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From Virtual to Conceptual: The use of Computer Graphics in Learning Matrices

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ABSTRACT

Context: The growing presence of computer graphics in everyday life highlights the need to articulate school mathematical concepts with real technological applications, especially in High School, where content such as Geometric Transformations and Matrices are little explored in significant situations. **Objectives:** To investigate how the concepts of Geometric Transformations used in Computer Graphics contribute to the learning of operations with Matrices and to analyze the students' understanding of using digital resources. **Design:** Qualitative study of an applied nature, structured in a Learning Unit based on the Theory of Meaningful Learning and the Three Pedagogical Moments. **Environment and participants:** The research was developed with 30 students from the 3rd year of high school in a public school in Santo Ângelo/RS, distributed in two classes, who fully participated in the activities. **Data collection and analysis:** Pre-and post-test questionnaires, written records, field diary, productions in the GeoGebra software and statistical analysis (McNemar's test) were used to verify conceptual advances. **Results:** The students demonstrated a greater understanding of the operations with Matrices when relating them to the Geometric Transformations in GeoGebra, indicating engagement, autonomy and significant learning, with a statistically significant difference between pre and post-test ($p = 0.002$). **Conclusions:** The articulated use of GeoGebra and Computer Graphics enhances the teaching of Matrices, favors conceptual visualization, promotes meaningful learning and indicates promising ways to integrate technology and Mathematics in High School.

Keywords: Geometric Transformations; Arrays; GeoGebra; Computer Graphics; Meaningful Learning.

Do Virtual ao Conceitual: o uso da Computação Gráfica na aprendizagem de Matrizes

RESUMO

Contexto: A crescente presença da computação gráfica no cotidiano evidencia a necessidade de articular conceitos matemáticos escolares a aplicações tecnológicas reais, especialmente no Ensino Médio, onde conteúdos como Transformações

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Geométricas e Matrizes são pouco explorados em situações significativas. **Objetivos:** Investigar como os conceitos de Transformações Geométricas utilizados na Computação Gráfica contribuem para a aprendizagem das operações com Matrizes e analisar a compreensão dos estudantes ao utilizar recursos digitais. **Design:** Estudo qualitativo de natureza aplicada, estruturado em uma Unidade de Aprendizagem fundamentada na Teoria da Aprendizagem Significativa e nos Três Momentos Pedagógicos. **Ambiente e participantes:** A pesquisa foi desenvolvida com 30 estudantes do 3º ano do Ensino Médio de uma escola pública de Santo Ângelo/RS, distribuídos em duas turmas, que participaram integralmente das atividades. **Coleta e análise de dados:** Utilizaram-se questionários pré e pós-teste, registros escritos, diário de campo, produções no *software* GeoGebra e análise estatística (teste de McNemar) para verificar avanços conceituais. **Resultados:** Os estudantes demonstraram maior compreensão das operações com Matrizes ao relacioná-las às Transformações Geométricas no GeoGebra, indicando engajamento, autonomia e aprendizagem significativa, com diferença estatisticamente significativa entre pré e pós-teste (p = 0,002). **Conclusões:** O uso articulado do GeoGebra e da Computação Gráfica potencializa o ensino de Matrizes, favorece a visualização conceitual, promove aprendizagens significativas e indica caminhos promissores para integrar tecnologia e Matemática no Ensino Médio.

Palavras-chave: Transformações Geométricas; Matrizes; GeoGebra; Computação Gráfica; Aprendizagem Significativa.

INTRODUCTION

Nowadays, we encounter digital images everywhere. They are part of our daily lives and are found in most of our daily routines. There are countless places where digital images can be easily found, for example, on television, in movies, on cell phones, in video games, on websites, among others. However, before these images are viewed by us, there is a whole process of creation, and it is precisely at this point that we find the function of Computer Graphics. As theoretical support for Computer Graphics, mathematical concepts such as Matrices and their operations and Geometric Transformations are used.

Dante (2013) explains that images are actually formed by small points, elements of a matrix. When a graphics program changes the position, reflects, rotates, or changes the scale of the image, it is actually changing the position of the points that form the image. This is all done by operations with matrices, and in computer graphics, this is called geometric transformations.

However, many high school textbooks do not cover this content on Geometric Transformations; others, however, cover the concept but do not present its applicability. The Curriculum Guidelines for High School (Brazil, 2006) indicate that the study of Geometric Transformations in the plane is another opportunity to work with mathematical concepts in a complementary way in algebraic and geometric studies. Therefore, Geometric Transformations can be represented algebraically in the form of operations with Matrices.

The choice of topic was based on the fact that the content of Geometric Transformations is not always addressed in high school mathematics classes, despite being a topic of great importance in numerous areas of knowledge such as medicine, architecture, civil construction, among others, as well as possible areas of knowledge for many students after completing high school.

In general, the concepts of mathematics studied in high school are taught based on textbooks that prioritize the algebraic development of concepts. However, some teachers rely on the contextualization and application of concepts to teach them. In the case of this work, we are interested in verifying how the concepts of Geometric Transformations, used in Computer Graphics, can contribute to the learning of operations with Matrices.

The general objective of this work was to analyze the contributions of the concepts of Geometric Transformations, used in Computer Graphics, to the learning of operations with Matrices. In order to achieve the research objective and answer the problem, a Learning Unit (LU) was developed, structured according to the Three Pedagogical Moments (TPM) methodology (Delizoicov; Angotti & Pernambuco, 2011), whose proposed activities made use of the GeoGebra software.

The concepts underlying the work, the methodology used, and the results obtained are presented below.

TEACHING MATHEMATICS AND INFORMATION AND COMMUNICATION TECHNOLOGIES (ICT)

Students often have difficulty learning the concepts studied in the areas of Natural Sciences and Mathematics, according to the International Student Assessment Program (PISA) indices on mathematics learning outcomes in Brazil. The indices presented in this program indicate that mathematics has a higher failure rate than other areas. These learning difficulties can be seen in the difficulty in solving the problems proposed, in conceptual poverty, in the lack of contextualization, and in the inability to apply the concepts studied in everyday situations (Fiolhais & Trindade, 2003, p.259). Getting students to understand the structure and/or process implicit in everyday laws/phenomena and enabling them to generate meaningful learning requires working with its meaning. In addition, making them critical and fascinated by the phenomena and techniques of their daily lives so that they can intervene in them is one of the trends in teaching today, along with the use of ICT (robotics, augmented reality, digital games, web research and interaction, online teaching, among others).

One way to change these results would be to modify mathematics learning activities and make them more dynamic, so that they can inspire greater autonomy and leadership in students in their studies, given that mathematics is present in our daily lives.

Furthermore, the reality for most high school students today is immersion in digital technology. It is everywhere: on the street, in schools, in clubs, in bars, in restaurants, etc. Students have access to it through school computer labs, personal computers, and mobile technologies such as cell phones and tablets.

In contemporary society, with the advent of personal computers and rapid changes in technology and media, basic knowledge in most areas is rapidly expanding and changing. This makes it essential to prepare students to deal with the proliferation and explosion of information and other rapid technological changes and to adapt to different professional fields. In addition, the job market needs people who are capable of researching, questioning, performing their tasks competently, taking initiative, and solving problems. Such skills require people to have developed and make use of critical thinking abilities (Halpern, 1999, p.69). These abilities allow individuals to solve problems and make rational decisions (Halpern, 1999, p.69).

On the other hand, ICT emerged in the 20th century as a resource for innovation in formal and informal education. It enables the integration of mobile and fixed multimedia, capable of meeting contemporary educational goals, which require fluidity, interactivity, and accessibility to knowledge, within the proposed scope and specificities of teaching science and mathematics. According to the general National Curriculum Guidelines (DCN) for Basic Education, in Chapter I (item VII), it determines that the forms of curriculum organization should ensure:

> Encouraging the creation of teaching and learning methods using information and communication technology resources, to be incorporated into everyday school life, in order to bridge the gap between students who learn to receive information quickly using digital language and teachers who have not yet mastered it. (Brazil, 2010, p.18).

According to the PCNs, ICT encompasses "[...] different means of communication (print journalism, radio, and television), books, computers, etc.” (Brazil, 1998, p. 135), and its purpose is the acquisition, storage, processing, and distribution of information by electronic and digital means, such as radio, television, telephone, and computers, among others, which emerged in the 20th century as resources for innovation in formal and informal education. These enable the integration of mobile and fixed multimedia, capable of meeting contemporary educational purposes, which presuppose fluidity, interactivity, and accessibility to knowledge.

It should be clear, however, that the use of these technological tools aims to arouse curiosity and interest in the specific topic. They seek to improve learning and the appropriate use of available technological means. They also seek to make students active participants in the construction of knowledge.

Traditional forms of learning are losing ground and meaning in a context of constant innovation and transformation, making it essential to embrace ICTs in a school setting to explore the potential of the various tools currently available, such as software, blogs, webquests, wikis, virtual communities, concept maps, virtual libraries, social networks, and virtual learning environments (VLE), as possible options for teaching and learning for students in basic education, used simultaneously with face-to-face contexts.

When we focus on the idea of “innovative teaching practices,” we are thinking about interactive classes, addressing issues such as: class time scheduling; the activities used in class, sparking students' interest in study and research; structuring the class with a view to learning outcomes and, preferably, meaningful learning and the development of talents in the areas of Science and Mathematics.

USE OF SOFTWARE IN MATHEMATICS EDUCATION

Currently, digital materials, such as educational software, are considered auxiliary resources in various areas of teaching. Due to technological advances, students are increasingly connected and using various digital resources. Thus, to spark students' interest in their studies and facilitate learning, teachers have chosen educational software to assist in their activities and enhance the learning of the concepts taught. The PCN states that “the effective use of computers in the classroom also depends on the choice of software, based on the objectives to be achieved and the concept of knowledge and learning that guides the process” (Brazil, 1998, p.44).

Among the many educational software programs for teaching mathematics, GeoGebra (http://www.geogebra.org/) is the one most teachers use because it is easy to navigate and install, as well as being free. According to Brandt & Montorfano (2007), GeoGebra software is an alternative tool in teaching practice and can provide greater accuracy and speed in certain actions. This technological resource should lead students to understand their geometric constructions, reinforcing the knowledge they have already acquired in the classroom and promoting new discoveries.

GEOMETIC TRANSFORMATION AND MATRICES

The National Common Core Curriculum (BNCC) highlights the importance of using geometric transformation concepts to "analyze different human creations such as civil constructions, works of art, among others." (Brazil, 2016, p. 101). This document also indicates that learning about this topic is important for "interpreting and representing the location and displacement of a figure on the Cartesian plane, identifying isometric transformations, and producing enlargements and reductions of figures." (Brazil, 2016, p. 93). Another highlight of the BNCC for the topic of Geometric Transformations is that:

> Movement and position are present in the location of numbers on straight lines, figures, or configurations in the Cartesian plane and in three-dimensional space; direction and sense, angles, parallelism and perpendicularity, isometric geometric transformations (which preserve measurements) and homothetic geometric transformations (which preserve shapes), and data distribution patterns. The use of maps, GPS, and other resources involves the observation and study of this pair of ideas. Investigative activities with dynamic software that interrelate movement and position can also promote the development of these ideas, which are important in cartography and in the daily movements of ordinary citizens. Because we live in a connected world with cell phones in our hands, geolocation devices, cable TV, surveillance cameras, etc., the study of movement and position has many purposes in various areas (Brazil, 2016, p. 97).

Given this, learning about the concepts of geometric transformations is important. With this knowledge, students can change the position or size of an original geometric figure, form another geometrically equal or equivalent figure, among other things.

Historically, the representation of sets of numbers in the form of matrices appeared in the 19th century, although the Chinese were already solving some types of calculation problems using tables around 2500 BC (Dante, 2013). In 1850, English mathematician Arthur Cayley (1821–1895), one of the pioneers in the study of these calculations, began to publicize the name "matrix," initiating demonstrations of its applications (Moura, 2014). Arthur Cayley developed a work on the relationship between Geometric Transformations and Matrices when he tried to verify whether the commutative law of multiplication was always valid, that is, whether it would be possible to have an Algebra in which ·a·b was different from b·a. Arthur Cayley found a possible answer to his question in Matrix Algebra.

The emergence of Matrices is linked to transformations of the type $\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases}$ in which a, b, c, d are real numbers and can be imagined as applications that take the point (x, y) to the point (x', y'). The preceding transformation is completely determined by the four coefficients a, b, c, d. The so that it can be symbolized by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, which we call a matrix (square, of order 2). The history of Matrix multiplication is due to the composition of Geometric Transformations. Therefore, considering the transformations, according to Eves (2004), if in the transformation $\begin{cases} x'' = ex' + fy' \\ y'' = gx' + hy' \end{cases}$ it can be shown, though elementary algebra, that the result is the transformation $\begin{cases} x'' = (ea + fc)x + (eb + fd)y \\ y'' = (ga + hc)x + (gb + hd)y \end{cases}$. This leads to the following definition of the product of two Matrices: $\begin{bmatrix} e & f \\ g & h \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea + fc & eb + fd \\ ga + hc & gb + hd \end{bmatrix}$. Thus, the definition of the product of Matrices arose from the composition of two Geomatric Transformations.

The PCNs for the 3rd cycle (Brazil, 1998, p. 65) point out the importance of Geometric Transformation for "solving problem situations involving flat geometric figures, using procedures of decomposition and composition, transformation, enlargement, and reduction." And those for the 4th cycle (Brazil, 1998, p. 81) indicate that they serve to "interpret and represent the location and displacement of a figure on the Cartesian plane."

The PCNs also point out that Geometric Transformations are important for the classroom, especially in the use of teaching resources, and emphasize that

> Activities involving the transformation of a figure on a plane should be prioritized in these cycles because they allow for the

development of geometric concepts in a meaningful way, in addition to giving this study a more “dynamic” character. Currently, there is software that explores problems involving figure transformations. It is also interesting to propose situations to students in which they compare two figures, where the second is the result of the reflection of the first (or translation or rotation), and discover what remains invariant and what changes. Such activities can start from the observation and identification of these transformations in tapestries, vases, ceramics, tiles, floors, etc. (Brazil, 1998, p. 124).

The geometric transformations in the plane covered in high school textbooks are: **reflection**, **scaling, translation, and rotation**. The use of GeoGebra software as a teaching resource can help develop each transformation and its matrix representation. In this work, the geometric transformations of shearing and homothetic were not covered. Below, we describe the geometric transformation of the reflection type. The other types of geometric transformations were constructed with the students in a similar way.

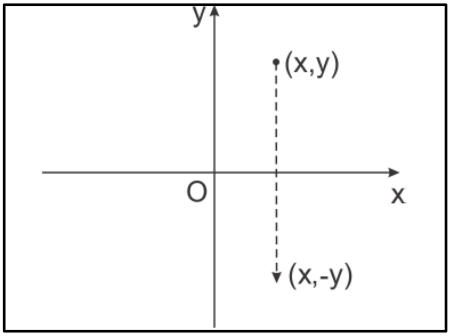
GEOMETRIC TRANSFORMATION: REFLECTION

The following text is based on the books: Dante (2013)[1] and Gonçalves (2013)[2]. The transformation of reflection around an axis, or mirroring, applied to an object, produces a new object that is as if the previous object were seen reproduced by a mirror, positioned on the axis around which the mirroring takes place. In the case of a 2D (2-dimensional) reflection, the mirror can be considered on the vertical or horizontal axis. Reflection in relation to the x-axis: The transformation $\boldsymbol{T}: \mathbb{R}^{\mathbf{2}} \rightarrow \mathbb{R}^{\mathbf{2}}$ given by $\boldsymbol{T}(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x}, -\boldsymbol{y})$ is called reflection in relation to the x-axis (Figure 12). In Matrix form there is: $\boldsymbol{T}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \boldsymbol{x} \\ -\boldsymbol{y} \end{pmatrix}$. In this case, it is said that the matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix} = \boldsymbol{M}_{\mathbf{1}}$ represents the reflection transformation with respect to the x-axis (Figure 1).

[1] DANTE, Luiz Roberto. **Matemática:** contexto & aplicações. 2. ed. São Paulo: Ática, 2013.
[2] GONÇALVES, Haniel Soares. **A importância das matrizes e transformações lineares na computação gráfica.** 2013. Dissertação (Mestrado) – Universidade Federal de Goiás, Instituto de Matemática e Estatística, 2013.

Figure 1

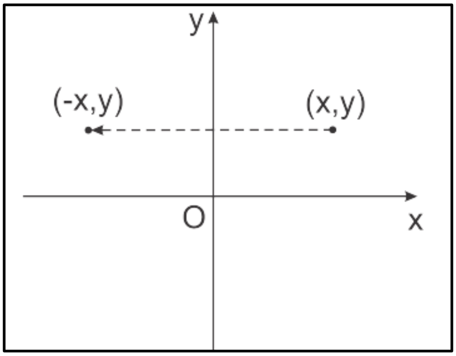
Transformation reflection in relation to x-axis ***(****Gonçalves, 2013).*



Reflection in relation to the y-axis: The transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $T(x, y) = (-x, y)$ is called Reflections in relation to the y-axis (Figure 2). In the Matrix form, there is: $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$. In this case there is the Matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = M_2$ which represents the transformation reflection in relation to the y$-axis$ (Figure 2).

Figure 2

Reflection transformation in relation to the $\boldsymbol{y - axis}$ *(Gonçalves, 2013)*

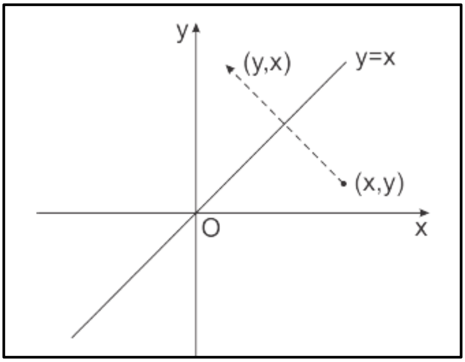


<u>Reflection in relation to the line $y = x$:</u> The transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $T(x, y) = (y, x)$ is called reflection around the line $y = x$ (Figure 3). In Matrix form, there is: $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$. In this case, it is said that

the Matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = M_3$ represents the reflection transformation in relation to the line $y = x$ (Figure 3).

Figure 3

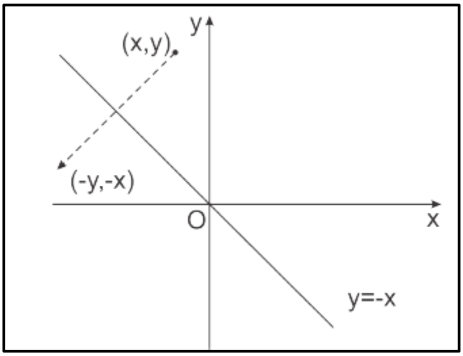
Reflection in relation to the line $\boldsymbol{y = x}$ *(Gonçalves, 2013).*



Reflection in relation to the line $y = -x$: The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (-y, -x)$ is called reflection in relation to line $y = -x$ (Figure 4). In Matrix form, there is; $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}$. In this case, it is said that the Matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = M_4$ represents the reflection transformation in relation to the line $y = -x$ (Figure 4).

Figure 4

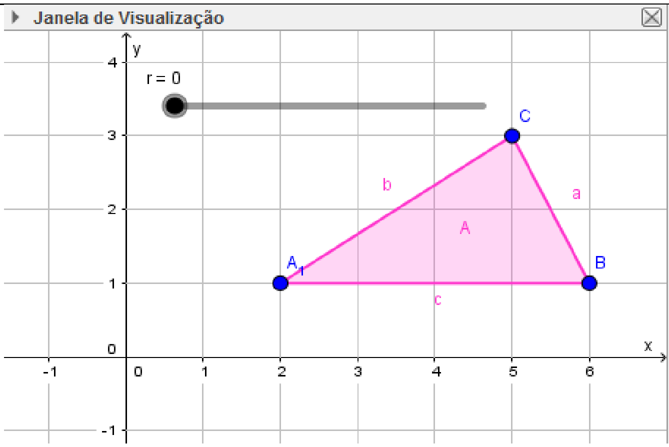
Reflection in relation to line $y = -x$ *(Gonçalves, 2013)*



Tables 1, 2, 3 and 4 (below) present some reflections done using GeoGebra software and the Matrix representations to present to the students.

Table 1

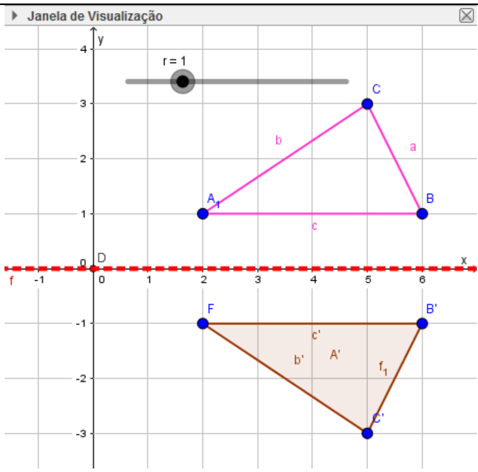
Geometric and Matrix Representation of Triangle A (Adapted from Dante, 2013).

Triangle A $(r = 0)$	
Image developed at GeoGebra sotware	
Matrix Representation	Vertices of figure A: $A_1 = (2,1);\ B = (6,1);\ C = (5,3)$ Matrix associated with vertices of this figure is: $A = \begin{pmatrix} 2 & 6 & 5 \\ 1 & 1 & 3 \end{pmatrix}$

Points (x, y), which make up the vertices of the geometric figures, were expressed in the rows and columns of the matrices. Table 2 geometrically represents the reflection of triangle A in relation to the $x - axis$, which resulted in triangle A', developed in GeoGebra.

Table 2

Geomatric Representation of the reflection o triangle $\boldsymbol{A}$ *in relation to the* $\boldsymbol{x-axis}$ *(Adapted from Dante, 2013).*

<table>
<tr><th colspan="2">Reflection in relation to $\boldsymbol{x-axis}$ $(\boldsymbol{r=1})$</th></tr>
<tr><td>Image developed in GeoGebra</td><td>
</td></tr>
<tr><td>Matrix Representation</td><td>Vertices of figure A: $A_1=(2{,}1)$; $B=(6{,}1)$; $C=(5{,}3)$

Vertices of figure A': $F=(2,-1)$; $B'=(6,-1)$; $C=(5,-3)$

Matrix associated to the vertices of figures:

$$A=\begin{pmatrix}2&6&5\\1&1&3\end{pmatrix} \text{ e } A'=\begin{pmatrix}2&6&5\\-1&-1&-3\end{pmatrix}$$
The reflection that takes A in A' is indicated by:

$$A\to A', \text{ that is, } A=\begin{pmatrix}2&6&5\\1&1&3\end{pmatrix}\to A'=\begin{pmatrix}2&6&5\\-1&-1&-3\end{pmatrix}$$
Observe that the reflection is in relation to $x-axis$. It is obtained the Matrix of A' multiplying matrix A by matrix $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, that is:

$$\begin{pmatrix}1&0\\0&-1\end{pmatrix}\times\begin{pmatrix}2&6&5\\1&1&3\end{pmatrix}=\begin{pmatrix}2&6&5\\-1&-1&-3\end{pmatrix}$$</td></tr>
</table>

Table 3 geometrically represents the reflection of triangle A in relation to line $y=x$, which obtained triangle A_1', developed in GeoGebra.

Table 3

Geometric Representation of the Reflection of triangle $\boldsymbol{A}$ *in relation to line*

$$\boldsymbol{y = x}$$

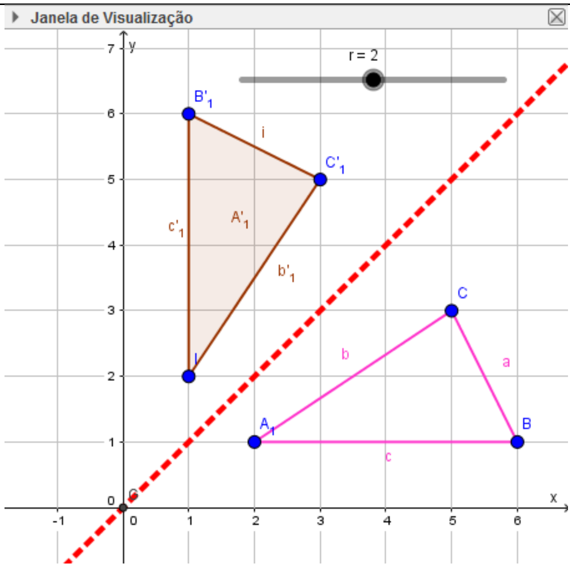
Reflection in relation to line $\boldsymbol{y = x}$ $(\boldsymbol{r = 2})$	
Image developed in GeoGebra	

Table 4, the matrix representation of the reflection of triangle A in relation to line $y = x$ that obtained the triangle A_1' is represented.

Table 4

Matrix Representation of the reflection of triangle $\boldsymbol{A}$ *in relation to line* $\boldsymbol{y = x}$ *(Adapted from Dante, 2013).*

Matrix Representation	Vertices of figure A: $A_1 = (2,1)$; $B = (6,1)$; $C = (5,3)$ Vertices of figure A'_1: $I = (1,2)$; $B'_1 = (1,6)$; $C'_1 = (3,5)$ Matrix associated to vertices of figures:

	$A = \begin{pmatrix} 2 & 6 & 5 \\ 1 & 1 & 3 \end{pmatrix}$ e $A'_1 = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 6 & 5 \end{pmatrix}$ The reflection that takes A in A'_1 is indicated by: $A \rightarrow A'_1$, that is, $A = \begin{pmatrix} 2 & 6 & 5 \\ 1 & 1 & 3 \end{pmatrix} \rightarrow A'_1 = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 6 & 5 \end{pmatrix}$ Observe that the reflection is in relation to a line. The matrix of A'_1 is obtained by multiplying the matrix of A by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, that is: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & 6 & 5 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 6 & 5 \end{pmatrix}$

Similarly, other forms of geometric representation in relation to the x and y axes and the lines y = x and y = -x were developed in GeoGebra software and their corresponding matrix representation. As an example of geometric transformation, the reflection of other types: scale, translation, and rotation, were also worked on. The tutorial with all the geometric transformations developed in this research can be found at: http://www.ufn.edu.br/site/ensino/mestrado/programa-de-posgraduacao-em-ensino-de-ciencias-e-matematica/producoes/.

COMPUTER GRAPHICS

Computer graphics is an area of computer science that studies and develops techniques and algorithms for generating digital images using computers. According to Dante (2013), a computer monitor functions as a matrix (table) with information (colored dots shown on the screen, or pixels) stored in rows and columns.

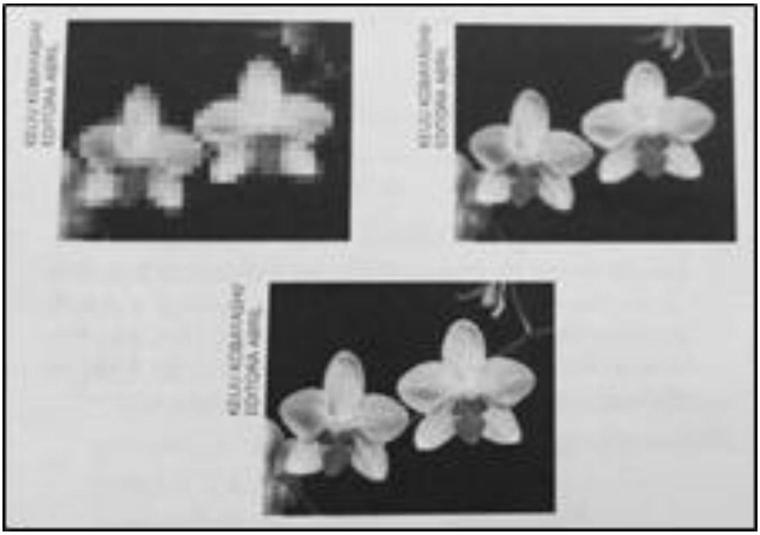
Gomes & Velho (1998, p. 1) define Computer Graphics as "a set of methods and techniques for transforming data into images using a graphics device." Today, Computer Graphics is present in almost all areas of human knowledge, from the development of a new car model to the development of electronic games.

In the section on Computer Graphics, the textbooks present several digital images of the same object. Their goal is to demonstrate that the intersection of a row and a column on the screen forms a node, called a *pixel*[3]. The greater the number of pixels, the higher the screen resolution and the sharper the images. The greater the number of pixels, the higher the screen resolution and the sharper the images.

Figures 5 and 6 show the same images, but with different resolutions (number of pixels). In these figures, you can see well-defined (high resolution) or distorted (low resolution) images (Figure 5). Resolution depends on the number of pixels, i.e., the greater the number of rows and columns in a digital image, the greater the number of pixels and the higher the resolution, and vice versa. In Figure 6, the first image has 27 rows and 33 columns, while the third image has 1,645 rows and 2,008 columns, in accordance with the studies by (Dante, 2005).

Figure 5

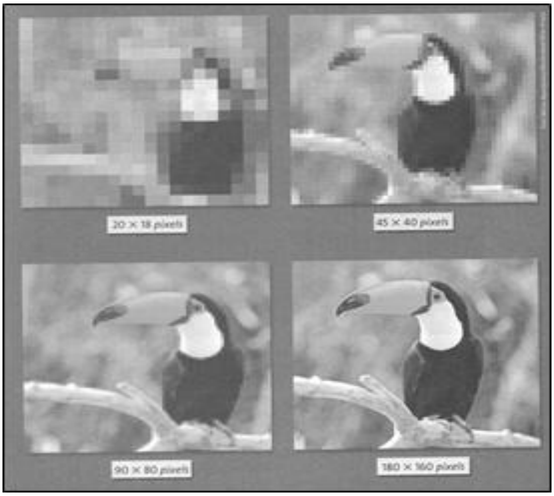
Image sharpness on the computer (Dante,2013, p. 73)



[3]Pixel - A pixel is the smallest point, and thousands of pixels together form a digital image.

Figure 6

Three different resolutions (Dante, 2005, p. 240).



According to Dante (2013), an image on a computer screen with a resolution of 800 x 600 has 800 x 600 = 480,000 pixels distributed across 800 columns and 600 rows. When a graphics program changes the position, reflects, rotates, or changes the scale of the image, it is changing the position of the pixels that form it. This is all done by matrix operations; it is called geometric transformations. Therefore, digital images created on computers are obtained by numerical representations and transformed through matrix operations, projected on the computer monitor.

LEARNING UNITS (LU) ON MATRICES

To study the subject of Geometric Transformations, a set of teaching and learning activities was organized using GeoGebra software. The choice of activities was based on Ausubel's (2003) Theory of Meaningful Learning (TML) and organized according to the Three Pedagogical Moments (TPM) methodology developed by Delizoicov, Angotti, and Pernambuco (2011).

Meaningful learning, according to Ausubel (2003), is a process of interaction between new and prior knowledge, that is, a process through which new information relates to a relevant aspect of the individual's knowledge structure, that is, it involves the interaction of new information with a specific knowledge structure, which is defined as a subsuming concept existing in the individual's cognitive structure (Moreira, 1999b). According to Ausubel, the storage of information in the human brain is a “highly organized process, forming a kind of conceptual hierarchy in which more specific elements of knowledge are linked to (and assimilated by) more general and inclusive concepts, ideas, and propositions” (Moreira, 1999a, p. 13).

As knowledge is developed and taken in, the learner's cognitive structure changes. The simple memorization of formulas and concepts, for example, is a type of learning in which new information is stored arbitrarily, without interacting with what already exists in the student's cognitive structure and contributing little or nothing to conceptual elaboration and differentiation (Moreira, 1999a), but it can also modify the subsumers.

These changes in cognitive structure can occur in two ways: through Progressive Differentiation and Integrative Reconciliation. Progressive Differentiation consists of the process of assigning new meanings through the successive use of existing subsumers, while Integrative Reconciliation is based on eliminating apparent differences and integrating new knowledge into existing subsumers, in addition to generating new meanings (Moreira, 2012).

In order for meaningful learning to occur and new knowledge to be generated, teaching must be approached from the most general to the most specific, in order to promote Progressive Differentiation and Integrative Reconciliation. The most general does not mean knowledge in its final form (formal, abstract, and sophisticated), but rather the most general concept to the most specific, such as presenting the concept of matrices (more general) and then working with the concepts of matrix operations (more specific).

In view of these aspects, some studies indicate that the TMP methodology (Delizoicov, Angotti & Pernambuco, 2011) has proven effective for the development of meaningful learning (Bulegon, 2011; Schons, 2017), given that in **PI,** real issues or situations that students know and witness and that are involved in the topics are presented, so that they are challenged to express what they are thinking about the situations. The purpose of this moment is to provide students with critical distance when faced with interpretations of the situations proposed for discussion and to develop subsumers about the topic studied. The second stage: **OC,** is when the concepts necessary for understanding the topics and the initial problematization are studied. The third stage, **AC,** is when the knowledge that has been incorporated by the student is analyzed. According to Bulegon (2011), teaching activities can be developed at each stage using various types of teaching materials, such as experimental activities, lectures and dialogues, debates, text reading, exercise lists, software, hypertext, hypermedia, learning objects, among others.

Table 5 shows the structure of the Learning Unit (LU) developed, as well as the resources, theme, and objectives developed in each stage. Table 5

LU on Matrices

Methodology	Date	Theme	Objective	Resource
PI	Activity 1 (50 min)	- Computing; - Computer Graphics;	Verify previous conceptions.	Questionnaire
	Activity 2 (50 min)	- Matrices; - Geometric Transformations	Verify knowledge on Matrices and Geometric Transformations	Test
OC	Activity 3 (50 min)	- Matrices.	Define the concept of Matrices;	Whiteboard; Marker; Photocopies; slide presentations
	Activity 4 10 rounds (50 min each)	- Geometric Transformations - GeoGebra;	Define the concept of Geometric Transformations Explain GeoGebra; Show Geometric Transformations with GeoGebra.	GeoGebra software.
	Activity 5 (50 min)	- Computer Graphics.	Define the concept of Computer Graphics.	Research.
AC	Activity 6 (50 min)	- Geometric Transformations; - Computer Graphics; - GeoGebra.	Verify the comprehension on the concepts worked in the activities and the contributions of GeoGebra and the operations with Matrices to solve these concepts.	Questionnaire.
	Activity 7 (50 min)	- Matrices; - Geometric Transformations; - Computer Graphics;	Verify the learning obtained after the activities.	Test.

Activity 1: The first activity was conducted using a pre-test questionnaire consisting of 13 questions on the topics of Information Technology and Computer Graphics to verify whether students had prior knowledge of these topics.

Activity 2: The second activity was conducted through a pre-test consisting of eight questions on the content of Matrices and Geometric Transformations to assess students' knowledge of these concepts. Students used pencils, pens, and paper to complete this test.

Activity 3: The third activity was carried out using a whiteboard, marker pens, and photocopies. The historical context of the emergence of matrices was presented using slides; the concept and types of matrices, as well as operations with matrices, were presented on the whiteboard.

Activity 4: The fourth activity began with defining the concept of Geometric Transformations using GeoGebra software. First, GeoGebra's functionality, tools, and applications were explained.

Next, we constructed the Geometric Transformations within the aforementioned software, using flat shapes such as triangles and squares, relating the vertices of these shapes to the Matrices. After that, we replaced the triangles and squares with any shapes, limiting their size to the points marked in the previous shapes, and represented them in their respective Matrices. Thus, the students began manipulating GeoGebra, constructing the Geometric Transformations and the corresponding Matrices. This activity was produced in eXe Learning, which is available on the UFN[4] website.

Activity 5: In the fifth activity, students researched on the web, in the school's computer lab, what Computer Graphics is and how the concepts of Matrices and Geometric Transformations can be identified in this subject.

Activity 6: The sixth activity was conducted through a post-test questionnaire consisting of eight questions on the concepts of geometric transformations, computer graphics, and GeoGebra software. To complete this post-test, students used pencils, pens, and paper, as they had in the pre-test.

Activity 7: The seventh activity was carried out through a test consisting of eight questions on all the content covered in the LU on Matrices.

ANALISIS AND DISCUSSION OS RESULTS

The results presented below were obtained during the application of the seven teaching and learning activities on matrices planned in this study, as described in Table 3. The activities were developed with 11th-grade students

[4] Available at: http://www.ufn.edu.br/site/ensino/mestrado/programa-de-posgraduacao-em-ensino-de-ciencias-e-matematica/producoes/ (Ano 2017) - Luana Pereira Villa Real - Transformações geométricas: aplicação de matrizes na computação gráfica.

from a state school in the municipality of Santo Ângelo/RS, in the morning shift. Class A had 19 students and class B had 21, totaling 40 participants. However, the sample analyzed consisted of 30 students, as only these students fully completed all the stages and activities planned in the experiment.

The instruments used in data collection were questionnaires and tests (before and after the development of the LU), the teaching practice diary, completed by the researcher at the end of each activity, the students' answers written in their notebooks, always collected at the end of each activity, and the file of activities performed by the students in the GeoGebra software, always saved at the end of each activity. To preserve the students' identity, they were identified as E01 to E30.

Activity 1 (pre-test questionnaire) aimed to assess students' prior knowledge of computer science and computer graphics, since according to Meaningful Learning Theory (MLT), the factor that most influences learning is what the student already knows. Based on the identification of the students' knowledge, we developed activities that would stimulate existing knowledge or create new knowledge in the students.

The results of the initial questionnaire (pre-test) show that students have basic computer skills, i.e., they know how to use tools and software, but they use computers to browse social media and the web for general research. Students did not have much knowledge about Computer Graphics.

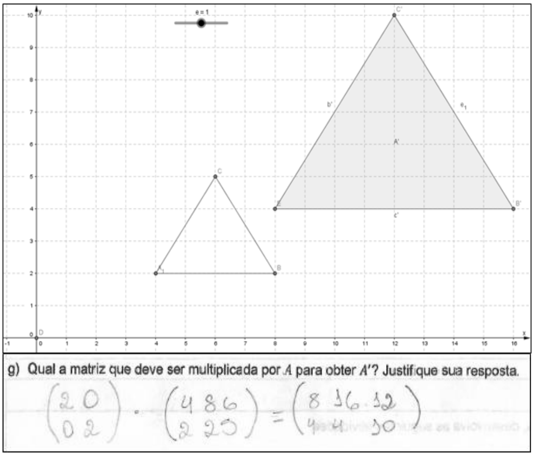
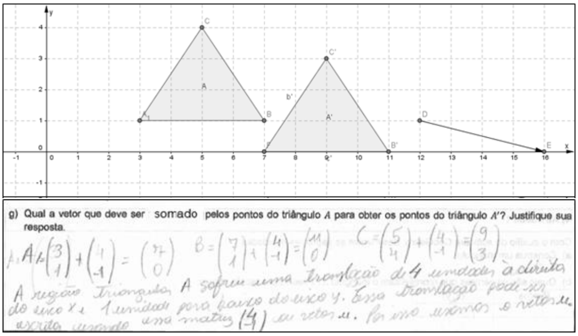
Activity 3 dealt with the concept of matrices and their operations. This concept was explained by the teacher through a slide presentation, starting with the emergence of matrices, their definition, and operations. When doing written exercises on this topic, using consumables such as pencils and paper, it was observed that the students had difficulties with Matrix operations, as they frequently asked how to solve addition, subtraction, and multiplication with Matrices. In addition, the students had initial difficulties understanding the concept of Geometric Transformation.

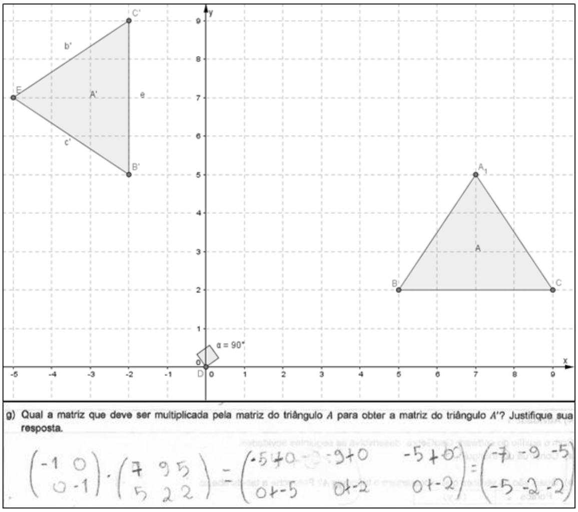
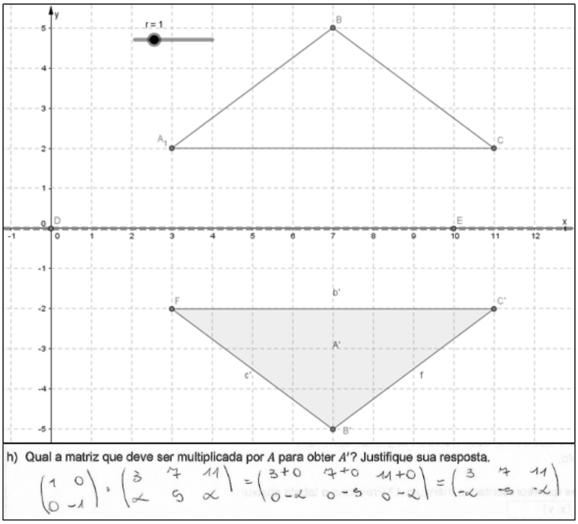
In this sense, students were able to test the features of the GeoGebra software, construct and manipulate the flat figures (triangle and square) formed in activity 4. For each figure constructed and manipulated, students wrote matrices with the points of their vertices. No student changed the order of the matrix, such as 3x2 to 2x3.

The students understood that the number of columns in the matrix was associated with the vertices of each flat figure; for example, a triangle has three vertices, so the number of columns in the Matrix would be three. We also aim to develop reflection, scaling, translation, and rotation of geometric figures and identify operations with Matrices. Table 6 shows some demonstrations developed by students in this activity.

Table 6

Demonstration of activities developed by students using GeoGebra

Scale - Expansion

Translation


Rotation

Reflection


When carrying out activities using GeoGebra software, students showed engagement, motivation, and creativity in using the software and relating the images created to the construction of meanings of matrices and their operations. They commented that it was positive and interesting to have started building knowledge of geometric transformations through activities with the software. For them, first seeing the image and then identifying the elements that indicate the use of matrix operations in the images made this knowledge meaningful and allowed them to understand the relationship between the matrices and the vertices of the figures and the countless variations that the images can obtain in the plane. It was noticed in all activities that some students were more autonomous, using the software tools and performing the constructions with the help of the tutorial developed in GeoGebra, organized in eXe Learning and available to students. Others, more restrained, waited for the researcher to perform the construction steps.

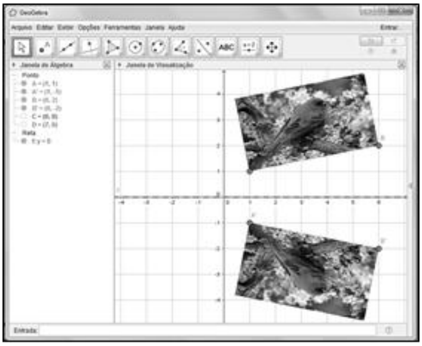
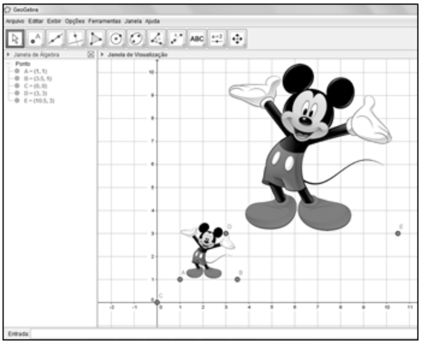
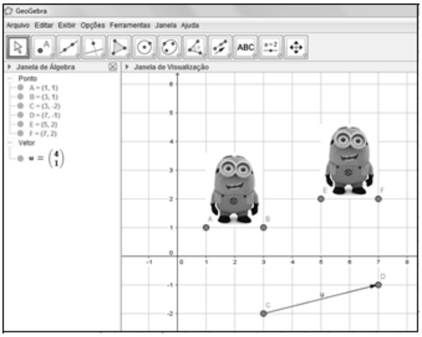
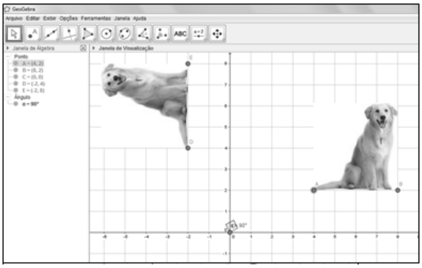
Given these results, it can be inferred that students find it easy to use digital tools, and that this adaptation aids in the learning of mathematical concepts. However, it has been observed that students often do not use digital tools for studying and learning mathematics outside the classroom if they are not encouraged to do so.

In activity 5, students had to research the concept of computer graphics on the web. Subsequently, a debate was held on the findings in order to enrich the research and develop students' skills in analysis, synthesis, and presentation of results. According to them, it was the first time they had done an internet research activity at school and had contact with a way of doing research on the web. Based on how this activity was carried out, students were able to conduct other research outside of class, since most of them have access to the internet at home or other places outside of school.

Table 7 presents examples of the use of GeoGebra software in the study of Geometric Transformations that were developed by students in the classroom.

Table 7

Geometric Transformations

Reflection	Scale
	
Translation	**Rotation**
	

In the activities developed by the students, various images were used and points were associated with them in the GeoGebra software. By changing the points' locations, the students verified the geometric transformations studied, as well as the matrices related to them. Next, they analyzed the points in the "Algebra Window" and, in their notebooks, developed the corresponding Matrix.

As a final activity, students were asked to complete a questionnaire (activity 6) to assess their understanding of the concepts covered in class, the contributions of GeoGebra software, and matrix operations for solving these concepts. The answers showed that the students did not have a clear understanding of the concept of geometric transformations, with 94% of students responding that GeoGebra software facilitated learning and justifying this by saying, “because I didn't know how to use GeoGebra, and it facilitated learning because it was something practical and not just theoretical,” “because doing it and seeing it personally on the computer is better for student learning,” and “because it sparked my interest more and is different from what we do in class”.

When asked if the concept of geometric transformation contributes to learning matrix operations, most students (94%) answered “yes” and wrote why it contributed to their learning: "because in the 2nd year I only learned how to calculate matrices, and in the proposed activities I learned to identify the ordered pair of a geometric figure and then construct the matrix and solve the operations with matrices. I also learned what a row and column of a matrix were, as well as multiplication and addition with Matrices. “They also wrote, ‘Yes, it contributed, because with the figures I was able to understand better how to assemble a matrix, and with the construction of geometric transformations I was able to learn, with the matrix of the original figure, solving the operations with matrices gave the result of the matrix of the geometric transformation." Given these reports, it can be said that the concepts of Geometric Transformation contributed to the learning of operations with Matrices.

To verify the significance levels (p) of the activities developed in this AU, the results of the pre-test and post-test were analyzed using SPSS software based on the McNemar statistical test. It was found that of the 30 students who participated in the study, 26 had positive results, i.e., their post-test scores increased, 3 had negative results, and 1 had no change. Due to the p-value = $0.002 < 0.05$, we rejected the null hypothesis and, as a result, there was a significant difference when analyzing the results of the pre-test (6.48 ± 1.92) and post-test (8.19 ± 1.64), which allows us to conclude that the students had advanced their knowledge. In addition, it was found that the students had understood the content as figures from magazines, newspapers, websites, etc., were being analyzed and they identified what type of geometric transformation had occurred

In summary, it can be said that students had an intuitive understanding of matrices, but faced difficulties in identifying and understanding geometric transformations, such as reflection, scaling, translation, and rotation. This finding guided the development of teaching activities on this topic, which were planned based on TAS, acting as prior organizers to facilitate the assimilation of new concepts, so that students could establish relationships between their prior knowledge and new knowledge.

The use of GeoGebra software was decisive in this process, as it allowed students to concretely visualize the effects of matrix operations on geometric figures, promoting connections between prior knowledge and new knowledge. The results indicate that students developed meaningful learning, demonstrating greater autonomy, engagement, and understanding of the content covered. This experience reinforces the potential of digital technologies and critical methodologies as effective tools for teaching mathematics in high school.

FINAL CONSIDERATIONS

In this research, the main objective was to analyze the contributions of Geometric Transformations concepts, used in Computer Graphics, to the learning of Matrix operations.

This topic was chosen because the study of geometric transformations highlights the concepts of matrices, their operations, and their applicability in various areas of knowledge: medicine, architecture, civil construction, and several other areas, such as computer graphics, a topic present in the daily lives of elementary school students. This fact contributes to the development of meaningful learning.

Analysis of the students' responses in the pre-test showed that they had an intuitive understanding of matrices and types of geometric transformations, but had difficulty identifying them when presented together, which is the first condition for meaningful learning, according to Ausubel: learners must possess the necessary underlying concepts for new learning.

The way in which the activities were proposed in the AU on Matrices and GeoGebra, as a technological resource used to explore Geometric Transformations and facilitate the visualization of Matrix concepts and properties, proved to be potentially significant, as it allowed students to be active in the learning process and, consequently, protagonists of their knowledge. In addition, the GeoGebra software allowed students to create images and explore their movements, which contributed to enhancing learning and distinguishing between types of Geometric Transformations. This meets the second condition for meaningful learning: the material to be used for teaching must be potentially meaningful and relatable to the learner's knowledge structure in a non-arbitrary and non-literal way.

For the third condition of TAS to occur, the learner must be predisposed to learn, that is, to relate the new material in a way that is not arbitrary, but substantial to their cognitive structure. Throughout the development of activities using GeoGebra software, it was noticed that students demonstrated willingness, autonomy, and a great desire to learn the concepts of matrices and their operations, as they found meaning in what they were doing, in addition to perceiving the application of the topic of geometric transformations in something that is part of their daily lives, such as Computer Graphics.

It was clear that the students enjoyed working with the GeoGebra software, as it provided a dynamic visualization of Geometric Transformations and the identification of each vertex for the construction of Matrices. These enhanced aspects of progressive differentiation and integrative reconciliation regarding the concepts involved in this theme.

The research conducted demonstrated that the use of geometric transformation concepts, especially those present in Computer Graphics, is an effective strategy for promoting meaningful learning of Matrix operations in the context of Basic Education. By linking abstract mathematical content with concrete and technological applications, the study contributes to making teaching more attractive, contextualized, and aligned with the contemporary demands of school education.

The adoption of GeoGebra software as a teaching resource proved to be innovative, as it provided students with a dynamic, interactive, and visual learning experience. This approach breaks with the traditional expository teaching model and favors student protagonism, allowing students to construct and manipulate graphic representations, explore movements, and actively identify mathematical relationships. Such practices enhance progressive differentiation and the integrative reconciliation of concepts, as recommended by Ausubel's TAS.

In addition, it was observed that students demonstrated autonomy, interest, and a willingness to learn, which reinforces the importance of considering engagement and motivation as central elements in the teaching and learning process. The use of ICT, such as GeoGebra, proved to be a catalyst for this involvement, allowing students to connect school knowledge with real and meaningful situations, such as those present in Computer Graphics.

Given the results obtained, it is hoped that this research will inspire teachers and educators to explore interdisciplinary and technological approaches to teaching mathematics, promoting more meaningful and inclusive teaching practices that are connected to the students' reality. The integration of theory, practice, and technology represents a promising path for strengthening learning and training critical, creative individuals who are prepared for the challenges of the 21st century.

AUTHORSHIP CONTRIBUTION STATEMENT

The research presented here was developed by the first author, under the guidance of the second author. The first author developed the research (study, data collection, and analysis), and the second author reviewed and guided the entire process.

DATA AVAILABILITY STATEMENT

The data supporting the empirical development of this research is stored by the researchers, in accordance with ethical principles.

APPROVAL BY AN ETHICS COMMITTE

This research was not submitted to the Ethics Committee, as it was conducted with students at the institution where the first author was a professor, and the data collection was carried out in the year that Resolution No. 510/16 of the National Council of Ethics in Research (CONEPE) was issued.

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