

The mobilization of Advanced Mathematical Thinking processes in the development of questions for the Brazilian Public School Mathematics Olympiad

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ABSTRACT

Background: In this article, we present an investigation into the mobilization of Advanced Mathematical Thinking (AMT) processes in designing questions for the Brazilian Public School Mathematics Olympiad (OBMEP). **Objective:** To identify the AMT processes mobilized in creating OBMEP questions, based on an analysis of a semi-structured interview with the Exam Committee Coordinator and an analysis of the questions from the first and second rounds of the 2025 OBMEP. **Data collection and analysis:** Using a qualitative research approach, we analysed data collected via the interview, followed by an analysis of the Level 3 questions from the first and second rounds of the 2025 OBMEP. **Results:** The qualitative analysis of the interview and the questions was grounded in Tommy Dreyfus's theory; it revealed the mobilization of processes involved in representation—such as visualization, change of representations and translation between them, and modelling—as well as processes involved in abstraction, such as synthesis, generalization and rigorization. **Conclusions:** The findings indicate that OBMEP questions can foster the development of AMT, particularly by mobilizing processes of representation and abstraction in a manner distinct from most Mathematics questions designed for the high school context.

Keywords: Advanced Mathematical Thinking; Brazilian Public School Mathematics Olympiad; Mathematics Education.

A mobilização dos processos do pensamento matemático avançado na elaboração de questões da Olimpíada Brasileira de Matemática das Escolas Públicas

RESUMO

Contexto: Neste artigo, apresentamos uma investigação cujo objetivo foi identificar a mobilização dos processos do pensamento matemático avançado (PMA)

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na elaboração de questões da Olimpíada Brasileira de Matemática das Escolas Públicas (OBMEP). **Objetivo:** Identificar os processos do PMA mobilizados na elaboração de questões da OBMEP, a partir da análise de uma entrevista semiestruturada realizada com o coordenador do comitê de provas e de questões das provas de 1ª e 2ª fases da edição 2025 da OBMEP. **Coleta e análise de dados:** Numa abordagem qualitativa de pesquisa, foi realizada a análise dos dados coletados por meio de entrevistas e, a seguir, a análise das questões das provas de 1ª e 2ª fases do Nível 3 da edição de 2025 da OBMEP. **Resultados:** A análise qualitativa da entrevista e das questões foi embasada na teoria de Tommy Dreyfus e evidenciou a mobilização de processos envolvidos na representação, tais como a visualização, a mudança de representações e a tradução entre elas e a modelação, bem como de processos envolvidos na abstração, tais como a sintetização, a generalização e a rigorização. **Conclusões:** Apontam para a potencialização do desenvolvimento do PMA a partir das questões da OBMEP, especialmente, pela possibilidade de mobilização dos processos envolvidos na representação e na abstração de uma forma diferenciada da grande maioria das questões de matemática elaboradas no contexto do Ensino Médio.

Palavras-chave: Pensamento Matemático Avançado; Olimpíada Brasileira de Matemática das Escolas Públicas; Educação Matemática.

INTRODUCTION

It is in the school environment that scientific thinking is structured and formalized: systematic observation, hypothesis formulation, controlled experimentation, critical data analysis, and the construction of theoretical models are processes intrinsic to this thinking. Scientific thinking can be divided into elementary and advanced in terms of complexity.

One line of research in Mathematics Education that has gained prominence in studies of the processes of teaching and learning Mathematics, in both basic and higher education, is *advanced mathematical thinking*.

In the 1980s, in the United Kingdom, during the International Group for the Psychology of Mathematics Education, a group called the Advanced Mathematical Thinking Group was formed after some researchers discussed the need to study the dynamics of teaching and learning Mathematics in adulthood. At that time, the researchers introduced the concept of Advanced Mathematical Thinking (AMT) in contrast to Elementary Mathematical Thinking (EMT). The fundamental aspect that distinguishes these two types of mathematical thinking is that the advanced stops at the abstractions of mathematical concepts (Tall, 1995; Dreyfus, 2002).

Since then, several theorists have developed research that aims to conceptualise the AMT (Tall & Vinner, 1981; Gray & Tall, 1994). A detailed literature review (Torrente & Reis, 2023b) indicates that Tommy Dreyfus and

David Tall are the most prolific researchers on this topic. According to Dreyfus (2002, p. 26): “There is no clear distinction between many of the basic and advanced processes of mathematical thinking, but advanced Mathematics is more centred on the abstractions of definition and deduction”. Tall (2002) understands AMT as a process that mobilizes and articulates the cognitive structures arising from a wide range of mathematical experiences, fostering the emergence of new concepts that support and extend the network of formally established theorems.

Dreyfus (2002) points out that, from his perspective, AMT is not a single process but rather a series of interrelated processes, with visualization and generalization as the main processes, and representation and abstraction as key components. However, according to the author, such processes manifest in mathematical reasoning at both the elementary and advanced levels.

On the other hand, the Brazilian Public Schools Mathematics Olympiad (OBMEP) was created in Brazil exactly 20 years ago. Especially over the last decade, OBMEP has expanded remarkably, increasing the number of participants and becoming the largest scientific Olympiad in the country and one of the largest in the world in terms of participation, mobilizing millions of students in 99.8% of Brazilian municipalities (Torrente & Reis, 2023c).

Among OBMEP's objectives, the following stand out: to promote the study of Mathematics in Brazil; to identify young talent and encourage their entry into universities in the scientific and technological fields; and others (OBMEP, 2025, p. 91). Within this perspective, especially for the identification of "talents" for Mathematics, an initial analysis of the issues that integrate the different OBMEP tests, at the most varied levels and phases, points to the need to mobilize several AMT processes in resolution, both representation and abstraction processes (Torrente & Reis, 2023a).

Within this context, in this article¹, we present an investigation in which we aim to identify the AMT processes mobilized in the formulation of OBMEP questions, based on the analysis of a semi-structured interview carried out with its Exam Committee Coordinator and the questions of the 1st and 2nd phase tests of the OBMEP 2025 edition.

¹ Bibliographic production associated with the postdoctoral research in Mathematics Education of the 1st author, developed at the Federal University of Ouro Preto, Brazil, under the supervision of the 2nd author, and in international collaboration with the 3rd author.

To this end, we first summarise some researchers' ideas on AMT and its processes, followed by a brief history and recent contextualisation of OBMEP. Next, we present the investigation conducted, discuss its results, and draw some conclusions.

ABOUT ADVANCED MATHEMATICAL THINKING

Scientific knowledge can be understood as a framework of ideas essential to individual development. It constitutes a system of interpretation by which we organise, analyse, and attribute meaning to the phenomena and the natural and social reality that surrounds us.

The dividing line is not established simply by a given topic, but by the level of depth with which it is approached. It is perfectly possible to have an in-depth analysis of a fundamental concept and a simplified introduction to an advanced topic. What really establishes the distinction is the complexity of the tools and methods used to apply these concepts.

There are other equally important actions that enhance how we interact with knowledge, such as:

- — Classifying: Organising data and concepts into groups based on similarities and differences;
- — Conjecturing: Formulating hypotheses or making informed assumptions in the absence of complete evidence;
- — Inducing: Reaching general conclusions from specific observations or examples;
- — Analysing: Dividing a whole into its constituent parts to examine its relationships and functions;
- — Formalizing: Structuring ideas or systems in a rigorous and logical way, often through well-defined symbols or rules.

According to Dreyfus (2002), AMT is not limited to representing and abstracting information from the world, although these are among the most important points, as they enable the transition from one level of understanding to another and provide the basis for evolution.

However, other processes are relevant in the development of AMT. For example, according to Domingos (2003), the ability to visualize enables us to intuit and, more deeply, understand the genesis and structure of concepts, especially in Mathematics. In essence, visualization is the bridge that connects abstract or symbolic representation to intuition and discovery.

Since understanding is subjective, it also seems natural to us that each person develops a unique mental image for the same concept. It is also common for an individual to use multiple forms of representation, such as symbolic and mental representations, which complement each other and are combined as needed. Thus, from Dreyfus's (2002) perspective, the main result of this cognitive development is that the individual acquires multiple interconnected representations, and this interconnectedness allows such an individual to mobilize them simultaneously and switch between them with great efficiency. This process essentially involves the direct transition between different representations and the ability to “translate” information from one to the other.

Among the processes of representation, we highlight modelling, which is the act of building a mathematical structure to describe and analyse an object or event that is not intrinsically mathematical. According to Dreyfus, modelling “means the construction of a mathematical structure or theory that incorporates essential characteristics of the object of the system. This structure or theory, the model, can be used to study the behaviour of the object or process to be modelled” (2002, p. 30).

Thus, all these processes described so far (representation, change of representations and translation between them, and modelling) constitute the nuclear process of representation.

The second nuclear process described by Dreyfus (2002) is abstraction. Abstraction, fundamental for the mastery of certain skills in Mathematics, emerges from the articulation between generalization and synthesis. Such processes amalgamate to provide the necessary cognitive basis, making abstraction the most relevant foundation for the development of these competencies:

If students develop the skill consciously, making abstractions from mathematical situations, they will reach an advanced level of mathematical thinking. Achieving this ability to abstract should become the most important part of advanced Mathematics Education. (Dreyfus, 2002, p. 26).

The generalization process, according to Dreyfus (2002), can be understood as the mechanism by which the reach of a conclusion or rule is expanded. It begins with the detailed observation of cases, and by identifying common traits or recurring patterns, it is possible to extract a broader principle. This process is fundamental, as it enables knowledge acquired in a delimited

scenario to be applied and validated across analogous situations, transforming an isolated truth into a rule.

The second process that complements abstraction is synthesis. It consists of the union of distinct elements to form a unified concept. As stated by Brandenberg (2010), this stage is marked by the formulation of a theoretical model that groups and establishes logical connections among phenomena or principles that initially seem disconnected.

A point Dreyfus (2002) raises is that teaching often fails to give due value to the synthesis process. Teachers tend to create and present ready-made summaries and syntheses, which inhibit students from exercising this fundamental step. The pedagogical ideal would be to encourage students to value and practice synthesis. It is through this practice that they can break their dependence on standardised solutions and, more crucially, develop the ability to connect and articulate different mathematical concepts.

Thus, abstraction is connected to generalization and synthesis. However, although all require mental effort, none of these processes imposes on the student a cognitive challenge as intense as abstraction. At this point, students are encouraged to engage deeply with the relationships and structures associated with the objects of study to achieve a full understanding of them.

Thus, abstraction and representation are not isolated nuclear processes, but complementary and interdependent in the development of AMT. An abstract concept often emerges from the analysis of multiple concrete representations, while an already abstract concept can, in turn, generate new representations.

Next, we seek to present and briefly describe the processes involved in the AMT, according to Gereti and Savioli's (2015) interpretation, based on Dreyfus (2002):

Chart 1

Description of AMT processes, based on Dreyfus (2002)

Processes Involved in REPRESENTATION	
Symbolic Representation	A mathematical concept or object can be represented in writing, in the form of notations or symbols. However, there must first be a meaning associated with the mathematical concept or object being represented.
Mental Representation	The representation of a mathematical concept or object occurs in an individual's mind and is related to the set of

	concrete representations that person has of that concept or object.
Visualization	Through intuition and understanding, this process allows mental representations to be created.
Change of Representations and Translation Between Them	Navigating among different representations of a mathematical concept or object requires the ability to correctly connect them whenever necessary. Translation of representations refers to the transfer of information from one mathematical statement or property to another, as well as translation between languages (mathematical and verbal).
Modelling	The object or process to be modeled requires the development of a mathematical framework or theory that encompasses its characteristics.
Processes Involved in ABSTRACTION	
Synthesis	Use a combination of (distinct) mathematical objects and concepts, interrelating them in order to solve the problem as a whole.
Generalization	Based on specific cases, identify common characteristics so that the validity can be expanded. It may be necessary to include the formulation of other mathematical concepts.

We can also highlight another important process involved in abstraction: rigorization. According to Reis (2001), rigorization is a process of conceptualising intuitions; in this movement, some are maintained, and others are refuted. We note that this process is very peculiar to the construction of mathematical knowledge (Reis, 2009) and to the development of AMT, where rigorization is mobilized simultaneously and interdependently with the other processes involved in abstraction, especially generalization.

Since we present and describe the processes of the AMT, we believe it is appropriate to draw a brief comparison between Dreyfus's (2002) and Tall's (2002) perspectives on central aspects in the constitution of the AMT, such as the conception of understanding/thinking, the elementary versus advanced relationship and cognitive development/nuclear processes, as presented below:

Chart 2

Aspects of AMT according to Dreyfus (2002) and Tall (2002)

Aspect of AMT	Dreyfus (2002)	Tall (2002)
Concept of Understanding / Thinking	Understanding in the student's mind is an intricate sequence of activities and interactions among mental processes (representation, visualization, generalization, classification, induction, analysis, synthesis, abstraction, formalization). It is this interaction among processes that drives the development of Advanced Mathematical Thinking.	Advanced Mathematical Thinking uses cognitive structures derived from various mathematical activities to generate new ideas that consolidate and expand theoretical systems and theorems.
Comparison Elementary vs. Advanced	It is possible to approach advanced topics in a basic way. The distinction lies in the complexity and how it is handled. There is little distinction between Elementary Mathematical Thinking and Advanced Mathematical Thinking, with the latter focusing on abstractions of definitions and deductions.	The development of elementary mathematical thinking into advanced mathematical thinking in an individual is built upon "perceptions of" and "actions on" external objects. Many activities are common to both levels, but the possibility of formal definition and deduction is what distinguishes them.
Cognitive Development / Core Processes	The most important process is abstraction. A student reaches an advanced level by developing the ability to consciously make	Development occurs through two parallel processes: 1) From the visual / spatial to the formal / deductive; 2) From successive encapsulations

	abstractions from mathematical situations. Abstraction and representation can be considered complementary processes that move in opposite directions.	of the process to the concept, using symbolic manipulation. These processes inspire creative thinking based on formally defined objects and systematic proofs.
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In short, the AMT and the EMT are not seen as entirely distinct; rather, they are differentiated mainly by complexity and by how the thinking processes are managed. Thus, the transition between Elementary and Advanced Mathematical Thinking is considered a significant leap in mathematical cognition, characterised by some important changes:

— **Nature of Concepts**

- — EMT: It begins with the perception and action on objects in the external world. Concepts are often based on intuition, examples, and what is visual-spatial or algorithmic (the “how to”). Concepts are often defined intuitively or by examples;
- — AMT: It involves deductive and rigorous reasoning about mathematical notions that are not entirely accessible to the senses. Concepts are constructed from formal definitions and their properties are proven.

— **Rigor and Proof**

- — EMT: It focuses on describing and convincing (often by examples or empirical arguments);
- — AMT: It focuses on defining and proving in a logical and coherent way, using the definition as a basis for the deduction and construction of theorems.

— **Abstraction and Generalization**

- — EMT: Abstraction and generalization are present, but at more elementary and less in-depth levels;
- — AMT: Abstraction is a central and more complex process. It involves the ability to consciously make abstractions from mathematical situations and manipulate them as new mental objects. It also involves

a broader generalization and the quest to visualize the “generalizing totality” of an object.

— **Process and purpose:**

- — EMT: A tendency to see mathematical entities (such as numbers, operations) more as processes for doing something (for example, counting, calculating, among others).
- — AMT: Greater flexibility to see and manipulate these entities both as processes and as concepts (mental objects) on which one can think and apply logic.

Thus, although many of the core mental processes (such as representation and abstraction) are present in both, AMT is distinguished by complexity, formality, deductive rigor and the centrality of definition and proof in the construction and manipulation of mathematical knowledge.

ABOUT THE BRAZILIAN PUBLIC SCHOOLS MATHEMATICS OLYMPIAD

In Brazil, in 2005, the Government of the State of Ceará launched the project “*Linguagem das Letras e dos Números – Leituralizar e Numeratizar*” [Language of Letters and Numbers – Reading and Numeratising], through its Departments of Science and Technology (SECITECE) and Basic Education (SEDUC) (Barbosa, 2008, p. 383), in order to seek different ways that would enable significant changes in the teaching-learning process, in addition to an improvement in the quality of basic education in public schools in the state of Ceará. According to Barbosa (2008, p. 387), the objective of this project was “to correct deficiencies in formal education that affect citizenship and social inclusion, hindering scientific and technological growth and that of professional and higher education”.

Due to the successful experience in the state of Ceará and the success of the Mathematics Olympiads worldwide (Torrente & Reis, 2023c), the project aroused interest among students, educators, politicians, entrepreneurs, and the public. Following this interest, after meetings with several potential organisers, the then-President Luiz Inácio Lula da Silva announced that the Mathematics Olympiad would be a government project, to be named OBMEP – Brazilian Public Schools Mathematics Olympiad. Thus, OBMEP was officially launched on May 19, 2005, in Brasília, DF, by the then-President of the Republic, Luiz Inácio Lula da Silva, and the Ministers of Science and Technology, Eduardo Campos, and of Education, Tarso Genro.

In the educational scenario, its creators thought of a long-term project, and its good development would be “a definitive contribution to a leap in quality in public education and to open new horizons for students from less favoured classes” (OBMEP, 2025, p. 91).

Over the years, OBMEP's objectives have changed, and new ones have been incorporated. In the Initial Project, OBMEP had the following main objectives (OBMEP, 2005, p. 91-92):

[1] Encourage and promote the study of Mathematics among public school students;

[2] Identify young talent and provide opportunities for their entry into the scientific and technological fields;

[3] Encourage the improvement of public-school teachers, thus contributing to their professional development;

[4] Contribute to the improvement of Mathematics teaching in the public network;

[5] Contribute to the integration between public schools, federal universities, research institutes and scientific societies.

In the 3rd edition of OBMEP, in 2007, a new objective was incorporated: [6] Promote social inclusion through the dissemination of knowledge. In addition to this insertion, a new wording was made for the fourth objective: [4] Contribute to improving the quality of basic education.

In 2017, in the 13th edition of OBMEP, the seventh objective was added: [7] Promote the diffusion of mathematical culture. A new wording was made for the following objectives: [1] Encourage and promote the study of Mathematics in Brazil; [2] Identify young talent and encourage their entry into universities in the scientific and technological fields; and [4] Contribute to improving the quality of basic education, enabling the largest number of Brazilian students to have access to quality teaching material.

Over the years, OBMEP has demonstrated remarkable growth, acknowledge today as the largest science Olympiad in Brazil and one of the largest in the world in terms of reach and dissemination.

Regarding participants, since its 1st edition, OBMEP has registered a significant increase in the number of students and schools involved:

- — Beginning in 2005: The inaugural edition already had an impressive attendance of 10.5 million registered students. This initial participation involved 31,028 schools in 5,197 Brazilian municipalities, demonstrating significant national coverage from the beginning;
- — Expansion and consolidation: Over the following editions, the number of participants continued to grow, consolidating OBMEP as a prominent event on the educational calendar;
- — Current figures: The latest data show that OBMEP has reached peaks of participation, surpassing the mark of 18 million students. The 19th edition, held in 2024, had 18.5 million students from the 6th grade of elementary school to the 3rd grade of high school, registering a record number of schools and municipalities, reaching 99.9% coverage in Brazilian municipalities. The 20th edition, held in 2025, had 18.6 million students from the 6th grade of elementary school to the 3rd grade of high school, involving 57,222 schools (51,428 public and 5,794 private).

Currently, the OBMEP tests are administered at three levels: Level 1 (6th- and 7th-graders), Level 2 (8th- and 9th-graders), and Level 3 (high school students). Each level consists of two phases.

The 1st-phase test, consisting of 20 closed questions, is administered to all students from public and private schools. The 2nd-phase test, containing six open questions, was answered only by students classified in the 1st phase, who constitute approximately 5% of the total number of students enrolled at each school and level. It is up to each school to select the students with the best performance in the 1st phase to participate in the 2nd phase. In the end, the students with the best performances in the 2nd-phase test are awarded gold, silver, or bronze medals. As OBMEP medallists, those students receive junior scientific initiation scholarships from the National Council for Scientific and Technological Development (CNPq).

Therefore, given the importance of mobilizing the processes of representation and abstraction, characteristic of AMT, in the various processes of teaching and learning Mathematics, we justify an investigation into the possibilities of mobilizing such processes in the preparation of OBMEP questions, such as the one outlined below.

PRESENTING THE METHODOLOGICAL PROCEDURES OF THE INVESTIGATION

Our investigation, based on an interpretative approach, was qualitative and used document analysis (Bogdan & Biklen, 2013), considering the data-producing instruments described below.

Initially, in August 2025, we conducted a semi-structured interview with the Exam Committee Coordinator of the OBMEP, Prof. Dr. Pedro Mallagutti, an associate professor at the Federal University of São Carlos (UFSCAR), who holds a master's degree and a PhD in Mathematics². The interview with the Exam Committee Coordinator of the OBMEP was conducted in a semi-structured manner, based on a previously sent script, containing the following topics and subtopics:

- — OBMEP and the Exam Committee: contact with OBMEP; objectives of OBMEP; importance of OBMEP; coordination of the Exam Committee of OBMEP; work of the Exam Committee of OBMEP.
- — The formulation of OBMEP questions: general formulation principles; mathematical knowledge; representation processes; abstraction processes; expected levels of rigor.

Subsequently, we analysed selected questions from the 1st- and 2nd-phase tests of Level 3 (High School) of the 20th edition of OBMEP, administered in June and October 2025, respectively, in line with our research objective. The choice of questions was precisely to identify the AMT processes mobilized in their preparation.

PRESENTING AND ANALYSING THE RESULTS

We begin by briefly describing and analysing excerpts from the interview with the Exam Committee Coordinator of the OBMEP; then we present and analyse questions from the 2025 OBMEP tests, starting with the 1st-phase test and concluding with the 2nd-phase test, both at Level 3 (high school).

² The Informed Consent Form (ICF) was signed; however, no ethical evaluation of the research was requested by the competent bodies due to a time restriction. Furthermore, the authors exempt *Acta Scientiae* from any consequences, including full assistance and possible compensation for damages to any of the research participants, in accordance with CNS Resolution n. 510, of April 7, 2016.

In our analysis, we will not highlight the mobilization of symbolic and mental representation processes, as we consider them trivially inherent in the writing and interpretation of any component of mathematical theory, particularly in all the questions we will present and analyse.

THE INTERVIEW WITH THE EXAM COMMITTEE COORDINATOR OF THE OBMEP

We then present a brief description and analysis of only a few discussions raised by questions posed by the researchers/interviewers and their respective responses issued by the deponent that, to some extent, implicitly or explicitly translate their view on some of the AMT processes that can or even should be mobilized in the elaboration of OBMEP questions.

When asked about his contact with OBMEP and its objectives, the Coordinator highlighted a very peculiar characteristic of the “spirit” of OBMEP's questions:

They don't have to be questions with some insight. They must be smart questions. In fact, many students who do not do well in traditional Mathematics in school demonstrate creativity and strong logical reasoning on the OBMEP. It's not uncommon for this to happen (Interview with the Coordinator, August 2025).

We note, then, that creativity and logical reasoning are valued processes in the preparation of OBMEP tests, beyond the traditional knowledge of formulas and resolution procedures.

Regarding the general principles of developing OBMEP questions, the Coordinator pointed out that the members of the Exam Committee are free to propose questions in various contexts:

Newsstand comics, an original creation, or research that proves to be a different “thing”. [...] It turns out that it is not a book issue, which we commonly find in textbooks. It's never a matter of routine. We insist that it is not (Interview with the Coordinator, August 2025).

We emphasise, then, that one of the objectives of OBMEP is to ensure that its questions are contextualised and thus different from those traditionally presented in Mathematics textbooks.

Regarding the strategies for presenting the questions of the OBMEP test, the Coordinator pointed out that the questions obey “levels of difficulty”, the former being easier than the latter:

In general, the first 4 questions, 5 questions from the first sheet, are entrance questions, an invitation for the student to take the test. They are easier questions that they look at, think through a not-very-developed strategy, and they like to do it. In the next three questions, the step moves up, with the last page featuring much more difficult and challenging questions (Interview with the Coordinator, August 2025).

We note, then, that when presenting the questions by “levels of difficulty”, the expectation is that students solve the questions they consider easier to gradually feel challenged to solve more difficult ones.

Regarding the requirement of generalization in OBMEP questions, the Coordinator underscored that some questions begin by presenting a particular case and then present a more general case of a certain result:

In item (a), the question is more particular. In item (b), it is a more general form. In item (c), it is to make the general case (Interview with the Coordinator, August 2025).

We emphasise, then, that such questions require a generalization from particular cases that may take more general forms, as presented in the various items that make up a given question.

In view of the approach to OBMEP questions involving geometry, the Coordinator stressed the importance of visualization in the development of the students' geometric thinking levels:

In geometry, this is quite common as well, because Van Hiele's theory is widely used; it describes the levels of students' geometric thinking and helps them understand how they learn and develop their geometry knowledge. Geometry learning should be gradual, where intuition and reasoning are developed over time. That is, you need to visualize in the figures “things” that are not clearly drawn or that clearly do not exist (Interview with the Coordinator, August 2025).

We note, then, the appreciation of the processes of intuition and visualization in the elaboration of geometry issues, mainly seeking to explore more advanced levels of geometric thinking.

Regarding the approach of combinatorial analysis in OBMEP questions, the Coordinator pointed out that it is the "most difficult" area of Mathematics and that it also requires a generalization process:

Combinatorics is the most difficult area of Mathematics. If you want to complicate things, ask a question in combinatorics. First, we make a not-so-general case, and then the general one. That's how it works (Interview with the Coordinator, August 2025).

Then, generalization is a process mobilized in the formulation of questions of various mathematical content, based on specific cases that must be generalized sequentially.

Regarding the use of new theoretical approaches in OBMEP questions, the Coordinator pointed out that elements of graph theory have been explored in the last editions of the Level 3 tests (high school):

Now, we're using graph theory. We do not use that name, but based on the reasoning in the questions, the possibilities. We do it like this (Interview with the Coordinator, August 2025).

We note, then, that exploring elements of such a theory requires establishing relationships between the objects of a set and their modelling as structures to be visualized.

Regarding the preparation of the questions for OBMEP Mirim (ages 0–12), the Coordinator stressed the need to change the mathematical representations for children who are usually still in the literacy phase:

OBMEP Mirim demands more effort than the preparation for the older ones. You must communicate a Mathematics problem in three lines. You cannot assign a long text to a child who is learning to read. It must be a very short "thing". More visual. More playful. And it can't be an ordinary "thing." It's not about taking a pizza, dividing it into 4 slices, and asking for the fraction of each slice. That doesn't motivate anyone. It must be a "thing" that the student has a strategy to do, you know?! (Interview with the Coordinator, August 2025).

Thus, the need to adapt mathematical representations to students' educational levels for each test requires multiple representations in the presentation of the questions.

Again, referring to OBMEP questions that address geometry, the Coordinator pointed out the possibility of relating geometric contents with graphical representations of functions:

You are saying everything that we, in fact, try to do. I will give examples: the geometry questions that involve a kind of movement. Two triangles that are moving, one against the other. This generates a common area. This common area is a graphical representation of a function. So, while doing a dynamic in geometry with area calculation, you also need to interpret what will happen graphically, which will have a maximum or minimum point etc. (Interview with the Coordinator, August 2025).

We note, then, the possibility of modelling offered by some questions, as they require the synthesis of different contents, for example, geometric and algebraic, to solve a given problem.

The modelling in the OBMEP questions is also highlighted by the Coordinator, especially in the 2nd-phase tests, due to the characteristics of such tests, in addition to their differences in relation to the 1st phase tests:

I think the modelling process fits better in the 2nd phase. Because in the 1st phase, there is limited space in the test itself, and the test is very general. So, in the 2nd phase, I think I could do a modelling process in this gradual "spirit" to move towards a firmer result (Interview with the Coordinator, August 2025).

Thus, the preparation of the tests is guided by the objectives of each phase, and the 2nd phase is characterised by questions that mobilize more elaborate processes, such as modelling and rigor.

When addressing some demonstration processes in the OBMEP questions, the Coordinator highlighted finite induction, noting that the method can be developed without the characteristic rigor of mathematical demonstrations:

For some questions, for example, you see students using the finite induction principle very clearly. But a student who has not gone through this process will not do it with that rigor. Prove that it is valid for $n = 1$. But they correctly do the question using finite induction, in their language. Look, they must think like this: since I saw that it started this way and it

was always valid for the case ahead, it will always work. It is a “thing” that is transmitted. So, this level of abstraction and rigor must be considered. Because if we demand that abstraction be only for trained students, it will be very bad. We want them to have students who are coming to the abstraction of resolution (Interview with the Coordinator, August 2025).

We note, then, that abstraction is also mobilized in solving questions that require demonstrating results, even when the methods of demonstration are not formally rigorous.

Finally, the work on rigor is also addressed by the Coordinator, especially in the 2nd-phase tests, but at different levels for elementary and high school:

In elementary school, one already works on the theme; one already seeks a specific, adequate level of rigor in writing as well as in justifying the results. In high school, a little more is required (Interview with the Coordinator, August 2025).

Then, rigorization is a process considered when formulating specific questions, such that the “adequate level” of rigor relates not only to writing but also to the argumentation used in resolutions.

In the discussion of the results, we will revisit some of the questions addressed by the Coordinator in his interview to derive a more holistic view of the AMT processes mobilized in formulating OBMEP questions from his speeches.

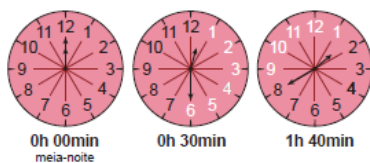
THE FIRST PHASE OF 2025 OBMEP LEVEL 3 TEST

The 1st-phase test of OBMEP Level 3 (high school) carried out in 2025 consisted of 20 closed questions, each with 5 options. As a matter of research, we chose to briefly present and analyse only five questions whose formulation requires evidence of the mobilization of certain AMT processes for their resolution. However, we emphasise that the resolutions to the questions will not be presented here and can be accessed from the same source as the questions. Question 6 is presented in Figure 1 below.

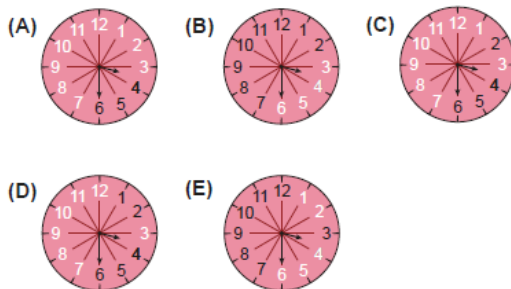
Figure 1

Question 6 of the 1st-phase test of 2025 OBMEP Level 3

A cor de cada número de um relógio alterna de preto para branco ou de branco para preto, cada vez que qualquer um dos ponteiros aponta para a marca da hora que esse número representa no relógio. À meia-noite todos os números ficam pretos e, depois, alternância de cores começa. Na figura, vemos as cores dos números do relógio em alguns horários:



Como estarão as cores dos números às 03h30?



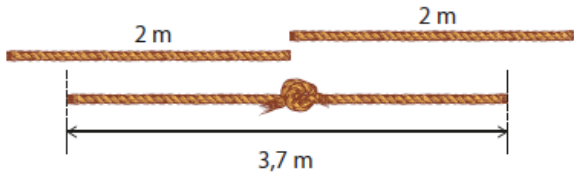
The color of each number on a clock alternates from black to white or from white to black every time either hand points to the hour mark that the number represents on the clock. At midnight, all the numbers turn black, and then the color alternation begins. In the figure, we see the colors of the numbers on the clock at certain times. What will the colors of the numbers be at 3:30 a.m.?

The formulation of this question shows that the resolution requires students to mobilize *visualization* to understand the relationship between the alternation of colours and the time indicated by the clock, and, initially, also *generalization* of the parity of the number of colour changes for some of the times presented. Question 9 is presented in Figure 2 below.

Figure 2

Question 9 of the 1st-phase test of 2025 OBMEP Level 3

Jairo tem muitas cordas com 2 metros cada uma. Quando ele amarra duas dessas cordas, ele obtém uma nova corda com 3 metros e 70 centímetros.



The diagram illustrates the process of tying two ropes. At the top, two separate ropes are shown, each labeled '2 m'. Below them, the two ropes are tied together at a central knot. A horizontal line with arrows at both ends spans the total length of the tied ropes, labeled '3,7 m'.

Se ele quiser uma corda com exatamente 20 metros, ele precisará cortar um pedaço da última corda amarrada. Qual é o tamanho, em metros, desse pedaço?

a) 0,95 b) 0,70 c) 0,45 d) 0,30 e) 0,15

Jairo has many ropes, each 2 meters long. When he ties two of these ropes together, he gets a new rope that is 3 meters and 70 centimeters long. If he wants a rope that is exactly 20 meters long, he will need to cut a piece from the last rope he tied together. How long, in meters, is that piece?

The conception of this question shows that the resolution requires the mobilization of *modelling* to algebraically represent the measurements of the strings and their pieces according to the data provided, and also *generalization* to obtain an equation that represents the total length of the string, thus allowing the size of the piece of the last string to be obtained. Question 12 is presented in Figure 3 below.

Figure 3

Question 12 of the 1st-phase test of 2025 OBMEP Level 3

Qual a soma dos algarismos do número
 $(10^{25} + 25)^2$.

a) 18 b) 19 c) 20 d) 25 e) 28

What is the sum of the digits of the number?

The formulation of this question shows that the resolution requires the mobilization of *changes of representations and translation between them*, involving numerical representations in the forms of powers and decimals, as well as *synthesis* in relation to the positional system of numbers, enabling different numerical representations. Question 18 is presented in Figure 4 below.

Figure 4

Question 18 of the 1st-phase test of 2025 OBMEP Level 3

Dizemos que um número é ordeiro quando tem exatamente três algarismos diferentes de zero que estão em ordem crescente ou decrescente de valor. Por exemplo, os números 237 e 921 são ordeiros, mas os números 231 e 315 não são. Qual é a soma de todos os números ordeiros?

- a) 93250 b) 93240 c) 93260 d) 93230 e) 93270

We say that a number is ordered when it has exactly three digits other than zero that are in ascending or descending order of value. For example, the numbers 237 and 921 are ordered, but the numbers 231 and 315 are not. What is the sum of all ordered numbers?

The formulation of this question shows that the resolution requires the mobilization of the *abstraction* involved in understanding the definition of an orderly number and also the *generalization* necessary to establish the representation of all orderly numbers, considering the symmetry properties of the decimal system, even if there is a limitation of three digits in the definition. Question 20 is presented in Figure 5 below.

Figure 5

Question 20 of the 1st-phase test of 2025 OBMEP Level 3

Seja $U = \{1, 2, 3, 4, 5\}$ um conjunto com 5 elementos e sejam A , B e C subconjuntos de U . Dizemos que (A, B, C) é uma tripla ordenada especial se $C \subseteq A \cup B$. Quantas são as triplas ordenadas especiais?

- a) 7^5 b) 5^7 c) 5^6 d) 6^5 e) 5^5

Let $U = \{1, 2, 3, 4, 5\}$ be a set with 5 elements, and let A , B , and C be subsets of U . We say that (A, B, C) is a special ordered triple if $C \subseteq A \cup B$. How many special triples are there?

The formulation of this question shows that the resolution requires the mobilization of the *abstraction* involved in understanding the definition of a special ordered triple, as well as the *synthesis* necessary to integrate concepts and relationships between set theory and combinatorial analysis in determining the number of special ordered triples.

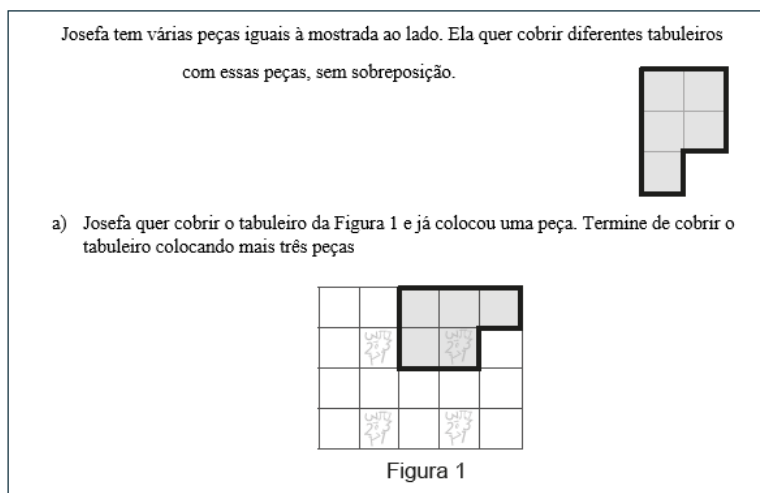
THE SECOND PHASE OF 2025 OBMEP LEVEL 3 TEST

The 2nd-phase test of OBMEP Level 3 (high school) carried out in 2025 consisted of 6 open questions, each with 3 or 4 items. As a matter of research, we chose to briefly present and analyse only 3 questions whose formulation requires evidence of the mobilization of certain AMT processes for their resolution. However, we emphasise that the resolutions to the questions will not be presented here and can be accessed from the same source as the questions.

Question 2 is presented in Figures 6, 7, and 8 below.

Figure 6

Question 2(a) of the 2nd-phase test of 2025 OBMEP Level 3



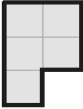
Josefa has several pieces just like the one shown here. She wants to cover different boards with these pieces, without overlapping them.

a) Josefa wants to cover the board in Figure 1 and has already placed one piece. Finish covering the board by placing three more pieces.

Figure 7

Question 2(b) of the 2nd-phase test of 2025 OBMEP Level 3

Josefa tem várias peças iguais à mostrada ao lado. Ela quer cobrir diferentes tabuleiros com essas peças, sem sobreposição.



b) Cubra o tabuleiro da Figura 2 com quatro peças de Josefa

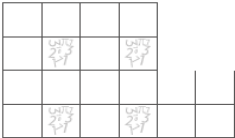


Figura 2

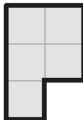
Josefa has several pieces just like the one shown here. She wants to cover different boards with these pieces, without overlapping them.

b) Cover the board shown in Figure 2 with four Josefa pieces

Figure 8

Question 2(c) of the 2nd-phase test of 2025 OBMEP Level 3

Josefa tem várias peças iguais à mostrada ao lado. Ela quer cobrir diferentes tabuleiros com essas peças, sem sobreposição.



c) Explique por que Josefa nunca vai conseguir cobrir o tabuleiro da Figura 3 com cinco peças

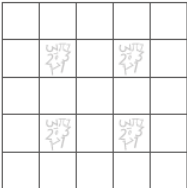


Figura 3

Josefa has several pieces just like the one shown here. She wants to cover different boards with these pieces, without overlapping them.

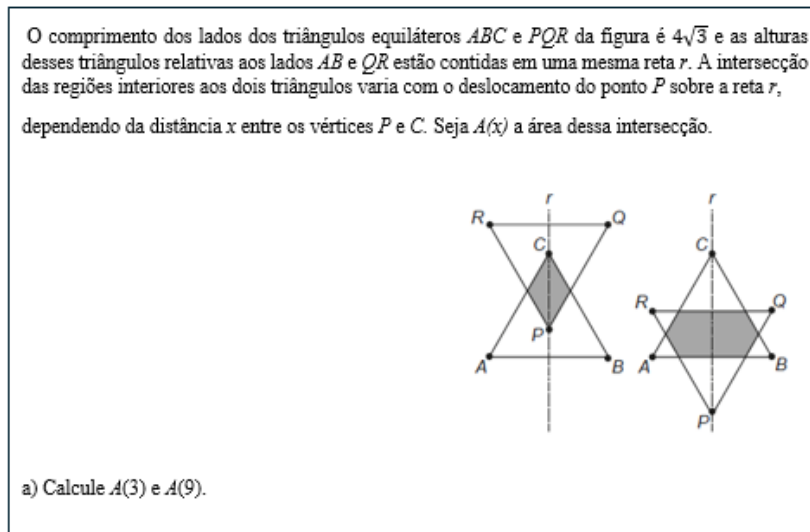
c) Explain why Josefa will never be able to cover the board in Figure 3 with five pieces

The preparation of this question shows that items *a* and *b* require the mobilization of the *visualization* necessary for the exact fitting of the pieces on the various boards presented, and item *c* requires *abstraction* in creating a justification based on geometric concepts such as translation, rotation, and reflection, particularly isometries, associated with a combinatorial restriction of filling in the space.

Question 3 is presented in Figures 9, 10, and 11 below.

Figure 9

Question 3(a) of the 2nd-phase test of 2025 OBMEP Level 3



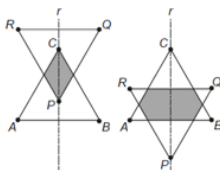
The lengths of the sides of the equilateral triangles ABC and PQR in the figure are $4\sqrt{3}$, and the heights of these triangles relative to the sides AB and QR lie on the same line r . The intersection of the regions inside the two triangles varies as point P moves along line r , depending on the distance x between vertices P and C . Let $A(x)$ be the area of this intersection.

a) Calculate $A(3)$ and $A(9)$.

Figure 10

Question 3(b) of the 2nd-phase test of 2025 OBMEP Level 3

O comprimento dos lados dos triângulos equiláteros ABC e PQR da figura é $4\sqrt{3}$ e as alturas desses triângulos relativas aos lados AB e QR estão contidas em uma mesma reta r . A intersecção das regiões interiores aos dois triângulos varia com o deslocamento do ponto P sobre a reta r , dependendo da distância x entre os vértices P e C . Seja $A(x)$ a área dessa intersecção.



b) Exiba uma expressão de $A(x)$ no intervalo $0 \leq x \leq 6$ e uma expressão de $A(x)$ no intervalo $6 \leq x \leq 12$.

The lengths of the sides of the equilateral triangles ABC and PQR in the figure are $4\sqrt{3}$, and the heights of these triangles relative to the sides AB and QR lie on the same line r . The intersection of the regions inside the two triangles varies as point P moves along line r , depending on the distance x between vertices P and C . Let $A(x)$ be the area of this intersection.

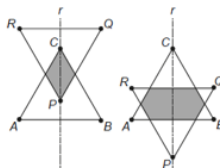
b) Write an expression for $A(x)$ on the interval $0 < x < 6$ and an expression for $A(x)$ on the interval $6 < x < 12$.

Figure 11

as point P moves along line r , depending on the distance x between vertices P and C . Let $A(x)$ be the area of this intersection.

c) What is the maximum value of $A(x)$ on the interval $0 < x < 12$?

O comprimento dos lados dos triângulos equiláteros ABC e PQR da figura é $4\sqrt{3}$ e as alturas desses triângulos relativas aos lados AB e QR estão contidas em uma mesma reta r . A intersecção das regiões interiores aos dois triângulos varia com o deslocamento do ponto P sobre a reta r , dependendo da distância x entre os vértices P e C . Seja $A(x)$ a área dessa intersecção.



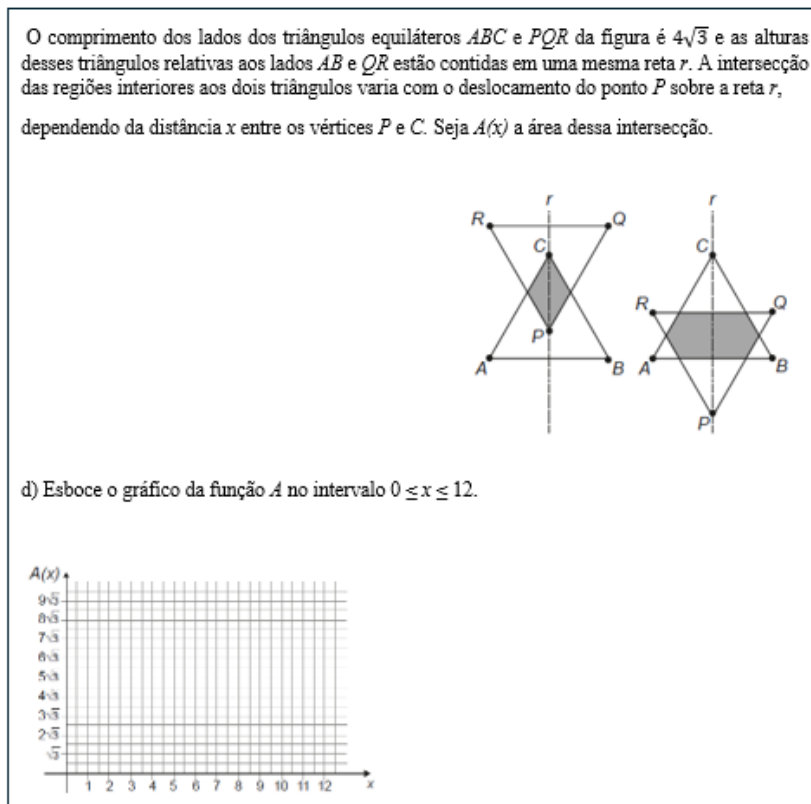
c) Qual é o valor máximo de $A(x)$ no intervalo $0 \leq x \leq 12$?

Question 3(c) of the 2nd-phase test of 2025 OBMEP Level 3

The lengths of the sides of the equilateral triangles ABC and PQR in the figure are $4\sqrt{3}$, and the heights of these triangles relative to the sides AB and QR lie on the same line r . The intersection of the regions inside the two triangles varies

Figure 12

Question 3(d) of the 2nd-phase test of 2025 OBMEP Level 3



The lengths of the sides of the equilateral triangles ABC and PQR in the figure are $4\sqrt{3}$, and the heights of these triangles relative to the sides AB and QR lie on the same line r . The intersection of the regions inside the two triangles varies as point P moves along line r , depending on the distance x between vertices P and C . Let $A(x)$ be the area of this intersection.

d) Sketch the graph of function A on the interval $0 \leq x \leq 12$

The formulation of this question shows that the resolution requires the mobilization: in items *a*, *b*, and *c*, of *generalization* to obtain the formulas of the height and area of the equilateral triangle, of *synthetisation* of geometry concepts to solve a problem of functions, and of *modelling* of the algebraic expression for the sentence that defines the area function; and, in item *c*, the *change of representations and translation between them* involving the algebraic and graphical representations of the area function.

Question 4 is presented in Figure 13 below.

Figure 13

Question 4 of the 2nd-phase test of 2025 OBMEP Level 3

- 4) Uma prova com 5 questões será aplicada a um grupo de alunos. Cada aluno receberá o resultado por meio de uma sequência de zeros e uns, indicando as questões que errou ou acertou. Por exemplo, a sequência [0 1 0 0 0] indica que o aluno acertou a segunda questão e errou as demais.
- a) Liste todas as possíveis sequências de zeros e uns que indicam exatamente duas questões corretas
- b) Explique por que, se a prova for aplicada a 33 alunos, pelo menos dois deles vão receber a mesma sequência de zeros e uns como resultado.
- c) Suponha que a prova seja aplicada a um grupo de alunos e que o total de questões respondidas corretamente seja 81. Explique por que podemos garantir que pelo menos dois alunos receberão a mesma sequência de zeros e uns como resultado

A test with 5 questions will be administered to a group of students. Each student's score is represented by a sequence of zeros and ones, indicating which questions they got right and which they got wrong. For example, the sequence [0 1 0 0 0] indicates that the student got the second question right and got the rest wrong.

- a) List all possible sequences of zeros and ones that indicate exactly two correct questions.*
- b) Explain why, if the test is administered to 33 students, at least two of them will receive the same sequence of zeros and ones as their result.*
- c) Suppose the test is administered to a group of students and the total number of questions answered correctly is 81. Explain why we can guarantee that at least two students will receive the same sequence of zeros and ones as their result.*

The formulation of this question shows that the resolution requires the mobilization: in item *a*, the *change of representations and translation between them* involving the matrix and combinatorial representations, in item *b*, the *abstraction* necessary for the use of the pigeonhole principles; and in item *c*, the *generalization* of the number of types of evidence from a given presented data, and the rigorization in writing and argumentation built throughout the justification.

DISCUSSING THE RESULTS

Initially, the brief description and analysis of the interview with the Exam Committee Coordinator of the OBMEP implicitly or explicitly conveyed his views on certain AMT processes that can, or even should, be mobilized in formulating OBMEP questions.

The first highlight is the general principle of valuing creativity and logical reasoning when preparing questions, as the Coordinator emphasised. In this context, the processes involved in representation tend to be valued, such as: *visualization* mobilized in questions associated with the development of geometric thinking and that present geometric structures and relationships; the *change of representations and translation between them* mobilized in questions that require algebraic, geometric, and/or graphical representations, present at all levels of evidence; *modelling* mobilized in contextualised questions and that use new theoretical approaches as elements of graph theory.

Another highlight made by the Coordinator was the importance of the “levels of difficulty” of the questions and tests in each phase of the OBMEP. In this context, the processes involved in abstraction tend to be valued, such as: the *synthesis* mobilized in questions that require the use of concepts and relationships of different contents for their resolution; the *generalization* mobilized in questions that start from specific cases to more general cases, as evidence for the approach to principles of combinatorial analysis; the *rigorization* mobilized in questions that require writing, arguments, and demonstrations to justify the results obtained or formulas presented.

In turn, the analysis of the questions of the 1st-phase test of Level 3 (high school) of OBMEP 2025 revealed that, even with “closed” questions, the increasing levels of difficulty also favour the valuation of both processes involved in representation and abstraction, often in the same question, such as: the *visualization* mobilized in questions that require the understanding of properties presented through examples and the *generalization* mobilized through the expansion of validity to more general cases; the *modelling*

mobilized in questions that require the use of a representation different from that presented in the data and the *generalization* mobilized through the extension provided by such representation; the *change of representations and translation between them* mobilized in questions that require the use of various representations and the *synthetisation*, mobilized through the composition of other mathematical concepts necessary for their resolution.

Finally, the analysis of the questions of the 2nd-phase test of 2025 OBMEP Level 3 (High School) revealed that the creation of "open" questions with several items favours the valuation of processes involved in the representation; however, the focus fell on the processes involved in the abstraction, in line with the analysis we made of the interview with the Exam Committee Coordinator of the OBMEP.

Even so, it should be noted that, because it requires more complex and justified writing on certain items, there is a potentiation of the mobilization of the indispensable *generalization* to solve questions that demand the extension of results to cases, in fact, general, an also of the *rigorization* in the argumentation and, sometimes, demonstration of results or formulas, which resignifies the results presented by Torrente and Reis (2023, a).

In conclusion, we present our considerations regarding our investigation and the set of its results and analyses.

FINAL CONSIDERATIONS

In this article, we present a study aimed at identifying the AMT processes mobilized in the formulation of OBMEP questions. It is worth remembering that, like Dreyfus (2002), we understand that AMT is not a single process, but rather a series of processes that are interrelated and, exactly, as they are mobilized in the development of a theory or mathematical activity, they contribute to the advancement of mathematical thinking from more elementary levels.

Furthermore, the investigation of complex cognitive processes in Mathematics Education requires a precise delimitation of its theoretical constructs in view of the observed school context. In the present investigation, the focus was on the development of AMTs, specifically at the high school level. It is important, however, to establish a fundamental epistemological differential: the classification of a cognitive activity as "advanced" or "elementary" does not constitute an absolute attribute of the mathematical object itself, but rather a relationship between the level of formalization required and the student's stage of development.

We also sought to highlight our option for classifying the AMT processes involved in representation and abstraction (Gereti & Savioli, 2015) so that, in our analysis of the qualitatively produced data, we could identify evidence of the demand to mobilize such processes in the formulation/resolution of the questions presented.

We must consider that the processes involved in representation are mobilized in the formulation of Mathematics questions, in general, since we understand that it is natural that the OBMEP questions provide such mobilization; however, we highlight the potentiation of the mobilization of *visualization* associated with the development of geometric thinking, the *change of representations and translation between them* covering algebraic, geometric and/or graphical representations, and *modelling* through contextualisation.

Also worth mentioning is the high potential of OBMEP's questions for the mobilization of the processes involved in abstraction, characteristic of more elaborate mathematical activities; particularly, in addition to the mobilization of *synthesis* through the fusion of concepts and relationships and *generalization* under restricted or generic conditions, we point to the mobilization of *rigorization* as a differential of the vast majority of Mathematics questions created in the context of high school.

Finally, we understand that one of the premises of OBMEP, which is to encourage the development of mathematical skills such as logical thinking and creativity, must be a principle not only for the formulation of Olympic questions, but of any activity that perspectives learning and the advancement of mathematical thinking and, in this context, must gain increasing relevance in Mathematics Education, research on the mobilization of the processes of representation and abstraction for the development of the AMT.

AUTHORSHIP CONTRIBUTION STATEMENT

Authors ACB and FSR were responsible for the conceptualisation of the research and its objectives, the development of the theoretical framework, and the methodological design. Author AD was responsible for developing the theoretical framework and the methodological design. FSR was responsible for supervising the postdoctoral studies. AD was responsible for supervision in international collaboration. All authors actively contributed to the production and analysis of the data, the discussion of the results, the revision of the manuscript, and approved the final version of the article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, ACB, upon reasonable request.

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