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Process of Semiocognitive Modeling in Mathematical Learning

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ABSTRACT

Background: Mathematical learning presents significant challenges, especially in the coordinated coordination of registers of semiotic representation and in the understanding of abstract concepts, with an often superficial apprehension of meanings and the prioritisation of algorithmic approaches to the detriment of conceptual understanding. **Objectives:** To present a structured and analytical synthesis of the conceptual essay on the "process of semiocognitive modeling". The objective of the original study is to propose a didactic construct focused on teacher planning that helps teachers overcome obstacles to textual and conceptual understanding as they solve mathematical problems and define concepts. **Methodology:** Qualitative, exploratory, and bibliographic research, structured in the form of an interpretative essay, focusing on the analysis and reflective reorganisation of existing knowledge about mathematical learning and teaching performance. **Data source:** Theoretical study based on specialised literature on Raymond Duval's theory of registers of semiotic representations and Guy Brousseau's didactic contract. **Data production and analysis**: The categories emphasised in the process of semiocognitive modeling emerged from the theoretical assumptions, and, from them, an area problem and a mathematical definition were analysed, with attention to the application of semiocognitive processes and the interrelationships among registers, cognitive strategies, and teacher mediation. **Results:** Semiocognitive modeling enables the coordinated articulation of different semiotic records, promoting conceptual understanding of mathematical definitions and problems and highlighting the importance of conversion, segmentation, and recontextualisation in effective learning. **Conclusions:** Learning mathematics depends on the teacher's planning, as they structure activities that mobilise semiotic registers, transforming definitions and problems into opportunities for the development of mathematical reasoning and the students' conceptual flexibility.

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Keywords: semiocognitive modeling; semiotic representation; mathematical understanding; mathematics teaching; didactic contract.

RESUMO

Contexto: A aprendizagem matemática apresenta desafios significativos, especialmente na coordenação coordenada de registros de representação semiótica e na compreensão de conceitos abstratos, sendo comum a apreensão superficial de significados e a priorização de abordagens algorítmicas em detrimento da compreensão conceitual. **Objetivos:** Apresentar uma síntese estruturada e analítica do ensaio conceitual acerca do "Processo de Modelização Semiocognitiva". O objetivo do estudo original é propor um constructo didático focado no planejamento docente que auxilie os professores a superarem os obstáculos de compreensão textual e conceitual na resolução de problemas e definições matemáticas. **Metodologia:** Pesquisa qualitativa, exploratória e bibliográfica, estruturada na forma de ensaio interpretativo, com foco na análise e reorganização reflexiva de conhecimentos existentes sobre aprendizagem matemática e atuação docente. **Fonte de dados:** Estudo teórico baseado em literatura especializada sobre a Teoria dos Registros de Representação Semiótica de Raymond Duval e o Contrato Didático de Guy Brousseau. **Produção e análise de dados**: As categorias enfatizadas no processo de modelização semicognitiva emergiram dos pressupostos teóricos e a partir delas foram analisados um problema de área e uma definição matemática, observando-se a aplicação dos processos semiocognitivos e a inter-relação entre registros, estratégias cognitivas e mediação docente. **Resultados:** A modelização semiocognitiva permite a articulação de forma coordenada de diferentes registros semióticos, promovendo a compreensão conceitual de definições e problemas matemáticos, evidenciando a importância da conversão, segmentação e recontextualização na aprendizagem efetiva. **Conclusões:** Aprender matemática depende do planejamento intencional do professor, ao estruturar atividades capazes de mobilizar os registros semióticos, transformando definições e problemas em oportunidades de desenvolvimento do raciocínio matemático, e da flexibilidade conceitual dos estudantes.

Palavras-chave: modelização semiocognitiva; representação semiótica; compreensão matemática; ensino de matemática; contrato didático.

INTRODUCTION

Mathematics learning is a complex field of study and is often marked by significant challenges for students at various levels of education. The difficulty in moving between different representations of the same concept, the superficial apprehension of abstract meanings, and the prevalence of algorithmic approaches at the expense of conceptual understanding are problems widely documented in the specialised literature (Hiebert & Wearne, 1992; Schoenfeld, 1992). In classrooms, mathematics teaching has historically prioritised the memorisation of formulas and the mechanical execution of procedures, downgrading the construction of meaning and the ability to understand to the background. However, mathematics, in its essence, transcends mere symbolic manipulation; it is fundamentally a problem-solving science, in which definitions and theorems are not isolated statements to be memorised but rather tools for structuring reasoning, argumentation, and understanding of real and abstract phenomena in the world (Tall & Vinner, 1981).

The persistence of this gap between the formalisation of mathematics and the students' ability to apply, interpret, and contextualise this knowledge in different situations is a central problem in the didactics of mathematics. Studies have indicated that, even with proficiency in a register of semiotic representation (e.g., algebraic), many students fail to convert or coordinate information with other registers (e.g., graphic or verbal), which prevents a full understanding of the underlying concept (Duval, 2006; Pasa et al., 2025a; Moretti et al., 2022). This inability or difficulty in mobilising different registers of semiotic representation is directly linked to limitations in understanding and solving mathematical problems.

In this context, the semiocognitive theory developed by Raymond Duval (1995, 2006) offers a robust theoretical framework and a powerful analytical lens for investigating the cognitive processes underlying mathematical understanding. Duval (2003) postulates that mathematical objects, being non-perceptible, are accessible only through their semiotic representations. Mathematical understanding is not, therefore, by the direct "perception" of these objects, but by the coordination and articulation of semiotic representations linked to such objects. He describes what he calls the "paradox of representation," in which the very need to operate with representations can paradoxically obscure access to the mathematical object if there is no flexibility and fluidity between them.

The semiocognitive theory emphasises three crucial types of cognitive activity related to registers of semiotic representation: *formation* concerns the constitution of the register from a set of representations endowed with identification criteria and own production rules, which must be explained to enable their understanding and sharing by a group of subjects; *treatment* refers to an internal operation to the same representation register, in which, according to specific rules, one representation is transformed into another within the register itself, such as simplifying an algebraic expression, and the *conversion* of a representation, which corresponds to transforming it into another representation of the same object, but in a different record, such as transiting between the algebraic expression and the corresponding graph. Conversion, in particular, is considered by Duval (1995) as the core of mathematical understanding, as it requires the recognition of the same underlying conceptual object in distinct semiotic forms. The ability to operate fluidly in these three activities is a crucial indicator of a deep conceptual understanding of a mathematical object.

When mobilised to understand definitions and solve problems, the semiocognitive perspective suggests that proficiency does not reside solely in memorising the formal statement or in algorithmic application. Rather, it lies in the ability to form representations appropriate to the definition or theorem, to treat them in such a way as to extract information or make inferences, and to convert them in a coordinated way to other records that can facilitate application to a problem or the construction of a proof. Thus, understanding mathematics requires not only the mobilisation of registers but also strategic segmentation processes, which consist of decomposing the problem into manageable parts and recontextualising the segmented parts and applying prior knowledge to new situations. Such processes, fundamental to the understanding of texts, are intrinsically mediated by semiotic representations.

Given the complexity of mathematical learning and the analytical potential of semiocognitive theory, this article proposes a process of *semiocognitive modeling* in which the teacher's role is central, especially in their planning activities. These activities involve the treatment and conversion of representation registers, as well as the strategic processes of segmentation and recontextualisation to contribute to the construction of a conceptual understanding in mathematics, in the field of definitions and theorems, as well as in problem-solving contexts. The specific objective is to present and analyse the interrelationship between these semiocognitive elements, investigating how they manifest in students' ability to transcend the formality of mathematical concepts and achieve a functional, applicable understanding.

In this discussion, we highlight Guy Brousseau, one of the pioneers of the didactics of mathematics, whose fundamental contributions established essential conditions for teaching and learning. Brousseau (1986, 1996) developed a theory to understand the interactions of the triad teacher, student, and knowing. According to Brousseau, the relationship between the teacher and the student in the educational context is considered a special relationship, whose primary objective is learning, always mediated by knowing. However, this interaction is subject to a series of rules and conventions that function as contract clauses. This ruleset that regulates the relationships teachers and students maintain with knowing is called the “didactic contract”.

The didactic contract, as proposed by Brousseau (1986), is an analytical tool for investigating the relationships among the teacher, the student, and knowing. According to Brousseau (1996, pp. 50-51), “the didactic contract is the rule of the game and the strategy of the didactic situation”; that is, the set of behaviours expected from each party (teacher and student) constitutes a system of rules that defines the dynamics of this interaction. These rules comprise explicit aspects (those formulated verbally in the classroom) and implicit aspects (constructed historically and interpreted in the classroom context). In each part, the teacher and the student assume mutual responsibilities, for which they will be, in some way, responsible for each other.

From this model, it is worth highlighting that, in the classroom, the responsibilities of each member of the triad (teacher, student, and knowing) are precisely defined through the interaction among these elements. In the process of *semiocognitive modeling*, the teacher is responsible for creating strategies that enable students to access school knowing, as well as managing their participation in the teaching and learning process. This involves proposing questions and activities aligned with students’ profiles, presenting significant information at appropriate times, and demonstrating pedagogical intentionality grounded in prior planning, which is the emphasis of the *semiocognitive modeling*. These activities align with the teacher’s role in the didactic contract. In turn, the student is responsible for following the teacher’s guidelines, participating in the proposed activities, and adapting to the communication format established for each task. In this process, knowing serves as a mediator between the teacher and the student.

Anchored in these ideas, this article is configured as an essay, based on Meneghetti (2011), characterised by its reflective and interpretative nature, whose essence lies in the reorganisation of existing knowledge.

> The essay is characterised by its reflective and interpretative nature, in contrast to the classificatory form of science. [...] The test is a means of analysing and reflecting on the object, regardless of its nature or characteristics. The essayistic form is the way new knowledge, even scientific or pre-scientific, is incubated. [...] The essay always speaks of something already formed or, at best, of something that has already been there; it belongs, therefore, to its essence that it does not highlight new things from a void, but limits itself to ordering, in a new way,

> things that have already been alive at some point. (Meneghetti, 2011, p. 322).

Therefore, this essay is the result of exploratory and bibliographic qualitative research and is structured so that, initially, the structuring aspects of the process of *semiocognitive modeling* are presented, and then, through examples, this modeling is applied to a contextualised problem and to a mathematical definition. We argue that the process of *semiocognitive modeling* does not focus on isolated "content", but on the subject's activity on representations. To argue in favour of this approach is to argue that learning mathematics is to learn to transform representations and to recognise the same object in different possibilities of semiotic representation.

SEMIOCOGNITIVE MODELING FOR UNDERSTANDING IN MATHEMATICS

The process of *semiocognitive modeling* proposes a possible path for understanding definitions and solving problems. Understanding a text and its organisation involves, as indicated by Duval (1995, 2004) and emphasised by Pasa et al. (2025a, 2025b), the interaction between the knowledge expressed in the text, called cognitive content, and the way this text is reworded and structured. From this perspective, the apprehension of a problem cannot be dissociated from either its editorial organisation or the reader's degree of familiarity with the mobilised cognitive content. Thus, the understanding of a text - a problem formulated in the mother tongue or a definition or a theorem - results from the interaction between text and reader, involving two equally relevant axes: the characteristics and functionalities of writing, called redactional variables, which include the degrees and modes of explicitation of the cognitive content, as well as the discrepancies between the redactional organisation and the discursive organisation typical of spontaneous orality; and the characteristics of the reader, covering their knowledge base related to the cognitive content of the text, the extent of their vocabulary and their syntactic decoding skills, among others.

It is important to note that the editorial organisation of a problem can be changed without altering its cognitive content. This restructuring may be necessary whenever the reader does not readily understand the text. However, the perception of sufficiency or insufficiency of the writing of the same text varies with the readers' knowledge base.

Thus, when working with problems and definitions in the mother tongue in a school context, the teacher must take as a reference the students' specific knowledge base and consider the operations described by Duval (1995, 2004), evidenced in Pasa et al. (2025b), in their textual understanding model: the segmentation of the text into units and the recontextualisation of these segmented units.

In addition, we highlight the importance of segmentation and recontextualisation, as explained below, in the process of *semiocognitive modeling* for every change of representation register necessary for understanding and solving problems.

The dynamics of segmentation

Segmentation consists of unfolding the problem into meaning-bearing units throughout the reading process. It should be noted that the identification of these units depends directly on the reader's focus of attention and can affect both an isolated word and a broader utterance. Therefore, there are no predefined units that can be considered basic or universally valid for any act of reading.

In Duval's semiocognitive theory (1995, 2004), the discrimination of these significant units is a necessary condition for intellectual learning, since it enables the reader to recognise and organise the relevant elements of the text. Segmentation refers to the ability to break down a complex situation or mathematical statement into smaller, more manageable parts. For Duval, this means guiding the student to identify the significant elements of a problem or a definition. In the context of basic education, segmentation is not only the passive reading of the utterance but also the activation of strategies to "slice" the information and build local meaning.

Duval (1995, 2004) describes three modalities of segmentation that can be mobilised by the reader during reading, with a view to distinguishing and structuring the units of meaning present in the problem: *cognitive*, which organises the text according to units that trigger specific mathematical knowledge of the student; *propositional*, which separates the utterance into logical propositions independent of its cognitive content; and *functional*, which divides the text according to the communicative role of each part (introduce, define, question, request an action, etc.). With the segmented text, understanding requires recontextualising it.

The dynamics of recontextualisation

Recontextualisation involves integrating the units obtained through segmentation and establishing relationships among them to form a coherent set of meanings. This operation organises the text into knowledge networks or structures specific to the wording of the problem, enabling the necessary inferences to overcome omissions and resolve ambiguities. Just as there are different types of segmentation, there are also different ways to recontextualise.

Cognitive recontextualisation is based on the reader's knowledge of the situations, objects or themes covered in the text. *Redactional recontextualisation*, on the other hand, relates the segmented units according to the utterance's discursive logic, allowing us to recognise that different linguistic forms can express the same cognitive content and to favour greater flexibility in reading.

Thus, recontextualisation is the process of transferring and adapting previously acquired knowledge and skills to a new situation or problem. It is the bridge between what students already know and what they need to learn or apply. For the teacher, facilitating recontextualisation means creating opportunities for students to recognise patterns and analogies, and to apply concepts in different scenarios, thereby reinforcing cognitive flexibility. Understanding a text, whether a problem or a mathematical definition, always depends on the interplay between segmentation and recontextualisation; their combination, in its different modalities, allows the classification of different models of textual understanding.

By teaching students to segment and recontextualise a text, the teacher enables them not only to solve specific problems but to develop adaptive mathematical reasoning, using definitions and theorems as guides for analysing new situations. This approach aligns with the view that conceptual understanding arises from the active, flexible manipulation of knowledge across different contexts.

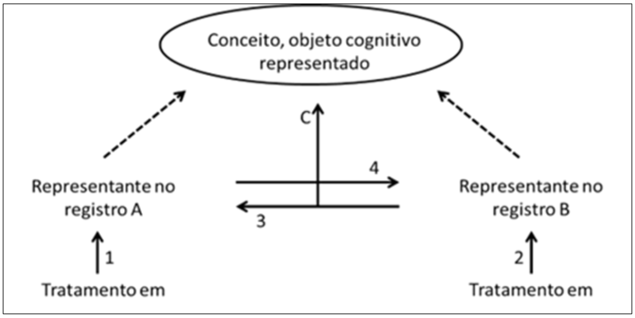
Fundamental hypothesis of learning and supporting representations

According to Duval (2012), mathematical learning occurs through the articulation between different registers of semiotic representation (RSR) of the mathematical object to be apprehended. As a precursor to the theory of registers of semiotic representations (TRSR), Duval (2003) highlights the relevance and diversity of RSR mobilised in mathematical activity, stating that the "originality of mathematical activity is in the simultaneous mobilisation of at least two registers of representation or in the possibility of constantly changing from one register to another" (Duval, 2003, p. 14).

From this perspective, an RSR can be understood as a system of semiotic representations that involves the three fundamental cognitive activities: education, treatment, and conversion, as explained above. According to Duval (2012), these three cognitive activities constitute the basis for constructing and understanding mathematical knowledge. Based on this understanding, it presents the fundamental learning hypothesis, which presupposes the coordination of at least two RSRs, as shown in Figure 1. Arrows 1 and 2 correspond to the internal transformations of a register, the treatments, and arrows 3 and 4 to the external transformations, i.e., the coordinated conversions, which lead to changes in the register, which are immediate or not, depending on the variations in semantic congruence. Arrow C represents understanding, assuming a coordination of two registers. The dotted arrows illustrate the distinction between the representative and the represented.

Figure 1

Fundamental learning hypothesis: structure of representation according to conceptualisation. (Duval, 2012, p. 282).



The fundamental hypothesis presented in Figure 1 is crucial to *semiocognitive modeling*, as it associates mathematical understanding with the coordinated conversion between at least two registers of semiotic representation of the mathematical object. However, a coordinated conversion is not always possible directly, and elements that contribute to it are necessary. These elements are presented here and are part of the process of *semiocognitive modeling*.

In the case of text comprehension specifically, coordinated conversion is also essential. Textual understanding, although based on the operations of segmentation and recontextualisation, also necessarily presupposes a change in the register of representation, since the passage of the text to be understood to the discourse that explains or analyses it cannot occur directly, i.e., it remains in the same discursive register. This transition requires an explicit deviation by a non-discursive representation, which functions as an *intermediate* or *auxiliary representation* between the original text and the interpretive discourse. It is through the cognitive work required by this deviation that it becomes possible to learn to understand texts and, analogously, solve mathematical problems.

Duval (1995, 2004) emphasises that learning to understand texts -in this study, centred on mathematical problems and definitions- is one of the main challenges of teaching and learning mathematics, since it requires the student to be able to interpret and extract information relevant to solving the problem in question. Reading a mathematical text is understood as a situated activity, shaped by context and the subject's cognitive domain. According to the author, the mathematical understanding of a text requires that the student be able to move between different RSRs, mobilising intermediate representations (Duval, 1995, 2004) or auxiliary representations (Duval, 2001).

The intermediate representations are characterised by containing most of the semiocognitive elements present in the main representation, but presented in a more structured way. When a mathematical text is presented to the student, whether in the form of a problem, definition or theorem, it is often found in a register of the mother tongue or in a symbolic notation that is not transparent to understanding. As Duval (1995, 2004) points out, in understanding a text, intermediate representations act as a link between the mother tongue and mathematical symbols, favouring understanding by making explicit the relations implied by the utterance. On the other hand, auxiliary representations are the ones that support the main representation, and contribute to constructing another representation. As Duval states, a main representation is "associated with one or more auxiliary representations to form a whole that constitutes a 'hyper-representation'", and in this set, the function performed by the auxiliary representation can be identified (Duval, 2001, p. 57).

The two types of representation described are crucial for understanding and solving problems, as well as for understanding mathematical definitions. Because of this, we call these representations *supporting representations* in the process of *semiocognitive modeling,* understanding them as essential instruments for constructing meanings.

The semiocognitive elements evidenced — segmentation and recontextualisation; formation, treatment, and conversion of RSR; and supporting representations, when mobilised — enable the understanding of the text (problem or definition) and serve as resources for resolution in the case of mathematical problems.

It is noteworthy that *semiocognitive modeling* is a process planned by the teacher and can be used in various contexts when it is necessary to change the register to understand and solve problems. In the case of utterances presented in the mother tongue, *semiocognitive modeling* begins with the selection and highlighting of significant elements (segmentation), followed by recontextualisation that enables understanding of the utterance and inference about its resolution. Then, it requires altering the register, for example, from the mother tongue to an algebraic representation, this process being accompanied by specific treatments in the new register, assisted by supporting representations. Depending on the characteristics of the problem, other record changes may be required, each involving steps of segmentation, recontextualisation, the use of supporting representations, and the application of corresponding specific treatments.

Next, we present two examples of *semiocognitive modeling*: one for a problem and one for a definition.

SEMIOCOGNITIVE MODELING TO UNDERSTAND AND SOLVE PROBLEMS - THE CASE OF THE MAXIMUM AREA

Understanding in mathematics permeates the integration and articulation of the previously explained aspects, forming what we call *semiocognitive modeling*.

Problem solving is a pillar of the teaching and learning process in mathematics, and understanding definitions and theorems is its formal basis. However, for the basic education teacher, the challenge is often to overcome the barrier between formal enunciation and student mobilisation. Duval's semiocognitive theory, although focused on RSR, engages deeply with metacognitive processes such as segmentation and recontextualisation, which are essential for understanding texts and, thus, for constructing mathematical knowledge.

We present a common problem that appears in high school textbooks to structure and exemplify the *semiocognitive modeling* proposed for understanding and solving problems, highlighting the possible interrelationships among semiotic elements and how they manifest in students' ability to transcend formality to achieve a functional, applicable understanding.

Problem involving maximum area (Adapted from Souza & Garcia, 2016, p. 126).

Pedro intends to fence a rectangular region on his farm to raise chickens. For this, he bought 78 m of canvas and intends to use it in order to obtain the largest possible area for the chicken coop. What should be the measurements of the sides of this chicken coop? What will be the maximum area of this chicken coop?

This problem appears in different contexts in textbooks, requiring the student to understand it before solving it by mobilising mathematical knowledge.

The dynamics of segmentation

Cognitive segmentation, as mentioned earlier, consists of decomposing the problem into significant units that directly trigger specific mathematical knowledge, so that each segmented unit is linked to a certain concept. This process can be promoted through teacher-elaborated questions that explain the concepts mobilised in the problem situation, such as the geometry of rectangles, area formulas, optimisation problems, perimeter calculation, the vertex of a parabola, or even trial-and-error procedures.

Questions such as: What is the shape of the land (a plot)? What does it mean to fence a plot? What is the perimeter of a rectangular plot? What does "dimensions" of a plot mean? What is the area of a plot? What does the value of 78 m refer to? Can two rectangles with the same perimeter have different areas? If applicable, name two of them; they are guidelines for the teacher's activity with the class.

Because it involves students' mobilisation of multiple concepts, understanding this type of problem requires the teacher's intentional pedagogical actions throughout the teaching process.

Another relevant form of segmentation in this case is *functional segmentation*, understood as the process of decomposing the problem into parts of the text and the communicative role each plays in the utterance. Such segmentation is fundamental to understanding the discursive structure of the problem, as it organises its constituent elements and explains the actions necessary to resolve it. In the case analysed, the first sentence introduces the situation and explains the task to be performed: "Pedro intends to fence a rectangular plot”. The second sentence presents the relevant data on the characteristics of the plot and the perimeter of the rectangle: “For this, he bought 78 m of canvas and intends to use it in order to obtain the largest possible area for the chicken coop”. Finally, the last two sentences present the demand of the problem: “What should be the measurements of the sides of this chicken coop? What will be the maximum area of this chicken coop?” is recontextualised. Thus, the objective is to determine the dimensions of a rectangular piece of land that maximises its area while respecting a previously established restriction on the perimeter.

This form of segmentation should favour students’ understanding of the specific function of each text component, enabling the coherent organisation of their resolution strategies. Therefore, the student must identify the object of study (the rectangular plot), the action involved (fencing), the condition associated with the perimeter (78 m of fence), the mathematical objective (maximising the area of the plot) and what is sought to be determined (the dimensions of the plot, i.e., length and width).

THE DYNAMICS OF RECONTEXTUALISATION

Recontextualisation, as already noted, involves understanding the relationships among the information units and integrating the previously identified segments. This activity is responsible for sustaining the multiple inferences necessary to understand a problem, allowing the reader to fill in informational gaps and resolve ambiguities arising from the reading process. In this sense, to understand the problem, two modalities of recontextualisation can be identified, which are promoted based on the teacher’s intention in teaching.

Cognitive recontextualisation primarily mobilises knowledge related to the situations, objects, or issues evoked by the problem, regardless of the degree of explicitness in its textual formulation. In other words, in this type of recontextualisation, the problem is understood through the student’s prior knowledge of the content or topic being addressed. Thus, the teacher prompts the student to cognitively reorganise the available information to structure the mathematical reasoning required to solve the problem situation.

In the case under analysis, cognitive recontextualisation implies considering, in an articulated way, the underlying mathematical concepts involved in the problem. In this case, if a rectangular plot is to be fenced with 78 m of fencing, then the plot has parallel sides of equal length, and the sum of all sides is 78 ($2x + 2y = 78$). As it is the area that will be maximum, the student needs to know how to calculate the area of a rectangle ($A = x.y$) and, finally, conclude that the area of the rectangle in question can be given by the function $A = x.(39 - x)$ or $A = -x^2 + 39x$, which is of the quadratic type and whose graph is a downward-facing concave parabola. That is, the function representing the area has a maximum at the vertex of the parabola.

Thus, we recontextualise the demand of the problem: “What should be the measurements of the sides of this chicken coop? What will be the maximum area of this chicken coop?” Thus, once recontextualised, the units of meaning enable the global understanding of the problem by mobilising mathematical knowledge such as: the definition of the area of the rectangle as the product between length and width; the typical structure of optimisation problems and quadratic function; the fact that, when replacing the variables, a quadratic function is obtained whose concavity allows the maximum or minimum value to be identified at the vertex. From this perspective, cognitive recontextualisation promotes the transformation of the original narrative, expressed in the register of the mother tongue, into an articulated set of abstract elements, mathematical relations, and an action plan subject to conversion activity, from the register of representation of the mother tongue to the algebraic and/or graphic registers.

The *redactional recontextualisation* explains the relationships established between the discriminated units through functional segmentation. This process allows the student to understand that different linguistic forms and discursive structures can express the same underlying logical organisation, that is, an equivalent cognitive content. In this way, the student stops fixating on a specific utterance model and begins to develop greater cognitive and textual flexibility. In didactic terms, it is as if the rewriting of the problem constituted a strategy for its own understanding.

This type of recontextualisation can also occur through the simplification of the utterance's structure, with a view to reducing the cognitive cost. Examples of this include using smaller numerical values or temporarily suppressing more complex aspects of optimisation, maintaining the central idea of maximisation while simplifying costs and numbers to facilitate the calculation and exploration of the previously presented problem. In addition, the resolution can be proposed through an investigative approach, based on successive attempts, assigning values to the variables x and y and calculating the corresponding area. Finally, this recontextualisation can be connected to the quadratic function content, highlighting that the maximum value of the function occurs at its vertex, thereby establishing an explicit correspondence between the problem situation and the formal mathematical treatment.

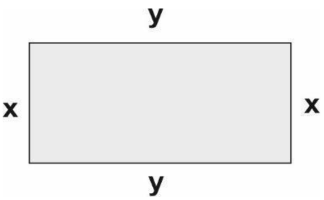
FUNDAMENTAL HYPOTHESIS OF LEARNING AND SUPPORTING REPRESENTATIONS

Understanding texts fundamentally involves the operations of segmentation and recontextualisation, which require decomposing the text into significant units and integrating them. This process requires changing the representation register, since understanding does not occur directly in the same register as the original text, but through supporting, often non-discursive, representations. According to Duval (1995, 2004), understanding mathematical texts is one of the main challenges of teaching and learning, as it requires the ability to extract relevant information and to move between different representation registers. In this sense, supporting representations play a central role in constructing meanings and resolving mathematical problems.

In the problem at hand, it is important to use representations that support inference of the problem's cognitive aspects. In this case, a geometric representation allows visualising the sides of the rectangle, while an algebraic representation, such as the one shown in Figure 2, allows inferring the function that represents the area.

Figure 2

Geometric representation of the rectangle of sides x and y.



The equations representing the perimeter and the area form a system

$$\begin{cases} 2x + 2y = 78 \\ A = x.y \end{cases}$$

that results in the following algebraic representations for the rectangle area:

$$A = x.(39 - x) \rightarrow A = -x^2 + 39x \qquad (1)$$

$$A - \left(+\frac{1521}{4}\right) = -(x - (+\frac{39}{2}))^2 \qquad (2)$$

Equation (1) is the function that represents the area as a function of the measure x of one of the sides of the rectangle. Equation (2) is obtained from (1), but its form allows the vertex of the parabola to be identified.

From such algebraic representations, it is possible to choose the empirical process using a tabular representation, replacing values of x in the domain of the function with the corresponding values and calculating the area. In this process, it is also necessary to segment and recontextualise, since significant units need to be clear to proceed in this way. For this, some questions can be asked, such as: What value can x assume, considering that it represents the measure of one of the sides of the rectangle at hand? How to calculate the area for specific values of x? Such questioning should allow the conclusion that $x \in R$ and $0 < x < 39$, since $x + y = 39$. Thus, we can construct a table of values.

Table 1

Assigning values of x and searching for the largest possible area.

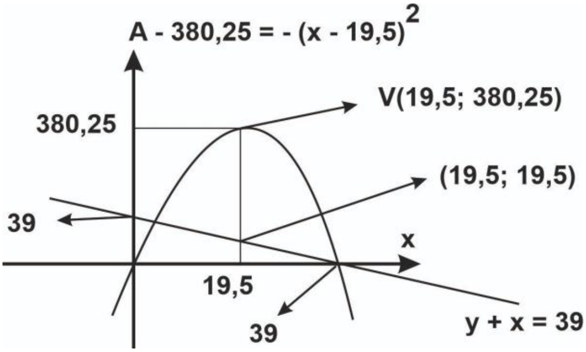
Values of x	**Values of y**	**Area values (A)**
1	38	38
2	37	74
5	34	175
10	29	290
15	24	360
20	19	380
25	14	350
35	4	140

This process, presented in Table 1, although intuitive and capable of providing an approximate value or an interval of x, whose area is the largest, does not solve the problem, since the value x that solves the problem is not an integer.

Another possibility is, given the clarity of the fact that the maximum value is the y of the vertex - y_V, using treatments, to find the roots of the area function and also the maximum point – vertex ($V(x_V, y_V)$). This is possible from the formulas $-x^2 + 39x = 0$ and $x_V = -\frac{b}{2a}$. However, we emphasise Moretti's (2003) idea — based on Duval (2011) — presented in Equation (2), whose outline of the parabola occurs through qualitative global apprehension when using translation as a global interpretation resource. In this case, the area represented by function (1) needs to be written in the reduced form (2), that is, $A - \left(+\frac{1521}{4}\right) = -(x - (+\frac{39}{2}))^2$, being (x_V, y_V) the coordinates of the vertex of the parabola, that is, $(19{,}5;\ 380{,}25)$. In other words, the parabola that graphs the area function $(A(x))$ in the problem is the translation of the "base" parabola $A(x) = -x^2$, 19.5 units to the right and 380.25 units upwards from the origin of the Cartesian system, presented in Figure 3, together with the function that represents the perimeter $(2x + 2y = 78)$.

Figure 3

Graphical representation of the area and perimeter.



A possible understanding of the situation again goes through segmentation and recontextualisation. In this case, the following idea can be proposed: imagine that an ant standing in $(0{,}0)$ begins to walk along the axis x toward $x = 39$. What happens to the area and the rectangle? Considering the domain and the fact that $x + y = 39$, a possible answer would be: when the ant is at the origin, i.e., in $x = 0$, there is no rectangle and, therefore, no area. As it begins to walk, for example in $x = 1$, the y becomes 38; a rather narrow rectangle is formed, but with a certain area. When it arrives in $x = 19{,}5$ cm, the area is maximum, and the rectangle becomes a square of 19.5 cm on a side. Starting at $x = 19{,}5$ cm and moving toward $x = 39$ cm, the area begins to decrease, and when it gets close to $x = 39$ cm, the rectangle again becomes very narrow, disappearing exactly at $x = 39$ cm.

In this maximum-area problem, *semiocognitive modeling* comprises several nonlinear steps. Figure 4 summarises and explains the various steps for understanding and resolving it.

Figure 4

Semiocognitive modeling for the maximum area problem.

Conversion or treatment	**Steps**	**Segmentation and recontextualisation**
	1) Problem statement in mother tongue	x
Conversion	↓	
	2) Drawing of the rectangle of sides x and y	x
Conversion	↓	
	3) Algebraic equations $A = xy$ and $2y + 2x = 78$	x
Treatment	↓	
	4) $A = -x(x-39)$ and $y + x = 39$	x
Treatment	↓	
	5) $A - \left(+\frac{1521}{4}\right) = -\left(x - \frac{39}{2}\right)^2$ and $y = 39 - x$	x
Conversion	↓	
	6) Sketch of the graphs of A and y	x
Conversion and treatment	↓	
	7) Solution	x
Conversion	↓	
	8) Drawing of the square of sides 19,5 cm by 19,5 cm and area 380,25 cm^2.	x
	Socialisation of the solution obtained	x

As shown in Figure 4, in *semiocognitive modeling*, segmentation and recontextualisation do not occur just once to understand the statement; they are necessary at each stage of conversion. It is noteworthy that the process of *semiocognitive modeling* enables coordinated conversions between different representation registers, and it is the teacher's role to plan teaching based on such semiocognitive elements. In addition, a problem of this type can serve as the starting point for another subject or content the teacher wants to address.

SEMIOCOGNITIVE MODELING TO UNDERSTAND A DEFINITION — THE CASE OF THE FUNCTION

In addition to understanding and solving problems, another difficulty encountered in the classroom is understanding mathematical definitions. Take as an example the definition of function, presented below according to a textbook:

> Given two non-empty sets A and B, a relation f of A in B is given the application name of A in B or a function defined in A with images in B if, and only if, for every $x \in A$ there is only one $y \in B$ such that $(x, y) \in f$ (Iezzi, 2004, p. 81).

This definition can be understood through a process of *semiocognitive modeling*, organised by the teacher, based on segmentation, recontextualisation, and the fundamental learning hypothesis, presented in Figure 1. Such a function definition is in the form of a theorem of the necessary and sufficient type, which more clearly characterises the conditions so that a relation can be a function. Thus, for the definition of a function, three propositions stand out, with A and B being two non-empty sets:

p: f is a function of A in B,

q: each element of A is uniquely related to some element of B,

r: all elements of A are related.

The dynamics of segmentation and recontextualisation will seek to highlight these three elements: **p**, **q**, and **r**.

The dynamics of segmentation

When the teacher presents the function definition "A relationship between two sets A (domain) and B (codomain) is a function if for each element of A there is a single corresponding element in B", it can explicitly encourage *cognitive segmentation* by proceeding as follows:

Cognitive segmentation 1: "A relationship between two sets A and B...": What is a relationship? What does it look like (ordered pairs, diagrams, graphs)? What are sets A and B?

Cognitive segmentation 2: "...if for each element of A...": This means that all elements of A must be associated with B. No element of the domain can be left out. This is the condition of necessity.

Cognitive segmentation 3: "...there is a single corresponding element in B.": this means that each element of A can only be associated with only one element in B, which is the condition of uniqueness that, in the context of functions, can be seen as a form of complementarity in the correspondence from A to B.

For *functional segmentation*, it is necessary to identify the communicative role of each part of the text, that is, what the discursive function of each segment is within the definition presented.

Functional segmentation 1: "A relationship between two sets A (domain) and B (codomain)" — discursive function: presents the central idea of the text, that is, that the function will be a relationship between two sets. This part indicates the basic structure of the concept. The role of this segmentation in the text is to introduce the object of study (the function) and to provide the context (set theory).

Functional segmentation 2: "If for each element of A" - discursive function: defines the condition of applicability of a function. This part explains that all elements of A, without exception, must participate in some form of association. The role of this segmentation in the text is to establish the reach (or scope) of the relationship.

Functional segmentation 3: "There is a single corresponding element in B" - discursive function: present the validity criterion for a relationship to be a function. This part clarifies the most important aspect of functions: uniqueness. The role of this segmentation in the text is to create the main restriction of the definition.

By segmenting a definition, the teacher helps students assign meaning to each part, avoiding empty memorisation and laying the groundwork for identifying properties crucial to the concept and behaviour of functions.

The importance of recontextualisation

As possibilities of procedures for *cognitive recontextualisation*, from the definition, we have:

Cognitive recontextualisation 1: Relationship between elements of two sets. A function is a relationship that connects an element of a set A to exactly one element of a set B. In intuitive terms, each input ($x \in A$) is associated with a single output ($y \in B$).

Cognitive recontextualisation 2: Principle of uniqueness: the idea of function consists in ensuring that for each x in A, there is only a single y corresponding in B. That is, no entry in the set A can be associated with more than one output in the set B.

Cognitive recontextualisation 3: Ordered pair (x, y) as representation: a function can be seen as a set of ordered pairs (x, y), where $x \in A$ and $y \in B$, and no value of x appears in more than one pair. This recontextualises the formal definition by the structure of the ordered pairs that make up f.

Also, for *redactional recontextualisation*, which organises the text, changing the way it is presented linguistically while containing the same semantic and mathematical content, considering the definition of function, we can have:

Redaction recontextualisation 1: "A function is a relationship between two sets A (domain) and B (codomain), such that each element of the set A relates to exactly one element of the set B, meaning that for any one $x \in A$, there is only one corresponding $y \in B$".

Redaction recontextualisation 2: "An application f, from A to B, is called a function if, and only if, each element of A associates (or 'points to') with a single element of B. In other words, no value of x in A is associated with more than one value of y in B".

Redaction recontextualisation 3: "Given a relationship f between two sets A and B, we can call it a function when, for every element x of the domain A, there is a single element y in the codomain B, so that the ordered pair (x, y) belongs to the relationship f".

As previously pointed out, segmenting and recontextualising a text, in this case, a definition, does not guarantee its understanding. Understanding is only consolidated through coordinated conversions — the fundamental hypothesis of learning — which are possible from supporting representations.

Fundamental hypothesis of learning and supporting representations

Conversion is the transformation of a representation of a mathematical object into a representation of the same object in a different register. Duval states that the true core of mathematical understanding lies in conversion, as it requires the student to recognise the invariance of the conceptual object despite changes in its semiotic form. Conversion difficulties are one of the main causes of failure in mathematics.

To address this failure, the teacher can propose activities that explicitly require conversion between registers, with a focus on the concept of function and its properties. From the definition of (verbal) function, a Venn diagram is a widely used supporting representation in textbooks that helps clarify what a function is. It can visually highlight the conditions of need and uniqueness for functions and non-functions. This activity helps the student to internalise the concept by requiring an active externalisation of their understanding.

In the cases presented in Figures 5 and 6, we have the conversion of the register of *representation of the mother tongue* ↔ *the Venn diagram representation*.

Figure 5

Representation of a function through the Venn diagram.

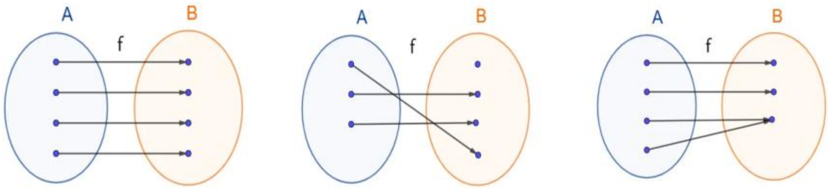


Figure 6

Representations of non-functions through the Venn diagram.

From this, the teacher can present examples such as:

- Problem: "Is the relationship that associates each natural number with its successor a function?"

- Conversion to Venn diagram: The student draws two sets and the arrows, converting the verbal description into a visual diagram. This allows them to visualise the conditions of the definition, verifying that each natural number has a single successor (need and uniqueness/complementarity).
- Contrapositive in conversion: present a Venn diagram (diagram register) in which an element of the domain (set A) has two arrows coming out of it (violating the uniqueness) or an element of the domain without any arrows (violating the need). Ask students to convert this representation register into a diagram in a verbal explanation (mother tongue register) of why it is not a function, using the definition and the contrapositive: "This relationship is not a function because, converting the diagram to the definition, the element 'x' of the domain, for example, has two distinct images 'y_1' and 'y_2' in the codomain, which contradicts the condition of 'a single corresponding element'". Or "the element 'z' in the domain has no image, contradicting 'for each element of A'".

By emphasising conversion, the teacher not only helps the student identify the function in several RSRs, but also understand the depth of its definition, including its uniqueness (complementarity) and how its absence characterises a "non-function" through the contrapositive. This strengthens the student's ability to develop a conceptual understanding and apply the concept of function flexibly in different situations and representations.

The role of the notion of the contrapositive in understanding definitions and theorems

Direct implication and contrapositive implication are equivalent propositions; that is, they have the same logical table of truth value:

$$\mathbf{s \Rightarrow t} \qquad \text{e} \qquad \mathbf{\sim t \Rightarrow \sim s}$$

are equivalent from a logical standpoint[1]. We have that: "~" is the logical negation operator, "**∧**" the disjunction operator, "**v**" the conjunction operator.

The double implication **s ⇔ t** is the same as **s ⇒ t ∧ t ⇒ s**, which, in turn, is equivalent to:

[1] In the following, "~" indicates the logical negation operator, "**∧**" the disjunction operator, "∨**"** the conjunction operator; **~(p∧q) = ~p** ∨ **~q**.

$$\sim t \Rightarrow \sim s \wedge \sim s \Rightarrow \sim t.$$

The definition of a function takes the form of a double implication argument.

With the propositions **p**, **q** and **r** previously defined (**p**: f is a function of A in B, **q**: each element of A is uniquely related to some element of B, **r**: all elements of A are related), and considering A and B two non-empty sets, we can rewrite the definition of function according to the following table.

Table 2

Arguments in the definition of function and contrapositive arguments.

Argument	Contrapositive argument
$p \Leftrightarrow q \wedge r$	$\sim q \vee \sim r \Leftrightarrow \sim p$
$p \Rightarrow q \wedge r$	$q \vee \sim r \Rightarrow \sim p$ if q does not occur or r does not occur or both propositions do not occur, ~p happens (f will not be a function)
and	and
$q \wedge r \Rightarrow p$	$\sim p \Rightarrow \sim q \vee \sim r$ ~p (f will not be a function) if q does not occur or r does not occur, or both propositions do not occur

The function defined in row 1 of Table 2 is equivalent to the definition contained in rows 2 and 3; the second column contains contrapositive arguments.

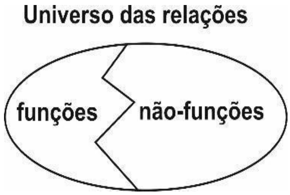
We can relate this definition extracted from a teacher's speech in the classroom:

> [...] When two arrows come out of the same element of A, or when any element of A has no arrow, it is not a function. The others are all functions. It is enough that you register these two counterexamples to determine whether it is a function or not (Sistema de Ensino Energia (2006) as cited in Moretti et al., 2012, p. 480).

In this verbalisation, the use of the contrapositive is identified as a representation of support, although there is confusion between a non-example, which is the exposed case, and a counterexample. There is still another fact embedded in this discourse: once the requirement of having two sets A and B not empty is fulfilled, and a rule that associates set A with set B, in the universe of relations, there are only functions and non-functions, as seen in Figure 7.

Figure 7

Universe of relationships



This dichotomy does not always hold; for example, even and odd functions: a function may not be even, but that does not mean it is odd. Let us analyse a situation closer to everyday life in order to understand the direct and contrapositive arguments for the argument “if it rains, then take the umbrella”, given as true:

- direct argument: if it rains, then take the umbrella;
- contrapositive argument: if you do not take the umbrella, then it does not rain.

After a certain time when it may or may not have rained, what can one conclude about the statement “the person will not get drenched by the rain at all”?

What can be said in the cases:

- if it rained: by direct argument, the person took the umbrella, so he/she did not get drenched;
- if it did not rain: examining the direct and contrapositive arguments, nothing can be said; that is, he may or may not have taken the umbrella, but it can be concluded that he did not get drenched because there was no rain;
- if he did not take the umbrella: it means, by the contrapositive argument, that it did not rain, so he did not get drenched either;
- if he took the umbrella: it is not known whether it rained or not; however, with rain or without rain, he will not get drenched because he took the umbrella.

In short, the person will not get drenched at all.

The teacher must assess students' accuracy in identifying violations and formulating justifications. Clarity in applying the contrapositive is an indicator of conceptual understanding.

From these discussions, the presentation of the definition of function in *semiocognitive modeling* can be systematised as shown in Figure 8 below.

Figure 8

Semiocognitive modeling for the definition of function.

Conversion or treatment	**Steps**	**Segmentation and recontextualisation**
	1) Representation of the definition of function in the mother tongue	x
Conversion	↓	
	2) Representation of a Venn diagram	x
Treatment	↓	
	3) Identification of relationships by viewing when it is a function or not.	x
Conversion	↓	
	4) Graphical representation	x
Treatment	↓	
	5) Identification in the graph of significant units - Visual variables	x
Conversion	↓	
	6) Examples and counterexamples	x
Conversion and treatment	↓	
	Understanding the concept of function, via definition	x

FINAL CONSIDERATIONS

The *semiocognitive modeling* presented here is characterised as a teaching process intentionally structured by the teacher in the classroom to favour students' learning. In this context, the didactic contract proposed by Brousseau (1996) establishes that the teacher needs to organise and manage the class dynamics to achieve the learning objectives. To this end, it is necessary to consider the didactic triangle as an analytical reference, which involves:

- the knowing to be taught and that makes up the school teaching curriculum;
- the student, who is the subject who places themselves in the process of learning concepts that are part of the school curriculum;
- the teacher, who has the function of managing the classroom and teaching the objects of knowledge with a view to conceptual learning.

The choice of topic to address must be made judiciously, as *semiocognitive modeling* requires explaining the semiotic systems that will constitute the teacher's proposed activities. In the case of the problem of the area of a piece of land, which seeks to determine the maximum area, the discursive systems of the mother tongue and algebraic language are involved, as well as the non-discursive Cartesian and geometric systems. It is, therefore, a seemingly simple problem, but it integrates four distinct semiotic systems.

This requires the ability to abstract the essential features of the problem, such as the linear relationship, the starting point, and the rate of change, and relate them to the conditions and conclusions of the mathematical properties. The ability to recontextualise these elements is an indicator of the understanding of mathematical knowledge.

Understanding a mathematical definition is evidence of the need for *semiocognitive modeling*. If comprehension were merely the storage of a statement in memory, the difficulties of transposition and application observed in students would be non-existent.

The difficulty in mathematics often does not lie in the complexity inherent in the mathematical object itself, but in students' inability to manage and articulate the various semiotic representations that convey it (Duval, 1995). Definitions, such as logical and operational structures, require this fluidity and mobility. The successful mobilisation of a definition into a problem indicates the effective mobility of the formation, treatment, and conversion of its representations, guided by the segmentation of the problem and the strategic recontextualisation of the definitions.

Therefore, the analysis of how students interact with mathematical definitions across multiple representations and strategies not only corroborates the relevance of *semiocognitive modeling* but also elevates it to a didactic power. For students to transition from memorisation to understanding and to flexibly mobilise concepts, teachers need to intentionally plan activities that stimulate this semiocognitive dynamic, making definitions a fertile field for the development of mathematical reasoning.

One of the primary roles of *semiocognitive modeling* is to contribute to coordinated conversions. The teacher's questions play this role because they go beyond what the problem addresses. For example, the question: What is the shape of the land? It suggests that a plot can have several shapes and, as a result, the formulas for the area and perimeter will differ. At first, the questions aim to understand the problem statement in the mother tongue to highlight the significant elements. In a second moment, once these significant elements (area, perimeter, sides, rectangle, etc.) are highlighted, the conversion to algebraic language occurs. In Step 6 in Figure 4, the conversion can be fully coordinated, which is one of the highlighted issues in this problem.

AUTHORSHIP CONTRIBUTION STATEMENTS

MTM was responsible for the study design, survey, and analysis of the sources. BCP and CMN contributed to the theoretical deepening and to the definition of the methodological procedures. All authors participated in writing the manuscript and analysing the data, and performed the final review of the text, actively participating in the preparation of the article and the approval of its final version.

DATA AVAILABILITY STATEMENT

Data sharing does not apply to this article, as it is based on research using publicly available bibliography.

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