1	TEP5	TEP7	TEP8
Recognition of the need for data							
Transnumeration	**5**	**6**	**10**	**15**	**1**	**5**	**12**
Classification			1				
Frequency calculation	5	2	6	7	1	2	9
Proportion calculation				1			
Graph/Tabular		2		4			1
Central Tendency Measurement Calculation		1	1			1	
Variability measurement calculation						1	
Other Calculations		1	2	3		1	2
Perception of the variation			**2**	**1**			
Model reasoning							
Integration of statistics and context							
Statement of the problem and questions							
Planning and design							
Data: Data collection							
Descriptive analysis of the data							
Conclusion: Interpretation and communication							
Critical evaluation of the results							
Statistical language							
Statistical Concepts		2		3		2	1
Vocabulary Comprehension							
Symbol Comprehension							1
Representations in tables and charts		2		4			1

As shown in Figure 2, transnumeration is the process with the greatest presence in textbooks. From Chick's (2003) perspective, the predominance of the calculation of the frequencies associated with the height of the bars that

make up the histogram is observed, which are linked to variables of the discrete quantitative type. An example:

> Calculate how many students are in the course. (TEP7-2016, p. 314)
>
> How many students wear number 34? (TEP5-2016, p. 288)
>
> What was the number of trips taken most often? (TEP7-2016, p. 327)

Another of the transnumerative techniques present in textbooks is graphing and/or tabular, i.e., building graphs according to the nature of the data and the frequency distribution, as exemplified in the following activities.

> Build a frequency table with the data shown. (TEP7-2016, p. 323)
>
> Build the histogram and the frequency polygon for each frequency table. (TES1-2016, p. 251)

On the other hand, less frequently, it is requested to calculate measures of central tendency and position, such as quartiles, for data summarised in a histogram of quantitative variables. Examples of these activities are shown below:

> What is the median? Interpret its value in the situation. (TEP7-2016, p. 323)
>
> Which is the third quartile? (TES1-2016, p. 267)
>
> Calculate the quartiles Q1, Q2, and Q3 of the test scores. (TEP8-2023, p. 190)
>
> What is the group's average age? (TEP7-2023, p. 200)
>
> What is the fashion? (TEP8-2016, p. 313)

In them, reference is made to the average concept (TEP8-2023), even though Triola (2006) has mentioned that this notion does not necessarily consider the arithmetic mean, which can generate lexical ambiguity. On the other hand, a type of thinking that occurs only in the textbooks of eighth grade of primary education and in the first grade of secondary education is related to the perception of variability, even though this concept is a cardinal notion in statistics, because it is essential for its existence (Watson et al., 2003).

> Which graph is best suited to represent the data in the table? Explain your choice. (TEP8-2016, p. 355)

Similarly, given a frequency distribution table associated with the discrete quantitative variable (number of mobile phones), the student is asked to answer the question:

> Which graph would allow the data distribution to be visualised? (TES1-2016, p. 249)

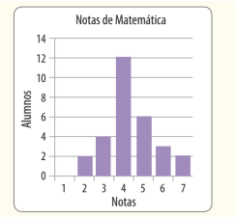
These interpellations invite students to make decisions when moving from the real system to the statistical system, which corresponds to one of the phases of the transnumeration process (Shaughnessy & Pfannkuch, 2002). This stage allows transforming data into representations, giving them meaning (Lee et al., 2014) to respond to problem situations.

However, the questions essentially focus on procedural activities, confirming Estrella's (2017) claim that the teaching of statistics is arithmetic. A scenario that, from the perspective of the authors of this article, would not allow a paradigm shift in the teaching of statistics, so that students acquire literacy and statistical thinking. On the other hand, it draws attention to the duality in the representation of continuous and discrete quantitative variables related to the process of transnumeration, given that in primary education textbooks, the histogram presents two structures: firstly, as simple bar graphs and, secondly, as a bar graph grouped indistinctly regarding the nature of the variable (discrete quantitative or continuous quantitative). Examples 1 and 2 are shown in Figure 3, where we note that the variable of interest is a discrete quantitative variable, represented as a rectangle with an amplitude, which could lead readers to assume that there is more than one piece of data.

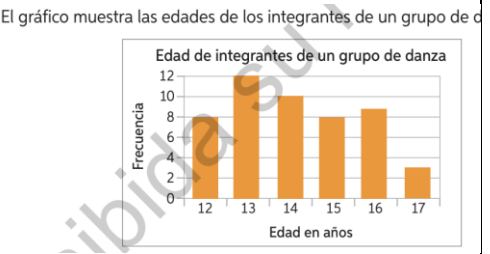
Figure 3

Examples of the presence of histograms associated with discrete quantitative variables

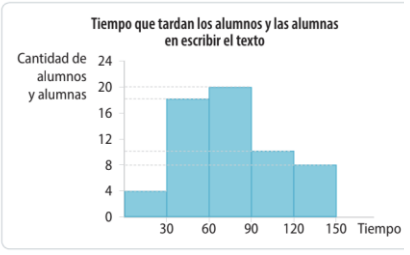
Example 1: TE7-2016 (p. 314)



Example 2: TE7-2023 (p. 200)



Example 3: TE8-2016 (p. 293)



Example 4: TE8-2023 (p. 180)



On the other hand, at the level of eighth grade of primary education, even though the variable time (in seconds) is of a continuous quantitative nature and the age (years) is of a discrete quantitative nature (Examples 3 and 4), the graph presents the form of a histogram according to the definition given by Triola (2006). On the other hand, in Examples 2 (TEP7-2023) and 4 (TEP8-2023), for the same type of variable, two types of graphical representations are shown, which could generate semiotic ambiguities for students. This fact could eventually generate tensions in the teaching and learning processes of this graphic representation, because, as Lee and Meletiou (2003) say, students have interpreted histograms as bar graphs, treating them as representing individuals rather than grouped data.

DISCUSSION

The histogram is one of the most cited graphical representations in statistical books, allowing the representation of data of a quantitative nature, whether continuous or discrete. However, they are not mentioned in the

learning objectives delivered by the MINEDUC in Chile. However, they are addressed in textbooks from the fifth grade of primary education onwards. These results show that, in some cases, this resource goes beyond what is explained in the ministerial guidelines (Table 1).

As mentioned, one of the skills students in the school system should develop is statistical literacy, which includes acquiring statistical concepts (Ben-Zvi & Garfield, 2004). That knowledge, as Rodríguez-Alveal et al. (2024) state, "will allow us to understand the messages behind the information that circulates in the media, such as the technical reports delivered by government agencies" (p. 2). In this sense, the textbooks under study introduce the definition of a histogram in eighth grade of primary education and first grade of secondary education, even though at the fifth grade of primary education the textbooks include activities related to this statistical graph. Definitions in which a didactic transposition is not evident, given that they are similar to those observed in specialised statistical texts, such as Triola's (2006) and Devore's (2008).

On the other hand, in the textbooks examined, two types of histogram representations are observed: one associated with discrete quantitative variables (simple bar graph) and another with continuous quantitative variables (grouped bar graph). Nevertheless, as noted by Parks et al. (2000), both representations are considered histograms, given that the variable is quantitative. This situation poses a challenge for the teachers of the school system, who must explain to students that, despite being visually different, the two graphs are, in essence, histograms, since they graphically summarise the data of a quantitative variable. It is worth noting that if they are not treated properly, they could lead the student to make incorrect connections when decoding the information summarised in them, a task that should eventually be addressed at the initial teacher education level.

Regarding the activities in the textbooks, we noted that they often invite students to read the graph literally without interpreting its information. Likewise, they request that the data be interpreted and integrated by performing operations on it. That is, they focus on the first and second levels of Curcio's (1989) taxonomy. What, from the perspective of the authors of this article, evidences a scarce vigilance of the publishers and the MINEDUC in the contents present in the textbooks, so that the students acquire skills related to statistical literacy.

On the other hand, students must make changes in representation, which is technically called transnumeration (Wild & Pfannkuch, 1999), in

which the calculation of frequencies stands out, according to the typology proposed by Chick (2003). In addition, there is little use of statistical concepts in the activities students must respond to. In short, they focus on performing calculations and memorising content and procedures. These practices, from the perspective of Lavy and Mashiach-Eizenberg (2009), would be insufficient to foster a deeper understanding of statistical concepts.

Finally, the present study provides empirical evidence on how textbooks address the introduction of one of the most important graphic representations in statistics, showing that there are pending tasks for teachers in the school system. Such tasks aim to prepare critical citizens, using the graphic information with which they interact daily in the media.

On the other hand, among the limitations of this study are the number of textbooks analysed and the consideration of only those distributed by the MINEDUC. With this, we propose, for future research, increasing the number of texts, including those accessed through trade and from other countries.

CONCLUSIONS

In this study, we investigated the activities related to histograms in primary and secondary education textbooks in Chile and how they support students' acquisition of literacy and statistical thinking. Through content analysis, we obtained the following conclusions. It is relevant:

- To introduce and promote activities in mathematics textbooks for the Chilean school system that enhance the reading and interpretation of histograms in contextualised situations.
- To deepen theoretical knowledge of the histogram as a representation of discrete and continuous qualitative data, taking into account that this representation poses challenges in Chilean primary education textbooks.
- To create monitoring policies for the textbooks delivered by the Chilean Ministry of Education year after year, regarding the activities they bring that are related to the histogram.
- To encourage teachers who are working in the school system to reflect, investigate, and understand the importance of graphical representations, particularly the histogram, when summarising quantitative information.
- To introduce activities related to contextualised data for the interpretation and critical reading of information from daily life.

AUTHORSHIP CONTRIBUTION STATEMENT

Conceptualisation, FRA; writing the manuscript, FRA; revision and editing of the manuscript, DDL and CGA; methodology, FRA and DDL; Analysis, FRA, DDL, and CGA; supervision, FRA, DDL, and CGA.

DATA AVAILABILITY DECLARATION

The data supporting the results of this study will be made available by the corresponding author, FRA, upon reasonable prior request.

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