

The construction of artefacts with friezes and rosettes: connecting mathematics with visual arts

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ABSTRACT

Background: Geometry plays a central role in mathematics education by fostering visual reasoning and spatial awareness. Within geometry, symmetry provides meaningful opportunities to connect mathematics with artistic expression and cultural heritage. However, research has shown that students and pre-service teachers often experience difficulties in understanding and applying symmetry concepts beyond procedural approaches. **Objectives:** This study aims to understand how pre-service elementary school teachers solve symmetry tasks, including the construction and decoration of artefacts, based on frieze and rosette patterns, and how these experiences contribute to the development of visual reasoning. **Design:** The study followed a qualitative, interpretative, and exploratory research design, focused on a didactical intervention, combining progressively challenging tasks involving paper-and-pencil activities, manipulatives, digital tools, and the design of artistic artefacts based on friezes and rosettes. **Setting and Participants:** The participants were 26 pre-service elementary school teachers enrolled in a master's programme in Primary Education at a public higher education institution in Portugal, attending a Mathematics Education course, who voluntarily participated in the study. **Data collection and analysis:** Data were collected through participant observation, written documents, artefacts produced by the participants, and photographic records. The data were analysed inductively using content analysis. **Results:** The findings indicate that engaging with multiple symmetry tasks and constructing artistic artefacts supported participants in revising some initial misconceptions, strengthening aspects of their understanding of symmetry transformations, and showing evidence of progression from predominantly intuitive visual approaches towards more analytical and justified reasoning. The tasks also promoted creativity and motivation, while encouraging participants to establish meaningful connections between mathematical concepts and artistic production. These results extend previous research by demonstrating how aesthetic-constructionist learning environments can simultaneously support mathematical understanding and artistic expression in teacher education. **Conclusions:** Integrating visual art-based activities into mathematics teacher education can promote visual reasoning, deepen

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conceptual understanding of symmetry, and motivate future teachers to view mathematics as a creative and cultural human endeavour. Engaging pre-service teachers in designing mathematically informed artefacts provides didactical experiences that can enrich geometry teaching through meaningful connections between mathematics and the visual arts.

Keywords: Connections; Visual arts; Friezes and rosettes; Artefacts; Teacher education.

A construção de artefactos com frisos e rosáceas: conexões entre a Matemática e as Artes Visuais

RESUMO

Contexto: A Geometria desempenha um papel central na educação matemática, promovendo o desenvolvimento do raciocínio visual e da orientação espacial. No âmbito da Geometria, as simetrias proporcionam oportunidades significativas para estabelecer conexões entre a Matemática, a expressão artística e o património cultural. No entanto, a investigação tem mostrado que alunos e futuros professores revelam frequentemente dificuldades na compreensão e aplicação de conceitos de simetria para além de abordagens procedimentais. **Objetivos:** Este estudo procura compreender o modo como futuros professores do ensino básico resolvem tarefas sobre simetrias, incluindo a construção e decoração de artefactos com base em frisos e rosáceas, e de que modo estas experiências contribuem para o desenvolvimento do raciocínio visual. **Design:** O estudo seguiu uma abordagem qualitativa, interpretativa e exploratória, centrada numa intervenção didática que combinou tarefas progressivamente desafiantes, envolvendo atividades de papel e lápis, materiais manipuláveis, ferramentas digitais e a conceção de artefactos artísticos baseados em frisos e rosáceas. **Ambiente e participantes:** Participaram no estudo 26 futuros professores do ensino básico, inscritos num mestrado em Educação Básica de uma instituição pública de ensino superior em Portugal, a frequentar uma unidade curricular de Educação Matemática, que aceitaram participar voluntariamente no estudo. **Recolha e análise de dados:** Os dados foram recolhidos através de observação participante, documentos, artefactos produzidos pelos participantes e registos fotográficos. Os dados foram analisados indutivamente através de análise de conteúdo. **Resultados:** Os resultados indicam que o envolvimento em diversas tarefas de simetrias e na construção de artefactos artísticos apoiou os participantes na revisão de algumas conceções iniciais, no aprofundamento de alguns aspetos da sua compreensão das transformações geométricas e na manifestação de formas de raciocínio que variaram entre abordagens visuais predominantemente intuitivas e formas mais analíticas e justificadas. As tarefas promoveram também a criatividade e a motivação, incentivando os participantes a estabelecer conexões significativas entre conceitos matemáticos e a produção artística. Estes resultados ampliam o conhecimento existente ao evidenciarem como ambientes de aprendizagem de natureza estético-construcionista podem apoiar simultaneamente a compreensão matemática e a expressão artística na formação de professores. **Conclusões:** A integração de tarefas baseadas nas artes visuais na formação de

professores de Matemática pode promover o raciocínio visual, aprofundar a compreensão conceptual das simetrias e motivar os futuros professores a perspetivar a matemática como uma atividade humana, criativa e cultural. O envolvimento dos futuros professores na conceção de artefactos matematicamente fundamentados proporciona experiências didáticas que podem enriquecer o ensino da Geometria através do estabelecimento de conexões significativas entre a matemática e as artes visuais.

Palavras-chave: Conexões; Artes visuais; Frisos e rosáceas; Artefactos; Formação de professores.

INTRODUCTION

Culture can enrich students' mathematical experiences by transforming learning into an intellectually and emotionally engaging process. Integrating cultural and artistic elements into mathematics education helps students recognize mathematics as part of a broader body of knowledge, connected to everyday life and other disciplines (Choutou & Potari, 2024; Diego-Mantecón et al., 2019; Vale, 2017). In teacher education, this perspective is particularly important, as future teachers should be prepared not only to teach mathematical content but also to foster creativity, critical thinking, and cultural awareness (Emmer, 2013).

Mathematics is widely recognized for its instrumental, formative, and cultural value. Current Portuguese curriculum guidelines emphasize the importance of establishing connections both within mathematics and across disciplines (ME-DGE, 2021). Among these, the relationship between mathematics and the visual arts offers a particularly rich context for interdisciplinary learning (e.g., Choutou & Potari, 2024; Diego-Mantecón et al., 2019; Stix, 1995; Vale, 2017; Vale & Barbosa, 2025). Geometry is especially well suited to this integration, as it enables learners to explore mathematical ideas through symmetry, pattern, and form while expressing them through artistic creation, promoting both spatial reasoning and aesthetic appreciation.

Geometric transformations occupy a central place in geometry because they help learners understand how figures change while preserving essential properties. In particular, research shows that incorporating symmetry in instruction supports problem solving, mathematical thinking and spatial abilities (Battista, 2007; Leikin et al., 2026; Mukuka & Tatira, 2024). Among these, symmetries provide a natural bridge between mathematics and visual arts, allowing learners to experience concepts such as balance, harmony, and structure through meaningful visual representations and forms of visual language used during the resolution of participants' tasks such as composition, colour, proportion, creative expression, and aesthetic decision-making.

Although previous research has highlighted the educational value of integrating mathematics and the visual arts (e.g., Choutou & Potari, 2024; Nutov, 2021; Sylviani et al., 2024), less attention has been given to how pre-service teachers understand and apply specific geometric transformations through artistic production, particularly friezes and rosettes. Existing studies have mainly focused on general geometric reasoning, artistic approaches to enhance engagement, or broader interdisciplinary practices, leaving limited understanding of how future teachers identify, classify, construct, and apply these symmetries in meaningful creative contexts. This gap is especially relevant in teacher education, where future teachers are expected not only to master geometric concepts but also to design interdisciplinary learning experiences that connect mathematics, culture, and artistic expression.

Responding to this need, the present study adopts an aesthetic-constructionist perspective (Papert, 1980; Sinclair, 2006, 2009), which combines the aesthetic appreciation of mathematical forms with the constructionist view that learning is enhanced through the creation of personally meaningful artefacts. This perspective provides a lens for examining how mathematical understanding develops through the creation, interpretation, and discussion of artefacts decorated with friezes and rosettes. More specifically, the study aims to understand how pre-service elementary school teachers solve symmetry tasks, including the construction and decoration of artefacts, based on frieze and rosette patterns, and how these experiences contribute to the development of visual reasoning.

MATHEMATICS AND ITS CONNECTIONS TO THE VISUAL ARTS

The relationship between mathematics and the visual arts is ancient and deeply embedded in cultural practices. As Kline (1964) argued, mathematics is inseparable from culture, functioning not only as a method but also as a language and an art form. Likewise, Hardy (1940) described the mathematician as a creator of patterns, emphasizing that mathematical activity involves both logic and aesthetic appreciation (Vale & Barbosa, 2025).

Recent research in mathematics education has revisited this relationship through the lens of affective and cultural dimensions of learning, emphasizing the role of aesthetic experience in mathematical meaning-making (e.g., Karavakou et al, 2023; Nutov, 2021). Through constructionist learning environments and digitally mediated artefact creation, students can experience mathematics as personally meaningful and emotionally engaging. Sinclair (2009) argues that aesthetic experiences – marked by curiosity, surprise,

appreciation, and joy – play a central role in how learners construct mathematical meaning. From this perspective, aesthetics in mathematics extends beyond decorative beauty, emerging through perception, emotional engagement, creativity, appreciation of mathematical structure, and meaningful interaction with mathematical ideas.

Current studies reinforce the educational value of integrating artistic expression with mathematical activity. Karavakou and colleagues (2023) argue that aesthetics plays an epistemological role in mathematical reasoning by supporting sense-making and creativity. According to Sylviani and colleagues (2024) combining artistic expression with mathematical exploration enables students to approach complex ideas in imaginative and personal ways, fostering positive attitudes and deeper engagement. Nutov (2021) found that pre-service teachers who engaged in creative artistic tasks—rather than passive analysis—demonstrated higher conceptual understanding and stronger emotional connection to mathematical ideas. These outcomes are further supported by Choutou and Potari (2024), who argue that the intersection of mathematics and the arts acts as a boundary-crossing space, fostering hybrid practices where learners can move between symbolic reasoning and sensory, aesthetic experiences.

Despite this potential, mathematics is still frequently taught as an abstract and isolated discipline. International and national curriculum frameworks therefore advocate teaching approaches that emphasize mathematical connections within the discipline and with other areas of knowledge (ME-DGE, 2021; NCTM, 2014).

A central element in building these connections is visualization. Defined as the creation, interpretation, and manipulation of images to support mathematical thinking (Presmeg, 1986), visualization enhances pattern recognition, spatial reasoning, conceptual understanding, and aesthetic appreciation, particularly in geometry (Vale & Barbosa, 2023b). These dimensions are especially important in teacher education, where future teachers are expected to develop flexible, relational, and culturally responsive understanding of mathematics—one that values creativity and interdisciplinary approaches. Research suggests that engaging pre-service teachers in artistic and mathematical design broadens their conceptions of mathematics while strengthening visualization, reasoning, creativity, and reflective practice (Diego-Mantecón et al., 2019; Nutov, 2021; Sylviani et al., 2024; Vale & Barbosa, 2023b).

Tasks are the vehicle through which these principles may be enacted in practice. Rich, open-ended tasks—particularly those with multiple solutions and cultural artefacts—challenge students to think deeply, reason visually, and appreciate the aesthetic dimensions of mathematical ideas (e.g., Presmeg, 1986; NCTM, 2014; Vale & Barbosa, 2023b). When such tasks involve the analysis and construction of cultural artefacts through a mathematical lens, they become powerful opportunities for fostering connections between mathematics and the visual arts in everyday life. In these contexts, students are not only learning mathematical content; they are also engaging in acts of cultural and artistic expression that cultivate aesthetic appreciation and visual reasoning.

The educational value of integrating mathematics and the visual arts is well established, however less is known about how pre-service teachers engage with specific geometric structures such as friezes and rosettes. Existing research has identified persistent difficulties in visualization, symmetry, and geometric transformations (Battista, 2007; Mukuka & Tatira, 2024), yet relatively few studies have examined how future teachers identify, classify, construct, and apply these structures through artistic artefact creation. The integration of mathematical and artistic activities can nurture an affective relationship with mathematics, which is a crucial dimension of meaningful learning (NCTM, 2014; Swoboda & Vighi, 2016). Within this convergence of reasoning, beauty, and personal expression, mathematics becomes more than a set of abstract ideas - it is experienced as a creative and culturally situated human practice. This view aligns with the aesthetic-constructionist perspective adopted in this study (e.g., Papert, 1980; Sinclair, 2006, 2009), which emphasizes that meaningful mathematical learning may emerge through the creation of personally significant artefacts. Through artistic and constructive exploration, learners integrate cognition, emotion, and imagination—revealing mathematics not merely as logical reasoning, but as a form of creative and aesthetic human expression (e.g., Choutou & Potari, 2024; Nutov, 2021; Vale & Barbosa, 2025).

DECORATIVE ART IN MATHEMATICS

Given the focus of this study, the following sections narrow the discussion to geometric transformations and decorative structures—particularly friezes and rosettes—as mathematical and artistic objects with pedagogical relevance in teacher education.

Geometric transformations and Visual Arts

Geometric transformations, particularly symmetries, play a central role in mathematics and in many real-world contexts. They reveal patterns and

spatial relationships that connect mathematics with art, architecture, engineering, and nature, offering meaningful opportunities to develop visualization, spatial reasoning, and problem-solving skills (Hall, 2018; Vale & Barbosa, 2023b; Veloso, 2012).

Despite their importance, students often struggle with geometric transformations, showing limited conceptual understanding and difficulty applying them in new contexts. Research consistently reports difficulties arising from the abstract nature of transformations, challenges in visualizing movement and orientation, and misconceptions regarding congruence and similarity (Battista, 2007; Clements & Battista, 1992; Hollebrands, 2003). Such challenges are often rooted in underdeveloped spatial reasoning, limited exposure to visualization, and traditional teaching practices that privilege procedures over conceptual understanding (Mukuka & Tatira, 2024; Vale & Barbosa, 2023b). Transformations require mental manipulation of figures— a demanding process for those with underdeveloped spatial visualization skills (Battista, 2007). Students often struggle to imagine the movement of shapes, particularly when rotations or reflections across non-horizontal or non-vertical axes occur (Clements & Battista, 1992). They may also confuse different transformations— mistaking a reflection for a rotation or failing to recognize the properties that remain invariant, such as size and shape (Battista, 2007; Veloso, 2012). Some even believe that a transformation alters fundamental properties of figures, for instance assuming that a reflection changes the size of a figure (Hollebrands, 2003).

To address these challenges, researchers advocate learning environments that combine multiple representations, including manipulatives, dynamic geometry software, and artistic exploration (Battista, 2007; Laborde, 2003; NCTM, 2014). Such approaches help learners visualize transformations, explore their properties, and relate them to meaningful contexts, although technology alone does not guarantee conceptual understanding without appropriate pedagogical guidance (Laborde, 2003).

Artistic activities provide particularly rich contexts for learning geometric transformations. Creating tessellations, decorative patterns, and other visual compositions encourages learners to apply translations, rotations, reflections, and symmetries in authentic situations, strengthening visualization, creativity, and conceptual understanding (Eisner, 2002; Hall, 2018; Hetland et al., 2013; Nutov, 2021; Vale, 2017; Winner et al., 2013). By integrating mathematical reasoning with artistic creation, these approaches enable learners to experience geometric transformations as meaningful, creative, and

aesthetically significant rather than as isolated procedural techniques. Through these tasks, learners internalize transformation concepts intuitively and develop a greater appreciation of the beauty and structure inherent in mathematics (e.g., Eisner, 2002; Nutov, 2021; Vale, 2017; Winner et al., 2013).

Friezes and Rosettes in Decorative Art

This section illustrates, through concrete examples, how symmetry emerges in decorative arts and architectural elements, demonstrating the pedagogical, aesthetic, and cultural potential of engaging with friezes and rosettes in mathematics education.

Throughout the world and across history, symmetry has been a fundamental geometric feature, present in both natural forms and human creations. It is one of the most prevalent visual structures, appearing in painting, architecture, tiling, textile arts, and in patterned organization of plants or animals (Hall, 2018; Teixeira, 2015; Veloso, 2012). Figure 1 illustrates this ubiquity with familiar examples. The concept of symmetry underlies these elements and is powerful in both geometry and the visual arts because of the balance, proportion, and harmony it conveys (Vale & Barbosa, 2025).

Figure 1

Images with at least one symmetry



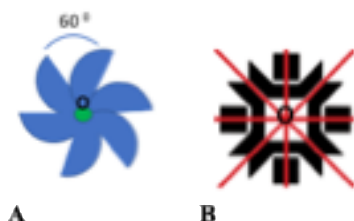
As Veloso (2012) notes, one of the defining features of decorative art is the “repetition” of elementary units of a figure – whether flowers, trees, geometric shapes, or other motifs– and the way this repetition is organized reflects a deep sense of harmony and structure. These images involve different geometric transformations— reflection, rotation, translation and glide reflection – with symmetry as a central property. Because these concepts are highly visual, they can be explored concretely through examples from decorative art.

Although these mathematical objects often can be approached intuitively, they offer fertile ground for exploring key geometric concepts. By analysing and creating these symmetric figures, learners uncover meaningful relationships between mathematical reasoning and aesthetic composition. The mathematical elements emphasized here are friezes and rosettes, which display patterns that acquire a specific meaning, different from those in the numerical domain (Veloso, 2012). The underlying law of formation governing each pattern enables learners to understand how it is constructed and therefore how it can be extended or reproduced mathematically. Through this process, learners experience symmetry not as an abstract property but as a tangible, culturally situated expression of mathematical thought, reinforcing their appreciation of how mathematics permeates the world around them (e.g., Choutou & Potari, 2024; Hall, 2018; Nutov, 2021).

Let us now examine concrete examples of rosettes and friezes. Rosette patterns are defined by rotational symmetry. Following Leonardo da Vinci's classification, rosettes fall into two types: those without reflection symmetry – cyclic (Figure 2, A), and those with reflection symmetry– dihedral (Figure 2, B). Image A has no axes of symmetry because no reflection (in the plane) leaves it invariant; in other words, it lacks reflection symmetry; however, it displays six rotation symmetries. Image B illustrates a rosette with both reflection symmetry (four reflection symmetries) and rotation symmetry (four rotation symmetries). In both cases, the images are examples of figures with symmetry, rosettes, in which several geometrically equal modules are identified and repeated, using the minimum motif (Vale & Barbosa, 2025).

Figure 2

Types of rosettes



These rosette types frequently appear in cultural artefacts, particularly in architecture. Figure 3 shows a rosette window exemplifying the interplay of rosettes embedded in religious and historical symbolism. Identifying symmetries and geometric relationships within these structures helps students

transition from an intuitive to a formal understanding of geometric transformations (Hall, 2018; Martin, 1982; Teixeira, 2015).

Figure 3

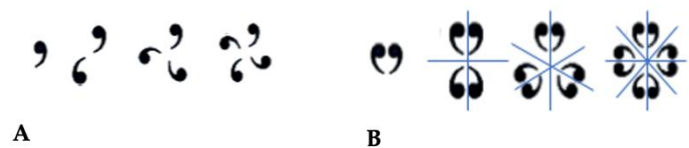
Rosette window of St. Bartholomew's Church (Ireland)



Figure 4 summarizes both rosette types through illustrative examples: those with only rotation symmetry– cyclic rosettes (A), and those combining rotation and reflection symmetry– dihedral rosettes (B) (Hall, 2018; Martin, 1982; Teixeira, 2015).

Figure 4

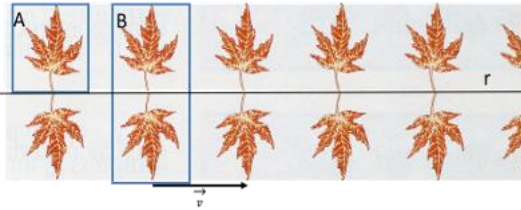
Rosette symmetries (A. Cyclic B. Dihedral)



Similarly, friezes provide valuable opportunities to study different geometric transformations - isometries. In Figure 5, the arrangement of leaves between two parallel horizontal lines can be imagined as extending indefinitely in both directions, with a motif identified that repeats endlessly. This pattern exhibits translation symmetry: translating the plane by vector- v maps the figure onto itself, although no single point remains fixed. The composed motif B is generated by successive translations of vector- v in both directions, while B itself originates from the simpler motif A by a horizontal reflection across axis r . The result is a frieze.

Figure 5

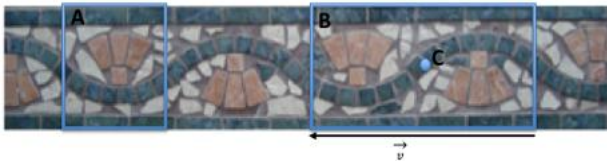
Example of a frieze



Friezes are common in tilework, but they can also be found in churches, buildings, visual arts, and artefacts such as baskets, textiles, and furniture. Figure 6 shows a frieze from kitchen tiles. If we consider A as the minimum motif, it originates motif B by a 180° rotation with centre C. This motif is then repeated by translation, for example according to vector v , generating the presented frieze.

Figure 6

Example of a frieze from kitchen tiles



A frieze is therefore a band with a motif that repeats indefinitely, characterized by translational symmetries in a single direction. It may also include additional symmetries beyond translation. Depending on the combination of different symmetries (reflection, rotation, translation and glide reflection) underlying the frieze, it can be classified into seven types (Hall, 2018; Martin, 1982; Teixeira, 2015). Figure 7 illustrates these seven types of friezes.

Figure 7

Example of the seven types of friezes



Friezes and rosettes integrate various geometric concepts beyond symmetry, including properties of geometric figures such as regular polygons, circumferences, and angles. These concepts offer an accessible and engaging context for exploring mathematics, particularly geometry, beyond the classroom. Observing and analysing such arrangements in everyday environments like architecture, exhibitions, and museums allows students to experience mathematics as tangible and culturally embedded. These experiences inspire creative engagement and provide authentic opportunities to apply geometric knowledge through the design and construction of artistic artefacts or interdisciplinary projects. The universal presence of symmetry thus makes it a powerful and accessible concept for educational purposes. As Nutov (2021) argues, engaging students with symmetric designs—especially those inspired by cultural artefacts—supports spatial reasoning, creative thinking, and the development of mathematical language.

In teacher education, such activities also prepare pre-service teachers to design tasks that bridge formal properties with visual perception, promoting the integration of mathematical reasoning and artistic creativity, as emphasized in international frameworks (NCTM, 2014). This connection exemplifies the aesthetic-constructionist view of learning, where conceptual understanding and artistic making become mutually reinforcing.

METHODOLOGY

Methodological options, Context and Participants

This study followed a qualitative methodology using an interpretative exploratory approach (Erickson, 1986; Miles & Huberman, 1994; Yin, 2003) conducted within initial teacher education, according to the guidelines of the Declaration of Helsinki, and approved by the Institutional Review Board of Universidade Politécnica de Viana do Castelo. It involved 26 pre-service teachers enrolled in the first semester of the first year of a Master's Degree Course in Primary Education (ages 6-12), in a course entitled Current Themes in Mathematics, which addresses topics in Mathematics Education relevant to future professional practice. The didactical experience took place within the module on Geometry, particularly focused on Geometric Transformations (symmetries), and consisted of a sequence of progressively challenging tasks that provided the context for analysing participant's approaches.

We aimed to understand how pre-service elementary school teachers solve symmetry tasks (identifying, classifying and creating) through the construction and decoration of artefacts based on frieze and rosette patterns,

and how these experiences contribute to the development of visual reasoning. Particular attention was given to the mathematical and artistic elements emerging from participants' productions and reflections within an aesthetic-constructionist learning environment. Accordingly, the study was guided by the following research questions: (1) How can we characterize future teachers' performance and manifestations of visual reasoning during the construction of the different symmetries and artefacts based on friezes and rosettes? (2) What main difficulties future teachers experience when working with friezes and rosettes?

From a theoretical and methodological standpoint, the study was grounded in an aesthetic-constructionist perspective, combining Papert's constructionism (1980) —which emphasizes learning through the design and creation of personally meaningful artefacts —with the aesthetic dimension of mathematical learning proposed by Sinclair (2006) who highlights the role of beauty, perception, and emotion in the development of mathematical understanding. Within this framework, learning is conceived as an act of making that unites cognitive, visual, and affective dimensions. The research design privileged learning by making, creativity, and reflection, viewing the process of artefact construction as a pathway for the emergence of mathematical ideas with artistic awareness.

The participants worked in small groups during class sessions, which encouraged collaboration and discussion. The researchers were also the course instructors, assuming a dual role as facilitators and participant observers. This dual role enabled close observation of the learning process and the collection of rich qualitative evidence of students' engagement, reasoning, and responses to the integration of mathematics and the visual arts. All participants took part voluntarily, having given informed consent prior to the study. Data were anonymized, and all artefacts and photographic records were used exclusively for research and educational dissemination purposes. The study complied with institutional ethical guidelines and received approval from the relevant ethics committee, ensuring participants' confidentiality.

The didactical experience

The didactical experience was developed within an active learning approach that, beyond the cognitive dimension, valued the social and physical dimensions of learning (Edwards, 2015; Vale & Barbosa, 2023a). It aimed to foster key 21st-century skills - critical thinking/problem-solving, creativity, communication and collaboration (the 4Cs) (e.g., ME-DGE, 2021; WEF, 2016)– through inquiry, exploration, and discussion. The tasks were designed

to allow multiple solutions and different levels of cognitive demand (Leikin, 2023) progressing from procedural reasoning to conceptual understanding.

The experience took place during five two-hour lessons, conducted at the Math Education Laboratory (LEM). This space (Figure 8) provided flexible conditions for autonomous and collaborative work, equipped with several resources (bibliographic, manipulative, technological).

Figure 8

The LEM space



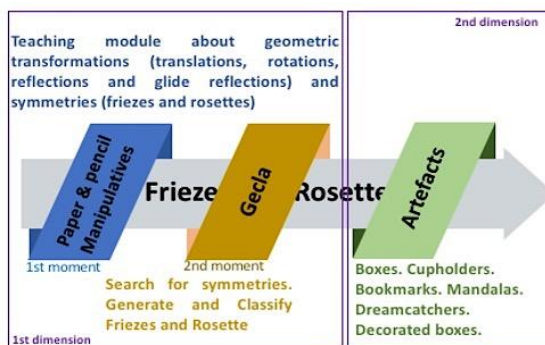
It is important to state that these students had only a brief experience with this topic during their compulsory mathematics education. Many students did not remember having studied geometric transformations; others reported having a vague recollection of symmetries, while others stated that they had not studied this topic at all. This is a topic that many teachers have some reluctance to approach, largely due to insufficient knowledge and confidence in the subject matter.

The didactical intervention was organized into two complementary dimensions (Figure 9). *The first dimension* unfolded in two moments. In the first moment, after the introduction to geometric transformations and symmetries, students worked with paper and pencil and manipulatives to consolidate the identification, characterization, and properties of reflection, rotation, translation, and glide reflection—focusing especially on friezes and rosettes. There was an emphasis on content knowledge, since students' prior scientific knowledge about these topics was often superficial or, in some cases, non-existent, as expressed by them during the classes. Throughout this moment of the didactical experience, various examples of photographs and images from different contexts—namely art, everyday life, and culture—were used, as well as different manipulative materials (e.g., paper and scissors; pattern blocks; MIRA) to illustrate, identify, justify, explore, and construct the various transformations being addressed. The trajectory moved from visual and concrete representations to more abstract and formal ones, supporting the

development of multiple representations and reasoning processes. In the second moment students were introduced to Gecla, a digital tool chosen because of its specific features for detecting and classifying symmetries and for generating friezes and rosettes. Three features of the software were used: Find symmetries, Generation of rosettes, friezes and patterns, and Generation of rosettes, friezes and patterns. This moment encouraged students to explore digital environments to manipulate geometric structures, compare patterns, and identify underlying transformations. *The second dimension* of the experience focused on the construction of artefacts inspired by everyday objects, decorated with friezes and/or rosettes, translating the mathematical-artistic connection into a tangible creative task. Each group was asked to “Be the artist” conceiving an artefact, design its decoration based on the studied symmetries, and use mathematical concepts to create friezes and rosettes in balanced and artistically pleasing compositions. This allowed them to contact with real-world examples of these concepts (bookmarks, cupholders, decorated boxes) verifying the usefulness and applicability of mathematics and establishing connections with everyday life. Students were also required to write an individual short report explaining the construction process and the mathematical characterization of their work (definitions, symmetry groups, minimal motifs, generating elements, vectors, axes, and angles when applicable) and a brief reflection on the proposed experience. The evaluation criteria included: the accuracy of the construction according to the conditions established; the correct identification and description of the geometric concepts involved; the creativity of the proposal; and the aesthetic quality of the artefact.

Figure 9

Summary of the didactical experience



From a didactical standpoint, the first dimension mainly supported conceptual consolidation and visual reasoning, with tasks of progressively higher cognitive demand, whereas the second dimension emphasized decision-making, justification, and communication in an artistic context. These design choices are consistent with the aesthetic-constructionist perspective that underpins the study, in which learning occurs through the creation of personally meaningful artefacts and the cultivation of artistic experiences throughout the process.

Data collection and analysis

Data collection documented the learning processes and outcomes emerging throughout the didactical experience. Consistent with the qualitative, interpretative and exploratory nature of the study (Erickson, 1986; Miles & Huberman, 1994; Yin, 2003) multiple sources of data were used to ensure a rich and contextualized understanding of pre-service teachers' reasoning, actions, and reflections. The data corpus included observations, written productions (task resolutions, written reports, researchers' fieldnotes), artefacts, and photographic records. These sources complemented one another, allowing for methodological triangulation and greater interpretative validity.

We used descriptive and content analysis methods (Miles & Huberman, 1994) to examine all the data to identify patterns in participants' performance and difficulties, they encountered during the construction of friezes and rosettes. Data were analysed iteratively, combining thematic categorization, with interpretative reflection, to capture both learning products and the underlying reasoning processes. The coding process involved repeated readings of the collected materials and the progressive organization of evidence into analytical categories aligned with the research questions. Written productions, artefacts, observations, and fieldnotes were examined comparatively to identify recurrent patterns, convergences, and difficulties across participants' responses. Coding remained flexible and iterative throughout the analysis, allowing categories and interpretations to be refined as new patterns emerged from the data.

The analytical categories were directly informed by the research questions and comprised two main categories: performance during the construction of artefacts and the main difficulties experienced in working with friezes and rosettes. Table 1 summarizes the analytical framework adopted.

Table 1*Analytical framework*

Analytical category	Subcategory	Indicators
Performance	Mathematical accuracy	Correct; partially correct; incorrect; missing solution
	Visual reasoning	Visual recognition and interpretation of patterns and symmetry relationships; spatial visualization of geometric transformations; recognition of invariant properties; coordination between visual representations and formal geometric reasoning; use of mathematical language to describe and justify visual and geometric relationships; progression from intuitive visual approaches to more analytical and justified reasoning
Difficulties	Characterization of friezes and rosettes	Identification and characterization of the symmetries involved; Identification of the minimum motif; identification of the generating motif; identification of reflection axes; identification of rotation centers and angles; identification of translation vectors; identification of glide reflections
	Construction using symmetries	Accurate application of geometric transformations in the construction of friezes and rosettes; coordination of transformations in the construction of artefacts

Performance was analysed through two complementary subcategories: mathematical accuracy and visual reasoning. Mathematical accuracy concerned the correctness and completeness of participants' responses and constructions and was classified as correct, partially correct, incorrect, or missing solution. Visual reasoning concerned the ways in which participants used and coordinated visual and geometric information when identifying, interpreting, and constructing friezes, rosettes, and their underlying transformations. The indicators included visual recognition and interpretation of patterns and symmetry relationships, spatial visualization of geometric transformations, recognition of invariant properties, coordination between visual representations and formal geometric reasoning, and the use of mathematical language to describe and justify visual and geometric relationships. Given the sequential

nature of the didactical experience, progression from predominantly intuitive visual approaches towards more analytical and explicitly justified reasoning was also considered as an indicator of visual reasoning. The category difficulties focused on the main obstacles encountered by participants and comprised two subcategories: characterization of friezes and rosettes, and construction using symmetries. The corresponding indicators included difficulties in identifying minimum and generating motifs; identifying and characterizing the symmetries involved, including reflection axes, rotation centres and angles, translation vectors, and glide reflections; and accurately applying and coordinating geometric transformations when constructing friezes, rosettes, and artefacts.

Consistent with the aesthetic-constructionist perspective underpinning the study, the aesthetic dimension was considered as a complementary interpretative dimension rather than as a subcategory of performance or difficulties. Following Sinclair (2009), aesthetic engagement was interpreted from explicit evidence in participants' productions and reflections, including attention to visual harmony and balance, intentional visual composition, creativity and originality, expressed appreciation of the aesthetic qualities of the artefacts and the creative process, and the integration of mathematical precision and artistic expression. This distinction allowed the aesthetic dimension to inform the interpretation of participants' experiences without treating aesthetic qualities as indicators of mathematical performance or visual reasoning.

The unit of analysis was each participant or group response to a task or question, considered together with the associated written explanation, visual production or artefact and, when available, observational evidence related to that response. Written productions, artefacts, observations, fieldnotes, and photographic records were examined comparatively. The coding process involved repeated readings of the data and their organization according to the categories, subcategories, and indicators presented in Table 1. Coding was iterative, allowing categories and interpretations to be revisited and refined through comparison across tasks and data sources.

Throughout the process, the researchers engaged in collaborative analysis to strengthen interpretative credibility and reduce potential bias. The data corpus and emerging interpretations were discussed regularly between the authors, who collaboratively reviewed coding decisions and analytical categories. Whenever different interpretations arose, these were revisited through discussion and comparison with the empirical evidence until consensus

was reached. Reflexive notes were shared and discussed to ensure coherence between the evidence and the theoretical framework. This analytical stance, grounded in the aesthetic-constructionist perspective, viewed the artefacts and the learning processes as expressions of both mathematical understanding and aesthetic experience. Hence, the qualitative interpretation emphasized the interplay between cognitive, visual, and emotional dimensions of learning, as reflected in participants' discourse, artefacts, and actions.

FINDINGS AND DISCUSSION

To interpret the evidence from the two dimensions, the analysis considered two complementary categories that emerged from the data: performance and difficulties. The category of performance refers to the ways in which pre-service teachers explored, represented, and communicated their mathematical and artistic ideas, while difficulties concern the main conceptual, visual, or procedural challenges that became apparent during the tasks.

The examples presented in this section were selected based on their analytical relevance and representativeness of patterns identified across participants' productions. In particular, examples were chosen to illustrate both successful mathematical performances and recurrent difficulties, as well as variations in participants' conceptual understanding, aesthetic engagement, and problem-solving strategies throughout the didactical experience.

First dimension of the didactical experience

The first dimension was developed during the classes and aimed to consolidate and deepen pre-service teachers' understanding of geometric transformations - reflections, rotations, translations, and glide reflections - and symmetries, friezes and rosettes. We used an approach with these future teachers that closely mirrored what they would later implement with their own students. So, we gradually progressed from a concrete to a more abstract and formal approach of the contents under study, through a sequence of exploratory tasks combining paper and pencil activities, manipulatives and digital tools. The main goal was to help pre-service teachers identify, represent, and describe the symmetries underlying friezes and rosettes, linking observation, visual reasoning, and formalization.

At the beginning, most participants relied on visual intuition to recognize symmetric configurations in familiar objects and artistic compositions. Figure 10 shows examples of free paper and pencil tasks where students explored reflections, rotations, and translations by sketching geometric figures, identifying axes and centres of rotation.

Figure 10

Examples of free paper and pencil tasks

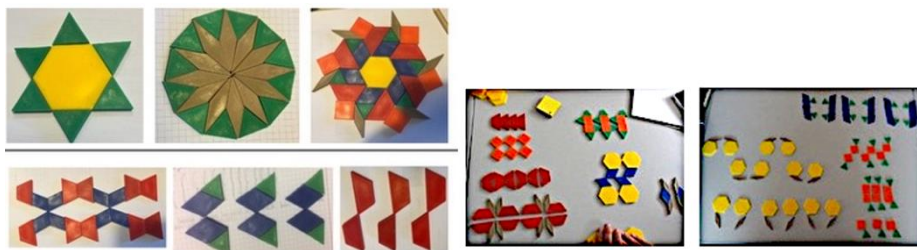


These early representations show careful observation and attention to visual balance, but also reveal conceptual difficulties, mainly in identifying invariant properties. One student, for example, used a compass to draw his rosette (first image in Figure 10), but not all of them demonstrated artistic sense and refinement in their constructions, although all used colour in their work-

The use of pattern blocks as manipulative (Figure 11), freely provided a familiar tangible context to explore symmetries. Students combined, rotated, and reflected the coloured polygons to produce repeated patterns, discovering through manipulation how transformations could be composed to generate continuous designs. As a result, many productions appeared. The physical features of this material contributed to the aesthetic appreciation of the students' constructions, and students appreciated it more than drawing, because they were easier to use.

Figure 11

Examples of free tasks with manipulatives



This practical engagement supported the development of students' performance, as they moved from trial-and-error experimentation toward intentional construction and verbal justification of choices. Using this material students ventured to create both simple and more complex rosettes. The difficulty was not so much in the identification of a cyclic or dihedral rosette, but in recognizing the minimum motif that generated the rosette – as seen, for instance, in the last image of the first row in Figure 11. This difficulty had

already been identified in previous situations of drawing. The last two images in Figure 11 show the work of the students who attempted to create all seven types of friezes. All students were able to create a free frieze when asked. However, they struggled to characterize symmetries constructed by their colleague, which became evident when they had to characterize the colleague's work.

Discussion within small groups and collective sharing moments helped students confront and revise their ideas. Through this process, they refined their language and representations, progressively developing the ability to describe symmetry using geometric reasoning rather than mere perception.

The construction phase - both with paper and pencil and with concrete materials - allowed students to establish relations of equivalence between different types of representations. This contributed to the development of their conceptual knowledge by establishing connections between representations of different nature.

Progressively, participants extended their work to other tasks in which they were required to solve problems and provide written explanations. Figure 12 illustrates two examples of those tasks focused on rosettes.

Figure 12

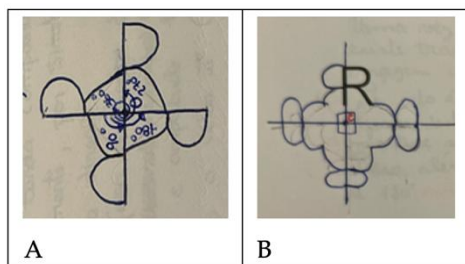
Two examples of rosette tasks

1. Use the R image for create a rosette of your choice. Then identify clearly the rosette you create.	R	2. Identify and characterize all the symmetries of the image of a rosette.	
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As expected, the first task proved somewhat difficult for these students, as it involved creating a rosette. This was especially true for those who attempted to draw a dihedral rosette which includes both rotation and reflection symmetries. As a result, many students opted for simpler versions, mainly the type of Figure 13. The tasks were only partially solved, because half of the students were unable to characterize the drawn rosette. Although the drawings were not always precise, most of them managed to solve the task successfully, with minor errors, particularly related to marking the rotation angles.

Figure 13

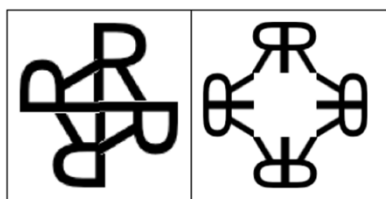
Two solutions of rosettes



Technology further supports conceptual refinement. The difficulty in drawing by hand, led one group to use the computer to create their rosette after several unsuccessful attempts (Figure 14). Digital precision allowed for more accurate constructions and for exploring rotational orders difficult to achieve manually. As they explained, “we can't get it to look the way we want”. So, the computer allowed them to go beyond the drawing limitations, and even to create two solutions. Digital manipulation promoted reflection on the relationship between tool and concept—students could adjust angles dynamically and observe invariance. Nevertheless, the majority continued to privilege the design over its mathematical justification, confirming that technological precision alone does not ensure conceptual depth.

Figure 14

Rosette created with computer

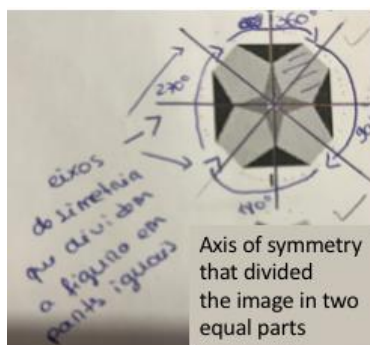


In the second task shown in Figure 12, the rosette was provided, and students were asked the same questions. They had no difficulty identifying the rosette as being dihedral. They correctly identified the four reflection symmetries by their axes and also the four rotation symmetries, corresponding centre and angles. However, they struggled to indicate the rotation angles in the rosette and to associate them with the minimum motif that generates the rosette, as illustrated in Figure 15. These difficulties suggest challenges in coordinating

visual perception with formal geometric reasoning, particularly when interpreting rotational relationships within decorative structures. As highlighted in the literature on spatial reasoning and visualization (Battista, 2007; Sinclair, 2009), understanding geometric transformations requires more than recognizing visual regularities; it involves mentally coordinating movement, invariance, and symmetry relationships. This difficulty was also evident in participants' written reflections: "I knew the figure repeated around the centre, but I was not sure how to determine the rotation angle." This situation was frequently connected with the difficulty to identify the minimum motif that generates the rosette.

Figure 15

A rosette task solution



The next two tasks (Figure 16) focused on friezes. The two tasks addressed similar questions, however, in the first task, students were required to create a frieze, whereas in the second the frieze was provided. Task 1 was an open-ended task, allowing students to be creative in drawing friezes of their choice. Task 2, although not being open-ended, allowed for multiple approaches.

Figure 16

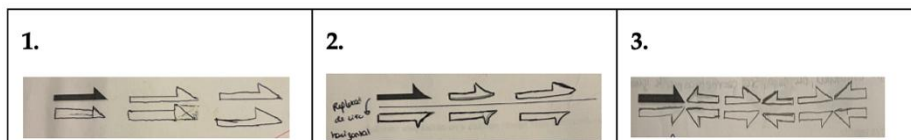
Two tasks with friezes

T1. Use the given motif for create a frieze with at least two symmetries. Identify and characterize all the symmetries used. 	T2. Consider the following frieze  Identify the minimum and the composed motif. Characterize all the symmetries that generate the frieze.
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The diversity of proposals is visible in Figure 17, showing several examples of friezes, but none was completely correct. Twenty-two students presented the simpler friezes, particularly those that used reflection symmetry. The first image in Figure 17 is incorrect, as it does not represent a frieze. The students stated that it was constructed using a reflection with horizontal axes followed by a translation. The second and third images in Figure 17 are two friezes. The students identified the symmetries, but did not specify each of the transformations involved. In particular, they failed to identify the motif that generates the frieze. These last two examples are representative of the general performance of the group.

Figure 17

Some students' solutions

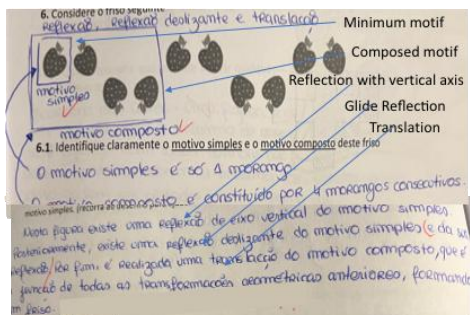


Participants' uncertainty regarding translation vectors and generating motifs suggests that repeated visual patterns alone did not always translate into formal mathematical characterization. This finding reinforces previous research indicating that future teachers may recognize symmetry intuitively while struggling to articulate its mathematical properties explicitly, highlighting the importance of connecting visualization with formal geometric language. As one participant reflected, "I could identify the symmetry visually, but characterizing the vector and the transformations was more difficult."

In the second task, students' performance was similar to that observed in task 1. Although they were asked to identify the minimum motif and the motif that generated the frieze, most were unable to do so. Figure 18 offers a particularly expressive example of a correct solution, with a successful identification of the motifs and a clear description of the type of frieze and its symmetries, however failing to illustrate the position of the axes and vectors. Even so, the work reflects a growing capacity to connect what is drawn and what is reasoned, a central aim of this first dimension.

Figure 18

Example of a correct solution



Particularly noteworthy was the reaction of participants to a non-routine visual task (Figure 19) which, although not directly centred on friezes and rosettes, addressed fundamental symmetry concepts that supported their later construction. This open-ended task involved a higher level of cognitive demand (Presmeg, 1986) and allowed multiple solutions, inviting creative and divergent thinking. This particular task was purely visual and demanded good visualization skills, flexibility, and solid knowledge of symmetries.

Figure 19

Task involving symmetries

T3 The image A has 5 squares. Add to image A the four isolated squares, observed in image B, so that the resulting figure has: 1. reflectional symmetry 2. rotational symmetry. Present two different solutions for each case 1 and 2.	
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The range of responses, illustrated in Figures 20 and 21, include both correct and incorrect solutions, that illustrate the varying levels of difficulty experienced while working on this task. Twenty-four participants solved the first question correctly and with confidence, although four presented only one possibility. While the second—requiring rotation symmetry—proved substantially more challenging to identify and create, since five of the 26 students succeeded. The others evidenced incorrect answers suggesting that they might have benefited from the use of manipulatives.

Figure 20

Some students' solutions - question 1

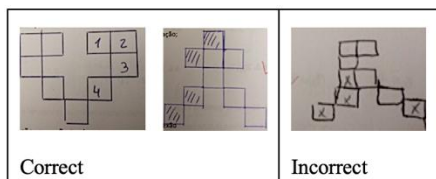
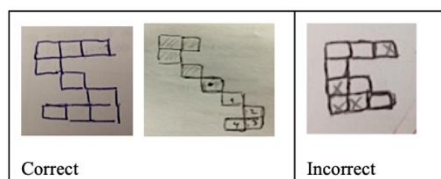


Figure 21

Some students' solutions - question 2



These performances also reveal aspects of participants' conceptual understandings of geometric transformations. In several cases, participants appeared able to recognize symmetry visually but struggled to justify the transformations mathematically. This finding suggests limitations in spatial visualization and in understanding invariant properties under transformation, particularly when moving from visual recognition to formal geometric reasoning (Battista, 2007; Hollebrands, 2003; Presmeg, 1986; Vale & Barbosa, 2023b). Such difficulties reinforce previous research indicating that geometric transformations are frequently understood procedurally rather than relationally.

Overall, during this teaching module, pre-service teachers demonstrated ability to create both friezes and rosettes and to identify the associated symmetries. However, we observed some difficulties, particularly in the characterization of the symmetries used in each case, whether with friezes or rosettes – such as specifying rotation angles, translation vectors, axes of reflection, minimum motifs, and composed motifs. The most common error concerned the incorrect identification of rotation angles in rosettes, an aspect that requires time for conceptual accommodation and practice using different resources and tasks. The difficulties observed in identifying axes of reflection and formally characterizing symmetries also point to limitations in spatial visualization and in recognizing invariant properties of symmetrical structures. From an aesthetic-constructionist perspective, these challenges are particularly

relevant because meaningful mathematical creation depends not only on visual appeal but also on the capacity to recognize and justify the geometric organization underlying artistic forms. For future teachers, these difficulties may also signal fragile pedagogical content knowledge regarding transformational geometry. Table 2 summarizes the work developed by the students during this first dimension.

Table 2

Summary of pre-service teachers' performance and difficulties in the first dimension of the didactical experience with friezes and rosettes

Task/resources	Performance	Main difficulties	Progression
Paper and pencil tasks	Most participants created friezes and rosettes; demonstrated visual observation and use of geometric tools and concepts	Limited precision in formal characterizing symmetries	Gradual shift from intuitive representations to more analytical reasoning through discussion and feedback
Manipulatives (pattern blocks)	Participants constructed and explored simple and complex rosettes and friezes; explored composition of transformations; revealed creativity, aesthetic awareness and engagement	Difficulty characterizing symmetries in peers' constructions	Increased intentionality in construction and verbal justification of geometric ideas
Digital tools (Gecla)	More precise constructions; ability to generate alternative solutions	Tendency to privilege visual design over mathematical justification	Awareness of the relationship between tools and mathematical concepts
Problem-solving tasks	All participants solved the tasks with some minor errors, or with an incomplete characterization of symmetries	Some difficulties in obtaining a complete characterization of friezes and rosettes; rotational symmetry was more challenging to identify and construct	Progressive connection between drawing and reasoning; revealed variability in spatial reasoning and visualization skills

Beyond documenting students' performance and difficulties, the progression summarized in Table 2 also suggests changes in how participants reasoned about geometric transformations. The presence of both predominantly visual identification and more explicit geometric justification reveals different manifestations of participants' reasoning, even though some concepts—particularly those involving rotational relationships—remained challenging.

Second dimension of the didactical experience

The second dimension of the didactical experience focused on the construction of an artefact decorated with friezes and/or rosettes and the elaboration of a report detailing the work developed. This stage marked the transition from exploratory classroom tasks to the design and realization of personal creations, where the concepts previously explored were integrated into a tangible product (Figure 22). It represented a synthesis of knowledge and an opportunity for participants to express their understanding through both mathematical reasoning and artistic composition.

Figure 22

Some artefacts created by the participants



The participants displayed artistic sense, producing a variety of objects such as boxes, cupholders, bookmarks, dreamcatchers, mandalas, and bags. The artefacts revealed a diversity of techniques, materials, and resources –from paper and pencil to collage, paper folding, drawing, and digital designs – demonstrating the participant's ability to adapt mathematical ideas to different expressive forms. Some artefacts closely followed the task's design guidelines, illustrating the connection between mathematical rigour, artistic awareness and usefulness.

The artefacts showed clear aesthetic concerns related with art decoration, possibly due to the additional time afforded by completing the work at home or the fact that the task was part of the unit course assessment. This

proposal led to a positive reaction from the pre-service teachers, who valued this approach as a meaningful way to engage with the mathematical contents learnt. Besides reporting enjoyment, some participants explicitly referred to visual and aesthetic concerns in their reflections. For example, one student noted that he “wanted the artefact to look organized and harmonious, not only mathematically correct,” while another mentioned that “choosing the colours and arranging the friezes carefully made the work feel more complete and visually balanced.”

As expressed in the written report, they enjoyed completing this work, appreciating the opportunity to combine geometry with creativity and personal expression. Several students explicitly emphasized that constructing an artefact made mathematical ideas feel more concrete and meaningful. For instance, one participant highlighted that “working with friezes and rosettes in an artefact helped us understand symmetry in a different and more meaningful way”, while another emphasized the satisfaction of “creating something aesthetically pleasing while applying mathematical concepts”. These comments suggest that participants did not merely enjoy the activity; rather, they engaged with mathematics through a combination of visual, emotional, and creative experiences, dimensions that Sinclair (2006, 2009) associates with aesthetic engagement in mathematical activity.

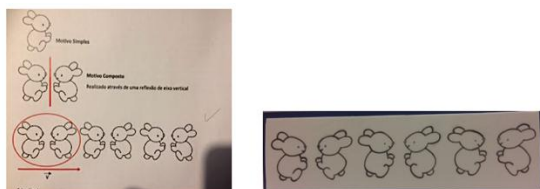
Regarding the mathematical content, students revealed caution: 24 of the 26 participants chose simpler decorative motifs to minimize failures in the mathematical characterization of the friezes and rosettes. This cautious approach also reveals aspects of participants’ mathematical reasoning. By intentionally selecting simpler motifs, many pre-service teachers appeared aware of the relationship between visual complexity and the precision required for mathematical characterization. Rather than representing a limitation alone, these choices may reflect an emerging understanding of the geometric constraints involved in constructing and classifying friezes and rosettes, particularly regarding symmetry relationships, repetition, and transformational accuracy. In this sense, students’ decisions suggest increasing awareness of the need to coordinate artistic creativity with mathematical rigor. To fill the decorative space, they did not intentionally use patterns (which were not contemplated in the syllabus of this unit course) and instead repeated the friezes to cover blank spaces (Figure 22, second line, the three boxes, black, blue, white). The written report formally explained all the processes and options involved in the construction and decoration of the artefact.

Beyond documenting students' engagement with the proposal, the written reports and artefacts also provided insight into how participants mobilized geometric ideas in new contexts. Rather than merely reproducing decorative patterns, students were required to justify the mathematical organization of their artefacts, including the identification of symmetry groups, minimum motifs, axes, vectors, and rotational relationships. This process of adapting geometric transformations to personally meaningful objects suggests that participants were progressively able to connect formal geometric ideas with creative expression and everyday contexts, moving beyond isolated task performance toward applied geometric reasoning.

The analysis of the written productions showed that only ten of the twenty-six participants clearly described the procedures, with a correct and complete characterization of the elements used and the transformations operated. Moreover, some of the incorrections detected during classes had been overcome. For example, the bookmark (Figure 23) includes a complete and accurate identification of the motifs used, as well as the identification of the translation vector that generates the frieze. The explanation is supported by clear visual representations, showing conceptual mastery and the ability to articulate geometric reasoning effectively. This improvement suggests that participants were progressively refining their understanding of geometric transformations through iterative construction, feedback, and justification.

Figure 23

A visual explanation of a frieze

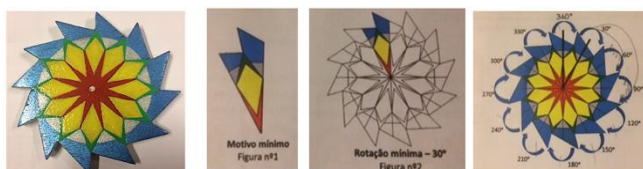


However, other reports revealed errors and misconceptions, particularly in the case of rosettes, and, in other cases, no explanation was given at all, a recurring situation also observed during the classes. Figures 24 and 25 show two examples of rosettes, one used on a cupholder (Figure 24) and another on a cloth bag (Figure 25). The cupholder, which was decorated with a rosette, while visually appealing and correctly structured, shows inaccuracies in the identification of rotation angles. The participants clearly recognized the twelve rotational symmetries and described them in writing but misrepresented their

amplitude in the diagram. This mismatch between verbal and visual reasoning points to the persistence of conceptual fragilities, even among otherwise competent performances.

Figure 24

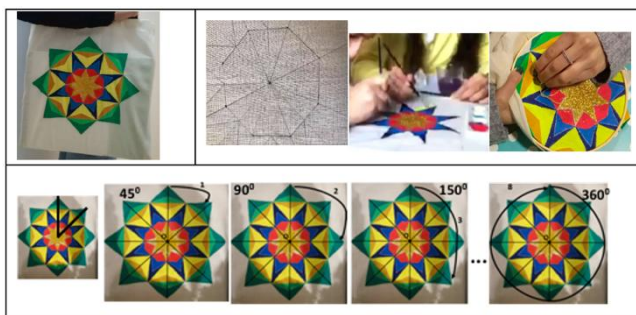
Example of a cupholder rosette



A different case is shown in Figure 25. Here, the participants correctly identified and represented the elements that characterize the eight rotational symmetries of the rosette, including images that show some of the steps used during the construction process. As the saying goes “an image is worth a thousand words”. This artefact demonstrates high performance and an awareness of the design process, evidencing reflective understanding and an ability to connect the mathematical and aesthetic dimensions of the task.

Figure 25

Example of a cloth bag rosette



The analysis of the artefacts and the respective written reports showed diverse approaches and degrees of mastery. Some previously observed difficulties persisted, while others were overcome. In nine of the twenty-six reports, the procedures were clearly described, and the elements and transformations operated were accurately and completely characterized. However, in other cases, errors and misconceptions persisted, particularly in rosettes, where they identified the rotations but not the angles. Also, in certain

reports the participants did not provide explanations to support their work. These results suggest that these students need to further develop their spatial and visual skills, as well as reinforce previous work on isometries - reflection, rotation, translation and glide reflection - that can contribute to overcome some of these difficulties. Overall, all participants achieved a satisfactory level of performance. Table 3 summarizes the work developed by the students during this second dimension.

Table 3

Summary of pre-service teachers' performance and difficulties in the second dimension of the didactical experience in creation and construction of artefacts

Task/resources	Performance	Main difficulties	Progression
Construction of artefacts	All participants produced artefacts integrating friezes and/or rosettes with aesthetic quality; participants clearly described procedures used	Preference for simpler decorative motifs to reduce risk of mathematical errors	Demonstrates awareness of mathematical constraints and task requirements
Identification of symmetries	Most of participants correctly identified symmetries in their artefacts	Incomplete or missing explanations in some reports; indicates uneven development of mathematical concepts.	It suggests the need for further development of spatial and visual reasoning
Integration of mathematical and artistic ideas	Artefacts revealed creativity, aesthetic awareness, and engagement	Some conceptual fragilities remained despite visually successful constructions	Confirms the importance of combining visual and formal reasoning concepts
Overall performance	All participants achieved a satisfactory level of performance	Some misconceptions persisted from the classroom phase	Reflects partial consolidation of geometric concepts

The results also suggest that the second dimension promoted stronger engagement and greater attention to the aesthetic aspects of mathematical work. Beyond completing the proposed tasks, participants appeared increasingly invested in refining their artefacts and connecting mathematical rigor with

visual composition, as reflected in the time devoted to revising details, improving decorative features, and refining the mathematical categorization of friezes and rosettes. This increased involvement was reflected in students' written reports. As one participant stated, "Compared to the paper-and-pencil activities, this task was more motivating because I had to think carefully about how to organize the motifs and make the artefact look balanced and pleasing to the eyes, making my own decisions." We also could read the thoughts of two groups that illustrate the general feeling of all class: "This project has been a valuable addition to our journey as future teachers, as it has introduced us to a different approach to this mathematics content than what we are accustomed to, as well as various ways to apply it with our future students"; and "The development of this project allowed us to deepen our knowledge about friezes and rosettes, while also promoting interdisciplinarity with the visual arts, particularly painting and drawing. Another positive aspect was the opportunity to develop our creativity and critical thinking alongside Mathematics, making it more motivating and enjoyable."

These reflections suggest not only positive affective engagement but also growing attention to aspects such as visual harmony, coherence of decorative elements, and the integration of mathematical precision with creativity. Importantly, participants' comments indicate that they particularly valued the opportunity to combine mathematics with artistic creation and to apply geometric concepts in a personally meaningful artefact, making mathematical ideas feel more concrete, creative, and relevant. In Sinclair's (2009) terms, these reactions may be interpreted as expressions of aesthetic engagement. At the same time, participants' productions revealed that aesthetic and visual success did not necessarily coincide with full mathematical formalization, since several pre-service teachers still experienced difficulties in explicitly characterizing the symmetries involved. This tension suggests that the ability to create visually coherent and aesthetically meaningful artefacts may precede, rather than automatically imply, conceptual mastery of geometric transformations.

CONCLUDING REMARKS

This study examined the performance of future elementary teachers (ages 6–12) in identifying, classifying, and constructing friezes and rosettes, as well as applying these concepts to the artistic decoration of artefacts. Grounded in an aesthetic-constructionist perspective (e.g., Papert, 1980; Sinclair, 2006, 2009), the study explored how mathematical reasoning and artistic expression can be meaningfully integrated. The findings suggest that combining

mathematical and artistic tasks provided opportunities for participants to mobilize mathematical understanding and visual reasoning while engaging with the aesthetic dimensions of mathematical activity.

The didactical experience comprised two complementary dimensions. The first focused on exploring and consolidating isometries through paper-and-pencil activities, manipulatives, and digital tools. The second extended this knowledge to the design and decoration of artefacts, enabling participants to apply geometric concepts in meaningful creative contexts. Together, these stages structured the experience as a progression from guided exploration to more autonomous creation, consistent with the view that mathematical knowledge may emerge through the interplay of action, perception, and reflection (Hardy, 1940; Presmeg, 1986; Sinclair, 2006, 2009).

During the first stage, participants generally succeeded in constructing and identifying symmetries, although difficulties remained regarding rotational angles, minimum motifs, generating vectors, and formal mathematical justification. These findings are consistent with previous research showing that students often struggle to develop a deep understanding of geometric transformations (Clements & Battista, 1992; Veloso, 2012). The participants' work across different representations suggests that the used resources were beneficial in their engagement with the geometric concepts, highlighting their complementary role (Choutou & Potari, 2024).

The second stage revealed significative engagement and creativity. Participants' artefacts reflected both mathematical understanding and personal aesthetic expression, although visually successful productions did not always correspond to accurate conceptual understanding. This finding reinforces the importance of combining creative production with explicit discussion of the underlying mathematical concepts. From an aesthetic-constructionist perspective, designing artefacts provided meaningful opportunities to visualize, organize, justify, and communicate mathematical ideas (Karavakou et al., 2023; Sylviani et al., 2024; Vale & Barbosa, 2025).

Beyond its didactical contribution, the study provides empirical insight into how pre-service teachers mobilize geometric knowledge and visual reasoning across a didactical experience that moves from the exploration of symmetries through multiple representations and resources to their application in the creation of artistic artefacts. We believe that the contribution of the study lies not only in the design of the experience itself, but also in showing how this sequence made visible different manifestations of visual reasoning and

difficulties that persisted when mathematical ideas, particularly related to geometric transformations, were applied in creative production.

Although this study does not claim that aesthetic experience directly produces conceptual understanding, the findings suggest that integrating artistic and mathematical activities can foster meaningful engagement with geometry while encouraging greater attention to its visual, structural, and creative dimensions (Sinclair, 2009). In this sense, mathematics teacher education may benefit from approaches that combine creative production with mathematical reasoning, providing future teachers with opportunities to explore geometric ideas through creative production and to consider more engaging, interdisciplinary learning environments. Such experiences also encourage future teachers to recognize mathematics as a cultural, creative, and communicative discipline (Emmer, 2013), enabling them to move beyond procedural teaching towards learning environments that promote creativity, critical thinking, and meaningful connections between mathematics and the world around them.

LIMITATIONS AND RECOMMENDATIONS

This study should be interpreted considering its qualitative and context-specific nature. It was conducted within a single institutional context with one cohort of pre-service teachers enrolled in the same Master's programme. Consequently, the findings are not intended to be statistically generalizable but rather to provide an in-depth understanding of participants' experiences and difficulties in learning about friezes and rosettes. In addition, the researchers' dual role as teacher educators and researchers may have introduced interpretative bias. To minimize this possibility, participation was voluntary, students were informed that participation would not affect course assessment, data were analysed only after the completion of the didactical experience, and interpretations were discussed collaboratively. Finally, the study focused on a specific learning experience and therefore does not provide evidence of the long-term development or retention of participants' conceptual understanding.

The findings nevertheless suggest important implications for mathematics teacher education. Teacher preparation programmes may benefit from providing future teachers with opportunities to explore geometric transformations through multiple representations, including visual, manipulative, digital, and artistic approaches. Activities involving the construction and analysis of artefacts decorated with friezes and rosettes can promote meaningful engagement with geometry, particularly when combined with explicit attention to mathematical language, justification, and discussion.

At the same time, teachers should avoid assuming that visually successful productions necessarily reflect conceptual understanding and should encourage learners to explain the mathematical reasoning underlying their work.

Future research could investigate the long-term impact of integrating mathematics and visual arts on conceptual understanding and classroom practice. It would also be valuable to examine how pre-service teachers later incorporate these experiences into their own teaching and how different forms of pedagogical mediation influence the relationship between visual intuition, aesthetic engagement, and formal geometric understanding.

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AUTHORS' CONTRIBUTIONS STATEMENTS

AB and IV collected and made a preliminary analysis of the data, wrote the first version of the theoretical framework and the research methodology. AB and IV also designed and discussed all parts of the article, reviewed contributed to the final version of the manuscript and approved the final version of the article.

DATA AVAILABILITY STATEMENT

The corresponding author, IV, will make the data supporting the results available upon adequately justified request.

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