

Mathematics teacher educators' professional learning in a design-based community of practice

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ABSTRACT

Background: A persistent conceptual gap exists regarding the professional development of mathematics teacher educators (MTEs) in higher education. **Objectives:** To characterise and interpret the learning opportunities and expertise development of eight MTEs within a collaborative network. **Methodology:** This qualitative-interpretive study employs design-based research (DBR) over a 28-month period. **Context and Participants:** The sample comprises eight MTEs from diverse Brazilian universities, organised into two subcommunities of practice (Foundations and Teaching Practice/Supervised Practicum) within the research group ForMatE (*Formação Matemática para o Ensino*). **Data collection and analysis:** The corpus involves qualitative triangulation contrasting minutes of plenary meetings (2024–2026), the material reification of two professional learning tasks (PLTs), and eight individual interviews analysed through a matrix of interactive scenarios. **Results:** Transitioning to an authentic collaborative model allowed MTEs to voice vulnerabilities, deconstruct academic hierarchies, and problematize historical teaching practices. Facing logistical constraints and ethical tensions in simulating protocols via Artificial Intelligence, the community innovated theoretically by expanding the design toward representations of practice. Plenary meetings functioned as a locus of global legitimation, fostering mutual scaffolding between pure mathematicians and mathematics educators. **Conclusions:** The dialectical movement of designing and validating tasks generated a shared intelligence that materialised the principle of inverse symmetry, turning the community into the very exploratory teaching model it advocates.

Keywords: Mathematics teacher educators; Design-based research; Communities of practice; PLOT model; Professional learning tasks.

Aprendizagem profissional de formadores de professores de matemática em uma comunidade de prática baseada em design

RESUMO

Contexto: Há uma lacuna conceitual persistente quanto ao desenvolvimento profissional de formadores de professores de matemática (FPMs) no ensino superior. **Objetivos:** Caracterizar e interpretar as oportunidades de aprendizagem e o

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desenvolvimento da expertise de oito FPMs em uma rede colaborativa. **Metodologia:** Investigação qualitativa-interpretativa apoiada na *design-based research* (DBR) ao longo de 28 meses. **Contexto e Participantes:** Oito FPMs de diferentes universidades brasileiras, organizados em duas subcomunidades de prática (Fundamentos e Prática/Estágio) no grupo ForMatE (Formação Matemática para o Ensino). **Coleta e análise de dados:** O *corpus* envolve a triangulação qualitativa confrontando atas de reuniões plenárias (2024–2026), a reificação material de duas tarefas de aprendizagem profissional (TAPs) e oito entrevistas individuais, analisadas por meio de uma matriz de cenários interativos. **Resultados:** A transição para um modelo colaborativo autêntico permitiu aos formadores externalizar vulnerabilidades, desconstruir hierarquias acadêmicas e problematizar práticas históricas de ensino. Diante de contingências logísticas e de tensões éticas ao simular protocolos por meio de inteligência artificial, a comunidade operou uma inovação teórica, expandindo o design para o conceito de representações da prática. As plenárias funcionaram como um locus de legitimação global, promovendo o suporte mútuo entre matemáticos puros e educadores matemáticos. **Conclusões:** O movimento dialético de projetar e validar tarefas gerou uma inteligência compartilhada que materializou o princípio da simetria inversa, transformando a comunidade no próprio modelo de ensino exploratório que defende.

Palavras-chave: Formação de formadores; Design-based research (pesquisa baseada em design); Comunidades de prática; Modelo PLOT; Tarefas de aprendizagem profissional.

INTRODUCTION AND BACKGROUND

Over the past few decades, research in mathematics education has devoted substantial efforts to understanding the professional development and learning opportunities of pre-service and in-service mathematics teachers currently working in primary and secondary education. However, this significant volume of research contrasts with a persistent conceptual and empirical gap in the specialist literature: the education and expertise of mathematics teacher educators (MTEs) themselves. As leading authors in the international arena (such as Ponte, 2014; Even & Ball, 2009; and Cochran-Smith, 2003) point out, it has historically been naively assumed that a professional with a solid background in pure mathematics or with experience in primary/secondary teaching is automatically qualified to take on the complex role of “teaching how to teach mathematics”. This lack of formal, intentional spaces for MTE education results in pedagogical isolation within university departments, where initial education practices are often inherited or reproduced without underlying critical theorisation.

This issue becomes even more challenging when analysed from the perspective of curriculum contingency and the historical divide between “academic mathematics” (taught in advanced university courses) and “school

mathematics” (taught in basic schools). The collaborative network research project funded by CNPq (Universal, 2023), through the ForMatE – *Formação Matemática para o Ensino* [Mathematics Education for Teaching] research group, emerged precisely to address this threefold gap: the need to make teaching practice the starting point in initial education; the urgency of intentionally linking academic and school mathematical knowing; and, fundamentally, the design of spaces that promote the professional learning of the very teacher educators involved in this process.

To underpin and support this formative ecology, the project adopted design-based research (DBR) as its methodological approach, implemented through the design, implementation, and refinement of professional learning tasks (PLTs) based on the PLOT model (Ribeiro & Ponte, 2020) and on the construct of teachers’ noticing (Llinares, 2013). Although PLTs are originally intended to develop undergraduates’ noticing, this article argues that the complex process of collectively conceiving, challenging, reifying, and evaluating these teaching artefacts itself provides a powerful and unique setting for the professional development of the teacher educators involved.

From the sociocultural perspective of social learning established by Etienne Wenger (1998), learning does not take place in an individual vacuum, but in the continuous negotiation of meanings within a community of practice (CoP). This study refuses traditional views that reduce teacher education to the mere passive appropriation of theories.

In line with Ribeiro and Llinares’s (2025) theoretical-analytical framework, we adopt the premise that MTEs’ professional learning is characterised and driven by their active participation in interactive co-creation scenarios. According to a previous study, the development of teacher educators’ expertise is manifested through three fundamental analytical traits: the ability to problematise what has been assumed; seeking a constant balance between the cognitive, social, and affective aspects of interactions; and the modelling and performance of expert theoretical leaders who play the role of “brokers” or conceptual mediators.

Given this scenario, this article proposes an intentional reversal of the classic investigative perspective: the focus shifts from the student teacher to an examination of teacher educators’ subjectivity, tensions, and identity transformations. The scientific relevance of this study lies in uncovering the “black box” of subgroup meetings and discussions, treating them as primary, documentational living data (Fiorentini & Oliveira, 2013).

To this end, the study seeks to answer the following research questions:

1. *In what way do social interactions and collaborative work within subgroups enable the negotiation of meanings and the questioning of traditional practices among teacher educators?*
2. *What tensions and contingencies (epistemological, didactic, and affective) emerge in the process of reifying PLTs in initial teacher education, and how do subcommunities overcome them?*
3. *How do the heterogeneity of profiles (senior-novice, pure mathematics-education) and the mediation of local leaders (brokers) shape the participation trajectories of teacher educators in the community?*

In answering these questions, this study aims to characterise and interpret the learning opportunities of eight teacher educators from different Brazilian universities, divided into two subgroups (Foundations and Teaching Practice/Supervised Practicum). By combining the material reification of the *Biscuits* (Algebra) and *Triangles on the Grid* (Geometry) PLTs with the reflective testimonies and plenary dynamics generated between 2024 and 2026, this article aims to advance theorisation on how inverse symmetry or formative isomorphism (as problematised by Oliveira and Bueno (2013)) can become an embodied and transformative practice for higher education in mathematics.

THEORETICAL FRAMEWORK

The sociocultural perspective of communities of practice

To understand the professional development and learning opportunities of mathematics teacher educators (MTEs), this study draws on the sociocultural perspective of social learning, with the central framework of communities of practice (CoPs) formulated by Etienne Wenger (1998). From this perspective, learning is not reduced to a purely individual process of cognitive accumulation or passive absorption of manuals; it is interpreted as an act of social participation in a collective's practices and in the continuous negotiation of meanings around a common interest. According to Wenger (1998), a CoP is structured around three interdependent pillars: domain, community, and practice.

The *domain* forms the basis of the community's identity, defining a common ground of interest that unites its members and specifies their purposes. In the context investigated, the domain is defined by research and the design of initial teacher education scenarios for mathematics teachers, supported by exploratory teaching. *Community* refers to the web of social relationships,

interactions, and discussions that generate a sense of belonging and mutual cohesion. Within the community, members support each other, share vulnerabilities, and validate their actions. Finally, *practice* refers to the shared repertoire of material and symbolic resources that the group develops over time (tools, languages, templates, artefacts, and narratives).

The evolutionary dynamics of a CoP are based on the dialectic between participation and reification (Wenger, 1998; Llinares, 2002). Participation involves lived experience, action, and integration into the group's social interactions. Reification, in turn, is the process of giving shape and materiality to abstract concepts, translating the community's discussions into 'things' or physical artefacts. In teacher educators' training, professional learning occurs through the design and use of these artefacts, shifting individuals' sense of belonging from peripheral participation to central, active participation at the heart of the community.

Characteristics of Interactive Scenarios in Teacher Educators' Training

Although the Wengerian framework offers a macro lens for interpreting social relationships, the investigation into the expertise and learning of mathematics teacher educators revealed the need for specific analytical categories that take into account the microsystem of scientific and didactic discussions. To this end, this study adopts the analytical framework established by Ribeiro and Llinares (2025), which characterises the learning opportunities of MTEs based on three fundamental features of interactive scenarios:

1. **Problematising what has been assumed:** This category examines how immersion in a co-creation environment encourages teacher educators to question their own historical certainties and practices entrenched in higher education. It involves deconstructing hierarchical identities and moving away from traditional knowledge-transmission models of teaching to adopt open, investigative approaches to mathematical and pedagogical knowledge.
2. **Seeking a balance between cognitive, social and affective aspects:** This dimension recognises that the successful design of formative material (cognitive dimension) is inextricably linked to the management of institutional time and curriculum syllabuses and regulations (social dimension), as well as to addressing insecurities, fear of peer judgement, and teachers' health conditions and workload (affective dimension). The teacher educator's learning is reflected in their ability

to adapt and fine-tune the design to keep the community operational and sustainable.

3. Modelling the expert MTE's role as a broker: The concept of broker (or boundary mediator), which comes from Wenger (1998) and was refined by Ribeiro and Llinares (2025), characterises the action of theoretical leaders who introduce new elements and scientific connections into the group. The broker introduces specialised literature, strictly defines the adopted terminology, and resolves epistemological conflicts, serving as a bridge that anchors novice teacher educators' immediate practice within the contemporary frontiers of international research.

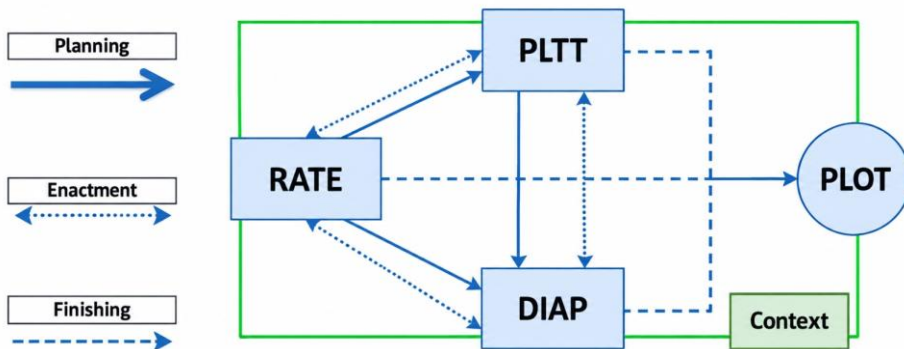
The PLOT Model: Professional Learning Opportunities for Teachers

The content and processes that constitute the domain and practice of the collaborative network investigated are theoretically guided by the framework of professional learning opportunities for teachers, widely known in the literature as the PLOT model, coined by Ribeiro and Ponte (2020). This theoretical-methodological model was designed to break away from linear or compartmentalised views of education. Its central purpose is based on two inseparable fronts: (i) organising and guiding the design of intentional formative processes and (ii) acting as an analytical framework to identify, reveal, and understand whether and how opportunities arise to deepen professional knowledge for teaching mathematics.

Inspired by Clarke and Hollingsworth's (2002) interconnected model of professional growth, the PLOT model adopts a strictly interactive and situated perspective on learning. The model's ecosystem is structured around the systemic integration of three main domains or dimensions, which operate in a connected manner across three operational phases: Organisation, Development, and Finalisation (Figure 1).

Figure 1

PLOT Model. Adapted from Ribeiro and Ponte (2020)



Role and Actions of Teacher Educator (RATE): This domain examines the educator's agency and role as the designer and orchestrator of the formative process. In the *conceptual dimension*, it involves the components of approximation (the intentional effort to strengthen ties and mitigate the discontinuity between academic mathematics and school mathematics) and articulation, which acts to stimulate and emphasise the inseparability between the purely mathematical and didactic dimensions of the professional knowledge necessary to lead the classroom (Ball et al., 2008; Rowland et al., 2005). In the *operational dimension*, it feeds into management (managing and supporting an exploratory teaching-learning environment) and the orchestration of productive didactic and mathematical discussions among all participants.

Professional Learning Task for Teachers (PLTT): PLTTs are the central mediating artefacts of the formative process. Its *conceptual dimension* requires the artefact to explore the teacher's specific professional knowledge and immerse it in the philosophy of exploratory teaching. Its *operational dimension* requires the coupling of a mathematical task for students (MTS), which must be characterised by a high level of cognitive demand and investigative potential (Boston & Smith, 2009; Smith & Stein, 1998), capable of triggering generalisation processes, and which must be associated with multiple practice records from real classrooms, structurally organised in the form of vignettes (student protocols, transcribed episodes, or video recordings).

Discursive Interactions Among Participants (DIAP): This domain focuses on the quality and nature of the discourse that emerges during task performance. Its *conceptual dimension* encompasses the promotion of

structured mathematical and didactic discussions, based on legitimate processes of argumentation and justification. Its *operational dimension* regulates the mobilised language, which requires the educator and the pre-service teacher to use precise mathematical and didactic terminology that is relevant and appropriate to the level of education being studied (Adler & Ronda, 2014; Radford & Barwell, 2016), fostering the transition from ritual discourse to exploratory discourse, and dialogic and integrative communication between educators and pre-service teachers.

As Ribeiro and Ponte (2020) assert, the amalgamation and binding of these three domains and their respective components break with traditional teacher isolation, creating the boundary conditions necessary for implementing professional learning opportunities (PLOs). In this article, the PLOT model is used from a mirrored perspective of inverse symmetry: we investigate how the complex process of MTEs acting in the *Organisation* phase (designing the components of the “Biscuits” and “Triangles” PLTTs) and in the *Development* phase (managing the orchestration and dialogical communication in the plenary) reverberates in the growth and learning of their own advanced mathematical and didactic knowledge.

Synthesis and Analytical Articulation: The Theoretical Coupling of the Study

The epistemological basis of this article does not lie in the isolated application of its reference points, but rather in the systemic and symmetrical coupling of its three dimensions. To make the theoretical framework that examines the empirical data in the Results and Discussion sections tangible, the concepts of communities of practice (Wenger, 1998), interactive scenarios (Ribeiro & Llinares, 2025) and professional learning opportunities (Ribeiro & Ponte, 2020) were merged into a single integrated analytical matrix, operating at complementary levels of granularity (Figure 2).

The macro concept of community of practice (CoP) offers a sociological lens for interpreting the social nature of the ForMatE group. From this perspective, general meetings and subgroup activities are no longer seen as mere administrative rituals, but rather as the living record of a community negotiating meanings around a common interest (the *domain*).

At this macro level, educators’ professional learning is assessed by their ability to *reify* abstract concepts from the literature into tangible tools for university teaching (the artefacts), generating a shared repertoire that breaks with the chronic isolation of higher education.

To refine this interpretation and understand *what* makes this community tick, the PLOT model (Ribeiro & Ponte, 2020) serves as a meso-level lens focused on didactic engineering. The PLOT framework regulates the structural quality of the tasks investigated, ensuring that the design implemented by the educators in the *Organisation* phase inextricably encompasses their role and actions of teacher educator (RATE), the high cognitive demand of the task combined with practice records (PLTT), and the induction of dialogic discursive interactions (DIAP).

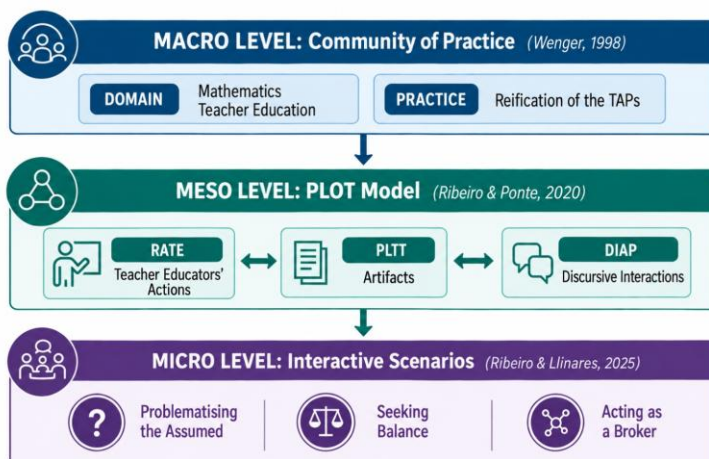
However, to capture professional learning at the exact moment it occurs — in the spark of plenary debates and conceptual conflicts — the matrix's micro-level lens is mobilised: the characteristics of Ribeiro and Llinares's (2025) interactive scenarios. This final analytical breakdown enables us to convert the practical tensions documented in the minutes and interviews into explanatory scientific categories.

Thus, when the article discusses the controversy over the use of artificial intelligence to generate protocols or the anxiety of educators due to the lack of teaching time at universities, the analysis draws on this integrated triad: it interprets how the community was forced to question what was assumed about the methodological rigour of DBR; how the subjects acted to seek a balance between the cognitive demands of design and the affective and social limitations of undergraduate education; and how senior educators acted as legitimate brokers at the blackboard, introducing the Van Hiele framework or fraction theory to stabilise the group's mathematical and didactic knowledge.

The articulation of this theoretical-analytical tripod safeguards the study's scientific consistency. It ensures that the results and discussion section is not merely a chronological description of statements, but rather a robust theorisation of how the co-responsible act of designing and validating formative tasks generates professional development and consolidates inverse symmetry in the education of mathematics teachers.

Figure 2

Integrated Analytical Matrix of the Study



METODOLOGICAL TRAJECTORY

Nature of the Research and Investigative Context

This study adopts a qualitative-interpretative approach and falls within the field of documental and analytical research (Fiorentini & Oliveira, 2013). The research context is within the collaborative network research project “*Aprendizagem profissional de formadores e de futuros professores de matemática: constituindo comunidades de prática e de investigação - AProFI*” [Professional learning of teacher educators and prospective mathematics teachers: establishing communities of practice and research], funded by CNPq (Universal 2023). The project encompasses mathematics teacher educators (MTEs) affiliated with six Brazilian higher education institutions distributed across different regions of the country (*A, B, C, D, E, F, G, H*), brought together in the ForMatE research group.

The longitudinal study follows the principles of design-based research (DBR) or development research (Cobb et al., 2003). The empirical focus of this article is on the curriculum design and validation activities carried out by the educators over just over two years of the project (March 2024 to July 2026). To carry out the research, the eight participating educators were organised into two specific subcommunities of practice. To protect the identity of the subjects and comply with the ethical principles of scientific research involving human

subjects, fictitious pseudonyms were adopted to preserve the teachers' original initials. Thus, the community was structured into Subgroup 1 (Foundations), consisting of Sara, Rebecca, Viviane, and Fabiana, and Subgroup 2 (Teaching Practices and Supervised Practicum), consisting of Renata, Jorge, Fernanda, and Leandro, ensuring analytical traceability without compromising anonymity.

In line with the methodological assumptions of design-based research (DBR) postulated by Cobb et al. (2003), the 28-month longitudinal study was organised into two iterative and feedback-driven design macrocycles:

- *Macrocycle I (March 2024 to July 2025)*: Focused on an exploratory study of the curriculum requirements of Brazilian degree programmes and on the preliminary design of the “Biscuits” (Algebra) and “Triangles on the Grid” (Geometry) PLTs within the subgroups.
- *Macrocycle II (August 2025 to July 2026)*: Marked by stress-testing and cross-validation of artefacts in plenary meetings (large group), culminating in conceptual refinement of tasks, mapping critical learning events, and rounds of reflective interim interviews.

Participants and Inclusion Criteria

The networked project involves teacher educators affiliated with eight Brazilian higher education institutions (HEIs)¹: UFOB, UFSJ, UNIFEI, UFPel, UP, UnB, and IFSP, in addition to the proposing institution (UFABC). This article focuses on a heterogeneous group of eight mathematics teacher educators (MTEs), intentionally and voluntarily invited during the structuring of the CNPq-funded proposal.

The inclusion criteria required participants to work directly with mathematical or didactic content components or with teaching practice/supervised practicum in their respective degree programmes. As detailed in Table 1, the community was intentionally shaped by the plurality of experiences — divided between senior and junior researchers — and by different fields of activity, bringing together lecturers in pure mathematics and

¹ UFOB: Federal University of Western Bahia; UFSJ: Federal University of São João Del Rey; UNIFEI: Federal University of Itajubá; UFPel: Federal University of Pelotas; UP: University of Pernambuco; UnB: University of Brasília; IFSP: Federal Institute of São Paulo; UFABC: Federal University of ABC.

mathematics education. Throughout the 28 months of longitudinal research, the group remained stable, with no records of dropouts.

Table 1

Profile of participating teacher educators.

Alias	HEIs	Probation Time	Role in the Project	Degree area
Sara	UFPEl	Senior (> 10 years)	FM or FDM	PM or AM and ME
Rebecca	UnB	Senior	FM or FDM	PM or AM and ME
Viviane	IFSP	Novice (< 5 years)	FM or FDM	PM or AM and ME
Fabiana	UNIFEI	Senior	FM or FDM	ME
Renata	UnB	Senior	TP or SP	ME
Jorge	UFOB	Senior	TP or SP	ME
Fernanda	UFSJ	Senior	TP or SP	ME
Leandro	UP	Novice	TP or SP	ME

Key: FM or FDM (Fundamentals of Mathematics or Fundamentals of Didactics of Mathematics); TP or SP (Teaching Practice or Supervised Practicum); PM or AM (Pure or Applied Mathematics); ME (Mathematics Education)

Constitution of the Corpus and Analysis Procedures

The corpus was constructed longitudinally over the two research macrocycles. The primary and secondary data were catalogued and organised into three main evidence matrices:

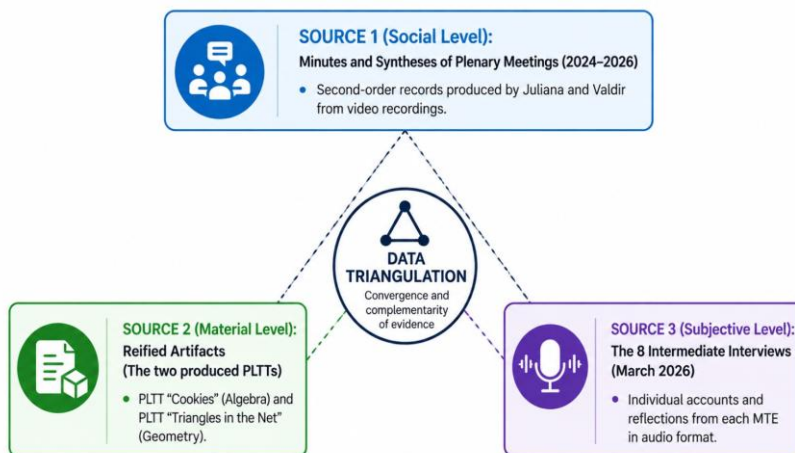
1. Audio and Video Recordings: A total of 20 plenary meetings of the formative community were recorded via a digital platform, totalling 60 hours of fully transcribed audiovisual recordings. Of this total, around one-third was dedicated to working subgroup meetings.
2. Semi-Structured Interviews: Eight individual interviews were conducted at the end of the macrocycles, based on a previously validated script that sought to capture the teacher educators’

perceptions of their participation in the CoPs and the project. The meetings generated around 9 hours of recordings, which were also transcribed for analysis.

3. **Written and Document Registers:** Document collection included 20 official minutes of meetings (used for triangulation and second-order validation), in addition to the preliminary and final versions of the two professional learning tasks (PLTs) coded in Tables 1 and 2.
4. **Asynchronous Interactions:** Participants' text messages and audio messages exchanged on WhatsApp, restricted to the refinement and stress-testing periods of the task design, were used only when teacher educators reported them during subgroup or plenary meetings.

Figure 3

Data Triangulation Matrix.



To process and interpret this volume of data, we adopted content analysis (Bardin, 2015), structured around pre-analysis, material exploration, and results processing phases through inference and interpretation. The analytical categories emerged from the confrontation between the theoretical framework of communities of practice and the practical tensions expressed by the educators.

The document data and interview transcripts were processed using systematic content analysis, with so-called critical events as units of analysis, a

term methodologically validated by Ribeiro and Llinares (2025). In the ecology of a CoP focused on the education of teacher educators, a critical event is defined as an episode of plenary interaction or an excerpt from an individual testimony marked by acute conceptual tensions, ethical conflicts, reviews of identity stances, or the need to negotiate meanings in order to resolve practical design impasses.

The data coding and processing procedures were guided deductively by the three categories of Ribeiro and Llinares's (2025) analytical framework, described in the theoretical framework of this study:

1. **Coding to *Problematisé what has been assumed* (C1):** We singled out from the corpus the textual fragments in which educators questioned the effectiveness of their own historical higher-education teaching practices (such as traditional lists for Calculus or mechanical abstract Algebra) or disrupted the traditional hierarchies between senior and novice teachers.
2. **Coding for *Seeking a balance between cognitive, social, and affective aspects* (C2):** We mapped episodes in which the group faced limits imposed by scarce teaching time and rigid university curricula, or expressed feelings of being overwhelmed, loneliness during data collection, fear of peer judgement, and serious personal health challenges.
3. **Coding to *Model the role of the specialist MET as a broker* (C3):** We isolated the plenary interactions in which local leaders or the macro coordinator intervened surgically to build conceptual bridges, veto ethical deviations (such as the use of AI-simulated data), or strictly demarcate the terminology adopted from the scientific literature.

Once coded, the critical events were compared with the material structure of the two PLTTs produced by the subgroups. This analytical cross-referencing made it possible to trace the genealogy of pedagogical issues and the frameworks for assessing tasks (such as the Van Hiele levels), showing how individual and collective subjective discussions and reflections from the meetings became practical, consolidated formative artefacts.

Regarding the group's relational dynamics, the research coordinator's role was not an external interventionist bias or discursive monopolisation, but an intentional methodological action inherent to the research design. From a design-based research (DBR) perspective, the researcher actively co-invests in and co-designs learning environments (Anderson & Shattuck, 2012).

Furthermore, within the communities of practice (CoPs) framework, the coordinator's role aligns with Wenger's (1998) concept of a broker (boundary spanner/connector). Far from centralising decisions, the broker's role is to introduce elements of one practice into another, conceptually challenge the group, and facilitate hybridisation between the theoretical knowledge of mathematics education and the practical knowing of higher education teaching. Thus, the coordinator's interventions acted as structural scaffolding, fostering the educators' progressive autonomy in refining the PLTs and overcoming cognitive conflicts, without undermining the horizontal, collaborative nature of the community.

Ethical Aspects and Data Protection

This study fully complies with the current resolutions of the National Health Council (*Conselho Nacional de Saúde* - CNS) for research involving human subjects. The macro project of which this study is a part, entitled "*Aprendizagem profissional de formadores e de futuros professores de matemática: constituindo comunidades de prática e de investigação*" [Professional learning of teacher educators and prospective mathematics teachers: establishing communities of practice and research], was approved by the Research Ethics Committee of the Federal University of ABC (UFABC), under Opinion N. 7.287.103 and CAAE: 82663424.6.0000.5594. All participants (teacher educators and undergraduates) voluntarily signed the Informed Consent Form (ICF). Risk mitigation protocols included strict confidentiality for audiovisual recordings. The asynchronous text and audio messages used as supplementary data were authorised through a specific consent addendum for storage and digital protection on encrypted institutional servers.

RESULTS

The Negotiation of Meanings and the Nature of Interactions in Subgroups: From Cooperative Work to Authentic Collaboration

To understand how the professional learning of mathematics teacher educators (MTEs) takes place within the project "Professional learning of teacher educators and prospective mathematics teachers: establishing communities of practice and research", it is first necessary to characterise the ecology of their interactions in the sub-communities. The empirical data reveal that the process of reifying professional learning tasks for teachers (PLTTs) required participants to make a qualitative shift in the way they conceive of collective work, moving them from a purely cooperative approach – based on

the administrative and fragmented division of tasks – to an authentic collaborative practice, characterised by close mutual engagement, horizontal listening, and the pursuit of theoretical consensus.

In the Foundations subgroup (focused on Algebra), this transition became a milestone in its members' professional development. In her interim testimony, teacher educator Fabiana theorises about this phenomenon by contrasting her experiences at different times in the community of practice (CoP). For the researcher, true collaboration implies a suspension of power relations and centralities, requiring the object of study (PLT) to be considered holistically and jointly:

There, I felt that we work collaboratively, not cooperatively [...] There's a balance of listening, speaking [...] the suggestions I make are accepted, as Sara's, Viviane's, Rebecca's [...] there's always consensus among us. Unlike the other group, where one of the people ended up monopolising the idea, and we ended up giving in [...] problem-solving requires everyone to think about the same thing, not to divide the task into parts (Fabiana, March 2026).

An in-depth analysis of this report shows that the term '*monopolising*' does not characterise the dynamics in force in Macrocycle II but rather serves as a fundamental analytical counterpoint established by the teacher herself to endorse the success of the new configuration. By pointing out that the current scenario is unfolding in a '*different way from the other group*', Fabiana evokes her residual experience of Macrocycle I (marked by discursive asymmetries) as a validation metric for the maturation of the current formative community. Far from indicating a step backwards or centralisation, the excerpt confirms the consolidation of a horizontal environment, guided by balanced listening, consensus and mutual support (co-design) in problem-solving, conceptually distinguishing genuine collaboration from a mere cooperative division of tasks.

From Ribeiro and Llinares's (2025) perspective, this balance of voices reflects the need to balance *the cognitive, social, and affective aspects* of the interactive setting. The emotional strength and social reciprocity established within the subgroup enable the cognitive advancement of design. This collaborative ecosystem has directly affected newcomers' participation trajectory. Having experienced this safe environment for negotiating meanings, Viviane reports the development of a pedagogical autonomy that encouraged her to break free from the constraints of traditional university teaching of Algebra:

The group gave me a lot of confidence in this regard [...] I was able to feel autonomous in the exchange with colleagues, to venture to make adaptations to the PLTs themselves [...] and not give in to following a more traditional algebra line as a teacher would do (Viviane, March 2026).

The tangible evidence of this horizontal collaboration was embodied in the very creation of the PLT “Biscuit”. As Viviane reported, Question 3 of the artefact, which challenges pre-service teachers to mathematically investigate the properties and algebraic implications of an incorrect “new addition” of fractions in the context of an integral domain, arose organically from asynchronous, informal dialogues with Sara via WhatsApp voice messages. The community's shared repertoire has therefore expanded beyond the barriers of institutional time, using everyday tools to maintain the flow of conceptual negotiation.

Symmetrically, the Teaching Practice and Supervised Practicum subgroup (focused on Geometry) expressed this same collaborative bond, described by the participants using the metaphor of an interactive “match”. Educator Renata interprets this dynamic as the culmination of the formation of a research community, where vulnerability and reconstructive criticism are embraced without defences or sensitivities:

Our subgroup was a good match. Our subgroup worked very well. I think we managed to support each other very well, but not just support each other, but also give each other space to learn, without having that fear [...] of one person finding out something and telling the other, of one person criticising the other. So, it worked out very well” (Renata, March 2026).

In addition, educator Leandro points out that joining this intertextual collaborative subcommunity acted as an antidote to the geographical and institutional isolation to which higher education teachers are often subjected. Mutual engagement in the development of the “Triangles in the Grid” PLT fostered professional enrichment based on reciprocity and the establishment of networking of knowings:

The opportunity that AProFI has given me to expand this network of ideas, of relationships with other people from different states – I think that's very enriching, because sometimes we get very entrenched in our professional realities [...] We also grow within this movement that is more collaborative” (Leandro, March 2026).

In both subgroups, we note that the educators' learning opportunity was intrinsically linked to the community's ability to generate a language economy. As Sara pointed out, as interactions matured, the group reified specific terms (such as “professional learning task” and “pedagogical mathematical practice”), eliminating the need for continuous conceptual redescrptions and allowing them to “cut corners” and focus on the didactic refinement of the artefacts.

Thus, Topic 1 shows that the transition to authentic collaboration in the subgroups was the fundamental boundary condition for the MTEs to problematise what had been assumed in their historical practices, laying the trust necessary to withstand the severe design tensions that would emerge later in the project.

Theoretical-Methodological Tensions in the Design of PLTs: The “Chaos of Protocols” and the Dialectic between the Authentic and the Simulator

As they moved from structuring social relationships to the concrete act of reifying tasks, the educators in both subgroups encountered the most complex methodological barrier in designing design-based research (DBR) in initial education: obtaining and selecting empirical evidence of students' thinking in basic education. While the original design of the project submitted to the CNPq assumed a linear flow of data collection, the practical reality of the degree programmes revealed tensions that shook the epistemological certainties of the community and forced the educators to *problematise what has been assumed* (Ribeiro & Llinares, 2025) regarding what constitutes legitimate formative material.

In the Teaching Practice and Supervised Practicum subgroup (Geometry), implementing the third part of the PLT, “Triangles on the Dot Grid”, generated what the teacher educator Jorge phenomenologically coined as the “chaos” of the protocol. Given the severe shortage of teaching time, the restrictions of rigid curricula and the lack of organic integration of some teachers in primary schools, the requirement to obtain genuine handwritten work from Grade 8 pupils created a crisis in terms of methodological viability in the community:

As important as official, accurate registers are [...] the model runs into the difficulty educators have in obtaining registers that can be used to develop the new PLTs. I think this could make the model unviable [...] When it comes to the protocol, that's when chaos starts to set in because

of the need for it to be a protocol in an original classroom context” (Jorge, March 2026).

This crisis scenario prompted the subgroup, at the end of 2025, to propose a disruptive technological solution at the CoP plenary: using artificial intelligence (ChatGPT) to synthetically simulate students' answers and mistakes. This episode was a genuinely critical learning event for the entire community. As mapped out in the group's minutes, educators Sara and Juliana led the ethical-scientific discussion, questioning the limits of control and the lack of theoretical grounding for purely algorithmic data. Coordinator Alessandro acted surgically as a broker, vetoing the proposal and establishing that the PLOT model requires the unpredictability and organic nature of exploratory teaching.

The interviews reveal that this tension generated learning opportunities with different implications for the educators' subjectivity. If, for Sara, the veto reaffirmed the uncompromising defence of empirical rigour, for Viviane, the clash served to awaken her investigative curiosity for the future post-project: *“It sparked a curiosity in me to think a little more about how to use artificial intelligence to help set up situations [...] There's a nagging feeling bothering me”* (Viviane, March 2026).

Resolving this impasse led to a significant theoretical refinement for the customisation of DBR in initial teacher education. The educators understood that, in order to make the PLOT model sustainable in teacher education, it was necessary to make the strict concept of "authentic classroom record" more flexible, moving towards the umbrella concept of "representations of practice", which allows the use of episodes, teaching vignettes, or simulated productions, as long as they are rigorously supported by the specialist literature on mathematics education. Educator Fernanda documents this conceptual and methodological shift the group experienced:

An important discussion in 2025 [...] was the idea that a record of practice doesn't necessarily have to be a record originating from actual teaching practice, and that it can be a representation of practice based on research results, right? So, this was a discussion that arose from our need and interest in putting the PLOT model into use in initial teacher education, but in this design, we ran into a difficulty: the lack of practice registers” (Fernanda, March 2026).

The dialectical dynamic between participation and reification within the CoP was not limited to the discursive level; it profoundly altered the internal

architecture of the educational materials produced. Table 1 illustrates the methodological refinement process carried out in the PLT “Triangles in the Grid” following the critical event of the ban on synthetic data generated by artificial intelligence at the 2025 plenary session.

Table 1

Methodological refinement process carried out in the PLT “Triangles in the Grid”.

PLT Component “Triangles in the Grid”	Initial Version (Design Proposal 2025)	Post-Plenary Reified Version (Final Design 2026)
Nature of the Practice Register	Fictitious response protocols generated synthetically via ChatGPT to simulate errors made by Grade 8 students.	Written vignettes and real handwritten protocols adapted and endorsed by consolidated findings from the research literature.
Scaffolding Tool for the Educator	Generic marking criteria based on the "right or wrong" dichotomy.	Analytical matrix of criteria strictly structured according to Van Hiele's levels of geometric thinking and Crowley's phases (1987).

Furthermore, the findings reveal that the refinement of the artefacts operated simultaneously on two of the three central dimensions of the framework by Grossman et al. (2009): if, on the one hand, the scarcity of data forced the community to redefine the nature of representations of practice, on the other hand, the collective construction of the frameworks of analytical criteria—based on Van Hiele (Crowley, 1987)—functioned as a legitimate exercise in the decomposition of practice, fragmenting the complexity of the teachers' professional perspective into specific components that can be studied and taught.

At the same time, in the Fundamentals subgroup (Algebra), the tension in the design of the “Biscuits” PLT was evident in the didactic-cognitive complexity of linking advanced mathematics with school mathematics. Sara

reports that the challenge lay in selecting, within real protocols, which representations of fractions (geometric, discrete or continuous) were salient to trigger prospective teachers’ professional *noticing*: “*It is challenging for us to look at that [...] and choose registers to put together the PLT questions [...] regarding the appropriate representations of fractions, why yes, why not, and how we might be able to use that [...] with prospective teachers*” (Sara, March 2026).

This didactic-cognitive tension experienced within the Fundamentals subgroup generated a similar movement of refinement in design. Table 2 summarises the structural changes made in the “Biscuits” PLT, highlighting the transition from a purely procedural approach to the mobilisation of higher-level algebraic structures.

Table 2
Structural changes carried out at PLT “Biscuits”.

PLT Component “Biscuits” (Algebra)	Initial Version (Design Proposal 2025)	Post-Plenary Reified Version (Final Design 2026)
Cognitive Focus of the Mathematical Task	Identification and correction of the classic error made by students in the mechanical addition of numerators and denominators $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$	Axiomatic investigation of the properties and restrictions of a "new addition" operating in the context of Integral Domains and Algebraic Structures.
Empirical Basis of the Evidence	Hypothetical discussions based on recurring questions included in the course syllabus.	Vignettes based on real handwritten protocols collected empirically in classrooms through partnerships with PIBID.

The final realisation of the two PLTs proves that the subgroups overcame these tensions by integrating scientific rigour with the immediate reality. The ‘Biscuits’ PLT reified the debate by transforming the classic error of adding fractions into an object of investigation of a higher algebraic structure (field and integral domain) supported by real handwritten protocols obtained through partnerships with PIBID. In turn, the “Triangles in the Grid” PLT

stabilised the “chaos of protocols” by structuring a robust second part based on Van Hiele's geometric theory and Crowley's (1987) phases, building an analytical framework of criteria that serves as scaffolding to guide the noticing of the educator and the undergraduate student.

Topic 2 therefore shows that the design difficulties and the friction with empirical and institutional reality forced the educators to produce new theoretical syntheses. By negotiating the boundaries between actual records and simulations, the MTEs developed their own professional perspective on task design, consolidating the community's shared repertoire.

Heterogeneity of Profiles, Participation Trajectories, and Local Leaderships: Vulnerability and Emotional Balance in the CoP

The third fundamental pillar shaping educators' professional development in AProFI concerns the dynamics of identity and the trajectories of participation developed within the community. The ecology of the ForMatE group is characterised by marked heterogeneity of profiles, creating tension and enriching the educational space by bringing together senior and junior researchers, as well as lecturers with different epistemological foundations—historically divided between the departments of 'pure' mathematics and the fields of didactics and mathematics education. Far from being an obstacle, the analysis of the interviews reveals that it was precisely this plurality that made it possible to disrupt rigid academic hierarchies and consolidate a horizontal learning environment based on mutual support.

In the Foundations subgroup, the process of *problematism* what has been assumed (Ribeiro & Llinares, 2025) manifested itself in the deconstruction of established identity stances. Senior teacher educator Fabiana explains, reflectively and brave, how working collaboratively with early-career members required setting aside a historical “academic arrogance”, making room for a two-way, horizontal flow of practical knowledge:

For me, the project made me realise that I can learn from people with less experience than me [...] I was somewhat arrogant in thinking that those with less experience might not have much to contribute. And for example, Viviane [...] taught me that. I learn a lot from her” (Fabiana, March 2026).

Symmetrically, in the Teaching Practice and Supervised Practicum subgroup, educator Jorge reports a similar shift in the reconfiguration of his professional identity. Jorge describes his process of maturation in the CoP as an exercise in decentralising power, learning to step back from a historical posture

of centrality to allow new voices and leaders to emerge and be legitimised in the small group, such as the one brought by Juliana's reified repertoire:

There's something that's kind of personal [...] For a long time in my life, I always wanted to be the person who was first in everything [...] who had to take the lead. But then, we grow up and we learn that the world doesn't revolve around us. And it's been a long time since I've tried not to take on this role of leading [...] I wait for the other person so that I'm not taking up their space” (Jorge, March 2026).

Under the analytical lens of Ribeiro and Llinares (2025), these identity transformations were directly supported by the community's ability to *seek a balance between cognitive, social, and emotional aspects*. The high-level cognitive scientific output (the two PLTs and their roadmaps) only proved to be sustainable because the CoP extended an affective and psychosocial safety net that accommodated its members' bodily and emotional vulnerabilities.

This phenomenon is poignantly evident in Fabiana's testimony, as she describes how ForMatE stood out from the competitive coldness of traditional mathematics departments by offering human support during her health diagnoses:

Beyond being teachers, we are human beings. It was a time when we really strengthened these bonds [...] we cared about each other. I told them that I was having health problems [...] and they asked [...] I've always worked in departments with mathematicians, and we don't have that” (Fabiana, March 2026).

Finally, local leaders, acting as theoretical brokers, regulated professional development and participation trajectories from the periphery to the centre (Wenger, 1998). While in the larger group, the role of the coordinator (*author*) was consolidated as the macro orchestrator who validates terminologies and resolves epistemological conflicts in the plenary sessions, in the small groups, the capillary mediation of teacher educators such as Sara (Foundations) and Fernanda (Supervised Practicum) emerged, as they were continuously consulted by their peers to calibrate the fidelity and suitability of the tasks to the PLOT model.

The conclusion of this third analytical strand reveals that the CoP of the project *“Professional learning of teacher educators and prospective mathematics teachers: establishing communities of practice and research”* succeeded in reifying a sense of collective belonging that culminated in the sharing of the same identity premise: the educator must experience the

hardships and joys of exploratory teaching to be able to teach it. Inverse symmetry, or formative isomorphism, is therefore consolidated not only as an abstract concept from the literature, but as a practice embodied in the professional development of this group of researchers.

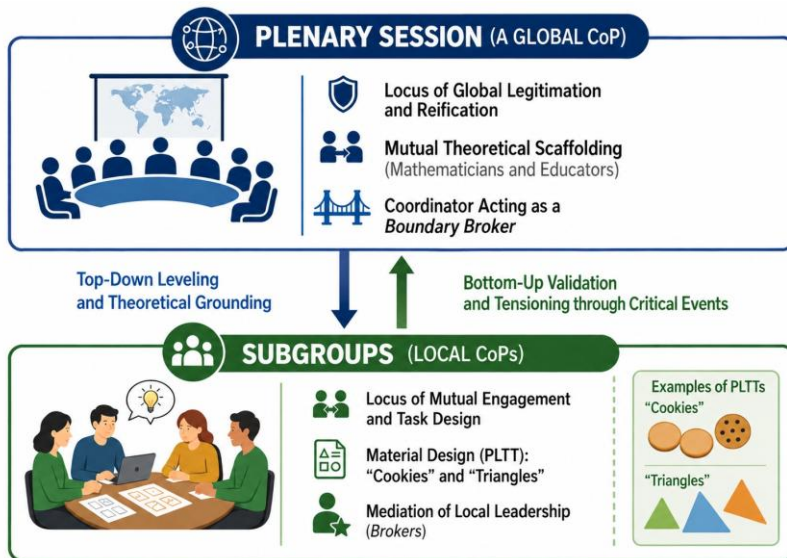
INTEGRATED DISCUSSION: THE PLENARY AS A LOCUS OF LEGITIMATION, ISOMORPHISM, AND REIFICATION OF THE COP

The isolated characterisation of the trajectories and tensions experienced within each subgroup, although fundamental, does not exhaust understanding of the AProFI project's learning ecology. Because both subcommunities shared the same sociocultural ecosystem and research field, the findings become homogeneous and scientifically consistent when analysing the dialectical movement between the small group and the expanded large group (plenary). From Etienne Wenger's (1998) theoretical perspective, the plenary functioned as the official space for negotiating second-order meanings, acting as a powerful regulator where local interactions were refined to expand the community's conceptual *domain*.

The ForMatE methodological flow established a back-and-forth dynamic: what was developed and rehearsed in the privacy of the subgroups needed to be publicised and submitted to the collective's scrutiny (Figure 4). This process of systematic sharing embodies what Wenger (1998) defines as the dialectic between participation and reification. As Renata rightly points out in her interview, the large group served as a conceptual 'umbrella' that aligned the community by developing a shared repertoire. Terms such as "professional learning task" (PLT) and "pedagogical mathematics practice" ceased to be abstract ideas and became reified objects that stabilised and accelerated communication in the CoP, resulting in a clear economy of language, as Sara validated.

Figure 4

The Dialectical Model of Interactive Scenarios and Reification of Professional Learning Tasks.



Empirical and analytical cross-referencing of local trajectories in the subgroups with macro movements in the plenary session enabled us to map a dynamic learning structure, which we consolidate as one of this study's main theoretical results. Figure 4 outlines the *Dialectical Model of Interactive Scenarios and Reification of Professional Learning Tasks*, which explains how MTEs' professional development is driven by a continuous flow of conceptual back-and-forth. The ecosystem operates in two symmetrical directions: from a bottom-up perspective, mutual engagement and practical tensions in the subgroups' micro-design feed the global plenary with real critical events, such as the chaos of protocols and the ethical debate on the use of artificial intelligence. In contrast, in a top-down movement, the plenary acts as a mechanism for conceptual alignment and overall legitimation. Under the macro-coordinator's mediation as a broker, this locus expands the group's theoretical boundaries, reifies the scientific vocabulary, and provides mutual support (scaffolding), converting the friction between pure mathematicians and educators into a collective intelligence that endorses the ultimate validity of the artefacts.

In this space for collective sharing, tensions within a group became learning opportunities for all, activating the three characteristics of interactive scenarios proposed by Ribeiro and Llinares (2025). The most emblematic example of this intersection was the historic debate on the nature of field data for initial teacher education (Topic 2). When the Supervised Practicum subgroup shared its concern about the “*chaos of real-world protocols*” (Jorge) in the plenary session and suggested synthetic simulation via artificial intelligence, the tension challenged the community to *problematise what has been assumed* about the methodological rigour of DBR.

Resolving this impasse required the macro coordinator to assume the theoretical role of a specialist broker (Ribeiro & Llinares, 2025; Wenger, 1998), building epistemological bridges to mediate the conceptual conflict. By rejecting synthetic data in favour of authentic exploratory learning while embracing the need for practical sustainability in design in the degree programme, the group reached a new theoretical consensus: expanding the original model to the concept of “*representations of practice*” (Fernanda). This movement accurately reflects the quest for a balance between cognitive, social, and affective aspects (*seeking a balance*), legitimising the artefact without stifling the viability of the work of educators in universities.

By rejecting algorithmic simulation in favour of representations of practice validated by the literature, the ForMatE group addressed a contemporary dilemma that puts pressure on international research. This move aligns with Even and Ball's (2009) warnings about the need to base teacher education on real-world, complex problems in teaching that cannot be reduced to purely linear or automated standards. Furthermore, by expanding the model to accommodate multiple forms of representation of practice, the community is responding to the call of Zaslavsky and Leikin (2004), who argue that the professional growth of the MTEs occurs precisely when they develop their own investigative perspective on mediation artefacts, learning to calibrate the level of cognitive demand in the face of the logistical contingencies of the university and basic school.

In addition, the plenary fulfilled the role of regulating the PLOT model (Ribeiro & Ponte, 2020) in the area of the teacher educator's role and actions. By working through the PLT “Biscuits” (Algebra) and “Triangles” (Geometry) structures, the educators consolidated their own professional perspective (*noticing*) on designing tasks with a high level of cognitive demand. The heterogeneity of profiles within the community—mathematicians and mathematics educators—enabled a process of mutual *scaffolding*: while the

content experts ensured advanced conceptual consistency (such as domain of integrity in the PLT Biscuits), the didactics experts refined the progression of Van Hiele's levels of geometric thinking (Crowley, 1987) in PLT Triangles.

Finally, the homogeneity of the findings is consolidated in the shared identity trajectory and the premise of isomorphism, or inverse symmetry, in teacher education (Fiorentini & Oliveira, 2013). As Fernanda succinctly summarised in her pitch, the group understood that, to prevent the prospective teacher from looking at the educator and thinking, “*he tells us to do at school what he doesn't do here*”, the CoP environment had to operate under the rules of exploratory teaching. Drafting in the subgroup, testing in the plenary, gathering input, and revising collectively created shared intelligence. The community itself has become the very model it advocates, proving that educators' professional development is, *par excellence*, an act of engaged, co-responsible, and scientifically grounded social participation.

CONCLUSIONS AND FINAL CONSIDERATIONS

Research into the professional development of mathematics teacher educators (MTEs) involved in the project “*Professional learning of teacher educators and prospective mathematics teachers: building communities of practice and research*”, between 2024 and 2026, made it possible to uncover and characterise the complex learning ecology that emerges when higher education lecturers engage in the collective reification of professional learning tasks (PLTs). By using the plenary minutes, artefact designs, and intermediary interviews as a documentation corpus, this study sought to provide elements to answer its three research questions, proposing reflections and important theoretical advances for the field of teacher educator education.

In response to the first research question, the results showed that collaborative work within the subgroups was the primary driver of negotiating meanings and destabilising traditional practices. The transition from a purely cooperative model (focused on the fragmented division of tasks) to authentic collaboration (based on the search for consensus and the encouragement of active and horizontal listening) allowed senior and novice educators to expose their vulnerabilities and build what the literature defines as *knowledge of practice* (Cochran-Smith & Lytle, 1999). This safe environment encouraged the MTEs to question what had been taken for granted in their historical institutional cultures, resulting in a break with the mechanistic teaching of algebra and the subversion of the traditional set lists of exercises entrenched in advanced calculus.

Regarding the second research question, the study revealed that the reification of PLTs in initial teacher education is persistently strained by severe curriculum and logistical constraints, characterised by the “chaos of real-world protocols” and the rigidity of university teaching schedules. However, the community overcame this epistemological impasse through a sophisticated effort to balance cognitive, social, and affective aspects. The main theoretical advance of this study lies in documenting how the CoP, in the face of the scarcity of field data and the ethical tension generated by the attempt to use artificial intelligence (ChatGPT), implemented a significant conceptual relaxation by expanding the boundaries of the original model (PLOT model) to accommodate what Grossman et al. (2009) define as “representations of practice” (vignettes and protocols adapted with strict grounding in the research literature), ensuring the feasibility and high cognitive demand of the tasks without stifling the sustainability of the work of undergraduate lecturers.

Regarding the third research question, the heterogeneity of the community's profiles shaped dynamic participation trajectories that altered traditional academic hierarchies. Immersion in the CoP helped mitigate discursive asymmetries among senior educators and fostered a more distributed power among established leaders, who consciously made room for the legitimation of newcomers, in line with Lave and Wenger's (1991) perspective. This movement was regulated and enhanced by the actions of local leaders who acted as conceptual brokers (boundary mediators) — a concept based on Wenger (1998) — enabling mutual scaffolding, according to the concept proposed by Bruner (Wood et al., 1976): the pure mathematicians injected conceptual rigour into the advanced algebra tasks, while the mathematics educators refined the didactic progression of geometry in light of Van Hiele's (1986) geometric development model.

Compared with previous studies in the international literature, which mostly focus on the structural challenges in teacher educator education or discuss the complexities of MTEs in the face of the bureaucratic isolation of their departments (Even & Ball, 2009; Ponte, 2014), this study goes further by offering an empirical model of a networked research community. The collective plenary has been shown to function as a space for overall legitimation, preventing subgroups from operating in parallel and fostering a cohesive and integrated theoretical articulation.

Despite its robustness, the research has the limitation that the sustainability of PLTs still depends heavily on voluntary engagement and the burden on individual lone researchers in the classroom—as shown by the

struggles of field data collection—which clashes with the physical limits of lecturers' working day at Brazilian universities.

One of this project's central contributions is the realisation of the principle of isomorphism, or inverse symmetry, in teacher education. By designing in the subgroup, testing in the plenary, welcoming contributions, and collectively reworking the artefacts, the community sought to mitigate the impasses of what contemporary literature characterises as a process of 'upside-down training', along the lines proposed by Oliveira and Bueno (2013). As a result, the group became a dynamic space for exploratory teaching that aligns with Ponte's (2014) perspective on this teaching approach. Looking to the future, the project points to a movement to multiply the CoP by shifting from material deliverables to creating local formative cells, generated in a decentralised manner by educators in their respective regions. This movement, based on the dynamics of social learning systems (Wenger, 2010), has the potential to disseminate the adopted theoretical framework and the construct of noticing, and to support reflections on ongoing and internalised professional development policies in mathematics education.

AUTHORSHIP CONTRIBUTION STATEMENT AND USE OF ARTIFICIAL INTELLIGENCE

AJR was actively involved in the theoretical conceptualisation of the study, the design-based methodological design, the curation and triangulation of qualitative data, the formal analysis, and writing the original draft, as well as subsequent review and editing of the manuscript. The author declares that he used artificial intelligence tools based on large-scale language models (LLMs) exclusively as academic co-piloting support during the writing process. AI was used in an assisted and supervised manner for stylistic and grammatical optimisation, visual structuring of conceptual diagrams, reducing analytical abstracts to meet the journal's word limits, and transposing and standardising bibliographic citations to the APA 7th edition international standard. The author assumes full scientific, ethical, and intellectual responsibility for the final content, interpretations, and conclusions presented in this article.

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DATA AVAILABILITY STATEMENT

The qualitative datasets generated and analysed during this study (minutes of plenary meetings and transcripts of individual interviews) are not publicly available in open repositories due to ethical and contractual restrictions, to protect the confidentiality and anonymity of the research subjects. However, the corresponding author may share anonymised analytical subsets or additional methodological information that do not compromise participants' identities upon formal request and justification for strictly academic use.

REFERENCES

- Adler, J., & Ronda, E. (2014). An analytical framework for describing teachers' mathematics discourse in instruction. *Proceedings of PME 38 and PME-NA 36*, 2, 9–16.
- Anderson, T., & Shattuck, J. (2012). Design-based research: A decade of progress in education research? *Educational Researcher*, 41(1), 16–25. <https://doi.org/10.3102/0013189X11428813>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
- Bardin, L. (2015). *Análise de conteúdo*. Edições 70.
- Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teacher preparation via a practice-based methods course. *Journal of Mathematics Teacher Education*, 12(2), 119–143.
- Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, 18(8), 947–967.

- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Designing experiments in educational research. *Educational Researcher*, 32(1), 9–13.
- Cochran-Smith, M. (2003). Learning and unlearning to teach to teach: What do we mean? *European Journal of Teacher Education*, 26(1), 5–17.
- Cochran-Smith, M., & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education*, 24(1), 249–305.
- Crowley (1987), M. M. (1987). The van Hiele model of the development of geometric thought. In *Learning and teaching geometry, K-12: Yearbook of the National Council of Teachers of Mathematics (NCTM)* (pp. 1–16). NCTM.
- Even, R., & Ball, D. L. (Eds.). (2009). *The professional education and development of teachers of mathematics: The 15th ICMI Study*. Springer.
- Fiorentini, D., & Oliveira, A. T. C. C. (2013). O lugar das práticas na constituição profissional do professor de matemática. *Zetetiké*, 21(40), 73–100.
- Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, 111(9), 2055–2100.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.
- Llinares, S. (2002). Los objetos matemáticos como ferramentas en el aprendizaje del profesor. *Revista de Educación de la Universidad de Granada*, 15, 115–132.
- Llinares, S. (2013). El "olhar profissional" do professor de matemática: perspectiva e desafios. In A. J. Ribeiro (Org.), *Formação de professores que ensinam matemática: reflexões e investigações* (pp. 13–33). Livraria da Física.

- Oliveira, A. S. de, & Bueno, B. O. (2013). Formação às avessas: Problematizando a simetria invertida na educação continuada de professores. *Educação e Pesquisa*, 39(4), 875–890. <https://doi.org/10.1590/S1517-97022013005000011>
- Ponte, J. P. (2014). Tarefas no ensino e na aprendizagem da matemática. In J. P. Ponte (Org.), *Práticas profissionais dos professores de matemática* (pp. 13–27). IEUL.
- Radford, L., & Barwell, R. (2016). Language in mathematics education research. In A. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 275–313). Sense Publishers.
- Ribeiro, A. J., & Ponte, J. P. (2020). Um modelo teórico para organizar e compreender as oportunidades de aprendizagem de professores para ensinar matemática. *Zetetiké*, 28, 1–20, Artigo e020027.
- Ribeiro, A. J., & Llinares, S. (2025). Becoming a mathematics teacher educator: Identifying Characteristics of learning opportunities, *Acta Scientiae*, 27(3), 1–37, 10.64856/acta.scientiae.8401
- Rowland, T., Huckstep, P. & Thwaites, A. Elementary Teachers' Mathematics Subject Knowledge: The Knowledge Quartet and the Case of Naomi. *J Math Teacher Educ* 8, 255–281 (2005). <https://doi.org/10.1007/s10857-005-0853-5>
- Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, 3(5), 344–350.
- Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press.

- Wenger, E. (2010). Communities of practice and social learning systems: the career of a concept. In C. Blackmore (Org.), *Social learning systems and communities of practice* (pp. 179–198). Springer.
- Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, 17(2), 89–100. <https://doi.org/10.1111/j.1469-7610.1976.tb00381.x>
- Zaslavsky, O., & Leikin, R. (2004). Professional development of mathematics teacher educators: Growth through practice. *Journal of Mathematics Teacher Education*, 7(1), 5–32.