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Interpretation of the equal sign by fourth-grade primary education students in open equality tasks

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ABSTRACT

Context: The interpretation of the equal sign is central to the development of algebraic thinking in the early years of primary education, especially in tasks involving open equations. **Objective:** To investigate the interpretation of the equal sign by fourth-grade primary school students (ages 9–10) when solving open equation problems. **Design:** The research adopted a qualitative, descriptive-interpretive approach. **Setting and participants:** The study was conducted in a primary school and included four fourth-grade students (ages 9–10), selected via convenience sampling. **Data collection and analysis:** Data were collected through individually administered, semi-structured interviews consisting of five tasks. Four of these tasks involved open equations, and one focused on exploring the generalisation of the interpretation of the equal sign. The analysis was performed using content analysis of the students' written and verbal work. **Results:** The results show that students interpreted the equal sign in three ways: as an operator, as operational numerical equivalence, and as relational numerical equivalence. **Conclusions:** It is concluded that, as students progress through the tasks and receive guidance from the researcher, they develop a relational interpretation of the equal sign, supported by the compensation strategy. This has direct implications for the teaching of early algebraic thinking and teacher education.

Keywords: School algebra; Primary education; Algebraic thinking; Relational thinking.

Interpretación del signo igual de estudiantes de cuarto año de educación primaria en tareas de igualdades abiertas

RESUMEN

Contexto: La interpretación del signo igual constituye un aspecto central para el desarrollo del pensamiento algebraico en los primeros años de la Educación Primaria, especialmente en tareas que involucran igualdades abiertas. **Objetivo:** Indagar en la interpretación del signo igual en estudiantes de cuarto año de Educación Primaria (9–10 años) al resolver tareas de igualdades abiertas. **Diseño:** La investigación adoptó un

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enfoque cualitativo de carácter descriptivo-interpretativo. **Ambiente y participantes:** El estudio se desarrolló en un contexto escolar y contó con la participación de cuatro estudiantes de cuarto año de Educación Primaria (9-10 años), seleccionados intencionalmente. **Recolección y análisis de datos:** Los datos se recolectaron mediante entrevistas semiestructuradas aplicadas de forma individual, compuestas por cinco tareas, de las cuales cuatro involucraron igualdades abiertas y una se orientó a la exploración de la generalización de la interpretación del signo igual. El análisis se realizó mediante un análisis de contenido de las producciones escritas y verbales de los estudiantes. **Resultados:** Los resultados evidencian que los estudiantes interpretaron el signo igual a partir de tres significados: operador, equivalencia numérica operacional y equivalencia numérica relacional. **Conclusiones:** Se concluye que, a medida que los estudiantes avanzan en las tareas y reciben orientación del investigador, progresan hacia una interpretación relacional del signo igual, sustentada en la estrategia de compensación, lo que presenta implicancias directas para la enseñanza del pensamiento algebraico temprano y la formación docente.

Palabras clave: Álgebra escolar; Educación primaria, Pensamiento algebraico, Pensamiento relacional.

Interpretação do sinal de igualdade por alunos do quarto ano do ensino fundamental em tarefas de igualdades abertas

RESUMO

Contexto: A interpretação do sinal de igualdade é fundamental para o desenvolvimento do pensamento algébrico nos primeiros anos do ensino fundamental, especialmente em tarefas que envolvem equações abertas. **Objetivo:** Investigar a interpretação do sinal de igualdade por alunos do quarto ano do ensino fundamental (9 a 10 anos de idade) ao resolver problemas com equações abertas. **Metodologia:** A pesquisa adotou uma abordagem qualitativa, descritiva-interpretativa. **Ambiente e participantes:** O estudo foi realizado em um ambiente escolar e incluiu quatro alunos do quarto ano do ensino fundamental (9 a 10 anos de idade), selecionados por amostragem de conveniência. **Coleta e análise de dados:** Os dados foram coletados por meio de entrevistas semiestruturadas individuais, compostas por cinco tarefas. Quatro dessas tarefas envolviam equações abertas e uma focava na exploração da generalização da interpretação do sinal de igualdade. A análise foi realizada por meio de análise de conteúdo dos trabalhos escritos e orais dos alunos. **Resultados:** Os resultados mostram que os alunos interpretaram o sinal de igualdade com base em três significados: operador, equivalência numérica operacional e equivalência numérica relacional. **Conclusões:** Conclui-se que, à medida que os alunos progridem nas tarefas e recebem orientação do pesquisador, desenvolvem uma interpretação relacional do sinal de igual, apoiada pela estratégia de compensação. Isso tem implicações diretas para o ensino do pensamento algébrico inicial e para a formação de professores.

Palavras-chave: Álgebra escolar; Ensino fundamental; Pensamento algébrico; Pensamento relacional.

INTRODUCTION

In recent years, the teaching and learning of school algebra has attracted growing interest within the scientific community, especially at the earliest educational levels (Castro & Molina, 2007; Brizuela, 2023). This interest arises, to a large extent, from the difficulties and high failure rates that secondary education students experience when studying algebraic content for the first time. Various investigations attribute these difficulties to a curriculum disconnect between arithmetic and algebra, as well as to teaching approaches focused on the mechanical manipulation of algebraic symbolism, which favour rote learning and a limited understanding of mathematical structures (Castro & Molina, 2007; Parodi & Lezama, 2017; Schliemann et al., 2011; Molina, 2009).

In response to this scenario, approaches have been proposed to introduce algebraic thinking at an early age and promote a progressive understanding of school algebra. In this context, the *early algebra* curriculum proposal suggests the integration of modes of algebraic thought from the first educational levels, giving an algebraic character to the teaching of arithmetic and establishing conceptual bases that facilitate the transition towards more abstract contents (Kaput, 2000; Molina, 2009; Escobar &Tirado, 2021; Pincheira & Alsina, 2021). This proposal does not imply advancing content typical of secondary education, but rather promoting skills focused on identifying and generalising mathematical structures and relationships present in arithmetic tasks (Blanton & Kaput, 2005; Escobar & Tirado, 2021).

From this perspective, relational thinking emerges as a key approach to developing early algebraic thinking by focusing on the study of expression structures and the properties of operations, prompting students to intentionally establish relationships between expressions (Molina, 2006; 2011; Fernández & Ivars, 2016). Among his main contributions are strengthening arithmetic and algebraic skills, reducing mechanical operations, and constructing a conceptual basis for the formal learning of algebra (Carpenter et al., 2003; Molina, 2006; Escobar & Tirado, 2021). These benefits have motivated its incorporation into various curricula. In the Chilean curriculum, for example, primary education students are encouraged to explore relationships between numbers and structures using the equal sign to represent situations of balance (MINEDUC, 2012; 2018). Similarly, while the Spanish curriculum emphasises the understanding of equivalence in expressions with unknown quantities (Ministerio de Educación y Formación Profesional, 2019), the American curriculum proposes that students understand the meaning of the equal sign and evaluate the veracity of simple equations from the third year of primary education (Common Core State Standards for Mathematics, 2010).

Despite these advances, the research shows that the understanding of the equal sign continues to be approached predominantly from an operational approach, interpreted as a signal to execute calculations rather than as an equivalence relationship between expressions (Parodi & Lezama, 2017; Kusuma et al., 2018; Napaphun, 2012; Wardat et al., 2021). Consequently, questions remain regarding how students at the earliest educational levels construct meaning of the equal sign in the context of the development of early algebraic thinking. In this scenario, the following research question was raised: How do fourth-year primary school students interpret the equal sign when solving open-ended equality tasks? How do the researcher's interventions favour interpreting the equal sign as relational equivalence among these students?

Based on these questions, the objective is to analyse how fourth-grade primary education students interpret the equal sign in tasks with open equals.

CONCEPTUAL FRAMEWORK

Algebraic thinking

Algebraic thinking is defined as the development of fundamental reasoning that may require the use of symbolic algebra (letters) as a tool; however, it does not depend solely on it. Likewise, it provides essential cognitive support for introducing students to formal algebra instruction, as it enables them to analyse relationships between quantities, attend to structure, study change, generalise, solve problems, model, justify, test, and predict (Kieran, 1996; Kieran, 2004). On the other hand, algebraic thinking can be defined as a type of mathematical thinking referring to indeterminate quantities (unknowns, variables, parameters or generalised numbers), which are treated analytically; that is, even if they are unknown, they are operated on as if they were known (Radford, 2018; Pinto et al., 2023). Early algebra aims to integrate algebraic thinking by promoting habits of thought and representation that enable the identification of the structural scope of mathematics (Blanton & Kaput, 2005; Escobar & Tirado, 2021). Among the main characteristics of algebraic thinking developed by children are the modification of mathematical expressions, the development of relational thinking and the construction of knowledge based on sets, their operations, and associated properties (Mejias & Alsina, 2020). The *early algebra* proposal argues that teachers at all levels should promote algebraic thinking, helping students attend to properties, relationships, and patterns present in various mathematical activities, even if they are not algebraic at first glance. The goal is to promote modes of algebraic thought rather than the mastery of procedures typical of algebra. From a practical perspective, the idea is to promote the observation of patterns, relationships, and mathematical properties in the classroom to cultivate habits of thought oriented toward the underlying structure of mathematics (Molina, 2009).

Relational thinking

Relational thinking can be understood as a type of algebraic thinking aimed at understanding the equal sign, reasoning with expressions and solving equations and inequalities (Pinto et al., 2023). It refers to attention to the structural numerical relationships present in mathematical expressions (Carpenter et al., 2005; Escobar & Tirado, 2021). Various authors (Molina & Castro, 2005; Castro & Molina, 2007; Molina, 2011) define it as an alternative cognitive activity to standard procedures, closely linked to algebraic work, especially the study and generalisation of patterns and relationships. This approach involves considering arithmetic expressions and equations as whole entities and applying numerical properties to relate or transform them (Carpenter et al., 2005; Fernández & Ivars, 2016). Likewise, it presupposes a relational vision of the equal sign, understood as a relationship of balance between the two sides of the equality (Fernández & Ivars, 2016).

Relational thinking promotes arithmetic and algebraic competencies, focusing not only on procedural mastery but also on understanding numerical properties and identifying relationships (Escobar & Tirado, 2021). Its focus is on the student's establishment of relationships between relevant data and prior knowledge, including those not immediately observable (Hernández, 2017). Among its main advantages is the reduction of mechanical calculation, which favours the recognition of properties, the manipulation of expressions, and the development of meaningful arithmetic, thereby providing a solid basis for the formal learning of algebra (Carpenter et al., 2003; Molina, 2006; Castro & Molina, 2007).

A person demonstrates relational thinking when they can examine mathematical ideas, identify relationships among them, and use these relationships to solve problems or make mathematical decisions (Molina et al., 2006). An example of this is the resolution of equations based on the relationships between expressions on both sides of the equal sign (Castro & Molina, 2007). In this sense, it is important to offer opportunities that foster their development, supported by specific resources such as Cuisenaire strips or numerical scales, which facilitate understanding the equal sign as equivalence (Fernández & Ivars, 2016). Since understanding the equal sign is a central axis of relational thinking, its implications in this approach are analysed below.

Equal sign

The equal sign (=) represents a mathematical concept that indicates an equality relationship between two expressions located on both sides of the symbol (Castro & Molina, 2007). In this sense, Carpenter et al. (2003) point out that it should be understood as a balance between quantities. For example, in the expression 12 + 10 = 11 + 11, a numerical equality is established, since, although the expressions are different, the sums of both sides are equivalent.

The equal sign supports various meanings. Parodi and Lezama (2017), building on Molina (2006) and Molina et al. (2009), describe both those recognised by the mathematical community and those used by students and textbooks. Among them, it can be interpreted as an activity proposal, a one-way operator, an action expression, a procedure separator, a conditional equivalence, a mathematical equivalence (numerical, symbolic, identity or by definition), the definition of an object, a functional relationship, a correspondence, an approximation or a value assignment.

From a didactic perspective, the equal sign can be approached from two perspectives: operational and relational (Molina, 2009). The first conceives it as a signal to perform a calculation and record a result, while the second understands it as an indicator of equivalence between expressions, which implies analysing the relationships and numerical properties involved (Molina, 2009; Burgell & Ochoviet, 2015; Parodi & Ochoviet, 2025).

Relational thinking is developed by working with equations and numerical sentences. Equalities correspond to true expressions that contain the equal sign and can be open or closed, as well as actions or non-actions. Sentences, on the other hand, are propositions that can be true or false (Molina, 2006; Castro & Molina, 2007).

Since the different meanings of the equal sign are not acquired intuitively, working with arithmetic expressions from a relational approach favours the discovery of patterns, the establishment of relationships, and the understanding of expressions as totalities (or whole entities), moving away from a merely operational interpretation (Molina & Castro, 2005; Castro & Molina, 2007).

In this research, the equal sign is approached as relational numerical equivalence, a meaning that satisfies reflective, symmetric, and transitive properties (Molina et al., 2006; Parodi & Lezama, 2017). This approach allows us to work on various relationships of the additive structure, such as compensation, understood as the need to balance the quantities on both sides of the equality when adding or subtracting the same quantity, thus promoting a relational interpretation of the equal sign in primary education students (Molina & Castro, 2005; Molina, 2009).

Generalisation

Generalisation is one of the fundamental practices for developing algebraic thinking. Mason (1999) states that when students enter primary education, they already possess innate abilities to perform and express generalisation processes.

Generalisation is defined as the action in which we note that some attributes of the mathematical task can vary or remain unchanged (Mason, 2017; Pinto et al.). 2023). Zapatera (2022) explains that generalising involves students exploring situations in which they identify certain behaviours, rules, and laws that can be expressed verbally or symbolically and applied in any situation. Enhancing these abilities from the earliest educational levels allows students to move away from arithmetic calculations and focus on the underlying structure and mathematical relationships (Blanton et al., 2011; Pinto et al., 2023). In the context of relational thinking, generalisation is closely related to this approach, since it favours and facilitates the algebraic character of arithmetic, focusing on the analysis of the structures underlying different expressions and enabling the identification of arithmetic properties (Trujillo et al., 2009).

Background

Relational thinking among students at the earliest educational levels has been the subject of research for some years; however, the body of knowledge available remains in its incipient stage. Among the pioneering studies, those of Molina (2006), Molina and Castro (2005), and Molina et al. (2006) stand out. These authors demonstrated the potential of third-grade students in primary education (8–9 years) to solve tasks involving relational thinking and generalise arithmetic properties, even though these are not explicitly promoted in formal teaching. These authors observed that students initially interpreted the equal sign operationally, but as the intervention sessions progressed, they progressed toward relational interpretation. They also attributed the difficulties in this understanding to teaching practices that focus on presenting the operations on the left of the equal sign and the result on the right.

Subsequently, Molina et al. (2009) delved into the difficulties of students at this educational level, showing that some ignored one of the terms of equality, as when responding 17 to the expression 14 + _ = 13 + 4. Along these lines, Knuth et al. (2011) argue that working systematically on the “equal sign” as equivalence since primary education favours the resolution of equations and the recognition of equivalences, even without resorting to explicit calculation. Napaphun (2012) found that students aged 10 to 12 understood the equal sign relationally, although they relied on calculating both sides of the equation to verify equivalence, highlighting the importance of promoting strategies such as compensation and the principle of equivalence.

Similarly, Kiziltoprak and Yavuzsoy (2017) showed that fifth-graders in primary education moved from a result-focused understanding to a relational interpretation of the equal sign, analysing numerical relationships without performing direct calculations. These authors conclude that relational thinking can develop early when teachers promote the exploration and discussion of relationships between numbers. In agreement, Matthews et al. (2012) found that students who conceive the equal sign as equivalence perform better on tasks involving equations than those who interpret it as an outcome indicator.

In contrast, research in secondary education shows that significant difficulties persist in interpreting the equal sign. Kusuma et al. (2018) found that although students recognise the equal sign as operational equivalence, they tend to apply mechanical procedures when solving equations, which indicates a superficial, calculus-focused understanding.

METHODOLOGY

Type of research

This research was qualitative because it sought to examine the meanings and definitions people attribute to specific situations. Moreover, it focused on how fourth-grade primary education students responded to mathematical tasks involving the interpretation of the equal sign. It was characterised by its subjectivity and flexibility. Besides, data collection is carried out using methods that are neither standardised nor purely predetermined, thereby granting interpretative richness (Hernández et al., 2014). In addition, this research was framed as a case study, as we sought to investigate in depth the work of students at the educational level indicated above (Stake, 1999).

Research subjects

The research subjects were four primary school students (aged 9–10 years) from a private subsidised establishment in the city of Talca, Chile. They were selected intentionally. We considered students with average academic performance in the course to avoid bias in their responses. At the time of the research, these students had no previous experience in tasks involving the interpretation of the equal sign. However, they had experience with addition and subtraction calculations involving numbers up to four digits, both written and mental, and with one-step equations and inequalities, and they demonstrated the ability to verbalise answers. This knowledge was key because it was necessary for them to tackle the proposed tasks.

Semi-structured interview and research tasks

The collection of information was conducted through semi-structured interviews. These act as facilitators in the construction of people's knowledge, allowing the interviewer to describe the situation from their own perspective and in their own words. In addition, it is versatile and allows various instances of dialogue (Villarreal-Puga & Cid, 2022).

For this research, we developed four tasks involving finding the unknown given an equality relationship, following Carpenter et al. (2003) and Molina (2006). Table No. 1 shows each task along with its statement and characteristic.

Table 1.

Assignments proposed to students in the fourth grade of primary education

Task No.	Task statement	Task characteristic
Task 1	5 + 17 = ☆ + 20	It is an open equality of non-action1 where the star (unknown) is at the first value on the right side of the equal sign.

[1] An open equality means that the expression is incomplete, i.e., that it has an unknown value. On the other hand, no-action refers to the fact that the expression can have operations on both sides of the equal sign or on neither of them.

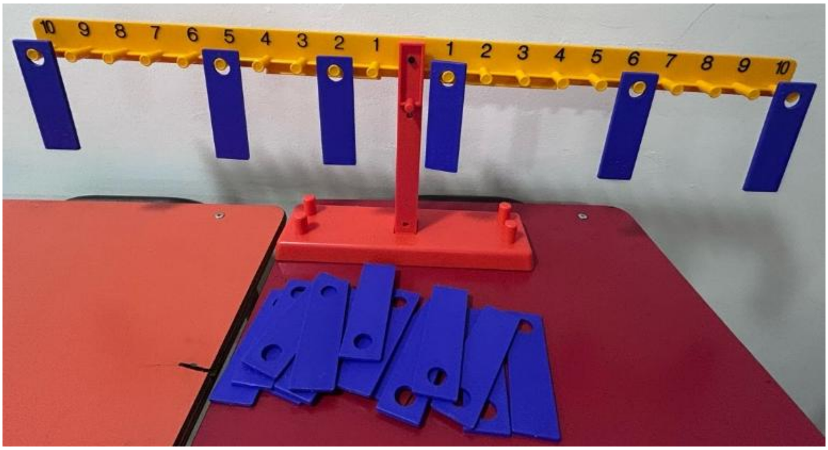
		Its structure attends to the open equality of the type a ± b = $\square$ ± d
Task 2	☆ + 25 = 10 + 30	It is an open equality of non-action where the star (unknown) is at the first value on the left side of the equal sign. Its structure attends to the open equality of the type $\square$ ± b = c ± d
Task 3	11 + ☆ = 10 + 12	It is an open equality of non-action where the star (unknown) is at the first value on the right side of the equal sign. Its structure attends to the open equality of the type a ± b = $\square$ ± d
Task 4	20 + 8 = 18 + ☆	It is an open equality of non-action where the star (unknown) is at the first value on the right side of the equal sign. Its structure attends to the open equality of the type a ± b = $\square$ ± d
Task 5	How do you always manage to find the unknown in each of the tasks? Can you explain it?	This question seeks to determine whether students can generalise the strategy worked on during the interviews.

Each task was presented individually to each of the four students during a semi-structured interview. Each interview was conducted in a classroom in their institution. The interview had no time limit, but lasted 20–30 minutes. During the interview, each student was presented with each of the four tasks and asked questions that addressed how to find the unknown associated with each task, justifying their approach to finding it. Once the equality questions were completed, they were asked a question that sought generalisation.

In addition, each student had instructions on a sheet of paper for performing the written calculations and a scale to corroborate them (see Figure 1).

Figure 1.

Image scale.



Analysis and categorisation technique

For the analysis of the information, we use the content analysis technique (Fernández, 2002). The transcript of each interview was prepared. Subsequently, each member of the research group analysed the transcripts and photographs of the students' productions to agree on the interpretation each student gave to the equal sign for each task. The categories were established based on: the conceptual framework of this research (Castro & Molina, 2007; Molina, 2009); the background that suggests investigating relational thinking (Parodi & Lezama, 2017; Kusuma et al., 2018; Napaphun, 2012; Wardat et al., 2021); and an a priori analysis of the students' responses, thus allowing to consolidate the categories of analysis on the interpretation of the equal sign. The consolidated analysis categories are shown in Table 2.

Table 2

Analysis categories for understanding the equal sign

Category	Subcategory	Description
Equal sign as an operator		Interpret the equal sign as an operator, that is, as a symbol in which on the left side there is a series of operations, and on the right side, their result.

		The student has a one-way view of the equal sign.
Equal sign as numerical equivalence	Operational	Interpret the equal sign as an equivalence between two representations of the same mathematical object, that is, that on both sides of the equal sign, the quantities add up to the same. Use arithmetic operations.
	Relational (compensation)	Interpret the equal sign as an equivalence between two representations of the same mathematical object, that is, that on both sides of the equal sign, the quantities add up to the same. Solve the task from a relational approach, establishing relationships between the values of both sides of the equality. The arithmetic compensation strategy is used.
Generalisation	It details	It answers a general question through a particular case about the interpretation of the equal sign.
	It generalises	It generalises the interpretation of the equal sign through indeterminate quantities (values).

RESULTS

The results of each of the four students' performances on each of the five tasks are shown below (Table 3). Each student was assigned a number to protect their anonymity and distinguish them from others (e.g., E1 denotes student 1 and so on).

Table 3

Students' responses to each research task.

Student	Task number				
	Task 1	Task 2	Task 3	Task 4	Task 5
E1	C2.1	C2.1/C2.2*	C2.1 / C2.2	C2.1/C2.2*	P
E2	C2.1/C2.2*	C2.1/C2.2*	C2.2	C2.2	G

E3	C2.1	C2.1/ C2.2*	C2.2*	C2.2	P
E4	C1/C2.1*	C2.1/ C2.2*	C2.2	C2.2*	P

Student responses to each research task. Note: C1: Equal Sign as Operator; C2: Equal Sign as Numerical Equivalence; C2.1: Operational; C2.2: Relational – Compensation. *Instruction; C3; (P) Details, (G) Generalises.

Task 1 results

Table 3 shows that, in Task 1, the four students predominantly interpreted the equal sign as operational numerical equivalence, using arithmetic operations to find the unknown. A representative example was E4's response. However, this student initially interpreted the equal sign as an operator (see Figure 2) and, with the researcher's guidance, later interpreted it as operational numerical equivalence, as shown below.

Figure 2

E4's response to Task 1

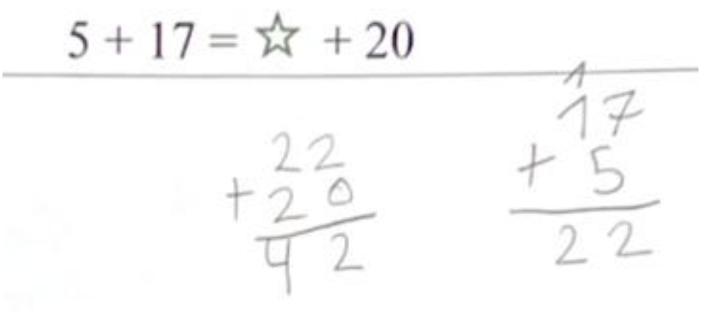
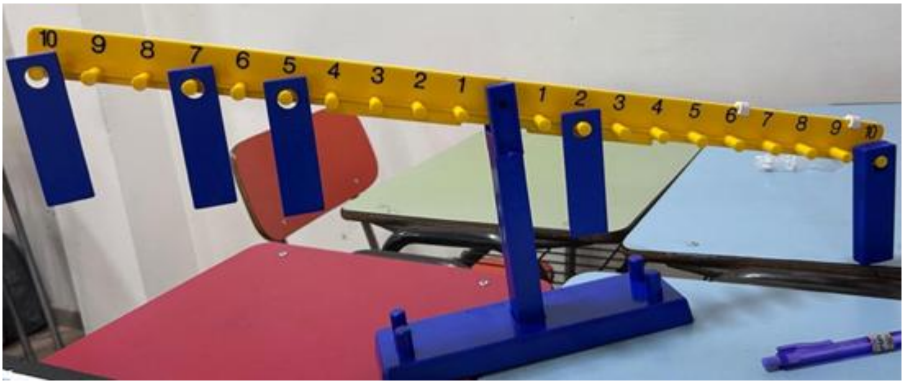


Figure 2 shows that E4 added the values of the first member (17 + 5) to find the unknown, yielding 22. Subsequently, said sum was added again with the known value of the second term on the left side of the equal sign (20), yielding 42 as a response to the task. On its side, E4 considered that the first of the two terms on the right of the equal sign corresponds to the sum of the terms that are on the left-hand side of said sign. In this way, E4 interpreted the equal sign as the result of an operation.

Next, and intending to guide E4 towards the understanding of his own answer and favour a correct interpretation of the equal sign, the interviewer (I) requested that, through a scale, he represent his procedure. The representation obtained by E4 is shown in Figure 3, which reports an inequality relationship.

Figure 3

E4's representation of the response in Task 1.



Once E4 noticed that the scale represented inequality rather than equality, the interviewer guided him with questions focused on the scale's balance and its link to the equal sign. This mediation was intended to facilitate E4's reinterpretation of the equal sign and the identification of its error, as evidenced by the following interview excerpt.

> I: What happened there on the scale?
> E4: It is not equal.
> I: What does what we have in the task mean? In other words, is 5 + 17 equal to?
> E4: 22
> I: The star plus 20? [...] Does it look unbalanced or balanced? Because it says that this (left side of the equal sign) must be equal to this (right side of the equal sign).
> E4: It must be in balance.
> I: How could we make it balanced? Removing 20 (from the right side of the equal sign)? (E4 previously, he had said "taking out the 20 from the right side of the equal sign)
> E4: No.

I: Why not? If, after removing 20, the scale is in balance, how much would the star have to be for it to yield the same result? If on this side (right), we already have 20.
E4: 2.
I: So, 5 plus 17 is equal to 2 plus 20? Is it in balance?
E4: Yes.
I: How would you do it then?
Next, E4 operates as shown in Figure 4.

Figure 4

E4's response in Task 1.

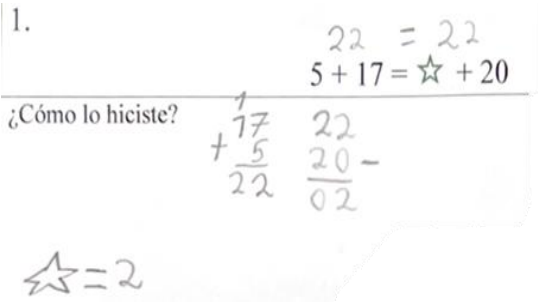
$$5+17=2+20$$
$$22=22$$

As seen in the interview fragment, E4 recognised that its initial answer was incorrect, since the value assigned to the unknown created an imbalance in the scale (see Figure 3). However, based on I's guiding questions, E4 understood that the sum of the unknown and 20 must equal the sum of 5 and 17, given the balance on the scale. This understanding is confirmed when E4 answers "2" as the value of the star to obtain 22 (see Figure 4). In this way, E4 interpreted the equal sign as an operational numerical equivalence.

On the other hand, in this same task, there was only one student (E2) who expressed an answer based on the interpretation of the equal sign as relational numerical equivalence with Is guidance, since his initial answer was based on the interpretation of the equal sign as operational numerical equivalence (see Figure 5).

Figure 5

E2's initial answer to Task 1.



As seen in Figure 5, E2 determined the first side of the equality by adding 5 and 17, obtaining 22. Subsequently, he subtracted from this result the known amount of the left side of the equality (20), which allowed him to obtain the value of the unknown (2).

Then I asked E2 some questions focused on transformations of quantities that maintain equality, such as: What happens to 17 to make it 20? What about the numbers? What should happen to the 5 so that there is a number that represents the star, and equality is maintained? In addition, he added pictorial representations (e.g., arrows; see the left image in Figure 6) so that E2 could visualise the necessary transformations that maintain equality and support an interpretation based on the compensation strategy. Faced with these instructions, E2 noted that 17 had to be joined by three to reach 20 and applied the same reasoning to 5, incorrectly obtaining the value 8 for the unknown (left image, Figure 6).

Figure 6

E2's answer to Task 1 and its verification.

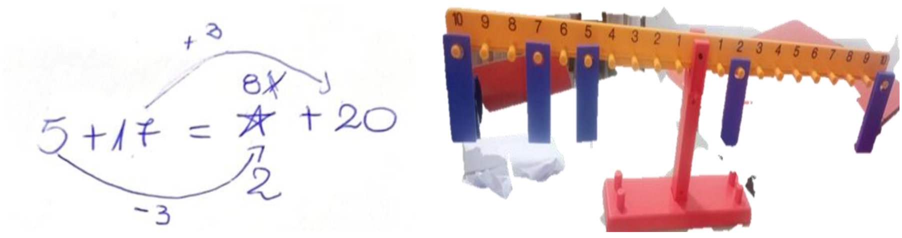
Given E2’s incorrect answer, the interviewer proposed corroborating it with the scale, which revealed an inequality (see the right image in Figure 6). The interview between E2 and I continued as shown below.

> I: Ok, on the second side, it gives you 20 plus 8; that is, we have to locate 28 on that side of the scale. On the other side, 5 and 17.
> I: What happened there? Is it the same?
> E2: It is not in balance.
> I: So, 5 plus 17 is the same as 8 plus 20?
> E2: No, it is not the same.
> I: [...] What could we do then so that the 5 and the star give us the balance? If we said that here (from 17 to 20), you add 3 [...]. Do you have to add 3, as you said?
> E2: No, I would subtract it. We subtract 3 from 5.
> I: And how much does it give?
> E2: It gives 2. (I was symbolically representing what E2 was mentioning, see left image, Figure 7)
> I: So, the 8 is no longer useful?
> E2: No.

At that time, E2 represented on the scale the numbers 10 + 7 + 5 on one side of the scale and on the other side 10 + 10 +2, to determine that 17 + 5 is equal to 20 + 2, see right image, Figure 7.

Figure 7

E2’s representation



Subsequently, to investigate whether E2 understood the compensation relationship between the quantities, I asked questions focused on the relationship between the terms of equality, specifically on how an increase in a term on one side of the equality corresponds to the term on the other side. E2’s answer is presented below.

> I: So, if you add 3 to one value of the sum, what would you have to do with the other value? Should you add 3 as well? Or subtract 3 from it?
> E2: Subtract 3.
> I: So that the balance is maintained, right?
> E2: Yes.
> I: And if you now subtract 5 from 17, what would you have to do with 5?
> E2: Seventeen minus 5?
> I: Yes. What do you do with that 5 to maintain equality? Do you add 5 or subtract 5?
> E2: We add it.

As seen in the previous fragment, with I's guidance, E2 understood that if there is an increase in a term on one side of the equality that corresponds to the term on the opposite side, the other term on that side of the equality must decrease to obtain the opposite term and thus maintain equivalence. Given this answer, E2 evidenced a compression of the equal sign as relational numerical equivalence.

Task 2 results

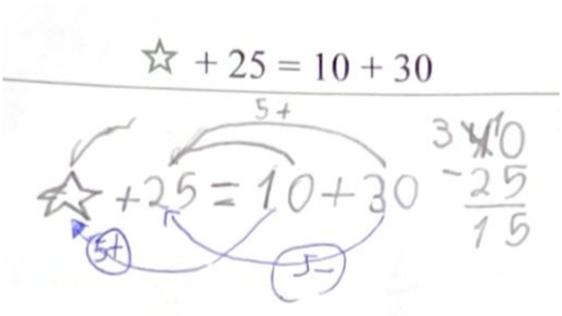
Table 3 shows that in Task 2, all four students transitioned from an interpretation of the equal sign as an operational equivalence toward a relational one. For example, initially, these students found the unknown by subtracting the sum of the two values on the right side of the sign from the known value on the left side of the sign ([30 + 10] - 25). Subsequently, with I's guidance, they solved the task using the compensation strategy. A representative example is E2's response, as shown in the following interview excerpt.

> I: […] Let's see, we better look at it backwards, what happens to the 30 to make it 25?
> E2: I'll subtract 5.
> I: Now, what happens to the 10? What would you have to do to get the star?
> E2: Add 5.
> I: And how much does the star give you there?
> E2: 15.
> I: Why do you get 15?
> E2: Because when I add 5 to 10, it gives me 15.
> I: The star, how much is it then?
> E2: 15.

As evidenced in the previous fragment, E2 solved the task by interpreting the equal sign as a relational numerical equivalence, relying on the compensation strategy. In this process, he related 25 and 30 by subtraction, subtracting 5 from 30 to obtain 25, and later added the same amount to 10 to determine the value of the unknown, which turned out to be 15. This procedure was corroborated by the interviewer's symbolic representation, which facilitated E2's understanding of the strategy (see Figure 8).

Figure 8

E2's response to Task 2



Another answer, based on the interpretation of the equal sign as relational equivalence, was provided by E1; instead of subtracting 5 from 30 to reach 25 (E2's answer), he did so by adding 5 to 25 to reach 30 (see Figure 9).

Figure 9

E2's response to Task 2.

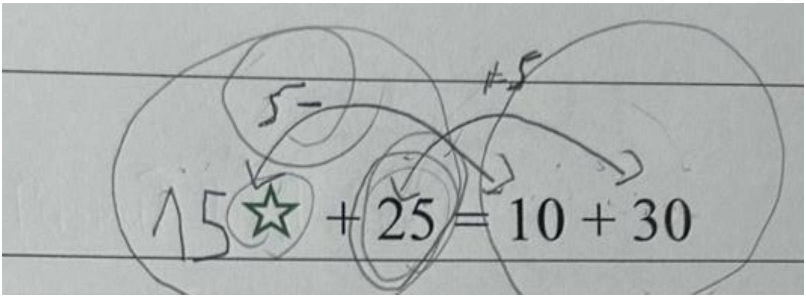


Figure 9 shows that E1 evidenced a relationship between the opposite terms of the equality. However, he did so with I's guidance, which used pictorial representations (curved lines) so that E1 could visualise the terms that were related. Below is the justification for E1's answer.

E1: The 25 was moved.
I: It was moved? To how much? What was moved? (The I joins 25 and 30 with a line).
E1: We add 5 so it equals 30.
I: And what would have to happen with the star? And on the east (left side), can we do the same, or not?
E1: No, because we did not know how many there are.
I: [...] Let's see, what happened from 15 to 10? Was there an addition or subtraction? (The I joins 15 and 10 with a line).
E1: 5 was subtracted.
I: So, from 15 to 10, you subtracted 5. And from 25 to 30, you added 5. Ok? And does it reach it or not?
E1: It would reach 5 because if it is subtracted, then it is added.

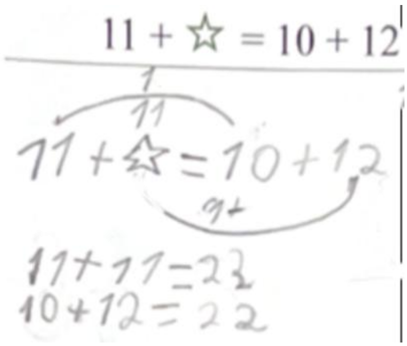
As seen in the previous fragment, E1 determined the value of the unknown by subtracting 5 from 15, obtaining 10, with I's guidance. He also corroborated his answer by recognising that, by relating a value on each side of the equality through addition, the two remaining values are linked through subtraction.

Task 3 results

Table 3 shows that all four students solved Task 3 by interpreting the equal sign as relational numerical equivalence. A representative example of this interpretation is E2, who solved task 3 using the compensation strategy, as shown in the symbolic representation in Figure 10.

Figure 10

E2's answer to Task 3.



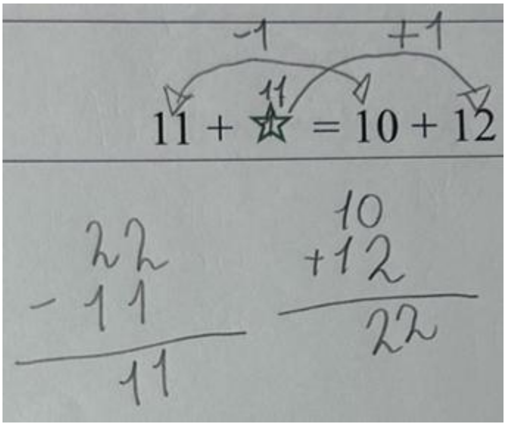
As seen in Figure 10, E2 showed that 11, corresponding to the first term of the left side of the equality, had to be subtracted by one unit to obtain the first term of the right side (10). He also recognised that the value of the unknown must be increased by 1 to reach 12, which enabled him to establish its value at 11. Subsequently, he verified his procedure by checking that the sums of both members of the equality matched. Below is an interview fragment illustrating E2's explanation during task resolution, carried out without the interviewer's intervention, thereby consolidating the compensation strategy for this task.

> I: […] Work with the terms (values) you know.
> E2: [...] I subtract 1 from 11 and add 1 to the star, and I get 12.
> I: So, how much would the star be?
> E2: 11. And 11 plus 11 equals 10 plus 12.
> I: Let's see, it corroborates. Add now.
> E2: 11 plus 11 equals 22, and 10 plus 12 equals 22.

On the other hand, an answer based on the interpretation of the equal sign as relational numerical equivalence, but with the orientation of I, was that of E1. For example, E1 determined that 10 is obtained from the subtraction of 11 minus 1 and that the value of the star must be added 1 to obtain 12, and thus maintain the balance. As seen in Figure 11. Note that I made pictorial representations (arrows) to guide the student on which terms to relate.

Figure 11

E1's answer to Task 3.



Below is an interview fragment showing the justification of E1's answer, who understood that the relationship between the first terms of both sides of the equality is established by a subtraction of one unit (11 - 1), while the second terms are linked through a sum of one unit (11 + 1). Given the above, E1 consolidated the compensation strategy.

> I: How did you do it?
> E1: [...] From 11 to 10, 1 is subtracted; and this, as it is also 11, is added one (to the unknown), so that there is 12.
> I: And do you think that's the same?
> E1: Yes.

Task 4 results

According to Table 3, all students found the value of the unknown by interpreting the equal sign as a relational numerical equivalence based on the compensation strategy. However, E1 initially interpreted the equal sign as an operational equivalence; with I's help, he solved the task by interpreting it as a relational equivalence.

Next, E2's answer is highlighted (see Figure 12), since, as in the previous task, it managed to maintain, without I's intervention, the interpretation of the equal sign as relational numerical equivalence and the consolidation of the compensation strategy.

Figure 12

E2's answer to Task 4.

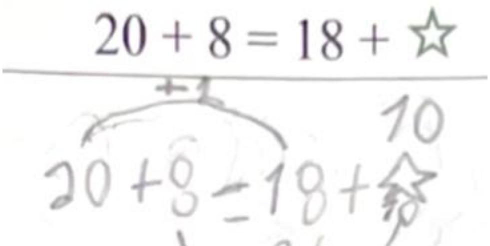


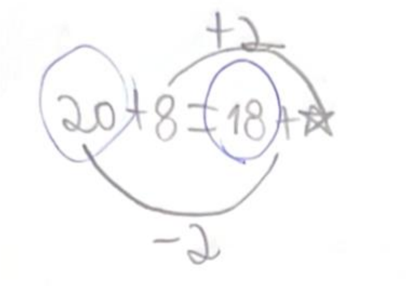
Figure 12 shows that E2 related 20 to 18 using a curved line, identifying a difference of 2. Subsequently, he linked the unknown to 8 and determined that adding 2 to 8 was necessary to find the unknown, yielding 10. Below is an excerpt from an interview in which E2 explains this procedure.

I: Explain how you did it.
E2: I would subtract 2 from 20, and add 2 to 8 to reach 10. Here, the star is worth 10 because when I added 2 to 8, I got 10.
I: And why did you add 2 to 8?
E2: Because 20 – 18 equals 2, and 2 plus 8 equals 10. Because if I added 20 plus 2, I wouldn't get 18; I'd get 22. And if I subtracted 2, I would get the same amount here (18). And 8 plus 2 equals 10. If 2 were subtracted from 8, the result would be 6.

This task also highlights E3's answer, since in the previous task, he had interpreted the equal sign as relational numerical equivalence with I guidance, but in this one, he did so autonomously, as shown in Figure 13.

Figure 13

E3's answer to Task 4.



For example, Figure 13 shows that E3 joined 20 and 18 with a curved line, and established that 2 (20 - 18) had to be subtracted. He then matched the unknown to 8 and determined that adding 2 (8+2) was necessary to find that unknown.

A fragment of the interview is shown below, in which the explanation of his answer is observed, and how he obtains the unknown (10) is shown. Then, it corroborates that the sum of the values on both sides of the equality is 28.

I: How did you do it?
E3: 20 minus 2 equals 18, and 8 plus 2 equals 10, and this is the unknown [...] gives 28 on both sides.

Task 5 results: Generalisation.

Table 3 shows that, in the general question, 3 students particularised (E1–E3–E4), while only one student generalised the interpretation of the equal sign as a relationship. Below, in the interview fragment, E3's answer is shown, exemplifying particularisation in response to the general question.

> I: Ok, and how do we always do it? How do we do it to find the unknown?
> E3: We had to find out the unknown. By adding and subtracting.
> I: What did you subtract and what did you add?
> E3: I added 17 plus 3, then subtracted 3 to find the unknown.
> I: What did you have to do that for? How did the strategy work?
> E3: First, we had to see 17 (the second value on the left side of the equality), and the second number (the second value on the right side of the equality) [...] we had 17, and how much was left to reach 20 was 3. Then, with the first numbers (the first value on both sides of the equality, 5, and the unknown, respectively), we subtracted 3 (from 5), getting 2.

As seen in the previous fragment, the student could answer the general question by considering the relationship between the first value on the left side and the first value on the right side. Analogously with the second values, both right and left of the equality. In the explanation of E3, it is observed that he required the use of particular (known) values to answer the general question.

Below, in the following interview fragment, we note that E2's response attends to the generalisation of the relationship between the quantities of equation.

> I: How do you always solve the task?
> E2: I made it so that with the first addition, I can add or subtract from the other first addition. For example, I subtracted 3 from 5 because when I add 3 to 17 (they are the second terms on each side of the equality of task 1) it equals 20, and when I subtract 5 minus 3 (second terms of the equality of task 1) it equals 2. And two is the value of the star. That's how you get it.
> I: And is that always so? For example, if I add to one amount and to the other, do I always have to subtract?
> E2: Yes.

I: So, I subtract and add the same number? Or were they going to be different?
E2: Equal.

As evidenced in the previous fragment, although E3 explained its reasoning from a particular case—by pointing out that "I added 3 to 17 and it gave me 20, and when I subtracted 5 minus 3, it is 2"—he could abstract a general rule. In particular, he explained that if a quantity is added to a term of the equality (for example, 17 + 3), the same quantity must be subtracted from the term that accompanies it. In this way, E3 understood that equality is maintained by the balance between the two sides of the equal sign. Consequently, although it required particularisation, he managed to extract a general rule from it to solve the task, which he could extrapolate to other similar instances.

DISCUSSION AND CONCLUSIONS

This research allowed us to characterise the understanding of the meaning of the equal sign in fourth-graders of primary education (9–10 years old) when solving non-action open-ended equality tasks. The results show that, in the initial tasks (1 and 2), students tended to interpret the equal sign in terms of operational meaning or numerical equivalence, finding the unknowns through arithmetic procedures and avoiding establishing relationships between the quantities involved. These findings align with previous research indicating that, at these ages, the equal sign is usually conceived as the result of an operation rather than as an equivalence relationship (Burgell & Ochoviet, 2015; Kiziltoprak & Yavuzsoy, 2017; Molina & Castro, 2005).

Likewise, we noticed that students do not intuitively understand the equal sign as relational equivalence but require explicit guidance from the researcher or teacher. In particular, we identified that interventions based on questions, such as What happens to the first value on the right and the first value on the left of the equality (in an analogous way with the second values), linked to pictorial representations such as lines (curves) that united these values, compelled students to compare and relate quantities, fostering, therefore, the relational understanding of the equal sign. These results support Molina's (2009) findings, which indicate the need to involve students in learning experiences that facilitate the recognition of relationships and fundamental properties, promoting relational thinking as a basis for learning school algebra (Castro & Molina, 2007; Molina, 2009; Burgell & Ochoviet, 2015; Parodi & Lezama, 2017).

A relevant finding was the performance of one of the students (E2), who maintained a relational understanding of the equal sign across tasks and generalised that relationship through a verbal representation. This result shows that early students possess the capacity to establish and generalise relationships between quantities, which aligns with previous studies on the early development of mathematical generalisation (Morales et al., 2025).

Regarding students' understanding of the equal sign, they moved from an initial level focused on operational equivalence to more sophisticated levels (medium and advanced), though mostly with the researcher's support. This progressive advance reinforces the idea that developing a relational understanding of the equal sign requires systematic instruction that favours that understanding.

In terms of teaching, the results suggest that using open-ended equality tasks is an effective strategy for promoting relational thinking and laying solid foundations for approaching first-degree equations, provided the teacher guides them. Likewise, the use of resources such as the scale proved to be a significant support for students in identifying errors, validating their procedures, and getting the solution to the proposed task.

Finally, the findings of this study reinforce the need to have prepared teachers, able to explicitly address the different meanings of the equal sign, especially that of relational equivalence, as suggested by national and international curricula (Castro & Molina, 2007; Molina, 2009; Burgell & Ochoviet, 2015; Parodi & Lezama, 2017; Matthews et al., 2012). Although this work was carried out with a small sample, it opens up future lines of research aimed at studying these phenomena in classroom contexts, with tasks that involve contextualised problems and with broader samples, as well as deepening the role of teaching intervention in the construction of meanings of the equal sign.

AUTHORSHIP CONTRIBUTION STATEMENT

The authors, RM, ML, JPF, and RG, conceived the research presented. All researchers collected the data and actively participated in the development of the theory, methodology, organisation, data analysis, and discussion thereof.

DATA AVAILABILITY DECLARATION

The data supporting the results of this study will be made available by the corresponding author, RM, upon reasonable request.

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