ACTA SCIENTIAE
ISSN: 2178-7727
DOI: 10.64856/acta.scientiae.8530

Basic Notions and Beliefs about Teaching Addition

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ABSTRACT

Background: Teachers' beliefs are among the most studied areas within the affective domain of mathematics education, due to their influence on pedagogical decisions. Despite their relevance, these beliefs are seldom systematically in initial teacher education. **Objectives:** This study examines whether and how pre-service teachers' beliefs about teaching addition evolve throughout their initial training. **Design:** This study follows a hermeneutic-reflective paradigm and adopts a descriptive-comparative mixed-methods design to understanding changes in prospective teacher's beliefs. **Context and participants:** Conducted at a Chilean university, within a primary teacher education program, the study used a convenience sample comprising two cohorts: 48 first-semester student and 47 in their final semesters. **Data collection and analysis:** Data were collected using a researcher developed questionnaire, validated through expert review and a pilot with second-year students (Cronbach's alpha = 0.87). It included Likert-scale and open-ended items on beliefs and basic notions about teaching addition. Quantitative data were analyzed using skewness and kurtosis (SPSS 22) to compare response distributions, while qualitative data were examined through content analysis. **Results:** Quantitative findings show that participants from both cohorts agreed on beliefs about teaching addition, particularly those emphasizing meaningful learning, prior experiences, and real-life applications. Qualitative analysis revealed that first-semester students relied predominantly on basic notions (e.g. adding or joining), often with ambiguous representations and premature or incorrect use of symbols. **Conclusions:** Beliefs about teaching addition show limited change during initial teacher training; instead, their internal structure and articulation become more refined over time.

Keywords: pre-service teachers; teachers' beliefs; arithmetic; mathematics education.

CONCEITOS BÁSICOS E CRENÇAS SOBRE O ENSINO DA ADIÇÃO

RESUMO

Contexto: As crenças dos professores estão entre as áreas mais estudadas no âmbito do domínio afetivo do ensino da matemática, devido à sua influência nas decisões pedagógicas. Apesar de sua relevância, essas crenças raramente são abordadas

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de forma sistemática na formação inicial de professores. **Objetivos:** Este estudo examina como as crenças dos professores em formação sobre o ensino da adição evoluem ao longo de sua formação inicial. **Design**: Este estudo segue um paradigma hermenêutico-reflexivo e adota um desenho descritivo-comparativo de métodos mistos para compreender as mudanças nas crenças dos futuros professores. **Ambiente e participantes**: Realizado em uma universidade chilena, no âmbito de um programa de formação de professores do ensino fundamental, o estudo utilizou uma amostra de conveniência composta por duas coortes: 48 alunos do primeiro semestre e 47 nos semestres finais. **Coleta e análise de dados**: Os dados foram coletados por meio de um questionário desenvolvido pelos pesquisadores, validado por meio de revisão de especialistas e de um estudo piloto com alunos do segundo ano (Cronbach 0,87). Ele incluiu itens em escala de Likert e de resposta aberta sobre crenças e noções básicas sobre o ensino da adição. Os dados quantitativos foram analisados utilizando assimetria e curtose (SPSS 22) para comparar as distribuições das respostas, enquanto os dados qualitativos foram examinados por meio de análise de conteúdo. **Resultados**: Os resultados quantitativos mostram que os participantes concordaram quanto às crenças sobre o ensino da adição, particularmente aquelas que enfatizam a aprendizagem significativa, experiências anteriores e aplicações na vida real. A análise qualitativa revelou que os alunos do primeiro semestre se baseavam predominantemente em noções básicas (por exemplo, somar ou juntar), muitas vezes com representações ambíguas e uso prematuro ou incorreto de símbolos. **Conclusões**: As crenças sobre o ensino da adição apresentam mudanças limitadas durante a formação inicial de professores; em vez disso, sua estrutura interna e articulação tornam-se mais refinadas ao longo do tempo.

Palavras-chave: professores em formação; crenças dos professores; aritmética; educação matemática.

INTRODUCTION

Teachers' beliefs are one of the most studied areas within the affective domain of mathematics education (Yang et al., 2020; Hidalgo et al., 2015). Despite the importance of beliefs in professional practice, they are rarely included in initial teacher training (ITT) by teacher educators (Montecinos et al., 2015). According to Hidalgo et al. (2015), ITT identifies and consolidates various systems of mathematical beliefs; hence the importance of studying them and characterizing the structure of these beliefs at the beginning and end of ITT.

Beliefs refer to personal and non-transferable truths held by each individual and stem from experience or imagination (Ponte, 1994), which people have encountered in their own process of teaching and learning mathematics (Casis, 2018). Studies such as those conducted by Cárcamo and Castro (2015); Dávalos et al. (2018); and Saadati et al. (2019) demonstrate that beliefs influence what occurs in the classroom, affecting both teacher performance and decision-making regarding the teaching and learning process. In this regard, Felbrich et al. (2014) examined the epistemological beliefs of prospective elementary teachers from an international perspective, investigating whether these beliefs were influenced by cultural patterns specific to each country, and demonstrating that the beliefs of these prospective teachers can be explained by the cultural patterns of the country to which they belong.

Meanwhile et al (2020) examined the relationship between beliefs regarding self-confidence in teaching both mathematics and literacy among pre-service elementary school teachers at four universities in Turkey. This study showed that the more self-confidence an individual demonstrates in teaching a subject area, the greater their sense of enjoyment and interest in working and teaching in that area. In the specific case of this study, it was found that most pre-service teachers demonstrated greater self-confidence in mathematics and, consequently, greater interest and enjoyment when preparing their lessons. The exploratory study by García-Moya et al. (2020) on mathematics and its teaching and learning among pre-service teachers concludes that beliefs differ between the constructivist approach to learning and the more traditional approach to teaching.

In the Chilean context, the study by Chandía et al. (2021) on the school-based beliefs held by pre-service teachers in FID for elementary education regarding how mathematics is taught and learned reveals that these beliefs can be modified through teaching experience. In this context and within these particularities, it becomes necessary to specify the content to be taught—believed to have a specific way of being taught—and to more precisely define the structure of the belief, indicating whether there is a theoretical construct that supports it or shares certain characteristics with the belief. Although the term "conceptions" is related to belief, Ponte (1994) describes them in a logic that is more cognitive than affective. Thus, preconceived ideas about purely mathematical aspects are better characterized as "conceptions" rather than "beliefs."

Addition is one of the first operations addressed in the Chilean curriculum (MINEDUC, 2013) and in other parts of the world. Difficulties in arithmetic stem from a combination of an individual's cognitive efforts and the way it is taught in school (Dowker, 2005). Consequently, Basic Notions (BN) are involved in the construction of knowledge associated with arithmetic (Vom Hofe & Reyes Santander, 2021). The literature on BNs indicates that they are necessary for the understanding, construction, and application of mathematical concepts. BNs provide mental images - they are mental representations – that we used throughout life to associate mathematics with reality. Thus, in the context of Initial Teacher Education (ITE) for elementary education regarding the trajectory of beliefs and knowledge associated with a course on the didactics of numbers and operations at a Chilean university, we pose the following research question: Do students' beliefs about the teaching of addition change during ITE?

To address the research question, this study aims to identify changes in teachers' beliefs regarding the teaching of addition among pre-service elementary school teachers. To achieve this, a questionnaire was developed to assess beliefs about the teaching of addition among students in their first and final years of training. The questions and, consequently, their analysis were based on beliefs and the NB; variations between first-year students and those in their final years of training were identified both quantitatively and qualitatively.

THEORETICAL BACKGROUND

To address the theoretical aspect of the study, we first present the affective dimension associated with belief and, second, the conceptual dimension of addition. In this section, we will focus on distinguishing between belief and conception.

Beliefs about the Teaching of Mathematics

According to Ernest (1989), beliefs that (i) conceive of the nature of mathematics are characterized by three types: the belief that mathematics is a continuous process of problem-solving, the Platonic view, and the instrumental view. Regarding beliefs that (ii) model the nature of mathematics teaching, this author identifies five different types of teachers and their teaching models: the coach, who focuses on skill transmission; the technologist, who emphasizes instruction in skill management; the humanist, who focuses on applied problem-solving; the progressive, who promotes personal learning; and the critical teacher, who teaches through discussion, research, and questioning. Regarding the beliefs that (iii) shape the process of learning mathematics, there is the belief that learning is achieved through memorization, practice, problem-solving, with the aid of technology, and based on communication.

Casis et al. (2017, 2020) and Castro et al. (2020) indicate that studies associated with the sociocultural dimension of mathematics education have focused on aspects that go beyond cognition and that influence both the quality of mathematics learning and students' academic performance. Over time, these works have developed a field of study known as the Affective Domain of

Mathematics Education. Its pioneers, McLeod (1989), Hart (1989), and Mandler (1989), establish three fundamental descriptors for this domain: beliefs, attitudes, and emotions.

Specifically, beliefs are understood as the "credibility of a conceptualization" (Colby, 1973, p. 253); for Hart (1989), the credibility of a conceptualization relates to the "acceptance, rejection, or suspension of judgment regarding a set of concepts and how these concepts interrelate with one another" (p. 41). In this same vein, Gil (1999, p. 50) considers beliefs to be a "set of notions to which one attaches a firm conviction, regarding them as truths." For McLeod (1992), beliefs correspond to the subjective experiences and knowledge (images) of the student and the teacher. For Hidalgo et al. (2015), students in FID associate mathematics, on the one hand, with knowledge and learning strategies and, on the other hand, with affective components related to personal experiences with mathematics, which have a strong affective component—mostly negative—and where self-concept and enjoyment of mathematics play a specific role.

However, people outside the school setting also hold beliefs about mathematics, whether due to the presence of this subject in daily life or the way they learned it; first-year students would fall into this category. Regarding the nature of beliefs, they are considered hypothetical constructs (Casis, 2018), since they cannot be directly observed in the individual but are rather inferred from their behaviours. Some researchers refer to belief systems, indicating that a belief never stands independently of others. For Rokeach (1968), a belief system is "a psychologically organized, though not necessarily logical, form of each and every one of the countless personal beliefs about physical and social reality" (p. 2). One could say that a person's belief system is characterized by the way they believe rather than by what they believe. Thus, a situation could arise in which two students might hold the same beliefs but have different belief systems; consequently, they would approach the same mathematical task differently and would have different ways of teaching the same concept. Within this field of study, the concept of "conception" emerges, which is often used interchangeably with "belief." We adhere to the distinction made by Ponte (1994), who states that conceptions constitute "underlying schemes of conceptual organization, which are essentially cognitive in nature" (p. 199). The term "conception" is thus reserved to refer to the ideas that an individual holds regarding specific mathematical concepts.

Basic Notions of Mathematical Knowledge

Basic Notions (BN) are mental images and form part of didactic content knowledge; they can be considered an alternative that unifies beliefs and mathematical knowledge. According to Vom Hofe and Reyes Santander (2021), the BNs of mathematical concepts began with Pestalozzi around 1800 and continued to be established in mathematics education, gaining prominence due to their association with mathematical modelling. Vom Hofe (1995) indicates that BNs (1) stem from the subject's experience—just as beliefs do—and allow for the construction and interpretation of mathematical concepts, (2) generate an appropriate mental representation for that concept, which aligns with mathematical knowledge, and (3) have the capacity to be applied to reality, recognizing structure in factual connections or through modelling.

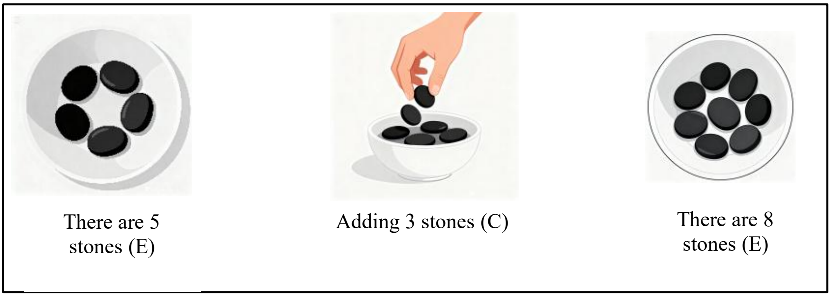
Since NB are actionable, they possess both static and dynamic characteristics; the focus is on the actual actions children can take to generate appropriate mental representations of the mathematical concept and its relationship to modelling (vom Hofe and Reyes Santander, 2021). NBs differ from what was proposed by Nesher et al. (1982) and Vergnaud (1982) in that they are not an arithmetic classification; rather, NBs are interpretations that originate from actions and mentally represent mathematical concepts, objects, definitions, operations, and relationships, providing appropriate constructs for translating the real world into the mathematical world.

Including the notions of static and dynamic, the NBs of addition are: adding objects starting with an initial state (E), followed by a change (C), and ending in a final state (E); dynamic joining, which has two static states and one dynamic state; and mental joining, which translates to three static states; and the fourth NB of completing, with two initial static states and one change (Reyes-Santander & vom Hofe, 2018).

- Adding E-C-E, the action of adding stones to a container (Figure 1), has an initial state (E), followed by a change (C) consisting of the action of adding stones, and ending in a final state (E). The set of steps can be represented mathematically by the expression "there are five stones in the container, three are added, and thus there are eight stones in the container," which is expressed mathematically as $5 + 3 = 8$.

Figure 1

Addition means adding.

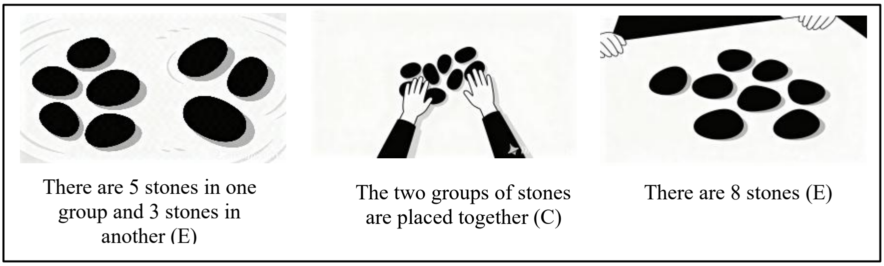


The actions performed by children that are associated with this NB are common in play situations; adding is a dynamic action and constitutes a sum. This action generates a mental representation that goes hand in hand with mathematics and allows children to model similar situations or transfer the same structure to other situations (vom Hofe and Reyes Santander, 2021).

- Combining E-E-C, the action of dynamically combining (Figure 2), involves having 5 stones in one group and 3 in another; the change occurs when these two groups are combined, resulting in the final state—a new set of 8 stones. The image is associated with the actions performed by the child and the mental mathematical representation of combining that is generated. The statement "there are 5 stones on one side and 3 stones on the other; if they are combined, there are 8 stones" is mathematically represented by the sum $5 + 3 = 8$.

Figure 2

Combining is addition.

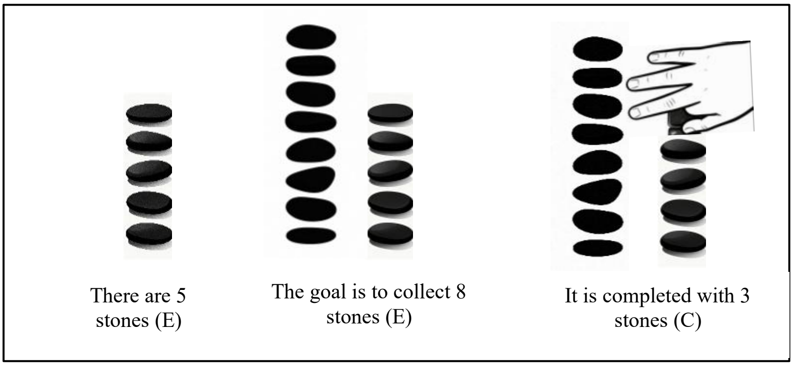


- Combining E-E-E, in the case of static combining, involves a mental union of two existing sets, and a new set is not concretely formed; it is not necessary to physically combine both groups to indicate the total number of objects.

- Completing E-E-C, or the reverse process required to provide a concrete solution to the question: How many squares are missing to make 8? (Figure 3). The strategy used in this case is the comparison of sets: the initial set of 5 squares and the final set of 8 squares. The state (E) is having the first 5 squares; the next state (E) is having 8 squares, and the change occurs by adding 3 squares (C). The answer given in this case is that 5 is completed with 3 to make 8, or that 3 squares are missing to make 8, and the addition associated with this action is 5 + 3 = 8.

Figure 3

Completing is addition.



The inverse operation that some students might perform to answer this question is subtraction: 8 - 5 = 3. Although the action related to addition is completing, the idea is “how much is missing,” where the student “wants to reach a certain quantity by completing,” and the student needs more objects.

NBs serve to establish guidelines for teacher training (Reyes Santander and Soto Andrade, 2023); they allow for the construction and explanation of mathematical knowledge based on students’ actions and generate mental representations that align with the mathematical concept. Other NBs that can be used in teacher training or for a lesson on addition are “advancing,” which relates to adding steps, and “joining,” which relates to putting things together. Vom Hofe and Reyes-Santander (2021) present the NBs in Spanish and as relevant content for teacher education.

METHODOLOGY

This study is grounded in the reflective hermeneutic research paradigm (Ríos Saavedra, 2005), which seeks, on the part of the researcher, to understand phenomena associated with the field of study. In our case, we investigate the NBs exhibited by students in the FID for primary education (EFIDP). To achieve our study objectives, we have opted for a mixed-methods approach.

The sample was selected on a convenience basis and consists of two cohorts—first semester and final-semester —of students enrolled in the Elementary Education program at a Chilean university.

To collect data, a self-designed questionnaire was used, which underwent expert validation and reliability analysis. Regarding validation, the instrument was first subjected to expert peer review by experienced academics; it was then subjected to a comprehension validation conducted by a group of second-year students from the same university, who provided feedback on:

- Technical quality (wording of the items)
- Representativeness (understanding of what the instrument aims to investigate)
- Structure (complexity of the instrument's structure)
- Time (time required to complete the questionnaire)

Regarding the reliability analysis, the results show that the questionnaire used has high consistency (Cronbach's alpha of 0.87). Regarding the structure of the instrument, it consists of two parts. First, it collects sociodemographic data on the participants (gender, age, type of school they graduated from, math-related courses they have already passed, and prior technical or university studies). Second, it explores the NB in three subsections:

(1) teaching by considering the subject's experiences to construct and give meaning to the mathematical concept.
(2) teaching by considering an appropriate mental representation for that concept.
(3) teaching by considering real-world applications, recognizing the corresponding structure in factual connections or through modelling.

Table 1 presents the dimensions of beliefs and NB, along with the questionnaire categories. In this mixed-methods study, the analysis of the Likert-scale questions is presented first, followed by the analysis of the open-ended questions. The study sample consisted of 48 first-year students and 47 students in their final years of the program. To present the results in greater depth and detail, only one of the two operations has been considered; although the study included questions on addition and subtraction in the closed-ended questions, the results shown here relate to addition in both the closed-ended and open-ended questions.

Table 1

Dimensions and Categories of the Questionnaire.

Dimension	Categories	Item	
(1) Meaning of the concepts of Addition and subtraction.	Beliefs about teaching addition.	A1. Theoretical mathematical foundations should be the most important. A2. I should work on the concept of "adding" before the + symbol A3. The child constructs knowledge for subtraction. A4. Children construct knowledge for multiplication.	Likert scale
		Describe the lesson you would teach to begin teaching addition	Open-ended
	Beliefs about teaching subtraction.	A5. This must have a meaning associated with the children's reality. A6. I should work on the concept of "taking away" before the + symbol	
	Beliefs about teaching addition and subtraction.	A7. The learning situation should emphasize mathematics. A8. The learning situation should focus on problems appropriate for the children's age. A9. My students' prior experiences would help me create appropriate situations.	Likert scale
(2) Conceptual representations.	Beliefs about teaching addition and subtraction.	B1. I should generate various mental images in children for these concepts. B2. I should pay attention to the learning of mathematical symbols. B3. Images or drawings associated with reality should be used. B4. One should work with the number line.	Likert scale
		Give an example of a teaching activity for addition.	Open-ended
		I think addition is…	Open

Dimension	Categories	Item	
(3) Modeling ability	Beliefs about teaching addition.	C1. The application of addition should play a very important role. C2. Application should occur in children's real-life situations. C3. Application should be reflected in math exercises. C4. Application is covered in subsequent math courses.	Likert scale
		Give an example of a real-life situation in which addition would be involved.	Open-ended

The quantitative analysis was conducted using SPSS 22, and statistical measures of skewness and kurtosis are presented to compare the distribution relative to the mean of the closed-ended responses from first semester and final semester. The qualitative analysis was conducted through discourse analysis (content), with discourse defined as the students' open-ended responses. In this analysis, words and phrases corresponding to the NB presented in the theoretical framework were selected, which are presented in Table 2.

Table 2

Study categories.

NB Dimension	Guiding phrases for the analysis
(1) Beliefs about the meaning of addition in teaching.	Addition is the act of adding. Addition is the act of putting together. Addition is the act of completing. Addition is a real action (with or without concrete materials). Addition is a mathematical operation. Addition is moving forward.
(2) Beliefs about representations in the teaching of addition.	Sums can be represented concretely (with or without the use of concrete materials). Sums can be represented pictorially. Sums can be represented symbolically. Sums can be represented on the number line.
(3) Beliefs about modeling in the teaching of addition.	Addition has mathematical applications. Addition has real-world applications. Addition has applications to children's situations. Addition has applications in exercises.

NB Dimension	Guiding phrases for the analysis
	Addition has applications at the following grade levels.

RESULTS AND ANALISES

Quantitative results

The quantitative results for the first dimension (1): Meaning of the concepts of addition and subtraction, show that the 95 participants agree with most of the beliefs proposed in the questionnaire for this dimension, given that their responses are close to the value 5, strongly agree. Table 3 shows the mean value, its error, and the standard deviation. To better visualize those requiring a more in-depth analysis across cohorts, we have highlighted those where the mean reflects a discrepancy and which have a higher standard deviation.

Table 3 shows a difference in the options chosen by students for “strongly agree” and “strongly disagree,” which were presented on the questionnaire’s Likert scale: statements A1. “Mathematical theory should be the most important,” A6. “I must work on the concept of ‘subtracting’ before the + symbol,” and A7. “The learning situation should emphasize mathematics.” These are the most interesting statements to present in the analysis for dimension (1) and to examine the differences between first semester and final semester, as they show the greatest variation.

Table 3

Mean, standard error (S-e), and standard deviation (S-d) by questionnaire items for dimension (1)

Categories	Item	Mean	S-e	S-d
Beliefs about teaching addition.	A1. Theoretical mathematical foundations should be the most important.	3.46	.084	0.82
	A2. I should work on the concept of “adding” before the + symbol	4.28	.088	0.86
	A3. The child builds knowledge for subtraction.	4.13	.089	0.86
	A4. The child builds knowledge for multiplication.	4.21	.107	1.04
Beliefs about teaching subtraction.	A5. This should have a meaning that relates to children’s reality.	4.78	.056	0.54

Categories	Item	Mean	S-e	S-d
Beliefs about teaching addition and subtraction.	A6. I must work on the concept of “subtracting” before the + symbol	2.98	.141	1.38
	A7. The learning situation should emphasize mathematics.	3.57	.088	0.86
	A8. The learning situation should focus on problems appropriate for the children’s age.	4.62	.069	0.67
	A9. My students’ prior experiences would help me create appropriate situations.	4.74	.082	0.80

Table 4 shows the skewness and kurtosis values for A1, A6, and A7. On the other hand, A1 and A6 are slightly skewed to the left, which would indicate that the group of all students would agree with the statements related to mathematical theory being the most important and that subtraction should be worked on before the addition symbol. In the case of A7, the group of all students is slightly skewed to the right, which means that all students would slightly disagree that the learning situation should emphasize mathematics. Regarding kurtosis, it is negative in all three cases, which suggests a relatively flat distribution. Since these data reflect the beliefs of the entire group, it is necessary to examine the cohorts separately and detail the structure of beliefs by distinguishing between first-year students and upper-level students.

Table 4

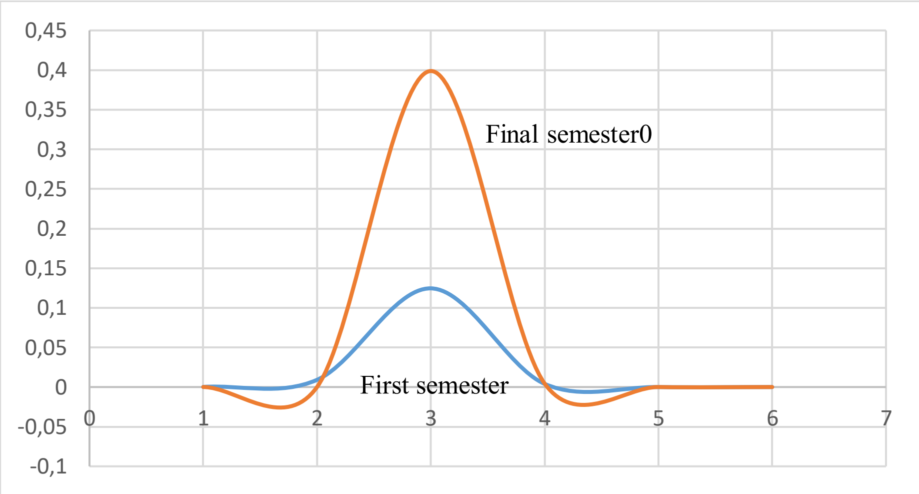
Descriptive statistics for skewness and kurtosis.

	Mean	Standard deviation	Variance	Skewness	Kurtosis
A1	3.46	0.82	0.68	-0.58	0.00
A6	2.98	1.38	1.89	-0.16	-0.80
A7	3.57	0.86	0.74	0.04	-0.63

In the case of responses indicating that a mathematical theoretical foundation should be the most important factor (A1), both first semester and final semester show slight agreement with this statement; that is, the belief in the need for a theoretical foundation when teaching addition persists throughout the FID. Figure 4 shows that upper-level students are more certain about being neutral—neither agreeing nor disagreeing—with this belief, whereas first-year students are distributed more evenly between agreeing and disagreeing.

Figure 4

Distribución de los datos del ítem A1 para primer año y cursos superiores.



Regarding responses to A6—whether the concept of "subtraction" should be taught before the + symbol—there is a difference in distribution between first-year and upper-level students (see Figure 5). This distribution highlights a difference between the beliefs of first semester students and those of final semester students. First semester students show a distribution where they both agree and disagree with teaching the concept of subtraction before the plus symbol, while final semester students are more in agreement with this belief. For upper-level students, there is greater consensus on the belief that the concept of subtraction should be taught before the plus symbol; this is not the case with the first-year distribution, as it behaves more like a belief—in fact, they do not believe that the concept of subtraction could be taught before the addition symbol.

Figure 5

Distribution of data for item A6 for first-year and upper-level students.

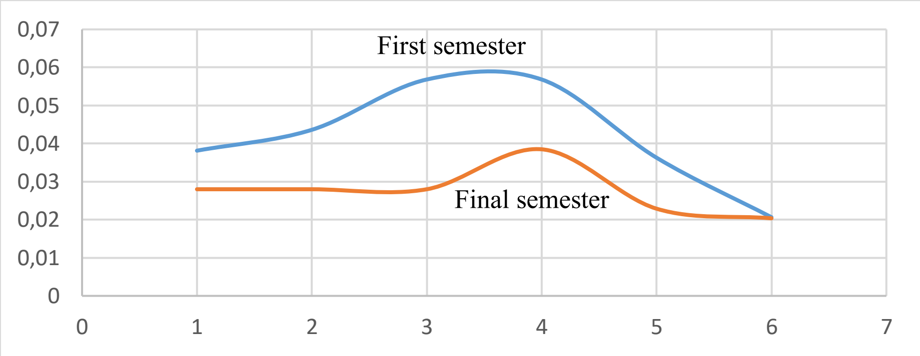


Figure 6 shows the distribution of responses to item A7, which reflect the belief that the learning situation should emphasize mathematics. For first semester students, this belief is closer to the average, while for final semester students, the distribution is flatter, and overall, they neither agree nor disagree with this belief.

Figure 6

Distribution of data for item A7 for first-year and upper-level students.

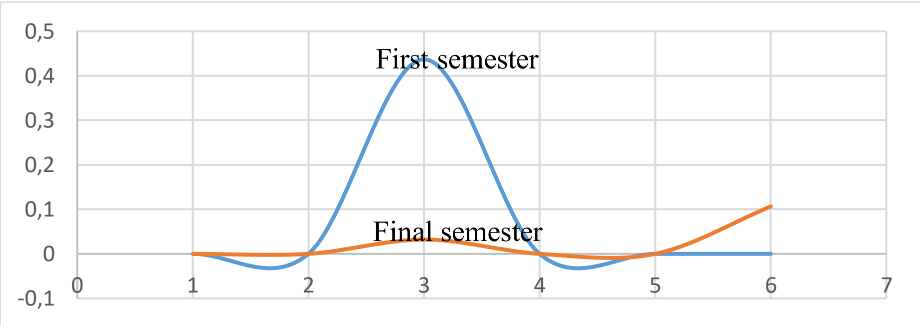


Table 5 shows the quantitative results of the mean, standard error, and standard deviation for dimension (2), concept representations. All students agree with these beliefs, as the mean for the entire group is very close to 5, which indicates complete agreement with the statements proposed in the item.

Table 5

Mean, standard error (S-e), and standard deviation (S-d) according to questionnaire items for dimension (2)

Categories	Item	Mean (max. 5)	S-e	S-d
Beliefs about teaching addition and subtraction.	B1. I should create various mental images for children regarding these concepts.	4.43	.82	0.82
	B2. I should pay attention to the learning of mathematical symbols.	4.01	.09	0.89
	B3. Images or drawings associated with reality should be used.	4.63	.08	0.73
	B4. You must work with the number line.	3.77	.11	1.07

Item B4 regarding the belief in using the number line for teaching addition has been highlighted in Table 5, as its mean indicates that not everyone agrees with this belief; its mean is 3.77, with a variance of 1.137, which suggests varied opinions. it has a negative skewness (-1.080) and a negative kurtosis (-0.587), which would indicate a curve that is slightly left-skewed and flat; that is, it rises slowly toward the mean and is not a normal curve.

Table 6 shows the quantitative results of the mean, standard error, and standard deviation for dimension (3), modeling ability. It can be seen that all students, both first-year and upper-level, agree with the beliefs that the application of addition should play a very important role (C1) and that the application should occur in children's real-life situations (C2), as in these cases the mean for the entire group of students is close to 5.

Table 6

Mean, standard error (S-e), and standard deviation (S-d) according to questionnaire items for dimension (3)

Categories	Item	Mean (max. 5)	S-e	S-d
Beliefs about	C1. The application of addition should play a very important role.	4.4	.09	0.89

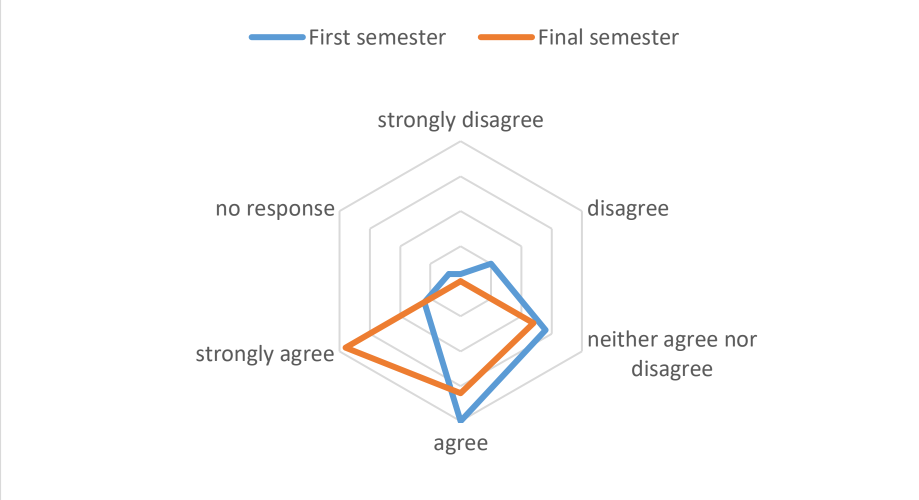
Categories	Item	Mean (max. 5)	S-e	S-d
teaching addition.	C2. Application should occur in children’s real-life situations.	4.6	.09	0.85
	C3. The application must be reflected in the math exercises.	3.69	.10	1.02
	C4. The application appears in the following math courses.	2.95	.13	1.27

Table 6 shows that students do not agree with the belief that the application appears in subsequent mathematics courses (C4) and that they differ in their belief regarding whether the application should be reflected in mathematics exercises (C3); therefore, an analysis of the normal curves is presented, differentiating between the two cohorts. This pattern is best illustrated by a pie chart showing the Likert scale options: strongly disagree, disagree, neither agree nor disagree, agree, strongly agree, and no response, as presented in Figure 8.

Figure 8 shows the distribution of responses to item C3, which reflect the belief that application should be reflected in math exercises. For first-year students, this belief is closer to the mean, meaning that students neither agree nor disagree with this belief; for students in higher grades, this belief has a flatter distribution, meaning that some hold this belief while others, to a lesser extent, do not. Note that there is a completely opposite situation between the cohorts, but with the same curves, for item A7 regarding the belief that the learning situation should emphasize mathematics. This aligns with the type of belief about mathematics and the difference in its application in learning situations and exercises.

Figure 8

Distribution of data for item C3 for first-year and upper-level courses.



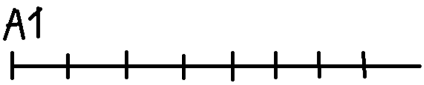
Qualitative Results

Regarding the results of the discourse analysis, it can be said that several beliefs related to pedagogical knowledge about teaching addition were identified. The following transcription of student responses, presented in Figure 9, illustrates the analysis process using an example of a response from an upper-level student describing their first lesson on teaching addition.

Figure 9

Example of analysis of student H3's results.

For the first lesson on addition, I would draw a number line [...] and place three different animals on it A1, A2 y A3.



Then the students would roll a die, and we would move the different animals forward.

Then we'd roll the die again, and I might say, "If we're at 3 and we rolled a 2, where on the line should we stop?" Then, with two dice, we'd add the numbers together.

In this analysis, the use of a teaching resource—such as the number line—is highlighted in bold; here, the NB of adding, represented by the action of moving forward, would be used. What is added to move forward are steps, beginning with several steps determined by rolling a die and continuing to move forward by adding more steps with a second roll. This student also adds a question that is not related to the sum symbol or a total, but rather to the action of moving forward and, therefore, to the characteristics of the NBs.

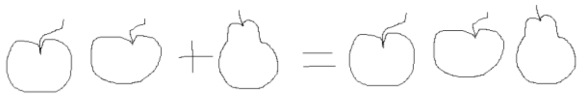
After analyzing each of the open-ended responses, it can be said that most of the FIRST SEMESTER and EFIDP2 students would use the NB of adding for the first type of addition. Very few, only 12 out of 95, would use the NB of dynamically combining; even fewer, 5 out of 95, would use the notion of statically combining; and only one would start with the NB of completing.

Figure 10 shows a response from a first-year student, in which one can observe the belief that numbers are represented by objects, without indicating that it is the quantity of objects that is being added. Furthermore, nothing is indicated about the counting required to represent quantities, and the + symbol is used in a confusing manner among the images of the fruits. Another confusing aspect of this student's belief is relating the addition to objects of different types without specifying the set on which the addition is being performed—the number of apples and the number of pears—resulting in a total number of fruits. In this case, it is assumed to be the set of fruits, but generally, when first teaching addition, quantities of objects of the same type should be added to facilitate understanding of addition and avoid confusion for the NB related to algebra.

Figure 10

Response from first-year student P35.

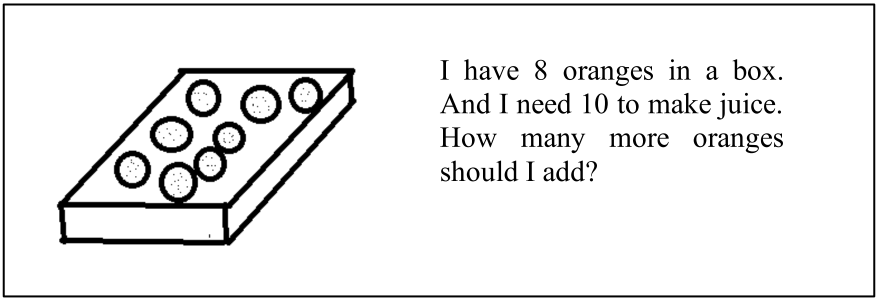
For the first class, I would represent numbers using objects—for example, apples and pears. Then, I would place two apples and one pear on the table and ask them, "How many fruits are there in total?"



Regarding the conceptual framework used, the student believes that addition is the act of bringing together objects independent of the set being worked on, although this action does not correspond to what is depicted, since bringing objects together requires the conceptual frameworks of quantity and counting—both of which are completely confused. This belief frequently emerged among first-year students and was represented in a similar way with objects and the incorrect use of the symbol in the figures (P2, P5, P15, P21, P22, among others). A mental representation different from that of bringing together is naively observed in other responses from FIRST SEMESTER, in which addition is perceived as adding; Figure 11 shows an example of a teaching activity for addition, in which a problem related to the mental representation of completing is presented.

Figure 11

Response from first-year student P6.



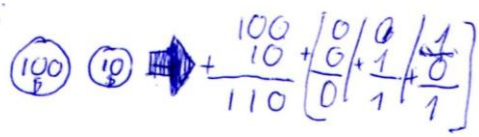
The concept that first-year students aim to develop in learning addition is how much is required to complete a given quantity. Here, "adding" is more about "completing"—how many oranges are needed to make 10. The reverse process would be to subtract 8 from 10 to determine the number of oranges required. This type of reasoning was rarely observed among both first-year students and those in higher grades, due to the difficulty inherent in such problems.

The responses developed by the EFIDP9 students are more complex and varied in terms of the proposed activity, the type of material to be used, the steps required, and the instructions to be given. Figure 12 shows the response of an upper-grade student that includes material, counting, grouping to characterize the first and second addends, and variations to introduce the addition algorithm.

Figure 12

Response from upper-grade student H23.

> *Para la enseñanza de la adición haría que los niños tengan bolitas y monedas y haría que hagan grupos de bolitas como ellos quieran y que cuenten cuantas bolitas tienen por grupo y que luego cuenten cuantas tienen en total, así explicándoles que acaban de sumar y para hacerlos más complejo una vez que ya aprendieron lo básico les haría sumar dos monedas a su elección.*
>
> *Les explicaría el sistema de suma*



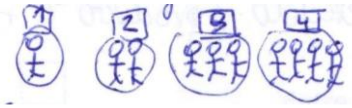
It can be inferred from the drawing that the student explains that the units are added first, followed by the tens and hundreds. The student works on counting and the concept of grouping, as by indicating that there are groups of marbles and then counting the total, they imply an action of grouping marbles to obtain the total. Thus, it can be said that this result shows that the student's initial teacher training has partially influenced their beliefs or that, in their training process, the children's needs regarding the generation of representations following the action have not yet been sufficiently reinforced, leading to an abrupt transition to working with coins and the addition algorithm.

The complexity of the activities presented by upper-grade students is also related to promoting enactive mathematics (Vom Hofe and Reyes Santander, 2021) in the sense that it includes group physical actions for children's learning. Figure 13 illustrates this emerging inclusion of enactive mathematics in the response of an upper-grade student, who proposes that quantities be organized into sets with a cardinality corresponding to the number of group members. This activity, which includes supporting materials such as posters with numbers to form groups, demonstrates an understanding of the connection between grouping and addition; that is, initial teacher training has influenced this student's beliefs in the expected manner.

Figure 13

Response from senior-year student H3.

Para la enseñanza de la adición a los alumnos les entregaría un número de acuerdo con la cantidad de integrantes.



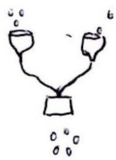
*Luego les pediría que se **juntaran**, sin desarmar los grupos, por ejemplo, de a 7 y veríamos con qué números de los grupos se formó el 7, por ejemplo, el grupo que tenía 4 y el grupo que tenía 3 alumnos. Y lograríamos llegar a la conclusión de que, si a 3 le **añadimos** 4, o viceversa, el resultado que se obtiene es 7. Podemos comprobarlo contando o en la cinta numerada.*

As in the previous case, the student uses language in a confusing way to describe the actions being performed, since they speak of bringing groups together and then of adding. The predominant action is that of bringing groups of people together and is related to the NB of dynamically bringing together. The action of adding people is generally related to people arriving at a place. Treating the NBs of adding and bringing together interchangeably could cause confusion among children, especially in the representations they form. Another concrete resource used by upper-grade students to respond to activities for the initial teaching of addition relates to the adding machine, in which objects are added and the first and second addends are represented concretely, as well as the total, which is obtained in a final container.

Figure 14

Response from upper-grade student H41.

*Para la enseñanza de la adición se invitaría a los niños a jugar a sumar elementos, mediante una especie de tobogán o cápsulas donde se **agregue** una cantidad a un lado y otra cantidad en la otra cápsula y luego se **junten**. Esta idea replica la forma de enseñar a sumar de la metodología Montessori.*



In this case (Figure 14), the upper-grade student uses the word “add” for addition and the word “put together” for the result. Thus, the NB of “add” is used solely for addition, distinguishing between the action and what is obtained as a result. This activity also showcases a novel concrete material that can be created by the students themselves; it highlights the structure of addition and the NB of "add" as both an action and the result of "putting together." Note that NBs are actions performed by children that generate mental representations for the symbol of addition.

CONCLUSIONS

The results obtained show that students' beliefs about the teaching of addition do not change substantially during the FID; what changes is the structure of the belief, which differs between first-year and final-year students. These results contribute to the concerns raised by Montecinos (2015) regarding the fact that studies on beliefs rarely focus on the FID, and they support the proposal by Casis (2018) that the decisions students make during the FID are influenced by their belief system and that this system is enriched during the FID.

The changes in beliefs regarding dimension (1) teaching of addition, detected in the quantitative analysis, are related to the importance given to theoretical grounding, to working on the number concepts of subtraction before the symbols of addition, and to focusing on mathematics in activities. For EFIDP9 students in upper-level courses, there is a tendency not to consider theoretical foundations important. According to Godino (2004), this is one of the idealistic beliefs with which mathematics teachers identify, in which the importance of theoretical learning in mathematics is emphasized and the acquisition and mastery of axioms and formulas for solving mathematical exercises are prioritized. In this sense, it can be said that the FID succeeded in changing the idealistic belief in mathematics from which FIRST SEMESTER students start, in which importance is given to the theoretical foundation of addition.

Regarding working on the NB of subtraction—such as subtracting before the addition symbol—upper-grade students adhere to this belief, whereas first-year students do not. For Piaget and Inhelder (1990), the reversibility of operations and actions is key to children's learning and the development of logical thinking. It is also considered within the national curriculum (Chilean Ministry of Education, MINEDUC, 2018) as a learning objective, in which students are required to demonstrate that addition and subtraction are inverse operations. From this, it can be inferred that students in higher grades conceive the NB of "adding" for addition and "subtracting" for subtraction as a development of the reversibility of operations and as a prerequisite for the introduction of symbols.

The results show that there are no changes in beliefs among the cohorts regarding dimension (2) of the representations of the concept of addition. Furthermore, there is a belief on which all students agree, concerning the use of the number line for teaching addition; here, it is also observed that students in higher grades are more likely to agree with this belief. National curriculum documents (MINEDUC, 2013) include a learning objective to develop modelling skills in third and fourth grade, which specifies the use of the number line to apply, select, and evaluate models involving the four operations and their placement on the number line and in the plane. Although the teaching of addition begins in first grade, it could be that the use of representations as a teaching aid in mathematics nationwide has influenced the development of this belief among both first-year and upper-grade students.

This study shows that there are some changes in beliefs regarding the modelling ability dimension, specifically related to the belief that application should be reflected in math exercises. First-year students neither agree nor disagree with this belief, indicating that they do not yet have a clear understanding of the application of addition or when it should be practiced. For students in higher grades, the distribution of this belief shows that, if they hold this belief—which indicates a change in the FID—students recognize that what they have learned about addition must be applied to real-world contexts, and this can be done through exercises. In this regard, the findings align with those obtained by Parra (2005), in which study participants unanimously agree on the need for mathematics with immediate application to operations. Thus, it can be said that upper-level students who hold this belief will have a stance similar to that of the actors, and that application is a goal shared by students in FID and by in-service teachers.

The qualitative results of this study show that first-year and upper-level students would use the "add" number sense for teaching addition, similar to what Vom Hofe and Reyes-Santander (2021) proposed regarding number senses and their teaching in the early years of schooling. Differences between the cohorts lie in the strategies and the type of material to be used; while first-year students present objects and representations—including making conceptual errors about addition and its representation—upper-level students indicate familiar or self-made materials, and their responses are more complex and varied. In this regard, it can be said that the work of the FID is crucial for the development of number sense and that it is necessary to address it from various angles to change the structure of belief.

We also observe that our results are consistent with the studies by Cárcamo and Castro (2015); Dávalos and Farfán (2018); and Saadati et al. (2019), regarding the fact that beliefs influence the decisions regarding the teaching and learning process made by students in elementary FID. In line with García-Moya et al. (2020), some of the beliefs expressed, particularly among first-year students, are related to the way they learned during their school years. The results align with those of Chandía et al. (2021), in that many of the beliefs held by students in their final years of training are the product of the educational process they have undergone for at least four years. In this regard, it is important to reevaluate the relevant beliefs expressed by students, particularly those in their final years, and contrast them with the sense of satisfaction they feel when preparing and conducting mathematics classes that incorporate number sense (Yilmaz and Turan, 2020).

The quantitative data show few differences between the beliefs of upper-level and first-year students. There are qualitative differences in the open-ended responses that include diagrams and explanations regarding the use of the material. The number concepts of "putting together" and "adding" can be observed in the open-ended responses of first-year students. Games play a primary role in first-year students' learning of addition, but there is no further description of the types of games.

It can be said that FID transforms beliefs but does not replace one set with another; rather, it deepens and broadens them, providing greater theoretical resources. In this regard, it is surprising to find upper-level students who consider the use of materials to be solely the teacher's responsibility and not that of the students themselves. This study highlights the need to include the NBs in teacher training programs for elementary education, specifically in lesson planning. As for how to conduct the first math class, most upper-level students prefer concrete actions, and very few of them would use a visual description. This is not the case for first-year students, who have different beliefs and vocabulary related to education.

AUTHORS' CONTRIBUTIONS STATEMENTS

P.R.S. conceived the study, secured funding, coordinated the project administration, conducted the investigation, provided study materials and resources, and supervised the research process. M.C.R. developed the methodology. P.R.S. and M.C.R. curated and validated the data, performed the formal analysis, designed the data presentation, and wrote the original draft. Both authors contributed equally to reviewing and editing the manuscript, participated actively in the discussion of results, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

Statement of Data Availability: Data supporting the results of this study will be made available by the corresponding author, [P. R. S], upon reasonable request.

Agencies. This research was carried out thanks to the UDLA-PIR Regular Project 202402 "Opportunities for the development of practical skills in initial teacher training", funded by the Universidad de Las Américas, Chile.

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