

The Rule of Three from the perspective of reverse Mathematical Modelling

Gleison de Jesus Marinho Sodré^a 

Saddo Ag Almouloud^b 

José Messildo Viana Nunes^b 

Renato Borges Guerra^b 

^a Escola de Aplicação da Universidade Federal do Pará, Estado do Pará, Brasil

^b Universidade Federal do Pará, Programa de Pós-graduação em Educação em Ciência e Matemática, Instituto de Educação Matemática e Ciências, Estado do Pará, Brasil

ABSTRACT

Background: Mathematics education has emphasised modelling as an activity that connects situations and models. However, little attention has been paid to the reverse process, which involves using and studying mathematical models to define and problematise the associated situations. **Objective:** To discuss, in light of the anthropological theory of didactics, reverse mathematical modelling as a methodological tool for teacher education. **Design:** We present a formative program grounded in the theoretical and methodological resources of this theoretical field. **Setting and participants:** Pre-service teachers in a mathematics education program at a public institution, engaged in the study of the mathematical model of the rule of three in contrast to mathematical situations. **Data collection and analysis:** An empirical sample of participants from the development of the formative program is analysed. **Results:** The results, which are localised and exploratory in nature, suggest the methodological relevance of reverse modelling for the study of mathematical situations and indicate that the use of computers can serve as a pedagogical condition conducive to the emergence of new research questions and changes in the quality of the participants' relationships with the rule of three. **Conclusions:** The empirical findings, though preliminary, suggest the potential of the rule of three, when applied through reverse modelling, for designing broader praxeological frameworks focused on teaching school mathematics. Furthermore, they encourage future research on the rule of three as a hypothesis for studying open-ended problems and for building new connections between mathematical models and real-world mathematical situations.

Keywords: Reverse mathematical modelling, Anthropological theory of teaching, Rule of three, Teacher education.

Corresponding author: Gleison De Jesus Marinho Sodré. Email: gleisonsodre@ufpa.br

A regra de três sob o enfoque da modelagem matemática reversa

RESUMO

Contexto: A educação matemática tem destacado a modelagem como uma atividade que inter-relaciona situações e modelos. Entretanto, pouca atenção tem sido dada ao processo inverso, que consiste na utilização e no estudo de modelos matemáticos para delimitar e problematizar as situações associadas. **Objetivo:** Discutir, à luz da teoria antropológica do didático, a modelagem matemática reversa como dispositivo metodológico para a formação de professores. **Design:** Apresenta-se um percurso formativo fundamentado em recursos teórico-metodológicos desse campo. **Ambiente e participantes:** Professores em formação de um curso de licenciatura em matemática de uma instituição pública, envolvidos no estudo do modelo matemático da regra de três em contraste com situações matemáticas. **Coleta e análise de dados:** Analisa-se um recorte empírico produzido com os participantes ao longo do percurso formativo. **Resultados:** Os resultados, de caráter localizado e exploratório, sugerem a pertinência metodológica da modelagem reversa para o estudo de situações matemáticas e indicam que o uso do computador pode atuar como condição didática favorável ao surgimento de novas questões de estudo e a mudanças na qualidade das relações dos participantes com a regra de três. **Conclusões:** Os achados empíricos, ainda que introdutórios, sugerem a fecundidade da regra de três, mobilizada por meio da modelagem reversa, para a concepção de organizações praxeológicas mais amplas voltadas ao ensino da matemática escolar. Além disso, estimulam investigações futuras sobre a regra de três como hipótese para o estudo de problemas abertos e para a construção de novas articulações entre modelos matemáticos e situações matemáticas.

Palavras-chave: Modelagem matemática reversa; Teoria antropológica do didático; Regra de três; Formação de professores.

La regla de tres desde la perspectiva del modelado matemático inverso

RESUMEN

Contexto: La educación matemática ha destacado la modelización como una actividad que interrelaciona situaciones y modelos. Sin embargo, se ha prestado poca atención al proceso inverso, que consiste en la utilización y el estudio de modelos matemáticos para delimitar y plantear como problemas las situaciones asociadas. **Objetivo:** Analizar, a la luz de la teoría antropológica de la didáctica, la modelización matemática inversa como herramienta metodológica para la formación del profesorado. **Diseño:** Se presenta un itinerario formativo basado en recursos teórico-metodológicos de este campo. **Entorno y participantes:** Profesores en formación de una carrera de Licenciatura en Matemáticas de una institución pública, que participan en el estudio del modelo matemático de la regla de tres en contraste con situaciones matemáticas. **Recopilación y análisis de datos:** Se analiza un fragmento empírico obtenido de los participantes durante el desarrollo del itinerario formativo. **Resultados:** Los resultados, de carácter localizado y exploratorio, sugieren la pertinencia metodológica del modelado inverso para el estudio de situaciones matemáticas e indican que el uso del

ordenador puede actuar como una condición didáctica favorable para que surjan nuevas cuestiones de estudio y se produzcan cambios en la calidad de las relaciones de los participantes con la regla de tres. **Conclusiones:** Los resultados empíricos, aunque preliminares, sugieren la utilidad de la regla de tres, aplicada mediante el modelado inverso, para la concepción de organizaciones praxeológicas más amplias orientadas a la enseñanza de las matemáticas escolares. Además, estimulan futuras investigaciones sobre la regla de tres como hipótesis para el estudio de problemas abiertos y para la construcción de nuevas articulaciones entre modelos matemáticos y situaciones matemáticas.

Palabras clave: Modelización matemática inversa; teoría antropológica de la didáctica; regla de tres; formación del profesorado.

INTRODUCTION: THE DIDACTICS OF REVERSE MATHEMATICAL MODELLING

The literature in mathematics education recognises the importance of mathematical modelling (MM) in school curricula and teaching practices, particularly for its role in solving problems of personal, professional, civic, and scientific interest, as noted by Krawitz, Schukajlow, Yang, and Geiger (2025).

Although Krawitz, Schukajlow, Yang, and Geiger (2025) suggest the lack of a univocal understanding of MM, we assume the theoretical-methodological assumptions of the mathematical modelling investigative cycle (Sodré & Guerra, 2018; Sodré, 2019) (MMIC), and, more broadly, of the anthropological theory of the didactic (Chevallard, 1989, 1999, 2005, 2019), or simply ATD, by situating that “much of mathematical activity can be identified with a mathematical modelling activity” (Chevallard, Bosch, & Gascón, 2001, p. 51), not as a simple dimension of mathematical activity, but as an MM activity in itself.

Under the theoretical lenses of ATD, Chevallard (1989) defines MM as a process between systems and models, structured in three stages: definition of the system to be studied, construction of the mathematical model and work in/of the model. The notions of system and mathematical model are functional and vary according to the institutional environment analysed.

From this perspective, systems can be understood as domains of reality (Chevallard, 1980, 2005), mathematical or extramathematical, and mathematical models as artificial constructions (Chevallard, 1989) aimed at the demagnification “: [...] of ‘*a piece of reality*’ by means of models that, far from being exact representations, turned out to be “machines” good at producing knowledge about the reality in question” (Chevallard, Bosch & Gascón, 2006, p. 2, emphasis added).

We assume that the introduction of minimum conditions, based on the didactics of the MMIC (Sodré & Guerra, 2018; Sodré, 2019), for the use, study, and development of mathematical models in conjunction with the study of situations may prove necessary, in our understanding, both in educational institutions and in teacher education, due to its “dual functional role as a didactic-methodological device: for the teaching-learning of MM and for teacher education” (Sodré, 2021, p. 17).

The praxeological organisation of the MMIC is described by six sequential tasks, namely:

- Task T_0 : Build an initial reference situation for the problem in context
- Task T_1 – Investigate the mathematical models that live in the school institution related to the problem in context
- Task T_2 : Find situations that can be associated with a mathematical model
- Task T_3 : Evaluate mathematical models
- Task T_4 : Develop a mathematical model
- Task T_5 : Disseminate and defend the mathematical model

MMIC upholds the characteristics of a formative path informed by ATD, in a context that requires knowledge or expertise action. This situation could be translated into one or more study questions (Q_i) for a class to address.

To find a useful answer (R)—if not adequate—to question (Q_0), the formative path is developed by the class from the institution of the study and investigation system, called the didactic system $S_D(X; P; Q_0)$, from which new situations and questions (Q_d) must be derived, and which, when judged to be of interest for teaching or for the formative process by the teacher (P), guide the iterative flow of creating new didactic systems $S_D(X; P; Q_d)$ until the class consensually judges that it has found the useful answer (R) appropriate to the study process.

MMIC shows the subcycle consisting of the types of tasks T_0 , T_1 and T_2 , which, in unity, Sodré, Ferreira, and Guerra (2022) called reverse mathematical modelling (RMM), because this structure allows for “the deconstruction and reconstruction of a mathematical model with constructions and reconstructions of situations” (Sodré, Ferreira & Guerra, 2022, p. 562). With this view, we understand RMM as a subcycle of MMIC, consisting of the tasks T_0 , T_1 and T_2 , in unity and, as such, oriented towards deconstruction and reconstruction in the face of the use or study of mathematical models, in

connection and integration with the construction and reconstruction of situations with mathematics.

RMM, as an integral part of MMIC, meets the interest of this investigation as a research methodology and teacher education in MM, as it highlights the study of situations with mathematics (Sodré, 2023, p. 205), in which this notion is understood here “from the notion of social practices with mathematics proposed by Chevallard (2005)” and, as such, goes beyond the terrain originally granted, i.e., the context of school mathematics teaching, by penetrating the set of uses of mathematics and infiltrating a multitude of institutional spaces in which mathematical knowledge is pertinent and its manipulation is observable.

The notion of situation in mathematics is adopted in this investigation because it appears almost transparent, if not absent, in the dominant cycles of MM. The study of the situation is present in all types of RMM tasks, particularly in the confrontation between the use of the model and the situation.

RMM didactics can highlight the role of situations in mathematics in relation to mathematical models and offer answers to the problem of the disarticulation of knowledge in school mathematics (Bosch, García, Gascón, & Ruiz Higuera, 2006). This creates the possibility of resuming content, questioning it, and integrating it into broader and more consistent situational knowledge organisations.

Finding answers to the problem of the disarticulation of knowledge in school mathematics is one of the objects of interest of ATD and, more broadly, of the didactics of mathematics, understood as the science of institutional conditions and restrictions (Chevallard, 2005, 2009). In this context, the path to delimit possible answers can be traced, in our understanding, through the RMM (Sodré, Ferreira, & Guerra, 2022; Sodré, Guerra, & Ferreira, 2026), in view of the study of the interrelationship between models and situations with mathematics.

The dialectical interrelationship between models and situations in mathematics can be developed through socially legitimised mathematical models, understood as generators of domains of reality (Chevallard, 1980, 2005). These models include those of interest for the teaching of school mathematics, such as the rule of three, which can fulfil relevant didactic functions in the connection and integration of mathematical and non-mathematical knowings. In this sense, Guerra and Silva (2009) and Silva (2017)

highlight the rule of three as an important instrument for modelling real situations.

We focus on this not inconsiderable scope of the rule of three, pointed out by Silva (2017), to favour the reconstruction of praxeological organisations aimed at developing mathematical models for studying related situations. Although the teaching of the rule of three is manifested based on the foundations of proportionality, as defended by Ávila (1986) and Lima (2001), we are interested in highlighting its potential as a structuring practical knowledge and independent of the mathematical theoretical discourse of proportionality, given that, “in many cases, the technological elements are even integrated into the technique [...], where the same expression has a double, technical, and technological function”¹ (Chevallard, 1999, p. 224, our translation).

School education and, not infrequently, textbooks reveal the making of the rule of three closely associated with the notion of proportionality, which seems to us to hide the “distance” between the rule of three and the theory of proportions under the historical-epistemological approach, as evidenced by Sarma (2002, p. 134): “With this rule on can easily solve several types of problems even without a knowledge of the general theory of proportion”. The rule of three and its social importance are felt by the need to make it accessible to ordinary people, since “the purpose of Hindu works is to provide a method that can be easily used by ordinary people” (Datta & Singh, 1938, p. 217, our translation).

The historical-epistemological traces (Datta & Singh, 1938; Sarma, 2002; Høyrup, 2012; Silva, 2011, 2017) of the rule of three reveal distances from the mathematical theory of proportions, even though this theoretical notion seems to have greater visibility, including in school teaching, to the detriment of the rule of three, since “the visibility of the knowing block increases, to the detriment of knowing-how-to-do [...] classically presented as a simple application of knowing”² (Chevallard, 1999, p. 226, our translation).

¹ Fragments of the text: En numerosos casos, incluso, algunos elementos tecnológicos están integrados en la técnica. [...] en la que el mismo pequeño discurso tiene una doble función, técnica y tecnológica (...).

² Fragment of the text: la visibilidad del bloque del saber aumenta, en detrimento del saber-hacer [...] classicamente como una simple aplicación del “saber”.

The understanding pointed out by Chevallard (1999), paraphrased in terms of the notions of the rule of three and proportionality, can reveal the supporting role attributed to the rule of three in the teaching and learning processes of school mathematics, as a mere application of the mathematical discourse of proportionality. Otherwise, proportion can be understood as a theoretical knowledge, that is, “a discourse whose first objective is to ‘rationally’ justify the technique [...], so that a rationality may seem unreasonable in another institution”³ (Chevallard, 1999, p. 224, emphasis added, our translation).

The *style of rationality* (Chevallard, 1999), legitimised by the academic mathematical institution, moves away from notions associated with the procedure of reduction to unity, perhaps due to the strangeness resulting from the comparison between quantities of different natures, although this procedure is widely accepted in different fields of scientific knowledge, in applied sciences, engineering, school physics, chemistry, geography, school mathematics, and in various human activities.

This strangeness in relation to the procedure of reduction to unity is also evidenced, according to Silva (2017), in some studies developed within the ATD, such as the ones by Bosch (1994), Bolea (2003), and Garcia (2005), which “do not deal properly with the praxeologies of the rule of three and its potential for the development of new praxeologies with other mathematical objects [...] for not finding meaning in the product or division of magnitudes, especially when they are different” (Silva, 2017, p. 170).

In this sense, we aim to highlight MMR as a methodological device for teacher education, understanding it as a necessary condition to deconstruct and reconstruct mathematical models in conjunction with the strategic role of situations with mathematics (Sodré, 2023; Sodré & Guerra, 2024), based on the use of the rule of three as practical knowing that preserves the idea of variation between magnitudes.

Under these conditions, we forwarded the research method and an empirical approach guided by RMM as part of a study and research path for teacher education (SRP-TE) in MM (Ruiz-Olarría, 2015; Barquero, Bosch & Romo-Vázquez, 2015; Sierra & Gascón, 2018; Florensa, Bosch & Gascón,

³ Fragments of the text: discurso cuyo primer objetivo es *justificar* “racionalmente” la técnica [...] de manera que una racionalidad institucionalmente dada podrá aparecer... como poco racional en otra institución.

2019; Almouloud, Nunes, Pereira & Figueroa, 2021; Guerra & Ferreira, 2022), with prospective teachers in initial education.

THE RESEARCH METHOD

From the ATD perspective, the SRP-TE seeks to create conditions for the development of activities supported by a functional epistemology of knowings (Barquero, 2009; Ruiz-Munzón, 2010; Serrano, 2013), based on the paradigm of questioning the world. Its objective is to familiarise prospective teachers with the SRP (Chevallard, 2005, 2013, 2019) as a methodological device for professional development (Almouloud, Nunes, Pereira, & Figueroa, 2021), considering that “what matters is not so much what the student will know in advance, but what he will be able to learn from his investigation in order to advance” (Ladage & Chevallard, 2010, p. 2, our translation).

Sierra and Gascón (2018, p. 96, our translation) highlight that the notion of SRP-TE derives from two complementary processes: “the questioning of the dominant epistemological model in a school institution around a field of mathematics to be taught [...] and the process of didactic transposition”.

According to Ruiz-Ollarria (2015), Barquero, Bosch, and Romo-Vázquez (2015), Sierra, and Gascón (2018), Florensa, Bosch, and Gascón (2019), and Almouloud, Nunes, Pereira, and Figueroa (2021), the SRP-TE is organised into five modules: **M₀**, **M₁**, **M₂**, **M₃** and **M₄**.

- **Module M₀: Explain the reasons for the SRP-TE.** It starts with a generating question of the Q₀-TE type: “How to teach a specific mathematical content?”, that is, how to teach the rule of three? The objective is to answer this question, at least partially, at the end of the study process. This first module is developed by searching for information available in research findings and curriculum guidelines. This is a strategic module in the SRP-TE, according to Ruiz-Ollarria, Bosch, and Gascón (2019) and Florensa, Bosch, and Gascón (2019), as it remains "open" and incorporates the results produced in the other modules;

- **Module M₁: Experience an SRP.** Prospective teachers are invited to act as students, to become familiar with the study process;

- **Module M₂: Analyse the SRP experienced.** After experiencing an interactive study environment, participants are invited to analyse the practices developed and reflect on them from a praxeological, didactic, and ecological perspective, considering the set of conditions and restrictions for their feasibility in the classroom;

• **Module M₃: Design an SRP as a didactic engineer.** From the experiences lived in the previous modules, prospective teachers are expected to build praxeological organisations for teaching;

• **Module M₄: Manage and try an SRP.** The objective is for prospective teachers in initial education to put into action, albeit partially, the management of an SRP in the classroom with students, considering the practices built or deconstructed in previously experienced modules, expressed through possible changes in the quality of relationships (Chevallard, 2005) established with the objects of knowing.

In the SRP-TE modules, it is necessary to consider that “the content of useful praxeological equipment is not defined in advance: we will discover it, although very partially, as we investigate [...]; it is very rare for knowledge [...] to *arise spontaneously for us*” (Chevallard, 2013, p. 8, emphasis added, our translation).

To delimit the SRP-TE, we considered an empirical study developed with four prospective teachers in initial education⁴ enrolled in the mathematics teaching degree course. It is, therefore, a reduced, situated corpus that allows us to produce analytical interpretations of the observed process and, for that reason, does not support broad generalisations. The investigation was guided by the following generative question:

- **Q₀-TE:** *How to teach the rule of three as a hypothesis for the construction and articulation of mathematical models, in contrast to the strategic role played by the situation in mathematics associated with it?*

In the light of ATD, “formally studying a generic question is, in general, working to give it an answer [...]. Studying a knowing, more precisely a praxeological entity, is actually studying some issues related to its structure, its

⁴The empirical data obtained from the teachers' manifestation in initial education took place in the context of a curriculum subject at a public institution of higher education. In this sense, the professors' registers highlighted here do not give their identities, images, and voices, thus ensuring the dignity and due protection of the participants in scientific research. For this reason, no prior permission was requested by the appropriate councils of the research project from which the work arises. Thus, we assume and exempt the Acta Scientiae from any consequences arising therefrom, including full assistance and possible compensation for any damage resulting from any of the research participants, as directed by Resolution n. 510, of April 7, 2016, of the National Health Council of Brazil.

functions, its genesis, etc.” (Chevallard, 2009, p. 16, emphasis added, our translation). In this direction, we convey the analyses and results produced with prospective teachers in initial education.

The data collected during the SRP-TE tentative include the speeches manifested by students and prospective teachers in the classroom, the descriptive records of the mathematical objects evidenced during school studies, the questions raised by the class and, more emphatically, the situations with mathematics revealed from the models generated, under the didactic hypothesis of the variation between magnitudes of the rule of three and the relationships established between them.

Under this didactic hypothesis, the rule of three was functionally assumed as a practical knowledge and, as such, endowed with potential for the study of situations with mathematics through the procedure of reduction to unity, the algebraic description of formulas, the use of the computer in the geometric construction of surfaces and, not least, the preservation of variation between quantities for the design and study of other mathematical models of greater didactic scope.

In this sense, the conceptual records produced by students and teachers regarding knowledge, as well as their statements during the study process, constitute the corpus analysed in this investigation. These records were interpreted in light of the theoretical-methodological references of ATD and RMM, seeking to highlight the strategic role of mathematical situations associated with the rule-of-three models that emerged in the classroom.

RESULT ANALYSIS

The empirical analyses were conducted in light of RMM, taken here as a theoretical-methodological device to interpret the relationship between the deconstruction and reconstruction of the mathematical model of the rule of three and the study of situations in emerging mathematics along the formative path. In this context, RMM allowed us to examine how the participants' answers, the questions formulated in the classroom, and the relationships established between quantities showed shifts in their relationships with their knowings and opened possibilities for integration with other objects of school mathematics.

Conducting an SRP-TE with teachers required defending the responses **(R)** produced during the module **M₄**, in which participants managed and

experienced a specific SRP in a class composed of 25 (twenty-five) students⁵ from the 9th grade of elementary school in a public school, continuing the work developed in the previous modules.

• **Module M₀: Explain the reasons of being for the SRP-TE.**

Guided by this module, the teachers expressed, in a consensual way, the following understandings, of which we highlight the speech excerpts from **P₁** and **P₃**:

- **P₁**: with the rule of three, we observed the study of functions, Thales' theorem, similarity, uses in physics (Hooke's Law, darkroom), in addition to the study using tables for the relationship between quantities, graphs, and the analytical-synthetic method [...];
- **P₃**: From the subject 'rule of three', it was possible to relate it to the construction of the first- and second-degree polynomials.

The highlights of the speeches of teachers **P₁** and **P₃** allow us to situate the strategic role of the rule of three in its relationship with the opening of didactic possibilities for the construction of different mathematical objects, such as the study of polynomial functions of the first and second degree, Thales' theorem, the similarity of triangles, and the variation between magnitudes through the compound rule of three.

These mathematical objects, resulting from the study processes developed using the rule of three, enabled the class to encounter different situations, such as the darkroom and Hooke's Law, which these teachers identified as mathematical situations relevant to school teaching.

From these manifestations, built consensually by the teachers, it is possible to interpret that the situations in mathematics, articulated with the mentioned mathematical objects, indicate didactic possibilities for integration

⁵ The empirical data obtained from the manifestation of students in the 9th grade of elementary school were collected in the context of a curriculum subject of a public institution of basic education. In this sense, the professors' registers highlighted here do not reveal their identities, images, or voices, thereby ensuring the dignity and due protection of the participants in scientific research. For this reason, no prior permission was requested by the appropriate councils of the research project from which the work arises. Thus, we assume and exempt the Acta Scientiae from any consequences arising therefrom, including full assistance and possible compensation for any damage resulting from any of the research participants, as directed by Resolution n. 510, of April 7, 2016, of the National Health Council of Brazil.

within the SRP, under the hypothesis of variation between magnitudes, as developed in the subsequent modules.

Modules M₁/M₂: Experience an SRP as a student and analyse it under praxeological, didactic, and ecological conditions

In line with modules M₁ and M₂ of the SRP-TE, two teachers said the following:

- **P₁:** Before, I saw the rule of three as a mechanical procedure: set up a proportion, cross-multiply, and find the unknown value. I thought it was a resource restricted to simple, everyday situations, such as calculating discounts, proportions in recipes, or solving a quick reasoning problem. It was something useful, but limited, almost without conceptual depth [...].
- **P₃:** I saw the rule of three as a quick and effective strategy to find unknown quantities, based on the proportionality relationship between what is sought and the information that is already known, for example, to convert units of measurement. Despite using this knowledge, I did not understand its importance and how it could directly relate to other important topics in school mathematics [...].

Teachers' speeches allow us to interpret, in a timely manner, indications of changes in the quality of relationships (Chevallard, 2005) established with the rule of three. In particular, the records of P₁ and P₃ suggest a shift from a more procedural and restricted understanding to another in which this knowledge becomes associated with different objects of school mathematics. It is, however, a situated interpretation, anchored in the manifestations produced during the formative process, rather than a conclusive demonstration of a stable transformation. In this sense, Professor P₂'s speech, presented below, offers additional elements for this analytical reading:

- **P₂:** To exemplify the steps of the analytical-synthetic method and the construction of algebraic, general and specific formulas, we will use the following question: If 12 workers build 32 metres of a wall in 8 days. How many workers are needed to build 16 meters of wall in 6 days?

In the first step, we must outline the rule of three in two lines: the hypothesis, which is the information we already know about the problem, and the question, which is what we seek to discover. In the

following table, we have the scheme that shows in the first line the hypothesis and in the second line the question.

Dias (d)	Muro (m)	Trabalhadores (t)
8	32	12
6	16	t

The analysis step consists of analysing the quantities of the question, verifying whether they are directly or inversely proportional and reducing them to unity to find the constant of proportionality k.

Dias (d)	Muro (m)	Trabalhadores (t)
$\downarrow$ $\frac{8}{8}$ 6	$\uparrow$ $\frac{32}{32}$ 16	$\uparrow$ $12 \cdot \frac{8}{32} = k$ t

Dias (d)	Muro (m)	Trabalhadores (t)
$\downarrow$ 1 6	$\uparrow$ 1 16	$\uparrow$ 3 = k t

This means that 3 workers build 1 meter of wall in 1 day, and we find the value of the constant, thus:

$$k = \frac{t_0 \cdot d_0}{m_0} = 3$$

From the algebraic manipulation of the constant, we can obtain the formula to find the value of t, which we call the general formula:

$$t = \frac{k \cdot m}{d} = 3$$

From there, we start a combination, integration, when we organise the specific formula that will be used to find the answer:

$$t = \frac{3 \cdot 16}{6} \rightarrow \text{combination}$$

$$t = 8 \rightarrow \text{answer}$$

With all the data and answers organised, the table is assembled, and the possibility of verifying the graph generated by the t function is open.

Consulting the teachers' speeches and the situational context analysed by **P₂** helped us identify the following elements:

- The arrangement of the data, as highlighted by Silva (2017), in the situational study of the rule of three, considering the variation between the quantities involved;
- The analysis of the context of the problem and the delimitation of the relationships between the quantities, whether direct or inverse, using arrows to represent these relations;
- The practical meaning of the constant of the compound rule of three, used based on the reduction to unity procedure;
- The construction of the algebraic formula in contrast to the study of the situation in context, based on the analytical-synthetic method experienced by teachers as a mathematical technological-theoretical foundation;
- The description of the steps of the analytical-synthetic method: the question to be answered, the hypothesis about the type of problem, the process of analysing, integrating, and obtaining an answer to the specific problem, as presented by Tilmant (1875).

Indications, here interpreted as changes in the quality of relationships, in the sense proposed by Chevallard (2005), established by teachers with the rule of three, considering the localised character of the corpus constituted by the speeches of **P₁** and **P₃**:

- **P₁**: Now, after building this map of knowings and realising all the possible ramifications, my point of view has changed a lot. The rule of three is no longer just an algorithm and has come to be understood as a starting point for several fields of mathematics and even other sciences;
- **P₃**: Despite using this knowledge, I did not understand its importance or how it could directly relate to other important topics in school mathematics, such as the construction of tables and graphs of algebraic functions;

Paraphrasing these prospective teachers' speeches from Chevallard's (2009) perspective, when a praxeology is available that incorporates this technique and allows the construction of new praxeologies for teaching, the relationship between the subject and the rule of three is transformed. It “then

becomes an object that we know can be reached by a path that we would never have imagined following otherwise”⁶ (Chevallard, 2009, p. 5, our translation).

The exploration of situational practices using the rule of three, supported by the reduction to unity procedure, created conditions, in our understanding, to highlight algebraic mathematical models, analyse the relationships between quantities, build numerical tables for the study of the relationships of growth and degrowth between quantities, and graphically represent the surfaces, based on the algebraic formula and the relationships established between them, in articulation through the interrelationship between model and situation with mathematics.

In addition, this looking at the functionality of the algebraic formulation sent by P_2 , in the context of a situation with mathematics explained by it, can “become suitable to prepare students to write and read other more or less simple physical or mechanical laws [...]” (Tilmant, 1875, p. 25, our translation), in addition, of course, to allowing didactic openings about the strategic role of the procedure of reduction to unity in the description of formulas.

Based on modules M_0 , M_1 and M_2 of the SRP-TE, teachers designed an organisation for the teaching of school mathematics, considering the use of the rule of three and the notion of variation between magnitudes as a guiding hypothesis for processes of transposition undertaken with students in the 9th grade of elementary school, according to modules M_3 and M_4 .

M_3/M_4 Modules: Design and experiment an SRP in the classroom

Based on the practices studied, analysed, and built in modules M_0 , M_1 and M_2 of the SRP-TE (Ruiz-Olarría, 2015; Barquero, Bosch, & Romo-Vázquez, 2015; Sierra & Gascón, 2018; Florensa, Bosch, & Gascón, 2019; Almouloud, Nunes, Pereira, & Figueroa, 2021), the prospective teachers, together with 25 students, 9th graders of an elementary public school, conducted the study processes throughout the school year. These processes were guided by the practical knowledge of the rule of three, seeking to explain, during classes, didactic tasks that promoted openings of the rule of three towards the construction of other mathematical objects, without losing sight of

⁶ Fragment of the text: [...] est alors un objet à quoi l’on sait pouvoir arriver par un Chemin. qu’on n’aurait pas imaginé d’emprunter sinon.

mathematical situations, in order to attribute meaning and significance to these objects.

One of the tasks (t_1) the teachers presented to the students was:

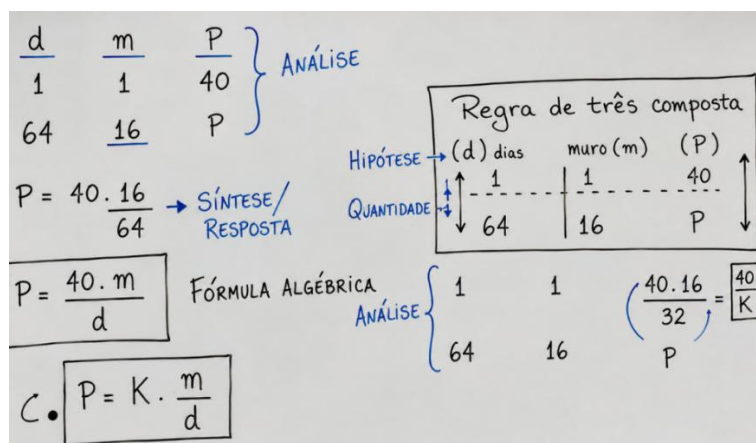
t_1 : Suppose 80 bricklayers build 32 m of wall in 16 days. How many bricklayers will it take to build 16 m of wall in 64 days? Under these conditions, determine:

- the constant of the relationships between the quantities;
- the steps of the analytical-synthetic method;
- algebraic formulas;
- the construction of the numerical table from the relationships between the quantities.

The answer constructed and highlighted by P_4 before the class was guided by the *style of rationality* (Chevallard, 1999) associated with the procedure of reduction to unity, as shown in Figure 1.

Figure 1

Teacher P_4 's record in the classroom: the use of reduction to the unit. (Research collection, 2025)

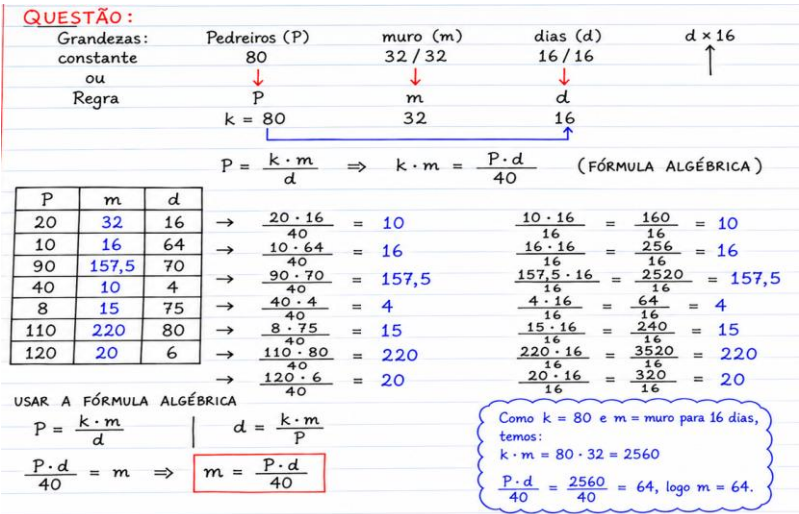


P_4 's manifestations reveal, locally, different ostensible objects, such as: data arrangements; the analysis of the relationships between the quantities of the problem considered in its context; the algebraic representation of the

formula of the compound rule of three; the calculation of the constant of the relationships between the quantities from the initial conditions of the problem, under the *style of rationality* (Chevallard, 1999) of reduction to unity; the steps of the analytical-synthetic method, in the sense proposed by Tilmant (1875); and the construction of numerical tables for the study of the relationships between the quantities.

The construction of the numerical tables enabled the study of relationships between the quantities by substituting numerical values into the resulting algebraic formula, as shown in Figure 2.

Figure 2
Construction of tables and algebraic formulas for the relationships between quantities. (Research collection, 2025)



Given the situation with the mathematics of the compound rule of three, expressed in Figure 2, from the construction of the numerical table, the representation of the algebraic formula described by $\left(p = 40 \cdot \frac{m}{d}\right)$ drew attention. In this specific case, (p) represents the number of masons, ($k = 40$) the constant of the relationship between the quantities, (m) the number of metres of wall to be built, and (d) the number of days.

Under this set of conditions, the class highlighted the relationships between the quantities involved in the problem in context at two different situational moments:

- In the mathematical situation given by the linear relationship between the variables number of masons (p) and size of the wall (m) to be built, the number of days (d) was adopted as a fixed value, in the sense of a numerical constant, and not necessarily as a variable;
- In the specific mathematical situation of the relationships between the quantities number of masons (p) and number of days (d), interpreted geometrically in the hyperbolic form, the variable wall size was considered (m) as a fixed value, that is, as a numerical constant and not exactly as a variable for this situation under analysis.

These interpretations point, to some extent, to didactic possibilities that can be explored through the ideas associated with the compound rule of three and the variation between magnitudes, when it is necessary to fix one variable as a numerical constant to establish relationships among the other variables involved.

On the other hand, the relationships between the quantities, when described in functional language, can serve as a heuristic starting point for discussing functions of various variables. In this case, the approach to themes such as the derivation or integration of real functions of two variables must be understood in a didactic-exploratory sense, rather than as an immediate formal consequence of the work carried out with the compound rule of three.

From this perspective, by numerically relating the number of masons to the size of the wall, the quantity "number of days" can be fixed as a constant for analysis, and vice versa. From these relational aspects, the algebraization of the compound rule of three can be interpreted as an initial didactic hypothesis to explore, at the introductory level, certain ideas associated with the study of functions of two variables.

Thus, any tasks involving first-order partial derivatives should be understood as prospective and exploratory developments of this didactic organisation, and not as results directly demonstrated by the empiricism presented here. Under this framework, the following situations can be highlighted:

- In relation to $x(f_x)$:, it treats y as a constant and differentiates it in relation to x ;
- $f_x = 6 \cdot x \cdot y$

- In relation to $y(f_y)$; it treats x as a constant and differentiates it in relation to y ;
- $f_y = 3x^2 + 5 - 3 \cdot y^2$;

In connection with the aspects above, the way of thinking and operating between the algebraized notions of the compound rule of three and the introductory study of real functions of various variables allows us to glimpse possible didactic links, as suggested in Figure 3, when taking the compound rule of three as a particular case within the universe of real functions of various variables.

Figure 3

*Didactic possibilities between the rule of three and the calculation study.
(Research authors, 2026)*



In any case, in the study of the relationships between the variables, “We are touching here the borderline that will separate what can be called “algebraic literacy” from “algebraic illiteracy”. Certain institutions [...] choose to 'spare' their students the necessary algebraic 'work'” (Strømskag & Chevallard, 2022, p. 384).

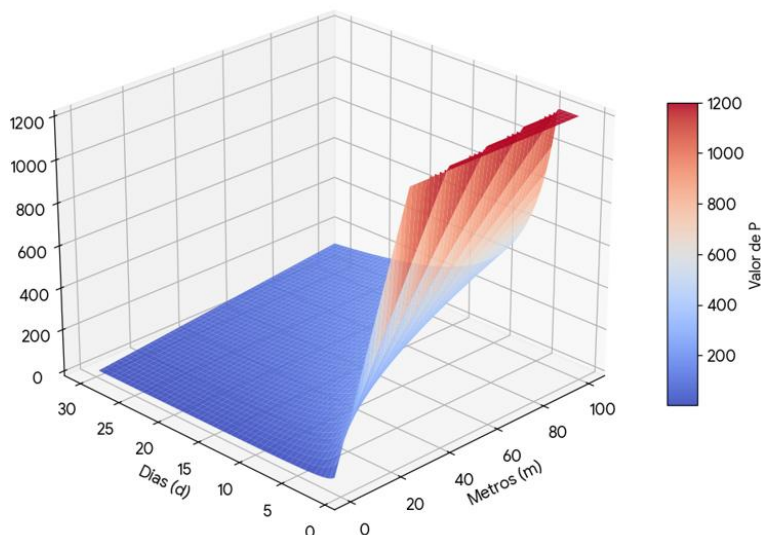
In view of the construction of the algebraic formula $\left(p = 40 \cdot \frac{m}{d}\right)$ and the study of the relationships between the variables, p_3 raised the following question (q_1):

- q_1 : How to graph the relationship between the quantities expressed by: $p = 40 \cdot \frac{m}{d}$?

The confrontation of the issue (q_1) by the class required the introduction of a didactic condition to the study process, in the terms of Chevallard (2009): the use of the computer. This condition proved relevant for enabling the geometric representation of the surface, whose construction and schematic organisation would be more limited without this resource. In addition to allowing graphical visualisation of the relationships between the quantities, the computer facilitated exploration of variations around the constant ($k = 40$), determined from the conditions of the situation and the mathematics under analysis, as illustrated in Figure 4.

Figure 4

Geometric representation of the algebraic relationship between $\left(p = 40 \cdot \frac{m}{d}\right)$. (Research collection, 2025)



From the perspective of didactic conditions, including the use of the computer by the class in the study of the relationships expressed by $\left(p = 40 \cdot \frac{m}{d}\right)$, it is necessary to consider that “the conditions cannot be listed a priori: their discovery is progressive” (Chevallard, 2009, p. 12, emphasis added, our translation).

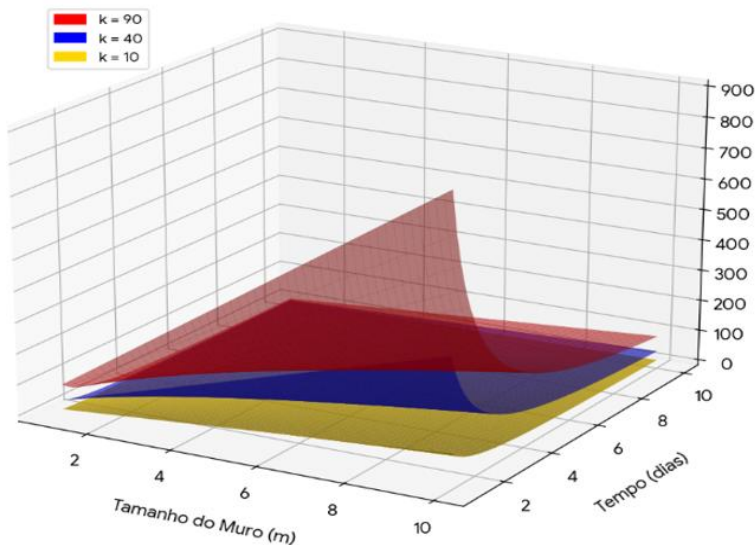
Thus, the study process mediated by the computer raised new questions:

- $P_3 \rightarrow q_{1.1}$: What happens to the graphical representation when the numerical value of the constant (k) changes, i.e., if this constant is a numeral less than 40? And if the value is greater than 40, how is it?
- $P_2 \rightarrow q_{1.2}$: How will the point referring to the number of masons (p) be represented in the graph, as a function of 220 meters of wall in 80 days?
- $P_2 \rightarrow q_{1.3}$: How will the point referring to the size of the wall to be built (m) be represented in the graph, as a function of the number of 90 masons in 70 days of work?

The use of the computer by the class of basic education students showed, in our understanding, its potential for the exploration of projections and analyses of the surface graph, especially in the study of the variation of the constant (k) between the quantities involved in this type of problem in context, as illustrated in Figure 5.

Figure 5

The impact of constant analysis (k) on the relationship between the quantities. (Research collection, 2025)



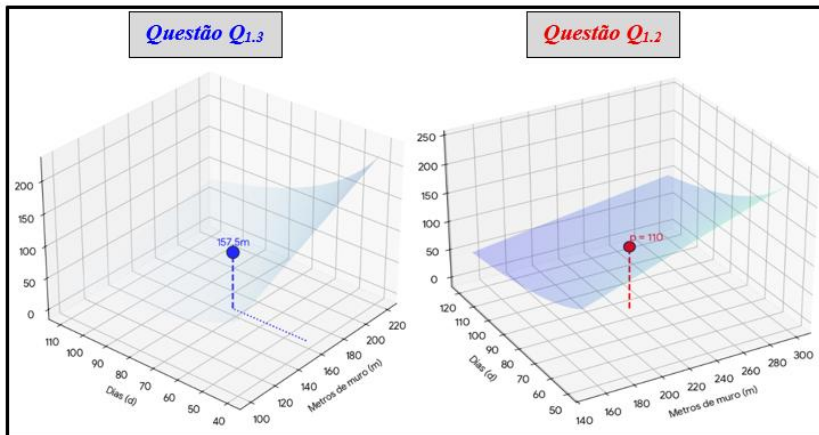
The variation of the constant ($\mathbf{k}$) gave rise to the following manifestations of the didactic systems $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$, respectively:

- $\mathbf{S}_1 (\mathbf{X}_{q_{1.1}}, \mathbf{P}_{3(q_{1.1})}, q_{1.1})$: When the constant ($\mathbf{k}$) is higher, more labour will be needed, depending on the demand for the size of the wall that will be built [...];
- $\mathbf{S}_2 (\mathbf{X}_{q_{1.1}}, \mathbf{P}_{3(q_{1.1})}, q_{1.1})$: The greater the number ($\mathbf{k}$), the greater the number of masons and the more metres of wall they make; as a result, the less time will be needed; we think that if we do not get it wrong, it is about the growth behaviour stretching or flattening [...];
- $\mathbf{S}_3 (\mathbf{X}_{q_{1.1}}, \mathbf{P}_{3(q_{1.1})}, q_{1.1})$: Teacher, looking at the graph, we can clearly see that the constant ($\mathbf{k}$) heavily affects the number of masons needed. When ($\mathbf{k}$) increases (like for $\mathbf{k} = 90$), the surface grows much faster, especially when the time is short; i.e., it needs a lot more people to account for it in the same period. With a smaller ($\mathbf{k}$) (like for $\mathbf{k} = 10$), the growth is much smoother, so the demand for masons does not grow as much, even when increasing the size of the wall. [...] the main conclusion is that the ($\mathbf{k}$) functions as a factor of “intensity” of the problem: it amplifies or softens the impact of the size of the wall and time. And it is clear that the behaviour is reversed over time: the more days, the fewer masons, but this effect becomes much more critical when ($\mathbf{k}$) is more expressive.

The specific mathematical situations analysed here allowed the study of the questions ($q_{1.2}$) and ($q_{1.3}$), related to the location of points on the surface of the graph, as shown in Figure 6.

Figure 6

Geometric representation of specific points on the surface. (Research collection, 2025)



The referral given by the class to the study of the questions $\{q_1, q_{1.1}, q_{1.2}, q_{1.3}\}$ not only revealed, in our understanding, the construction of qualities of relationships (Chevallard, 2005) with visual objects mediated by the computer but also showed how this didactic condition ($\mathbb{C}_1$) expanded the possibilities of performing relational calculations between the variables, as well as geometric visualisation of their growth and degrowth behaviours.

From this perspective, contact with such objects can favour different levels of approximation to a praxeology. As Chevallard (2009, p. 20, emphasis added, our translation) points out, “knowing a praxeology $\wp$ can be simply crossing with $\wp$, without more, or manipulating $\wp$ a little, or working a little, until further study”⁷.

Under this approach, the use of the computer signalled possibilities of articulation, here understood as an extension of human activities, as conceived by Tikhomirov's theory of reorganisation (1981, p. 278), according to which

⁷ Fragment of the text: rencontrer une praxéologie $\wp$, ce peut être simplement *croiser* $\wp$, sans plus, ou bien *manipuler* $\wp$ un tant soit peu, ou encore *travailler* $\wp$ quelque peu, voire l'*étudier* d'une façon plus approfondie.

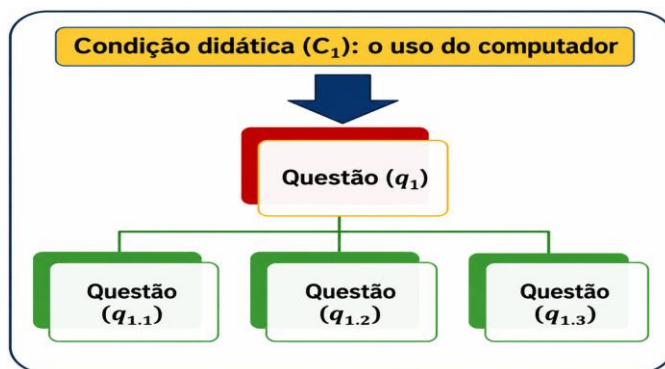
there are situations “in which a problem is solved jointly by humans and computer, that is, by the human-computer system itself”.

In the same vein, Tikhomirov (1989, p. 352) argues that the “interaction between the human operator and the computer assumes increasingly striking traits of joint activity - cooperation or rivalry”. The dynamics of the man-machine relationship showed the possibility of expanding the subject's action in the study of a situation using mathematics.

Thus, in addition to enabling the opening of new didactic perspectives “to process models with complex function terms or to reduce the calculation effort. Digital tools can perform a series of tasks in the teaching of mathematical modeling” (Greefrath & Vorhölter, 2016, p. 21, our translation), the use of the computer created conditions for the study of different issues arising from the geometric construction of the algebraic formulas associated with the compound rule of three. The questions, although situated in the field of the possible, could go unnoticed or unaddressed without the use of this didactic condition introduced in the study process, as illustrated in Figure 7.

Figure 7

The structuring role of computer use as a didactic condition. (Research authors, 2026)



With this look, the use of the computer as an integral and necessary entity in the activities developed by the class allows us to glimpse clues of articulations, albeit specific to the corpus analysed here, of possible changes in the epistemology of the rule of three as practical social know-how. Such connections present potential indications for the structuring of other

mathematical objects, including polynomials, motivated by the following questions:

- q_2 : How to calculate the Brazilian population in a given time frame?
- $q_{2.1}$: What is the average time of daily screen use on mobile devices?

The study of the question q_2 used real data on the Brazilian population for 2000, 2010, and 2020, corresponding, respectively, to approximately 174.7, 191, and 212 million inhabitants, made available by the Brazilian Institute of Geography and Statistics (IBGE). Figure 8 seeks to show the didactic relationships established between the years considered, the reference year and the corresponding population, approximately, in each analysed period.

Figure 8

Estimation of the population using the mathematical model. (Research collection, 2025)

ANOS	REF.	POPULAÇÃO	K_1	K_2
2000	0	174,7	1,63	
2010	10	191	2,1	0,0235
2020	20	212		

i) $\frac{191 - 174,7}{10 - 0} = 1,63$

ii) $\frac{212 - 191}{20 - 10} = 2,1$

iii) $\frac{2,1 - 1,63}{20 - 0} = 0,0235$

POLINÔMIO: $P(x) = y$

$y = y_0 + K_1(x - x_0) + K_2(x - x_0)(x - x_0)$

$y = 174,7 + 1,395x + 0,0235x^2$

$P(15) = 200,93$

$P(25) = 224,26$

A situation with specific mathematics was delimited for the study of the question (q_2), more precisely, with the objective of calculating, through a mathematical task, the numerical value of the polynomial obtained to estimate the Brazilian population in 2015 and 2025. The projection carried out in this time frame considered the variations between the quantities, described successively as follows:

- First variation: $k_1 = \frac{191-174,7}{10-0} = 1,63$;
- Second variation: that is, the variation of the variation between the quantities: $k_2 = \frac{2,1-1,63}{20-0} = 0,0235$;

The study of this situation with mathematics, in contrast to the polynomial mathematical model generated by the relations of variation between the quantities, led the auxiliary didactic system $S_3(X_{q_2}, P_{3(q_2)}, q_2)$ to the following manifestation:

- The model of the second degree polynomial is acceptable for the answers found, as it presents a good estimate for the population count, when considering the percentage values of the errors when comparing the answers found with the second degree polynomial in relation to the values provided by the IBGE for the population count.

The manifestations of the class were guided by the prospective teachers through the study and the construction, before the class, of polynomials of 1st and 2nd degrees. For this, the rule of three directly proportional was adopted as a hypothesis of support, according to which the ratio between the quantities and their variations remain constant. These constants, designated by k_1 and k_2 correspond, respectively, to the first and second variations, as shown in the records in Figure 9.

Figure 9

Algebraic description of the polynomials from the variation between quantities. (Research collection, 2025)

x_1	y_1
x_0	y_0

Se as grandezas são diretamente proporcionais,

Logo: $\frac{y_1 - y_0}{x_1 - x_0} = K_1 \Rightarrow K_1 = \text{primeira variação}$

$y_1 - y_0 = K_1 (x_1 - x_0)$

$y_1 = y_0 + K_1 (x_1 - x_0)$

$y = y_0 + K_1 (x - x_0)$

$y = y_0 + K_1 (x - x_0)$

Assim, a expressão: $y = y_0 + K_1 (x - x_0)$, pode ser escrita como:

$P(x) = y_0 + K_1 (x - x_0)$.

Para três pontos temos:

x_i	y_i	K_1	K_2
x_0	y_0	$\frac{y_1 - y_0}{x_1 - x_0}$	$\frac{y_2 - y_1}{x_2 - x_1} \cdot \frac{y_1 - y_0}{x_1 - x_0}$
x_1	y_1	$\frac{y_2 - y_1}{x_2 - x_1}$	
x_2	y_2		

$y_2 = y_0 + K_1 (x_2 - x_0) + K_2 (x_2 - x_0) (x_2 - x_1)$

$\vdots$

$y = y_0 + K_1 (x - x_0) + K_2 (x - x_0) (x - x_1)$

} Função polinomial de 1º grau

The experimentation with the SRP within the methodological structure of the RMM allows us to identify, from the manifestations of the class, the study of different mathematical situations (Sodré, 2023) and the displacement of the rule of three as practical knowing for broader forms of organisation of knowledge.

In this movement, the understanding of variations between quantities can be related, in an interpretative and didactic key, to ideas from Newton's

polynomial interpolation method by finite divided differences. This approach, however, should be understood as a formative analogy and a hypothesis of conceptual articulation, and not as a formal equivalence already established by the empirical analysis developed in this work.

From this perspective, the focus attributed to the rule of three, in the style of directly proportional rationality (Chevallard, 1999), and to the general formula of Newton's interpolation by divided differences can be correlated didactically in the following terms. To this end, we delimit, in an introductory manner, the data involving only two variations in the quantities, as shown in Chart 1.

Chart 1

Possible didactic relationships between the rule of three and Newton's interpolation. (Research authors, 2026)

Rule of three	Newton's interpolation by divided differences
First variation $k_1 = \frac{y_1 - y_0}{x_1 - x_0}$	First order $f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$
Second variation: the variation of the variation $k_2 = \frac{\frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0}}{x_2 - x_0}$	Second order $f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$

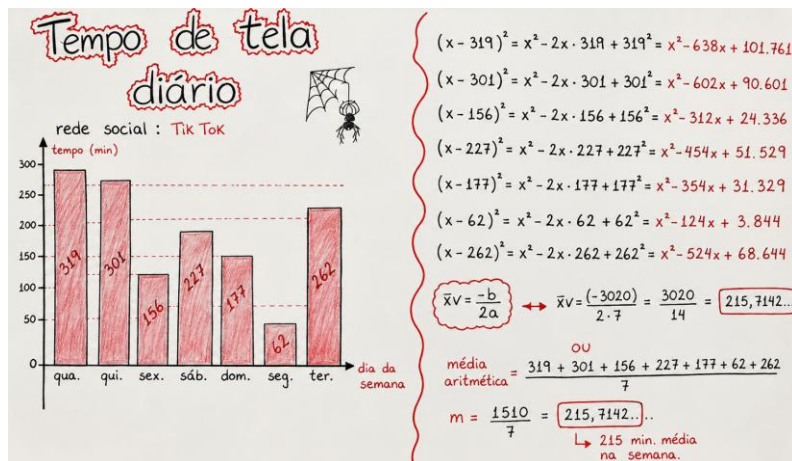
Mediated by this introductory context, and taking the coefficients $b_0 = f[x_0]$, $b_1 = f[x_0, x_1]$, $b_2 = f[x_0, x_1, x_2]$, ..., $b_n = f[x_0, x_1, \dots, x_n]$, the interpolator polynomial can be expressed by:

- $P_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \dots + b_n(x - x_0)(x - x_1) \dots (x - x_{n-1})$, in which $b_k = f[x_0, x_1, \dots, x_k]$.

The study of 2nd degree polynomials allowed the class to create conditions on the spot to answer the question (**Q2.1**), referring to the average daily time spent using screens on mobile devices, based on an application chosen consensually by the participants. The data obtained through this application are shown in Figure 10.

Chart 10

Calculation of the average screen time. (Research collection, 2025)



The description of the 2nd degree polynomial [$E(x) = (x - 319)^2 + (x - 307)^2 + (x - 156)^2 + (x - 227)^2 + (x - 177)^2 + (x - 62)^2 + (x - 262)^2 = 7x^2 - 3020x + 375.692$], opened didactic possibilities for the problematization of the *raison d'être* of notable products and the calculation of the value that minimizes the polynomial. This context has been interpreted here through a type of mathematical task that consists of determining the value that minimizes a function.

This polynomial draws attention because the value that minimizes the function can be interpreted, in this context, as an approximation of the arithmetic mean of the terms $\{319, 307, 156, 227, 177, 62 \text{ and } 262\}$. This didactic openness, based on minimizing the sum of quadratic errors, allows us to suggest a conceptual relationship with the least squares method.

More precisely, the arithmetic mean is a particular case of a least-squares fit problem: determining a constant value that best represents a set of data by minimising the sum of the squares of the deviations. In this formulation, it is appropriate to present this relationship as a specific, clearly delimited mathematical connection, avoiding treating it as a broad consequence of the empirical path developed in this work.

Thus, we obtain the following minimisation problem:

$$S = \sum_{i=1}^n (y_i - f(x_i))^2$$

Considering a constant function $f(x) = \mu$, the minimisation problem leads to the following expression:

$$S = \sum_{i=1}^n (y_i - \mu)^2 \rightarrow \mu = \frac{1}{n} \sum_{i=1}^n y_i \rightarrow \text{Arithmetic mean}$$

Considering a data set formed by five generic terms, $\{p, q, r, s, t\}$, the problem of determining the value of (x) associated with the vertex of the parabola that minimises the sum of the quadratic errors can be expressed as follows:

$$E_3(x) = (x - p)^2 + (x - q)^2 + (x - r)^2 + (x - s)^2 + (x - t)^2$$

$$E_3(x) = (1 + 1 + 1 + 1 + 1) \cdot x^2 - 2(p + q + r + s + t) \cdot x + (p^2 + q^2 + r^2 + s^2 + t^2)$$

$$E_3(x) = 5 \cdot x^2 - 2 \cdot (p + q + r + s + t) \cdot x + (p^2 + q^2 + r^2 + s^2 + t^2)$$

The value of (x) corresponding to the vertex of the parabola, denoted by (x_v) and given by: $x_v = -\frac{b}{2 \cdot a}$, which minimises the 2nd degree polynomial function, when $(a > 0)$, can be expressed by:

$$x_v = \frac{-(-2)(p + q + r + s + t)}{2 \cdot (1 + 1 + 1 + 1 + 1)} \rightarrow M_A = \frac{(p + q + r + s + t)}{5}$$

Ultimately, the empirical excerpt highlighted here suggests, in an introductory character, the methodological role of RMM in leading the class, during the study process, to meet mathematical situations placed in contrast to mathematical models organised under the didactic hypothesis of the rule of three, guided by the analysis of variations between quantities related to each other.

In this specific context, this hypothesis played the role of a starting point for the didactic exploration of different mathematical objects of interest for school education, among which the construction of numerical tables, the study of the relationships between quantities, the description of algebraic formulas, the analysis of geometric surfaces and the study of polynomials of 1st and 2nd degrees stand out. These explorations were developed under the institutional constraints of the elementary school class, as summarised in Figure 11.

Figure 11

RMM: the focus of situations with mathematics. (Research authors, 2026)

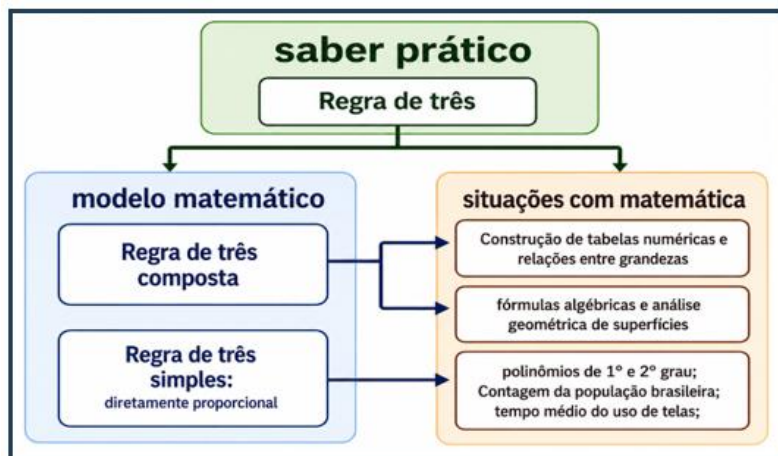


Figure 11 summarises and presents results interpreted as indications of changes in the quality of teachers' relationship with the rule of three, focusing on the interrelationships between mathematical situations and the model. In this context, the methodological dynamics of RMM created conditions, albeit insufficient, to encompass all the possibilities of study, to attribute meaning and significance to different mathematical objects that are often dissociated from a possible ecology of interconnected knowings. Thus, knowledge could be mobilised in several contexts, not as isolated knowledge dependent on specific situations, but as elements of a network of mathematical relationships.

The teachers' manifestations, guided by the invariant notion of variation between magnitudes and the relationships established between them, show, in an introductory way, localised and exploratory contributions to the essay of a possible answer to the problem pointed out by Bosch, García, Gascón, and Ruiz Higuera (2006) about the disarticulation of knowings in school mathematics.

FUTURE RECOMMENDATIONS: THE “OPEN PROBLEM” OF THE DIDACTIC TRANSPOSITION OF THE RULE OF THREE

This investigation aimed to highlight RMM as a methodological device for teacher education, understanding it as a necessary condition for the interrelation between model and situation, capable of favouring processes of deconstruction and reconstruction of mathematical models in close conjunction with the strategic and, in many cases, indispensable role played by situations with mathematics (Sodré, 2023; Sodré & Guerra, 2024). From this perspective, the preservation of the idea of variation between magnitudes was interpreted as an element of centrality, for producing, in the sense attributed by Chevallard (2005, p. 77, emphasis added, our translation) and reinforced by the empiricism built with teachers, a “contradictory balance *between past and future*: it is a *transactional object* between past and future”.

Although the rule of three is predominantly highlighted in the theoretical discourse on proportion as a style of rationality (Chevallard, 1999), its historical-epistemological trajectory (Datta & Singh, 1938; Sarma, 2002; Høyrup, 2012; Silva, 2011, 2017) is not limited to this interpretation. Its social relevance across scientific, educational, and professional fields highlights the strength and structuring role of the reduction-to-unity procedure, which remains undervalued in parts of the specialised literature, official curriculum documents, and some research in the field of ATD.

The empirical results found indicate that the idea of variation between magnitudes, interpreted in the study as directly proportional, occupied an organising position in the construction of polynomial mathematical models and in the approach to mathematical situations (Sodré, 2023), such as population projection and analysis of the average time of use of screens. These results can be understood as localised contributions of the RMM methodology (Sodré, Ferreira, & Guerra, 2022; Sodré, Guerra, & Ferreira, 2026).

It should be noted, however, that the possibility of progressively understanding the variation between quantities in conjunction with procedures like those mobilised by Newton's polynomial interpolation method constitutes, in this article, an interpretative hypothesis of didactic expansion, whose relevance should be deepened and examined in subsequent studies.

RMM favoured, among prospective teachers, changes in the quality of the relationships established with the rule of three (Chevallard, 2005), by allowing the articulation between this object and other knowledge of interest

for the teaching of school mathematics and, equally, different fields of knowledge, whose ways of thinking and acting are based, to a greater or lesser extent, on ideas associated with the rule of three.

Moreover, the introduction of the use of the computer, in the light of the theory of reorganisation proposed by Tikhomirov (1981, 1989), gave rise to new study questions, serving, in our interpretation, as an important mediator for the exploration of geometric representations and for the relational study of variations between quantities, aspects that would hardly be developed by the class with the same agility in its absence. Under another prism, the computer, understood as one of the study conditions in the sense of ATD, can be interpreted in light of Tikhomirov (1981) as an extension of human activities.

Ultimately, the processes analysed in mathematical situations, in contrast to the models of the rule of three, offer localised evidence compatible with the hypothesis formulated by Silva (2017), according to which the rule of three can play a structuring role in the school mathematical modelling of different mathematical objects.

The empirical findings described here, although preliminary, suggest the potential of this hypothesis for studying relationships between quantities and for structuring the 1st degree and 2nd degree polynomials that emerged during the investigative process. In this sense, the work encourages future investigations, from the perspective of the RMM, aimed at designing praxeological organisations of greater scope for the study of mathematical situations, considering the role of variation among magnitudes in the analysed context.

From these perspectives, the problem inherent in the field of investigation of the didactics of mathematics in which the ATD is inserted, related to the question of how to teach the rule of three, taking it as a providing hypothesis for the construction and connection of the interrelationships between mathematical models and situations, is here interpreted as a type of *open problem* (Gompertz, 2016; Aldon, 2021) of didactic transposition.

This interpretation stems from the potential this problem presents to spur new research programs and diverse study scenarios, involving not only mathematical objects within the reach of elementary schools but also other domains, such as logarithms, the exponential function, and financial mathematics, among others.

Perhaps for this reason, Chevallard (2019, p. 74, our translation) states that “problems of extension and complexity (or subtlety) are the main factors

and incentives of didactic transposition”. In this sense, the results presented here allow us to support, albeit in an exploratory way, the relevance of deepening the study of the rule of three as an object of didactic investigation capable of favouring new links between school mathematical knowings and expanding the possibilities of construction and reconstruction of models for the study of situations with mathematics.

AUTHORSHIP CONTRIBUTION STATEMENT

GJMS, SAA and JMVN conducted the investigation presented. SAA, JMVN and RBG oversaw the planning and execution, delimiting the methodological use of reverse mathematical modelling. GJMS and SAA planned the research question, objective, theoretical and methodological resources, analyses, and empirical results, as well as the investigation's recommendations and perspectives.

DATA AVAILABILITY STATEMENT

The data supporting this study and investigation will be made available by the corresponding author (GJMS), upon prior request.

REFERENCES

- Aldon, G. (2021). *Les problèmes ouverts du Rallye Mathématique de l'Académie de Lyon 2011-2020*. Lyon: IREM de Lyon; Rallye Mathématique de l'Académie de Lyon, 1-149.
- Almouloud, S. A., Nunes, J. M. V., Pereira, J. C. S., & Figueroa, T. P. (2021). Percurso de estudo e pesquisa como metodologia de pesquisa e de formação. *Revista de Educação do Vale do São Francisco*, 11(24), 426–466.
- Ávila, G. (1986). Razões, proporções e regra de três. *Revista do Professor de Matemática*, (8), 1–8.
- Barquero, B. (2009). *Ecología de la modelización matemática en la enseñanza universitaria de las matemáticas* (Tese de doutorado, Universitat Autònoma de Barcelona).
- Barquero, B., Bosch, M., & Romo-Vázquez, A. (2015). Mathematical modelling in teacher education: Dealing with institutional constraints. *Teaching Mathematics and Its Applications*, 34(4), 202–219.
- Bolea, P. (2003). *El proceso de algebrización de organizaciones matemáticas escolares* (Tese de doutorado, Universidad de Zaragoza).

- Bosch, M. (1994). *La dimensión ostensiva en la actividad matemática* (Tese de doutorado, Universitat Autònoma de Barcelona).
- Bosch, M., García, F. J., Gascón, J., & Ruiz Higuera, L. (2006). La modelización matemática y el problema de la articulación de la matemática escolar: Una propuesta desde la teoría antropológica de lo didáctico. *Educación Matemática*, 18(2), 37–74.
- Chevallard, Y. (1980). Mathématiques, langage, enseignement: La réforme des années soixante. *Recherches*, (41), 71–99.
- Chevallard, Y. (1989). *Le concept de rapport au savoir: Rapport personnel, rapport institutionnel, rapport officiel*. Université Joseph Fourier.
- Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, 19(2), 221–265.
- Chevallard, Y. (2005). *La transposición didáctica: Del saber sabio al saber enseñado*. Aique Grupo Editor.
- Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, 12, 71–114.
- Chevallard, Y., Bosch, M., & Gascón, J. (2001). *Estudar matemáticas: O elo perdido entre o ensino e a aprendizagem*. Artmed.
- Datta, B., & Singh, A. N. (1938). *History of Hindu mathematics*. Asia Publishing House.
- Florensa, I., Bosch, M., & Gascón, J. (2019). Study and research paths: A model for inquiry. *Educational Studies in Mathematics*, 102(2), 191–212.
- García, F. J. (2005). *La modelización como herramienta de articulación de la matemática escolar: De la proporcionalidad a las relaciones funcionales* (Tese de doutorado, Universidad de Jaén).
- Gompertz, A. (2016). *Les problèmes ouverts, une piste de différenciation pédagogique? (CMI)* (Dissertação de mestrado). DUMAS.
- Greefrath, G., & Vorhölter, K. (2016). *Teaching and learning mathematical modelling: Approaches and developments from German-speaking countries*. Springer.

- Guerra, R. B., & Silva, F. H. S. (2009). Reflexões sobre modelagem matemática crítica e o fazer matemático da escola. *Perspectivas da Educação Matemática*, 2(3), 95–119.
- Høyrup, J. (2012). *Jacopo da Firenze's Tractatus Algorismi and Early Italian Abbacus Culture*. Birkhäuser.
- Krawitz, J., Schukajlow, S., Yang, X., & Geiger, V. (2025). A systematic review of international perspectives on mathematical modelling: Modelling goals and task characteristics. *ZDM – Mathematics Education*, 57, 193–212.
- Ladage, C., & Chevallard, Y. (2010). Enquêter avec l'Internet: Étude d'une nouvelle condition du travail didactique. *Recherches en Didactique des Mathématiques*, 30(1), 85–115.
- Lima, E. L. (2001). *Meu professor de matemática e outras histórias*. Sociedade Brasileira de Matemática.
- Ruiz-Munzón, N. (2010). *La introducción del álgebra elemental y su desarrollo hacia la modelización funcional* (Tese de doutorado, Universitat Autònoma de Barcelona).
- Ruiz-Munzón, N., Bosch, M., & Gascón, J. (2015). El problema didáctico del álgebra elemental: Un análisis macro-ecológico desde la teoría antropológica de lo didáctico. *REDIMAT*, 4(2), 106–131.
- Ruiz-Olarría, A. (2015). *La formación matemático-didáctica del profesorado de secundaria: De las matemáticas por enseñar a las matemáticas para la enseñanza* (Tese de doutorado, Universidad Autónoma de Madrid).
- Ruiz-Olarría, A., Bosch, M., & Gascón, J. (2019). Construcción de una praxeología para la enseñanza en la institución de formación del profesorado. *Educación Matemática*, 31(2), 132–160.
- Sarma, S. R. (2002). Rule of three and its variations in India. In Y. Dold-Samplonius, J. W. Dauben, M. Folkerts, & B. van Dalen (Eds.), *From China to Paris: 2000 years transmission of mathematical ideas* (pp. 133–156). Franz Steiner Verlag.
- Serrano, L. (2013). *La modelización matemática en los estudios universitarios de economía y empresa: Análisis ecológico y propuesta didáctica* (Tese de doutorado, Universitat Ramon Llull).

- Sierra, T. A., & Gascón, J. (2018). Los recorridos de estudio e investigación en la teoría antropológica de lo didáctico. *Educación Matemática*, 30(2), 9–35.
- Silva, D. P. (2011). *Regra de três: Prática escolar de modelagem matemática* (Dissertação de mestrado, Universidade Federal do Pará).
- Silva, D. P. (2017). *A invariável prática da regra de três na escola* (Tese de doutorado, Universidade Federal do Pará).
- Sodré, G. J. M. (2019). *Modelagem matemática escolar: Uma organização praxeológica complexa* (Tese de doutorado, Universidade Federal do Pará).
- Sodré, G. J. M. (2021). Mathematical modelling and didactic moments. *Acta Scientiae*, 23(3), 96–122.
- Sodré, G. J. M. (2023). O descortinar da noção de situação em modelagem matemática escolar. *Educação Matemática Pesquisa*, 25(1), 200–226.
- Sodré, G. J. M., Ferreira, R. S. R., & Guerra, R. B. (2022). Modelagem matemática reversa. *Acta Scientiae*, 24(6), 552–583.
- Sodré, G. J. M., & Guerra, R. B. (2018). O ciclo investigativo de modelagem matemática. *Educação Matemática Pesquisa*, 20(3), 239–262.
- Sodré, G. J. M., & Guerra, R. B. (2024). Um cenário da matemática financeira escolar sob a modelagem matemática reversa. In S. A. Almouloud, R. B. Guerra, J. M. V. Nunes, & J. C. S. Pereira (Orgs.), *Grupo de estudos e pesquisas em didática da matemática: Contribuições para a educação matemática* (pp. 109–134). Editora Akademy.
- Sodré, G. J. M., Guerra, R. B., & Ferreira, R. S. R. (2026). Um modelo matemático e várias anomalias. *Educação Matemática Pesquisa*, 28, 1–26.
- Strømskag, H., & Chevallard, Y. (2022). Inquiry in mathematics education from the perspective of the Anthropological Theory of the Didactic. *Journal of Mathematical Behavior*, 65, Article 100934.
- Tikhomirov, O. K. (1981). The psychological consequences of computerization. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (p. 256–278). M. E. Sharpe.
- Tikhomirov, O. K. (1989). The theory of activity changed by information technology. In Y. Engeström, R. Miettinen, & R.-L. Punamäki (Eds.),

Perspectives on activity theory (p. 347–364). Cambridge University Press.

Tilmant, V. (1875). *Réforme analytique de la règle de trois ou règle d'or et notions d'analyse mathématique d'après Descartes, Pascal et Arnauld*. Librairie Classique de Paul Dupont.