Proposal of Enrichment Activities for Students with Indicative of High Skills/Giftedness in Mathematics

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ABSTRACT

The subject High Abilities/Giftedness has received little attention from the academic community in general and in Brazil. Thus, the potential and special educational needs of a significant part of the school population are ignored. In this work, we present some results of a research developed in a public institution of education in Brazil, aimed at high school students with high skills/gifted in Mathematics. The research, developed as part of a project that had a multi-professional team, had the purpose of elaborating, applying and evaluating an enrichment program to complement regular education through enriching activities and experiences. With theoretical support in Renzulli, it starts from the conception of giftedness according to Theory of the Three Rings and elaborates a program based on the Triadic Model of Enrichment to attend to the public of students. The elaborate enrichment program, entitled Kaleidoscopes and Geometric Constructions, aimed to offer students a wide variety of educational experiences with different methodologies to develop specific skills and to work on mathematical concepts involved in the construction of kaleidoscopes. During the workshops developed in the enrichment program, it was possible to observe the involvement and the interest that the students demonstrated with the accomplishment of the tasks.

Keywords: High skills/giftedness. Mathematics. Enrichment Program. High School.

Proposta de Atividades de Enriquecimento para Alunos com Indicativo de Altas Habilidades/Superdotação em Matemática

RESUMO

O tema Altas Habilidades/Superdotação tem recebido pouca atenção da comunidade acadêmica de modo geral e em particular no Brasil. Assim, o potencial e as necessidades educacionais especiais de uma parcela significativa da população escolar acaba sendo ignorada. Neste artigo, apresentamos alguns resultados de uma pesquisa desenvolvida em uma instituição pública de ensino no Brasil, direcionada para alunos do Ensino Médio com indicativo de altas habilidades/superdotação em Matemática. A pesquisa, desenvolvida no âmbito de um projeto que contou com uma equipe...
multiprofissional, teve o propósito de elaborar, aplicar e avaliar um programa de enriquecimento para complementar o ensino regular por meio de atividades e experiências enriquecedoras. Com aporte teórico em Renzulli, parte-se da concepção de superdotação segundo a Teoria dos Três Anéis, e elabora-se um programa baseado no Modelo Triádico de Enriquecimento, para atender ao público de alunos. O programa de enriquecimento elaborado, intitulado *Caleidoscópios e construções geométricas*, teve por objetivo oferecer aos alunos ampla variedade de experiências educacionais com diferentes metodologias para desenvolver habilidades específicas e trabalhar conceitos matemáticos envolvidos na construção de caleidoscópios. Durante as oficinas desenvolvidas no programa de enriquecimento, foi possível observar o envolvimento e o interesse que os alunos demonstraram com a realização das tarefas.

**Palavras-chave:** Altas habilidades/superdotação; Matemática; Programa de Enriquecimento; Ensino Médio.

**INTRODUCTION**

In Brazil, the high skills/gifted (HS/GF) public is part of the Special Education segment of the Inclusive Education Perspective. The term gifted, adopted in the 1990s, is defined, according to the National Policy of Special Education (1994):

> [...] As bearers\(^1\) of high skills/gifted learners who show remarkable performance and high potentiality in any of the following aspects, isolated or combined: general intellectual capacity; specific academic aptitude; creative or productive thinking; leadership ability; special talent for arts and psychomotor ability. (Brazil, 1995, p.17)

However, even though it is part of the target audience of Special Education in the perspective of Inclusive Education, it is possible to notice that in Brazilian public schools, students with HS/GF have not received the same attention as students with sensory impairments or with global disorders of development, although there are criticisms and restrictions on the service offered to these students.

There are myths and beliefs about the subject that are from the definition to the process of identification and specialized care that contributed to these individuals and their needs, which are neglected by the educational system. According to Mettrau (1994), it was believed for a long time that the gifted people, having above average intelligence, would be able to develop by themselves. Nowadays, we know that giftedness is an area of inclusive education and they need specialized educational services to meet their educational needs.

In this work, we present some pertinent discussions to the elaboration, application and evaluation of an enrichment program to complement the regular education through

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\(^1\) We are aware of the inadequacy of the term “carriers” used at the time of publication of the Guidelines.
enriching activities and experiences, carried out in the scope of a doctoral research. The enrichment program was offered to high school students with an HS/GF in Mathematics from a public teaching institution in Brazil who participated in a project funded by a governmental agency and under which this study was developed.

PROJECT IMPLEMENTATION

In order to carry out the research in a Brazilian public education institution, the Núcleo de Assistência a Pessoas com Necessidades Especiais (NAPNE) (Nucleus of Assistance to People with Specific Needs) was formed by a multi-professional team composed of psychologists, pedagogues, Libras (Brazilian Sign Language) interpreters, teachers and trainees who work from identification process to the attendance to students with special educational needs in that school environment, which includes those with high skill/giftedness. Our research was aimed at the students of the first years of High School (15-16 years-old) with high skills/giftedness in the areas of Exact Sciences and consisted in offering a Mathematics enrichment program in a complementary way to regular education through activities and experiences.

After several meetings of NAPNE studies and discussions, Renzulli’s conception of giftedness and his Three Ring Theory were adopted, according to which the individual is characterized with HS/GF if they present three fundamental characteristics: high capacity, commitment to the task and creativity (Figure 1) (Renzulli, 1986).

![Figure 1. Adaptation of the graphic representation of the giftedness definition. (Renzulli & Reis, 1997).](image)

The identification process of the students with HS/GF indicatives, under the responsibility of the NAPNE multi-disciplinary team psychologists, was also guided according to the Renzulli identification model, named Revolving Doors. In this model through the use of different instruments in the identification process, it is possible to
identify students with high indicative potential in the most diverse areas of knowledge (Virgolim, 2014).

In this project, the psychologists selected 19 students, most of those being of the male genre with two girls selected, and all of those were either 15 or 16 years old, who composed the group of students with indicative HS/GF, our Talent pool, as designated by Renzulli (Virgolim, 2014).

THE ENRICHMENT PROGRAM

The students with HS/GF have special educational needs and the school must be prepared to meet those needs and to provide learning opportunities so that these students can develop and enhance their skills. Enrichment programs are designed to promote stimuli and investigative and challenging experiences, which are compatible with the interests and needs presented by students with an HS/GF indication (Fleith & Alencar, 2007).

According to Jelinek (2013), enrichment programs is a set of activities and organized pedagogical resources – institutionally and continuously – to complement regular education. This conceptualization of enrichment program can still be complemented with Guenther’s definition:

[…] is an intentional and planned stimulation effort that seeks the growth of the child, broadening, deepening and complementing the basic school curriculum with knowledge, information and ideas that make it capable of greater awareness of the comprehensive context of each subject, discipline, or area of knowledge. (Guenther, 2006, p.67)

The Triadic Model of Enrichment is a proposal by Renzulli to serve the students of the Talent pool, identified by the Revolving Doors model from the conception of giftedness of the Three Ring Theory. This enrichment model is intended to encourage students to productivity through different themes, areas of interest and fields of study (Renzulli & Reis, 1997).

The Renzulli enrichment model is comprised of activities that comprise three types of enrichment, which are dynamically organized to accommodate different student skills:

• Type I – characterized by general exploratory activities that are designed to offer students a wide variety of activities and experiences so that they can have contact with different areas that are not always contemplated in the regular curriculum.

• Type II – aims to stimulate new interests based on motivating experiences that have been experienced with Type I experiences.
Type III – involves students who, from Type I and II, have aroused interest in a certain area and intend to dedicate time and effort to deepen their knowledge, to seek advanced content and, in fact, to assume the role of researcher in the learning process (Pérez, 2014).

In this sense, considering the Triadic Model of Enrichment by Renzulli, we elaborated an enrichment program having as central theme *Kaleidoscopes and geometric constructions*. Our program is composed of 10 workshops that were planned with Type I, II and III activities and that involved different didactic resources and different learning situations so that the student could develop their skills (Figure 2). As Fernandes (2014) highlights, the education of people with HS/GF must be linked to a challenging learning environment, involving the student in complex situations and leading him to improve his ability to think and decide.

![Figure 2. Workshop planning according to the Triadic Model of Enrichment. Adapted from Pérez (2014, p.545).](image)

The choice of the kaleidoscope theme is due to the fact that it is an object that allows the exploration of mathematical concepts and stimulates creativity for the creation and elaboration by the students of their own model. The booklet *Kaleidoscopes and Geometric Constructions*, which we elaborate to guide the work of the students, despite presenting tasks for each of the meetings, and defined clearly their objectives, resources to be employed, time of accomplishment and sequence to be followed, is a flexible proposal, subject to adaptations and contributions suggested by students. During the workshops, some changes were made to adapt them to the rhythm and the interest of the students, and to maintain the motivation for the final objective of the project that would culminate in the creation of a kaleidoscope model.

We sought in our meetings to make the correct use of mathematical language, since our target audience was composed of high school adolescents with potential for building
mathematical knowledge from a more formal language. We looked for a proposal for developing the student’s general abilities, evidencing their potential in some specific areas.

Our concern in the elaboration of activities included some care such as those highlighted by Chagas (2008) and other authors that point out some actions that should be observed in the care of children and adolescents with HS/GF, such as respect and mapping of skills, interests, learning styles and expression, planning activities that stimulate creative thinking, the use of research techniques, the training of social skills and the use of educational technologies.

The tasks were designed in order to offer students the opportunity to experience different learning situations through multiple didactic resources, allowing the development of specific skills to stimulate gifted behavior that could be related to Renzulli’s Three Rings theory. In planning the tasks, we focus on the development of specific skills such as those described in Chart 1 below:

Chart 1
Specific skills that are related to tasks.

<table>
<thead>
<tr>
<th>Specific Skills</th>
<th></th>
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<tbody>
<tr>
<td>Logical Ability – Mathematics – sensitivity to standards, order and systematization. It is related to the use of symbols and formal mathematical language.</td>
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<tr>
<td>Logical Ability – Deductive – the observation to establish generalizations and make inferences.</td>
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<tr>
<td>Manual Ability – ability to make coordinated movements with the hands.</td>
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<td>Abilities with technologies – ability to solve problems of information, communication and knowledge in a digital environment.</td>
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<tr>
<td>Space Visibility Skill – uses vision and imagination to locate in the world and manipulate different objects, both physically and virtually (GARDNER, 1994).</td>
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<tr>
<td>Expression Ability – ease to express yourself verbally and corporately.</td>
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<tr>
<td>Artistic Ability – ability to perceive and aesthetic awareness.</td>
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</tbody>
</table>

For each of the assignments offered to the students, we list the specific skills that we intended to develop or to encourage them to be revealed. It should be noted that the skills developed by the students are not limited to the specific skills listed in Chart 1. Other skills that can be applied to the domains of knowledge may also emerge in the course of workshops.

Considering the topic addressed in the workshops, most of the mathematical concepts discussed in the booklet relate to Geometry. In Chart 2, below, we nominally present the workshops with their objectives and resources made available. It should be noted that, in general, each workshop took place in a session with an average duration of 60 minutes.
Chart 2

Description of the tasks proposed in the booklet Kaleidoscopes and Geometric Constructions.

<table>
<thead>
<tr>
<th>KALEIDOSCOPES AND GEOMETRIC CONSTRUCTIONS</th>
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<tbody>
<tr>
<td><strong>Workshop 1 – Kaleidoscopes and Symmetries</strong></td>
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<tr>
<td><strong>Objectives:</strong></td>
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<tr>
<td><strong>Material:</strong> Booklet, Video, Pictures, Pencil, Eraser.</td>
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<td><strong>Workshop 2 – Isometries</strong></td>
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<td><strong>Objectives:</strong></td>
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<td><strong>Material:</strong> Booklet, Computer.</td>
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<tr>
<td><strong>Workshop 3 – Geometric constructions</strong></td>
</tr>
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<td><strong>Objectives:</strong></td>
</tr>
<tr>
<td><strong>Material:</strong> Booklet, Pencil, Eraser, Compass, Ruler.</td>
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<tr>
<td><strong>Workshop 4 – Dihedral kaleidoscope</strong></td>
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<tr>
<td><strong>Objectives:</strong></td>
</tr>
<tr>
<td><strong>Material:</strong> Booklet, dihedral kaleidoscope, Plastic Straws, Protractor.</td>
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<tr>
<td><strong>Workshop 5 – Mosaics</strong></td>
</tr>
<tr>
<td><strong>Objectives:</strong></td>
</tr>
<tr>
<td><strong>Material:</strong> Booklet, dihedral kaleidoscope, Colored cardboard paper, Scissors.</td>
</tr>
</tbody>
</table>
Workshop 6 – Triple dihedral kaleidoscope

Objectives:
To make a device that allows viewing symmetry.
To motivate the development of artistic skills and manual skills.

Material: Booklets, 3 mirrors 5 cm x 30 cm, Transparent plastic, Color Set Paper, Tracing paper, Colored beads adornments, Scissors, Hot glue, Adhesive tape.

Workshop 7 – The product

Objectives:
To launch the idea of the final goal of the project, i.e. the construction of a particular kaleidoscope model.

Material: Booklet.

Workshop 8 – Construction: Phase 1

Objectives:
To guide the construction of the product, acting as facilitator and intermediary of knowledge;
To motivate the development of specific skills such as logical-mathematical skills, manual skills, visual-spatial skills and artistic skills.

Material: Booklets, Computer, Mirrors, Cardboard paper, Beads adornments, Scissors, Hot glue, Transparent plastic, Laser keychain, Boxes.

Workshop 9 – Construction: Finalization

Objectives:
To guide the construction of the product, acting as facilitator and intermediary of knowledge;
To motivate the development of several specific skills such as logical-mathematical skills, manual skills, visual-space skills and artistic skills.

Material: Booklets, Computer, Mirrors, Cardboard paper, Beads adornments, Scissors, Hot glue, Transparent plastic, Laser keychain, Boxes.

Workshop 10 – Culminance

Objectives:
To create opportunities to develop the expression of skill.
To evaluate the final product produced by the students.

Material: Kaleidoscopes, Forms.

THE WORKSHOP DEVELOPMENT

To work the Symmetries and Isometries, we developed Type I and Type II activities, for example, according to the Renzulli Triadic Model of Enrichment (Renzulli & Reis, 1997), seeking to offer the students of the Talent Pool activities in different areas and experiences that are often not explored in the regular classroom. Specifically in these workshops, the proposal was to awaken new interests and creative thinking, aiming at the development of logical-mathematical, logical-deductive and spatial skills.

In examples and activities carried out by the students, we made use of images related to the Star Wars theme to get closer to the reality of the students and, thus, to make the...
activity more attractive. The proposal actually involved the students who had fun and interacted a lot with each new figure.

Geometry (Chagas, 2008) and different authors have pointed out the great interest of adolescents with HS/GF through electronic means and Internet and the involvement of students in tasks that use technological resources. Several authors suggest that these tools are used more effectively for the education and development of talents as a form of school enrichment and acceleration of studies. These authors also highlight the fact that these adolescents are native individuals of a technological, digital and informational world, bringing to the surface the need to use these technologies in educational practices (Figure 3).

As a didactic resource that attracts the interests of this young public with which we are working with, mathematical software has made it easier to visualize and to understand the geometric transformations in the plane, such as translation, rotation, reflection in a line, translation reflection or sliding reflection (Figure 4).
In another workshop, geometric constructions of different regular polygons such as triangles, quadrilaterals, pentagon, and hexagon with ruler and compass (Figure 5) were worked. The objective was to motivate the development of logical-mathematical skills related to the observation, analysis and comparison of geometric forms as well as the development of spatial and manual visual abilities. In addition, the constructions with the drawing material reminded the students of some properties that are implied when constructing the geometric figures with the software, since these must be known so that the figures can be traced on the paper with ruler and compass. This type of activity allowed arousing the interest (Type II) for different geometric forms that could be used in the construction of the particular model of kaleidoscope. Therefore, it is important the student to know the main regular polygons, their characteristics and construction.

The importance of exploring the geometric constructions in our workshops is also defended by Lima (1991 as quoted in Oliveira, s/d) when he considers the drawings of geometric figures an important part for understanding, fixation and creative imagination. The author points out that it is fundamental for the student to draw the figure by itself, searching for ways, imagining constructions, researching interconnections, forcing reasoning and exercising the mind. Kalter (1986 as quoted by Oliveira, s/d) also emphasizes that the teaching of drawing is essential so that there is no blocking of the capacities to plan, to design or to abstract, thus establishing a continuous relationship between visual perception and reasoning space.

In all workshops, we sought to respect the work rhythm and interests of each participant. In order to not discourage or even limit the student’s creativity, there was room for suggestions and creation of proposals in each of the tasks. On this workshop, for instance, the geometric constructions were motivated glimpsing the future pretension.
of building a kaleidoscope. We present bellow other geometric construction images suggested by the students.

Figure 6. Geometric constructions suggested by the students.

The first physical contact with a kaleidoscope, provided in our workshops, was with the dihedral model. In addition to the kaleidoscope, students received a protractor and plastic straws. Then they should handle the kaleidoscope by manipulating it on the protractor and the straws, positioning it at different angles and observing in the plane the formation of regular polygons (Figure 7).

Figure 7. Dihedral kaleidoscope.
Students explored different flat geometric figures, which provided the development of logical-mathematical skills, logical-deductive skills, spatial visibility skills and manual skills. Although not specifically related to this workshop, the general ability of communication and expression was also stimulated, since the pairs had to present their results to the classmates.

In the workshop of the Mosaics, we used the knowledge of geometry and the dihedral kaleidoscope that were worked previously. With cutouts of cardboard paper and color set of various colors, students were reunited in pairs and should make mosaics from flat geometric figures (Figure 8), make the photographic record using the cell phone and send it by email or WhatsApp to another pair that would be challenged to represent the mosaic received in the GeoGebra software (Figure 9). The activity was developed with much excitement and dedication. The students awakened a spirit of competitiveness and were highly motivated to do a good job – as they described as “very difficult” – to challenge the other pair. It was an activity that amused and involved them.

Figure 8. Mosaic proposed in the challenge.

Figure 9. Mosaic represented in GeoGebra.
The groups proved to be highly motivated with the activity and used a lot of creativity for the construction of several different mosaic with geometric figures (Figure 10).

![Mosaic construction](image)

**Figure 10.** Mosaic construction.

The launch of the last stage of the project consisted in the construction of a final product, that is, its own kaleidoscope model. This model should be idealized to be sold; thus, in addition to the project and the material to be acquired for the construction, the students should determine to which public it was destined, to elaborate an instruction manual of the use and the packaging that would pack it. All the tasks previously worked out at each meeting were the basis for preparing students for this phase of the project. As the Type III enrichment activity (Renzulli & Reis, 1997), students developed an authentic product for which they were being targeted during workshops.

The culmination occurred in the presence of family, friends, colleagues and evaluating teachers who appreciated the work. Students simulated a presentation situation for the purpose of marketing their product. Involvement with the task and creativity at each stage of product design was decisive for the results. They amazed the audience with the presentations, making the atmosphere interactive and relaxed.

The students used good arguments to justify the kaleidoscope model presented and were very confident about the quality of the product that was presented. They talked about the material used in the construction, the use of the product and the target audience they intended to achieve. They were concerned with showing every detail thought about the construction, not only on the product, but also on the packaging (Figure 11).
The researcher and the guests were very pleased with the results obtained through the students’ constructions and presentations. Parents were also pleased with the outcome, congratulating the project and the initiative and, at the same time, regretted the lack of opportunities such as this so that their children could further develop their skills.

**FINAL THOUGHTS**

By analyzing the assessments made by the students, the interviews conducted with the team involved in the project and listened to the audience that accompanied the final presentation, we are sure that our *Kaleidoscopes and Geometric Constructions* enrichment program offered students the opportunity to experience exploratory activities and have contact with a variety of educational activities and experiences, often unexplored in the regular teaching of Mathematics.

According to Renzulli’s Theory of Three Rings (1986), the conception of superdotation was confirmed in the course of the workshops, since the students presented, at times, characteristics of giftedness according to the rings: above-average ability, involvement with the task and creativity. This research was not concerned with identifying the student as gifted and then labeling him in some area of giftedness. Through our enrichment program, we seek to offer enriching experiences through different themes...
and didactic resources to stimulate the development of specific skills in the students who make up the Talent Pool.

In evaluating the program, throughout all the workshops, we dare to say that through the use of different methodological resources to work on the mathematical concepts involved in the construction of kaleidoscopes, we contribute to the development of the process of creative thinking and the development of general and specific abilities.

We agree with Renzulli (1999), when he presents the Enrichment Model for the whole school, as an opportunity to promote a more pleasurable and challenging learning for all students, seeking to achieve higher levels of knowledge in the most diverse areas of interest. This model starts from an inclusive idea so that all, without distinction, have opportunities to develop their abilities, once recognized the importance of the stimuli of the environment as decisive factors for the development of potentialities.

Recognizing the cognitive capabilities of all learners and the abilities that can be developed respecting the different learning styles, we recommend that the enrichment programs are offered to all students as an opportunity and incentive to reach their maximum potential (Rambo, 2018). On this perspective, we highlight the importance of the Enrichment Model for the whole School defended by Renzulli (1999) and we suggest it under an inclusive optic as an alternative to enrich the regular curriculum with investigative activities that allow the student’s autonomy and arouse the curiosity for different areas of interest. So as the Enrichment Model for the whole School suggests, we sought to broaden the learning experiences so that our participants had a good performance. Therefore, for future works, we suggest that the school is not divided in “gifted” and “not gifted” groups, but rather all students have opportunities to develop different abilities. Paraphrasing Renzulli (2004, p.121): The high tide elevates all ships.

AUTHORS’ CONTRIBUTIONS STATEMENTS

MCDR conceived of the presented idea. SHAAF supervised the project. MCDR and SHAAF developed the theory, adapted the methodology to this context, created the models, performed the activities, collected the data, analysed the data, discussed the results, and contributed to the final version of the manuscript.

DATA AVAILABILITY STATEMENT

Data supporting the results in the current study will be made available by the corresponding author, MCDR, upon reasonable request.
REFERENCES


