ACTA SCIENTIAE
ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6226

Understandings of perimeter and area mobilized with an exploratory approach in a lesson study

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Received for publication on 28 Oct 2020. Accepted after review on 6 Sep. 2021.
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Exploratory teaching, a theme increasingly investigated in mathematics education, has been examined as a pedagogical approach underlying the research lesson in lesson studies. **Objective:** The purpose of this article is to present and discuss the understandings of area and perimeter mobilized by 8th-grade elementary school students based on an exploratory approach, which was the basis for the research lesson in the lesson study. **Design:** The research was qualitative and interpretive, based on content analysis. **Setting and participants:** The activities were conducted in two classes, of two hours each, in which the students were invited to resolve an exploratory task about the topic "area and perimeter", which was prepared by a group of teachers participating in a cycle of a lesson study. The students were asked to explain their strategies, results, and conclusions. **Data collection and analysis:** The analysis was based on empirical material composed of the materials related to the solutions presented by the students and transcriptions of audio recordings of the students' discussions as they solved the tasks and of group discussions (the final moments of the research lesson). **Results:** The analysis presents aspects related to understandings of the topics area and perimeter, mobilized by the exploratory approach, which comprise the following categories: measure, mathematical operation and geometric property. **Conclusion:** The exploratory approach, underlying the research lesson, favored a deepening of understandings about area and perimeter, because it gave students the opportunity to explore and confront these concepts from different representations mobilized in an open task and to communicate their mathematical ideas and conclusions.

Keywords: Exploratory approach; Understandings of area and perimeter; Lesson study.

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Compreensões sobre perímetro e área mobilizadas a partir da abordagem exploratória em um estudo de aula

RESUMO

Contexto: A abordagem exploratória, tema crescentemente investigado em Educação Matemática, vem sendo examinado como abordagem pedagógica subjacente à aula de investigação em estudos de aula. **Objetivo:** O artigo dedica-se a evidenciar e discutir as compreensões sobre área e perímetro mobilizadas por alunos do 8º ano do Ensino Fundamental a partir da abordagem exploratória, a qual embasou a aula de investigação de um estudo de aula (lesson study). **Design:** Investigação qualitativa e interpretativa, baseada na análise de conteúdo. **Ambiente e participantes:** As atividades foram desenvolvidas em duas aulas, de duas horas cada, a partir das quais os alunos foram solicitados a resolver uma tarefa sobre os tópicos "área e perímetro", elaborada por uma equipe de professores participantes em um ciclo de estudo de aula, justificando as suas estratégias, resultados e conclusões. **Coleta e análise de dados:** A análise baseou-se no material empírico constituído dos materiais relativos às resoluções apresentadas pelos alunos e das transcrições das gravações em áudio das discussões durante a resolução da tarefa e da discussão coletiva (momento final da aula de investigação). **Resultados:** A análise explicita aspectos relativos às compreensões sobre os tópicos área e perímetro, mobilizadas a partir da abordagem exploratória, que compreendem as seguintes categorias: medida, operação matemática e propriedade geométrica. **Conclusão:** A abordagem exploratória, subjacente à aula de investigação, favoreceu o aprofundamento das compreensões sobre área e perímetro, pois oportunizou aos alunos explorar e confrontar estes conceitos a partir de diferentes representações mobilizadas em uma tarefa aberta e, também, comunicar suas ideias e conclusões matemáticas

Palavras-chave: Abordagem exploratória; Compreensões sobre área e perímetro; Estudo de aula.

INTRODUCTION

The approach to the concepts of area and perimeter is highly important, particularly in the final years of fundamental education[1], because they form the bases for the development of relations between concepts located in distinct subfields of mathematics, such as algebra, geometry, quantities and measures, which are presented in greater depth in high school. Moreover, studies that focus on the relations between these concepts form a well-established research

[1] The final years of fundamental education, also known as fundamental II, correspond to the second phase of fundamental education, and encompass the 6th to 9th grades.

line in mathematics education, and strongly contribute to the understanding of phenomenon associated to teaching and learning of these curriculum topics, and to establishing epistemological and methodological principles for these processes.

On the other hand, the approach to these concepts has proven to be complex due to difficulties in the educational process revealed by teachers, such as approaches characterized by an excessive use of formulas without an understanding of the concepts (Teles, 2007) and specifically, by difficulties presented by students in learning area and perimeter (Lima, 1991). These aspects have mobilized researchers around the world, who strive to understand difficulties associated to the processes of teaching and learning these topics (Artigue, 1990; Baltar, 1996; Facco, 2003; Pessoa, 2010).

In this context, D'Amore and Pinilla (2006) highlight that the approach to the concepts of area and perimeter in the final years of fundamental education is marked by learning difficulties among students. Their study indicates that difficulties students have understanding the relations between area and perimeter are of both a didactic and epistemological nature. They are didactic because the approaches frequently developed by teachers do not provide students the opportunity to identify and understand these notions (D'amore & Pinilla, 2006). The difficulties are epistemological because students who do not understand these concepts confuse them (Ventura, 2013; Zils, 2018), or by not recognizing the measurements of a geometric shape as one of its constituent elements (Baltar, 1996). Moreover, Teles (2007) adds that students have difficulties correctly using units of measure, while Melo (2003) points to difficulties in separating the concepts of area from perimeter. These difficulties, according to D'Amore and Pinilla (2006), accompany many students during their school trajectory, even in higher education.

Among students' most frequent difficulties in learning these topics, French (2004) locates the difficulty in separating area and perimeter and adds that this phenomenon may be associated to simple confusion between the terms that define these concepts. Other times, the difficulty is due to conceptual mistakes to the degree that students conceive perimeter and area as inseparable geometric properties, so that the variation of one of them implies a directly proportional variation in the other (French, 2004; Ventura 2013). From this perspective, Lopes et al. (2008) highlight fragilities associated to teaching and learning about area: in teaching, aspects of a didactic nature are indicated, such as the low amount of time dedicated to the theme and precocious teaching of the concept of area. The learning problems are highlighted by difficulties of conceptual understanding by students. Ventura (2013) also emphasizes conceptual difficulties related to area and perimeter and Zils (2018) presents difficulties associated to understanding and manipulation of definitions and concepts, mistakes about these definitions and in the use of formulas.

These aspects motivated us to develop a study with the objective to reveal and discuss understandings about area and perimeter mobilized by students in the 8th grade of fundamental education through an exploratory approach, which formed the basis for an research lesson within a lesson study. The analysis is part of an interinstitutional study conducted by the authors who have been dedicated to examining possibilities and challenges for implementation of lesson studies in the context of Southern Brazil. The lesson study, considering our proposal for an exploratory approach to the topics 'area and perimeter', involved eight teachers of mathematics in the final years of fundamental education, belonging to Rio Grande do Sul public schools. We focused on the 8th grade because according to the National Common Curricular Base (BNCC), which was recently instituted in the Brazilian educational system, at the end of that school year the students should be capable of determining expressions for calculating perimeter and area of flat geometric figures, analyzing relations between these concepts, and resolving common problems related to these topics (Brasil, 2017). Therefore, the 8th grade students must have developed conceptual, operatory and practical aspects, about area and perimeter.

In addition, we consider it important to investigate this issue in the context of the exploratory approach due to the possibilities it offers for presenting the thinking and strategies used by the students to express their understandings about these topics, by solving a problem that was carefully developed for this purpose. Finally, the representations and meanings attributed to the concepts of area and perimeter by elementary school students, particularly those in the later years, constitute the bases for deepening knowledge about geometry and algebra in the following school phases.

EXPLORATORY APPROACH, LESSON STUDY AND TEACHING PERIMETER AND AREA

The exploratory approach, also known as exploratory teaching, is a well-consolidated perspective that is increasingly studied in research in the field of mathematics education (Cyrino, 2015). In Brazil, there has been significant growth in the two past decades in studies about this theme, which has led to dissertations, theses, and scientific articles, influenced by the works of Portuguese researchers João Pedro da Ponte, Ana Paula Canavarro, Hélia Oliveira and Luís Menezes.

Ponte (2005) conceived the exploratory approach as a context in which students engage with tasks for which they do not have a strategy for immediate resolution. Therefore, they must construct their own strategies and turn to previous knowledge to resolve them. This approach, according to Ponte, represents a significant change in relation to teaching in which the teacher begins by previously demonstrating the method for resolution and then presents exercises for students to resolve.

Canavarro (2011, p.11), in turn, characterizes exploratory education as an approach in which students "learn based on serious work that they do with valuable tasks that cause to emerge the need for or the advantage of mathematical ideas that are systematized in collective discussion". Through involvement in exploratory, instigating, and challenging tasks, the students have an opportunity to see arise, with meaning, "mathematical procedures, concepts and knowledge and simultaneously develop mathematical capacities such as problem solving, mathematical reasoning and mathematical communication" (Canavarro, 2011, p.11). In this perspective, Oliveira, Canavarro and Menezes (2013) add that exploratory teaching assumes an interactive nature and, as such, "does not depend only on the nature of the mathematical task and on the objective for which it is proposed or of the students' previous experiences, but essentially on how they interact with the teacher and among themselves in various moments of the class (p. 49)".

The active and intense involvement of students in the realization of exploratory tasks is an important element in this approach (Ponte, 2005; Ponte & Quaresma, 2011). However, the moments of discussion, in which the students present the work conducted, express their conjectures and conclusions, present their explanations, and question each other, are valuable situations for mathematical learning, to the degree that the teacher seeks to clarify the concepts and procedures mobilized by the students, and to evaluate the arguments they present and establish relations within and beyond mathematics. That is, the "moments of discussion thus constitute fundamental opportunities for the negotiation of mathematical meanings and the construction of new knowledge" (Ponte, 2005, p.16), and to promote and improve communication in exploratory teaching (Rodrigues, Cyrino, & Oliveira, 2018). An exploratory teaching-learning strategy gives more value to reflection and discussion (Ponte, 2005). However, the success of this approach presupposes the role and action of the teacher, which begins with the discerning selection or elaboration of the task to be proposed for the students and with the delineation of the exploratory activity (Canavarro, 2011; Oliveira, Menezes, & Canavarro, 2013).

An exploratory task differs from an exercise and from a problem by its nature and its potential. An exercise fulfills the role of leading students to exercise and practice procedures and processes (Dante, 1998), to consolidate knowledge (Ponte, 2005), without needing to decide about the procedure to be used to reach a solution (Pozo & Angón, 1998) and frequently working individually. A problem, in turn, is the description of a situation by means of which one seeks something unknown and that is not previously established, having no algorithm or strategy to resolve it, therefore requiring initiative, creativity, and previous knowledge of strategies (Dante, 1998). Upon resolving problems, the mathematical capacities of students are challenged and they experience a taste for discovery (Polya, 1975). The exploratory task, in turn, is similar to the perspective of a problem, but adds the aspect of exploration, through which students take an active role by being involved with more open and structured activities and with a balanced level of challenge (Ponte 2005), prepared from the characteristics and needs of the class and with the purpose of addressing specific issues of learning. Thus, an exploratory approach can favor work with different representations about the topics ‘perimeter and area’, involving distinct understandings (measure, property and mathematical operation) and types of quantities associated to it, supporting the students’ learning and development.

In addition, according to Ponte (2005), an exploratory approach gives priority to the development of mathematical reasoning through challenging and open tasks, because exploratory tasks give students the opportunity to construct or deepen knowledge of mathematical concepts, procedures and representations. In this sense, students are invited to take an active role in the interpretation of tasks, in the representation of the information presented and in the conception and concretization of strategies for resolution, which they should be able to present and explain. The role of the teacher, in turn, is to promote a context for discovery, the negotiation of meanings, argumentation and collective discussion, leading students to develop mathematical reasoning and comprehension, as well as the ability to use it in various situations (Ponte, 2005). Ponte & Quaresma (2011) complement that the exploratory approach provides students significant learning experiences, which strengthen the development of mathematical reasoning and problem solving. In addition, these experiences promote mathematical communication through the valorization of work in pairs and groups and the realization of collective discussion at the end of class (Quaresma & Ponte, 2012).

Therefore, in an exploratory approach, learning is a simultaneously individual and collective process that results from the interaction of students with mathematical knowledge through interesting and challenging activities and also from the interaction with colleagues and the teacher, involving processes of negotiation of meanings (Bishop & Goffree, 1986; Canavarro, 2011; Oliveira, Menezes & Canavarro, 2013; Ponte, 2005). That is, it is an alternative to the model of transmissive teaching because it places students at the center of the process and understands learning as a phenomenon that results from a process (Estevam, Cyrino, & Oliveira, 2015).

The exploratory approach, moreover, characterizes the pedagogical perspective subjacent to lesson studies conducted in Portugal and Brazil (Richit, 2020; Richit & Tomkelski, 2020). The lesson study, which consists in a reflexive and collaborative process of professional development of teachers focused on teaching practices (Lewis, 2002; Quaresma & Ponte, 2019; Richit, Ponte, & Tomasi, 2021; Yoshida, 1999), arose in Japan in the early twentieth century in the Meiji government[2], when changes in the educational system became urgently necessary. This approach[3] became consolidated as a form of preparing teachers to develop their pedagogical practices (Isoda, 2007) and came to be broadly practiced in that country since then (Yoshida, 1999), and was disseminated to Western countries in the late 1990s (Richit & Tomkelski, 2020; Stigler & Hiebert, 1999).

The lesson studies conducted in Japan have a common nuclear structure with four steps: the *definition of the objective* for a class (the research lesson), the collaborative *planning* of this class, the teaching of the *research lesson* (which is accompanied by the other members of the participating team, who produce records about the students' actions when conducting the proposed tasks) and *reflection about the class* based on the registers produced by the observers (Lewis, 2002; Quaresma & Ponte, 2019; Richit, Ponte, & Tomasi, 2021).

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[2] The **Meiji** Era was the **first period of the Empire of Japan**, between 1868-1912. It was extremely important in the development of Japan, a time when it became one of the world's leading capitalist countries. It was marked by a period of political, economic and social transformations, including enactment of an Education Code (1972), which established teacher colleges (Isoda, Arcavi, & Mena-Lorca, 2007).

[3] The dissemination of the lesson study in Western countries took place in the late 1990s particularly through promotion of the book *The Teaching Gap* (Stigler & Hiebert, 1999) which gave credit to the problem solving structure of the Japanese lesson study, and especially to the process of professional development in which all Japanese teachers are involved, the success of the students in mathematics in the *Trends in International Mathematics and Science Study* (TIMSS) (Stigler & Hiebert, 1999).

Each one of the four steps that constitute a cycle of a lesson study presents some particular characteristics. In the definition of objectives for the research lesson, there is great concern with the needs and difficulties of students in relation to learning the curricular topic chosen to be addressed in the lesson study. The planning, which takes place around the elaboration of the research lesson based on previously defined objectives, presupposes collaborative work and discerning reflection that seeks to foresee the students' forms of thinking, their strategies for resolving the proposed tasks, the difficulties they will have, that which they will say during the class activities, etc. When teaching the *research lesson*, one of the members of the group concretizes the lesson planned for a class of students and the others, including the team that coordinates the process, observe and record the students' actions. The reflection step after the lesson, in which the group meets to discuss and reflect on that which was recorded in video and observed by the other members, also contributes to professional self-criticism (Richit, 2020; Richit & Ponte, 2017; Richit, Ponte, & Tomkelski, 2019).

An important aspect in the lesson study refers to the pedagogical approach subjacent to the research lesson: which is *structured problem solving* (SPS), which consists in promoting the learning of mathematics based on problem solving. SPS, according to Fujii (2013), consists in "teaching mathematics by solving tasks" focusing on a single task, which, when it is well chosen or prepared, allows important and new mathematical ideas to arise in group discussion. *Structured problem solving* is organized in four essential processes: presentation of the problem to the students, resolution of the problem by the students, comparison, and collective discussion of the solutions (*neriage*) and systematization of the lessons by the teacher (Fujii, 2013). This perspective is at the foundation of the lesson studies cycles promoted in Japan and has been presented in various studies around the world, to support better consolidated mathematics learning.

Using a perspective close to *structured problem solving*, the *exploratory approach* has been increasingly presented in reports about lesson studies. Ponte *et al*. (2014) highlight that the exploratory approach in mathematics education allows teachers to reflect on the practice in the classroom, while it seeks to provide students differentiated learning situations, improving learning by students.

In a form analogous to *structured problem solving*, the class based on the exploratory approach, according to Oliveira, Menezes, & Canavarro (2013), can be organized in four central moments: introduction of the task; realization of the task by the students through autonomous work; collective discussion of the task and of the solutions developed by the students, and finally a systematization of the mathematical learnings. This requires a careful and detailed planning process, through which the teacher must try to anticipate the possible difficulties and strategies adopted by the students in the realization of the proposed task (Canavarro, 2011; Ponte, 2005).

In addition, the planning consists in the selection or elaboration of tasks that are suitable to and enhance the explorations desired by the teacher. It is also necessary to organize the realization of the class, defining the duration of each step and the resources needed. Finally, for the collective discussion step, the teacher must, through observations of the autonomous work of students, select the solutions that can provide positive contributions to the debate (Stein *et al*., 2008).

Therefore, the characteristics associated to the exploratory approach, as well as the processes that influence the concretization of a successful approach, make it suitable as a basis for the research lesson (the third step of the lesson study). The research lesson, by emphasizing the autonomous work of students around tasks of an exploratory nature and the collective discussion of strategies, resolutions and points of view of the students, helps modify and qualify processes for teaching mathematics, favoring the learning of students (Richit & Tomkelski, 2020). In a similar manner, the prolonged and detailed planning of the research lesson, which encompasses the first and second step of the lesson study, favors the selection and preparation of instigating and challenging tasks, as well as the selection of resources suitable to the topic to be addressed in the lesson (Ponte et al., 2014). These aspects, in turn, corroborate the perspective of the detailed planning that precedes the exploratory approach (Ponte, 2005).

Finally, the notions of area and perimeter, which are central to the thematic unit quantities and measures, according to the National Common Curricular Base (BNCC), are closely correlated because they refer to measures. Ventura (2013) emphasizes that the concept of measure is inseparable from geometry, given that perimeter and area are measurable characteristics of certain shapes. Area is conceived as an extension of a surface that is measured in specific units, so that the measure of the area of a certain region is a real number that results from the comparison of this area with an area used as a unit. Thus, the area of a surface is identified with the measurement of this area (Albuquerque & Carvalho, 1990). Perimeter corresponds to the length of the line that defines the contour of a flat (or spatial) figure, according to these authors. And it is possibly because they are quantities of the same nature, that is, quantities associated to measurement, that they are frequently confused by students (Lima, 1991).

According to Serrazina and Matos (1996, p. 120), this difficulty can be minimized with tasks that simultaneously address these concepts and confront them, which helps correct mistaken concepts held by students, such as thinking "that if two shapes have equal areas they have equal perimeters and vice-versa. They also often think that the larger the area the larger the perimeter". Therefore, the approach to these concepts must allow the student to express through verbal, written, numeric, algebraic, or pictorial language, their understandings about these concepts and particularly to establish relations and distinctions between them. Moreover, based on the challenging tasks, developed to provide a significant context for the approach to mathematics, it is possible to help the students to shorten the gap between their personal knowledge and the formal knowledge of mathematics (Gravemeijer, 2005).

From this perspective, the form of developing the topics area and perimeter can be modified with the exploratory approach, subjacent to the investigative class of the lesson study, allowing the students to overcome some of the difficulties presented and above all, can provide us supports that allows us to understand the reasons associated to these difficulties based on different forms of representing and understanding these topics.

METHODOLOGY

The research[4], which had a qualitative and interpretive nature (Erickson, 1986) consisted in revealing and discussing understandings about area and perimeter mobilized by students of the 8th grade of fundamental education using an exploratory approach that was the basis for the research lesson in a lesson study. We were guided by the following question: What understandings of area and perimeter are mobilized by 8th grade students in an exploratory approach through an research lesson in a lesson study?

The lesson study was conducted in the second semester of 2019 and involved eight[5] math teachers in the final years of fundamental education, who work at three public schools in the state school system of Rio Grande do Sul.

[4] Approved by the Research Ethics Committee (CEP) of the Universidade Federal da Fronteira Sul, parecer número 3.997.760.
[5] They are: Adelle, Ellie, Filipa, Judy, Kadu, Maggie, Marie, Mateus – fictitious names.

The teachers received invitations, by email, that were sent to schools within the 15th Coordinating Education District of Rio Grande do Sul (15ª CRE), to participate in the lesson study, and according to their interest and availability, entered the process voluntarily. Ten teachers expressed interest in participating in the lesson study after receiving the invitation, and eight formally registered and attended the session, so it was not necessary to select the participants, considering that we had defined the maximum number at ten. The teachers, who were all accredited in mathematics, all had more than five years of experience in math education.

The lesson study was composed of twelve encounters of 2 ½ hours each, held every two weeks at the offices of the 15th CRE. During the planning sessions (8 meetings), the teachers were dedicated to preparing the research lesson, focused on the topics ‘area and perimeter’, supported by study of the mathematics curriculum guidelines, research results, and the analysis of activities presented in didactic materials, such as textbooks. The lesson study meetings and the research lesson were recorded in audio, transcribed and textualized.

The task, which was planned collaboratively by the participants, used as its context the construction of a mosaic with Tangram pieces to allow going more deeply into the selected topics, which had still not been studied in that school year, although the students were already familiar with them, because they had studied them in previous years. The choice of curriculum topic addressed was supported by the *curricular program* because the teachers sought to deepen the students’ understanding about area and perimeter. To do so, the research lesson sought to adjust itself to the schedule for the discipline. It also considered the *difficulties of the students*, given that the teachers understood it was essential to address these topics, about which the students presented distinct and frequent difficulties during their school trajectory (Richit, 2021, no prelo).

The research lesson, based on an exploratory approach, involved two classes of two hours each and was held with a class of 8th grade students in a public school in the municipality of Gaurama, in the north of Rio Grande do Sul state, because it is a class in which the teacher who taught this lesson works as a teacher. In this regard, we can clarify that the research lesson of a lesson study should preferentially be conducted with a class of students in which the teacher who voluntarily offers to teach it, serves as a teacher. The research lesson consists in a single intervention, which involves the lesson planned by the teachers, and can be organized in two moments, as we did in our investigation. The class, consisting of 12 students, was organized in six pairs (P1: Adam and Vitor; P2: Assis and Will; P3: Isis and Leon; P4: Jana and Paulo; P5: Samy and Nanda; P6: Sara and Carla – fictitious names), which worked autonomously to solve an exploratory task involving the curricular topics mentioned. The task prepared involved two activities (Activity 1 about perimeter and Activity 2 about area), which were conducted on subsequent days.

The empiric material of the investigation was constituted by records made by the eight teachers, and by transcriptions of the audio recordings of the discussions of the students during the autonomous work about the task proposed and the session for reflection. The resolutions of the task were also incorporated to the empiric material. The analysis, qualitative and interpretive (Erickson, 1986), based on content analysis (Bardin, 2003), established as units of reference the group of transcribed portions of the sessions of the lesson study and resolutions of the students that reveal aspects that characterize the understandings about area and perimeter. Next, based on the units of reference, the units of register were defined and finally the categories of analysis, which were denominated: *measure, mathematical operation and geometric property.*

UNDERSTANDINGS ABOUT AREA AND PERIMETER MANIFESTED BY THE STUDENTS BASED ON THE EXPLORATORY APPROACH

The analysis focused on aspects related to three understandings about the concepts of area and perimeter – that is measure, mathematical operation and geometric property – aspects that are supported in representative, verbal and textual elements produced by the students to resolve the task proposed for the research lesson.

Area and perimeter as measure

Based on the observations of the teachers and on the registers produced about the actions of the students during the autonomous work on the proposed task, it was possible to reveal, especially in the approach to the topic perimeter, an understanding associated to the perspective of *measurement of length,* taking as a starting point the idea of outline (explained in the description of the activity proposed to the students). The context in which this perspective emerged involved the first activity, which was organized as follows: after the students conducted a study on the web about mosaic art[6] and reported the results of the search to their colleagues, the teacher told them the legend of Tangram, a Chinese game that has been disseminated throughout the world. Based on this context (mosaic art and Tangram), the students were asked to create a mosaic artwork to decorate the classroom, using Tangram pieces, as described in activity 1.

Figure 1

Activity 1 of the task – perimeter.

RESEARCH LESSON: Activity 1 of the exploratory task

1) Using the Tangram pieces provided (each pair received various sets of pieces), create one or more decorative motifs (mosaic art) for the white pieces of cardboard (white cardboard squares) given to the group, using at least four (4) pieces to form the shape of each.
2) Then join the cardboard pieces to form a mosaic. Design the mosaic below.
3) Reproduce, in a drawing, the shapes (decorative motifs) defined by each cardboard.
4) We will color/decorate the cardboard cards highlighting the contour of the shape on each one.
5) Supposing that we would decorate the contour of each shape with golden thread, how many centimeters of thread would be needed? Explain your strategy.

With this activity, the pairs of students were able to explore the concept of perimeter from the perspective of measure. One aspect related to this perspective, which emerged during the autonomous work in pairs, refers to the idea of perimeter as a *unitary measure of the outline* of a geometric form or flat figure. This aspect was highlighted in the collective discussion (the final step of the research lesson), in which the pair of Isis and Leon (Pair 3) explained how to solve the first activity of the task. Isis[7], upon explaining how to

[6] Mosaics, according to historic records, arose with the Mesopotanians around 3,000 B.C.E. However, in the West, the Mayans and Aztecs were already working with moasics and for this reason, there are controversies about how it arose. The mosaic is a milennary decorative art form that combines small pieces of various colors to form a large figure. From Greek, the term mosaic (*mouseîn*) is related to muses. It represents the gluing of small pieces close together, forming a visual effect (whether a design, shape, representation) that involves organization, combination of colors, materials and geometric shapes, in addition to patience and creativity. (Adapted from Wikipedia by the teachers for the investigative class).

[7] We used fictitious names to protect the identity of the students and teachers mentioned in the text. The notation P1,P2.. is used to indicate the pairs, so P1 indicates pair 1 and so on.

determine the quantity of gold thread needed to outline the mosaic created, presented a procedure associated to the idea of unit length, as she stated:

> *I calculated the quantity of thread used on a ruler to measure the "size" of the outline of the shape. I was measuring each side of the shape and at the end reached the measure of 42 cm.* (Isis, P3, Dec. 2019).

The teacher who led the collective discussion requested that Isis explain better the procedure so that the colleagues understand the strategy adopted. She added:

> *First I measured one side and counted 9 cm. Then I moved the ruler to the second side and kept counting higher. I counted the centimeters until the end.* (Isis, P3, Dec. 2019).

The teacher Mateus, who observed the pair as they conducted the activity, confirmed that the procedure Isis presented was quite peculiar.

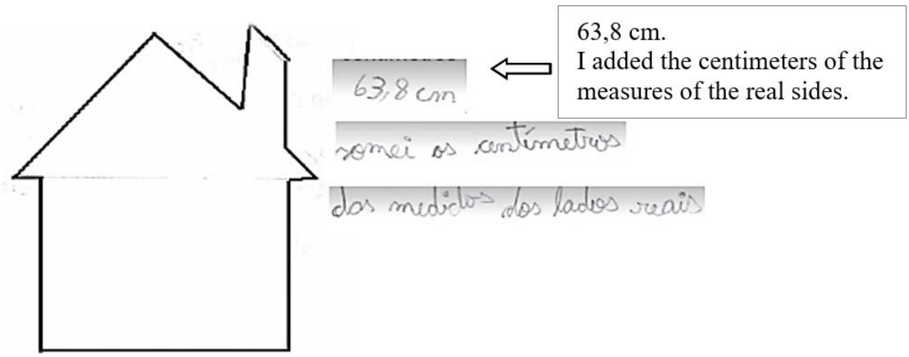
> *I realized that she calculated the amount of thread in a different manner. First she placed the ruler above the first side of the figure and counted the centimeters. Then, she moved the ruler to above the other side of the figure and began to count beginning from the amount found on the first side. For the first measure she reached 9 cm, then after placing the ruler over the second side of the figure she said out loud: 10, 11, 12, 13, 14.... and reached 17 cm. She then moved the ruler to the third side of the figure and said: 18,19…At the end of the measuring process she reached 42 cm. This was different! It is more common for the students to measure each one of the sides and then add the measurements. She did it differently.* (Mateus, Dec. 2019).

The procedure used by Isis revealed that, because the task requested the total amount of golden thread, this measurement (the measuring process) could not be divided, given that it sought to cover the contour with a single piece of thread. For this student, the outline of the figure could not be made with divided pieces of thread, but with a whole length that should be gradually placed over the outline of the figure, without dividing it. This aspect is relevant because it corroborates the notion of perimeter as a totality, which precedes the approaches based on the realization of mathematical operations and that is often supplanted in classroom practices.

Pair 2 (Assis and Will) used the strategy of *planning mosaic art* to represent the idea of contour, only designing the outside line of the artwork and indicating the total measure, a strategy that corroborated the aspect of integrality of the length of the contour. The figure below illustrates this aspect:

Figure 2

Representation of the contour of the mosaic. (Assis and Will)

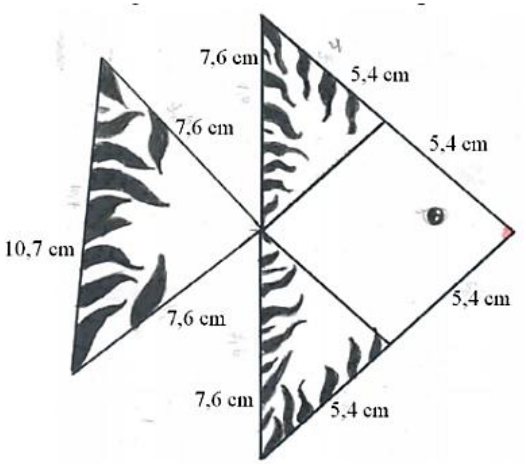


In relation to the amount of thread needed to cover the contour, although Pair 2 had indicated the total measure, when explaining the procedure used, the students revealed that they used the sum of the measures of the sides, a strategy confirmed by Adelle (the teacher who observed the pair during the research lesson). This strategy associates perimeter *to the unit measure obtained by the sum of the parts of the shape).*

Similarly, Pair 5 (Samy and Nanda) expressed the idea of perimeter as the *measure of the contour* of the art, obtained from the sum of the measures of the sides to associate each one of the sides of the figure to a measurement in centimeters (Figure 3). The procedure used by the pair predominated in the other pairs, but the striking aspect is the emphasis given to the lines of each Tangram piece used to compose the mosaic, given that for the lines on the interior of the artwork, they did not indicate any measure. This aspect reveals the pair's strategy to represent the contour of the artwork, indicating the understanding that the perimeter of the artwork corresponds to the outside shape, not considering the measures of the sides of the parts of the Tangram that are inside the figure. It also reveals the pair's ability to articulate the numeric and geometric representations of these elements (pieces of the Tangram and the artwork) to resolve the activity.

Figure 3

Mosaic made by pair 5. (Samy and Nanda)



The procedure used by this pair explored the perspective of perimeter as a measure of the external contour of the artwork, which was expanded as the students realized that it was necessary to *define a unit of measure* to calculate the total quantity of thread. That is, due to the need to use a measuring tool to conduct the activity, the students used a ruler, which led them to conclude that they had to define a unit of measure (centimeters). The pair 1, Adam and Vitor, emphasized this aspect when explaining their conclusion.

> *We realized a ruler was needed to calculate the amount of thread. Without a ruler it could not be done. And so we counted the centimeters. If I had used something else to measure, it would be a different measurement [referring to the unit of measurement]. In the first activity it was not necessary to count, because we only had to measure. To find the area was more difficult, because we could not measure. We could only count the small squares [referring to the squares in the grid placed as a suggestion on the back of the mathematic activity page] I thought it was different.* (Audio recording, P1, Dec. 2019).

This perspective also emerged in activity 2, about the study of area, which was presented as follows:

Figure 4

Activity 2 of the task – area.

RESEARCH LESSON: <u>Activity 2 of the exploratory task</u>

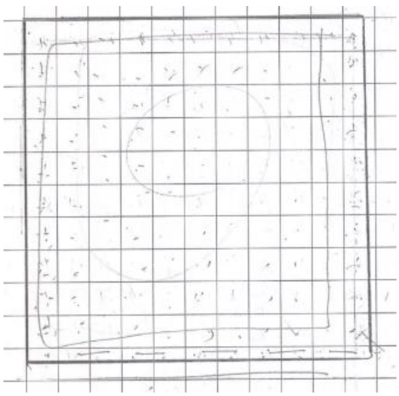
[After distributing to the pairs 4 square pieces of white cardboard, all the same size, the students were told to design a motif (shape), in such a way that by grouping the 4 sheets of cardboard they would form a mosaic.]

1) Choose one of the cartons. How many boxes are needed to cover the figure created in it? (You can use the checkered grid on the back).
2) How can we represent the idea of the previous question in mathematical language?
3) What is the blank space of the cardboard sheet? How did you reach this result?
4) What strategy did you use to determine how many small squares are needed to cover the mosaic?
5) How can we mathematically represent the previous question? Explain.
6) In mathematics, what is this result called?

When solving activity 2, Pair 3 (Isis and Leon), explored the perspective of measurement to determine the area. To do so, they used the checkered grid, which was suggested as a measuring tool in the presentation of the activity, over which they marked the region occupied by the mosaic sketched on the cardboard sheets distributed to the pairs (Figure 5).

Figure 5

Mosaic artwork of Pair 3. (Isis and Leon)



Upon explaining the procedure used to answer the first question of activity 2, Isis emphasized that it was necessary to count the small squares inside the artwork.

> *I drew the real figure on the graph paper. Then I realized that I only needed to count the squares inside the figure to know the amount.* (Isis, P3, Dec. 2019).
>
> *We used the grid that was on the back of the activity sheet, but we could have created something else, another material, to do this. It could be a grid with triangles. Then the unit would be small triangles. Then just count them.* (Leon, P3, Dec. 2019).

Although the mosaic art of Pair 3 was simple [they drew squares on the cardboard sheets and grouped them randomly], it allowed the pair to explore the notion of area as the measure of a region marked by a periphery, using as a unit of measure 1 cm squares. The grid on the back of the activity sheet was used as a measuring tool by all of the pairs.

In summary, the analysis revealed that the exploratory approach to the topics of area and perimeter allowed the students to explore these concepts as measuring processes. Based on the two activities that formed the basis for the exploratory approach, the pairs used different procedures: for the perimeter, they defined the tool to be used (ruler) and the unit of measure adopted. For area, the tool was suggested in the presentation (square grid) and the unit of measure was the small squares (each one 1cm per side). They thus realized that it was possible to create measuring tools to define area and use them to define a unit of measure to express it. Another important aspect related to estimating the area of the artwork refers to the possibility that the pairs explore this concept from the perspective of the measurement of the surface of the art, which is revealed in the procedure based on counting squares. Finally, the exploratory approach favored the autonomy of the students, revealing their understanding of the work to be conducted in the tasks, and gave them an opportunity to communicate their mathematical ideas in the group discussion (the final moment of the research lesson).

Area and perimeter as mathematical operation

This perspective was predominant in the solutions to the exploratory task (Activities 1 and 2). The analysis of the solutions reached by the pairs of students, of the representations used and the observations of the teachers during the research lesson indicate that the students explored the concepts of area and perimeter by conducting operations of addition and multiplication. Figure 6 below explains the strategy used by Pair 6 (Sara and Carla), which determined the perimeter of the mosaic using *successive additions*. In the same way, Figure 7, by Adam and Victor (P1), associated the concept of area to the operation of *multiplication of the measurements of the sides.*

Figure 6

Mosaic Art P6. (Sara and Carla)

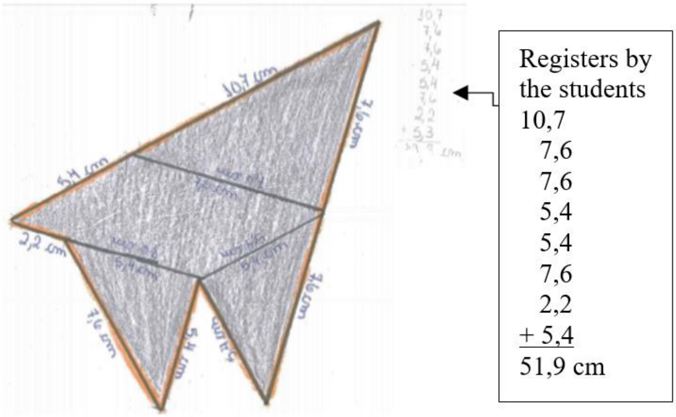
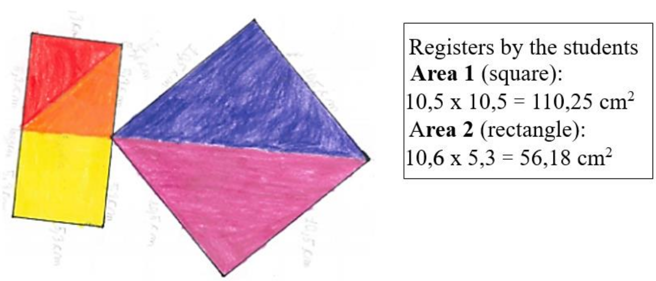


Figure 7

Mosaic Art P1. (Adam and Vitor)



The procedure used by pairs P1 and P6 is interesting because the students expressed *different representations* in the resolution of activities 1 (perimeter) and 2 (area). They first presented the Tangram pieces used to compose the artwork in their real dimensions. Then they presented the sizes of each part, explaining the unit of measure used (cm). Finally, to calculate the amount of golden thread needed for the perimeter, they made successive additions only of the outside measurements of the artwork, that is, of the measurements indicated on the perimeter (Figure 6 – orange line; Figure 7 – black line). And to calculate the area, Pair 1 multiplied the measures of the sides of each one of the shapes that composed the artwork (Figure 7). This activity led them to perceive the difference between unit of measure of length and unit of measure of area, because the elements considered were of a different nature, as were the mathematical notations used in each case, as Vitor indicted:

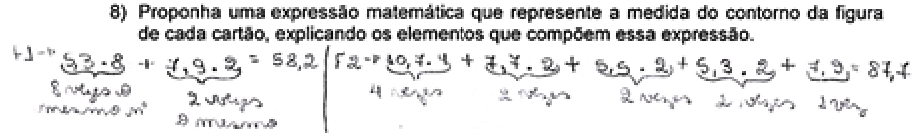
> *For the perimeter, we measured the sides of the figure, which are lines [referring to the segment of the straight line]. Then the result was in cm. [...]. For the area we had triangles and rectangles, so when we multiplied the base by the height, we weren't sure: we didn't know if it was only cm. [...]. And in the discussion we realized that it was the same as what we did last year, when we multiplied x by x and got x2. Then I understood why in the area it is cm2.* (Vitor, P3, Dec. 2019).

For example, when estimating the perimeter, the elements considered were all segments of straight lines, one-dimensional elements whose size is given by units of length. To estimate area, which involved polygons and flat geometric shapes, most pairs multiplied the measure of the base by the height, so that the unit of measure had to consider that the shapes used in the artwork had two dimensions.

The emphasis on the realization of the mathematical operations pointed to the previous assimilation, by some of the students, of the definitions of area and perimeter present in the didactic materials frequently used in the final years of fundamental education. This aspect was especially revealed in the context of realization of item 8 of the activity 1 (perimeter), in which the pairs were asked to propose a mathematical expression to represent the outline of the mosaic artwork created. Pair 5 (Samy and Nanda) proposed the mathematical expression in Figure 8.

Figure 8

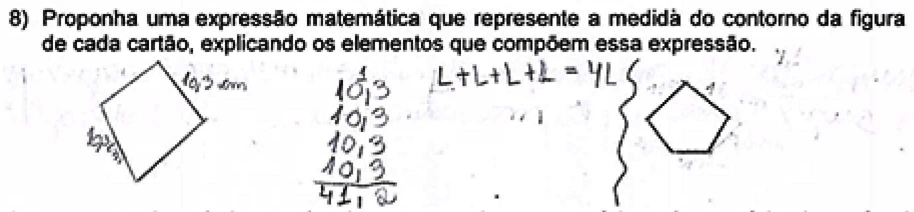
Mathematical expression to represent perimeter. (Samy and Nanda)



8) Propose a mathematical expression that represents the measure of the perimeter of the shape of each piece of cardboard, explaining the elements that compose this expression.

Figure 9

Mathematical expression to represent perimeter. (Isis and Leon)



8) Propose a mathematical expression that represents the measure of the perimeter of the shape of each piece of cardboard, explaining the elements that compose this expression.

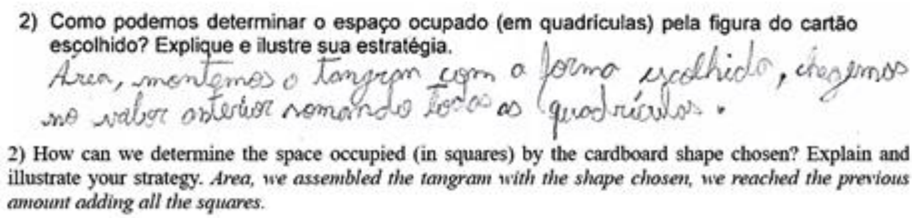
This expression is interesting because, in addition to associating perimeter to the realization of the successive additions, it revealed the *repetitive processes* identified by the pair to determine it. Upon finding that the perimeter of the art is constituted by the sum of the measures of the equal Tangram pieces that were used repeatedly in the composition of the art, the pair used the representation of the successive sums by means of multiplication, indicating the measure of the piece (5.3 cm, for example) and the number of times in which this measure was considered in the artwork (8 times, for example). Another equally important aspect is the fact that the students were able to abstract the notions of perimeter and area from the geometric shapes, representing them with *mathematical expressions that include addition and multiplications.* The procedure presented indicated the ability to generalize the mathematical process used to determine the perimeter of the art, which is revealed by the strategy of grouping successive additions by means of multiplication. It also expresses the ability to shift between the geometric representation of the artwork and the algebraic representation of the concept of perimeter. The representation below, from the pair Isis and Leon, corroborates this aspect.

The expression formulated by the pair Isis and Leon also presented the *generalization of the successive operations of addition by means of algebraic representation*, indicating primarily the representation of the dimensions considered (L + L + L + L) and at the end, by the fact that they involve sides of equal length, the algebraic representation is summarized by the expression 4L. The transition realized from the context suggested in the task indicates this pair's ability to represent the concept of perimeter in different ways.

Pair 1 (Adam and Will) associated the calculation of the area of the artwork to the realization of the successive addition of the small squares circumscribed by the outline of the mosaic, that is, by counting the squares inside the shape, as described in the strategy of Figure 10.

Figure 10

Description of the strategy used by Pair 1. (Adam and Will)


2) Como podemos determinar o espaço ocupado (em quadrículas) pela figura do cartão escolhido? Explique e ilustre sua estratégia.
Area, montemos o tangram com a forma escolhida, chegamos no valor anterior somando todos os quadrículos.

2) How can we determine the space occupied (in squares) by the cardboard shape chosen? Explain and illustrate your strategy. *Area, we assembled the tangram with the shape chosen, we reached the previous amount adding all the squares.*

By saying that after constructing the artwork with the Tangram "*we reached the previous amount adding all the squares*", the pair revealed that it initially used the mathematical operations to calculate the area and then used the square grid. The explanation of the strategy adopted by this pair corroborates the perspective of mathematical operation, although it raises an additional element by associating the concept of area to the operation of addition. This understanding reveals a broader comprehension of area given that the pair went beyond the idea of multiplying the measures of the sides of the artwork created and decided to use the operation of addition with the units of measurement adopted in the activity, that is, the squares in the grid given to the students.

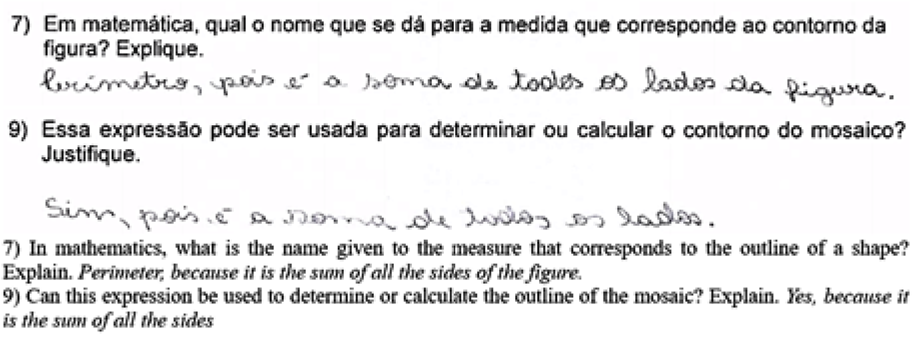
In summary, the aspects revealed in the analysis point to the possibility to allow students to expand their understandings about area and perimeter with an exploratory approach, giving priority to aspects related to ways of conceiving them. In addition to associating the concepts of area and perimeter to the operations of addition and subtraction, the exploratory approach led the students to move between the geometric representation of the artwork created to an algebraic representation of perimeter and area. They advanced to the degree that they proposed representations based on the associated properties of multiplication and addition, and especially by the fact that they represented the repetitive properties identified in the realization of the calculation of the perimeter. In addition, the exploratory approach allowed some pairs to broaden their understanding of area as a product of the sides and perceived the possibility to use addition of a unit of measure of the region filled in by the art. These aspects reveal possibilities to explore and deepen the mathematical generalization of the notion of perimeter based on the task proposed.

Area and perimeter as properties of geometric forms

The discussions of the students about what they should do in the task, and mainly the explanations that they gave for their conclusions, reveal that the pairs used the aspects related to understanding of area and perimeter as a specific property of the geometric shapes, something given *a priori*, which could validate the mathematical processes and results. This perspective was expressed in the resolution of questions 7 and 9 of activity 1. The explanation presented by Pair 5 (Samy and Nanda) illustrates this aspect.

Figure 11

Understanding of perimeter of Pair 5. (Samy and Nanda)


7) Em matemática, qual o nome que se dá para a medida que corresponde ao contorno da figura? Explique.

Perímetro, pois é a soma de todos os lados da figura.

9) Essa expressão pode ser usada para determinar ou calcular o contorno do mosaico? Justifique.

Sim, pois é a soma de todos os lados.

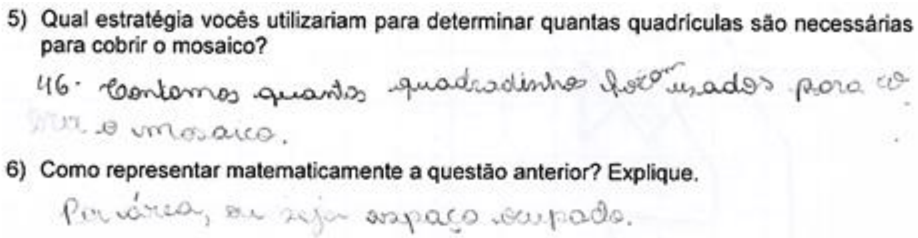
7) In mathematics, what is the name given to the measure that corresponds to the outline of a shape? Explain. *Perimeter, because it is the sum of all the sides of the figure.*

9) Can this expression be used to determine or calculate the outline of the mosaic? Explain. *Yes, because it is the sum of all the sides*

By defining the outline of a figure as perimeter, when explaining their procedures, all the pairs used the definition associated to this concept as a *property inherent to geometric shapes*. The conclusive answer “because it is the sum of all the sides”, present in the explanations of all the pairs, reveals the understanding of perimeter as a property that establishes an association between the dimensions of the sides of a figure or polygon, using addition. It therefore can be used in other situations or problems whose contexts involve elements of the same nature (sides that are straight segments). The task also allowed the pairs to explore the concepts of perimeter and area as particular properties of the measurement of the length of the ‘outline’ and of the ‘surface’.

Figure 12

Understanding of the pair about perimeter. (Samy and Nanda)



5) Qual estratégia vocês utilizariam para determinar quantas quadrículas são necessárias para cobrir o mosaico?

46. Contamos quantos quadradinhos foram usados para co... o mosaico.

6) Como representar matematicamente a questão anterior? Explique.

Por área, ou seja espaço ocupado.

Activity 2, by specifically addressing the concept of area, asked the pairs to deepen their understanding of the meaning of area, to the degree that the students were encouraged to express their understandings by explaining the procedures they took to determine the region circumscribed by the mosaic artwork.

In addition, they were encouraged to propose strategies to resolve and justify them based on the activities that led them to turn to concepts and properties previously studied and especially had the opportunity to communicate the mathematical understandings and learnings, as illustrated in the Figure 12.

By producing a mosaic artwork from Tangram pieces the students were involved in a task that challenged them to formalize, in a descriptive/discursive manner, their understandings about area and perimeter, explaining the elements and the operations used in this process.

The analysis thus indicates that the exploratory approach encourages the students to broaden their understandings about the concepts of area and perimeter, especially when they associate the concept of area to the space occupied by a certain shape and the counting of the units of measure used in the activity. It led them to associate these concepts to a process of measurement that supposes the identification/definition of the unit of measure associated, as well as a measuring tool. However, we understand that the indirect relationship of proportionality between area and perimeter was not explored much by the exploratory task proposed. In relation to the dynamics of the lesson, we understand that the exploratory approach favored the modification of the positions taken by the students in the classroom practices, to the degree that they took active roles, communicating their mathematical ideas and conclusions, questioning, reflecting, and complementing the ideas of colleagues.

DISCUSSION AND CONCLUSIONS

The exploratory approach was undertaken in a context that would allow the students to be involved in an instigating task, which involved the creation of a mosaic artwork with Tangram pieces, for which they would need to formulate their own strategies, using prior knowledge (Melo, 2003; Ponte, 2005) and present particular procedures for resolution and justifications in certain aspects. This approach represented a change in relation to teaching of these topics (Ponte, 2005; Quaresma & Ponte, 2012), to the degree to which the students were encouraged to explore and explain, in detail, the process that involved estimating the quantity of thread needed to outline the artwork (its perimeter) and its corresponding area.

The exploratory approach to the topics of area and perimeter, by their nature and characteristics, allowed the students to develop mathematical skills such as problem solving, mathematical reasoning and mathematical communication (Canavarro, 2011; Oliveira, Menezes, & Canavarro, 2013; Ponte & Quaresma, 2011), especially in the group discussion at the end of the research lesson (Fujii, 2013; Ponte *et al*., 2014) and through the explanations produced for the solutions presented by the pairs. By undertaking the task, (activity 1 – perimeter; activity 2 – area), the students were encouraged to propose strategies and procedures that would lead them to explore different mathematical representations for these concepts and thus realize the meaning (Canavarro, 2011) of aspects related to three understandings of area and perimeter (measure, mathematical operation and geometric property). These aspects were mobilized through the students' active involvement and through the discussion of the pairs when conducting the task (Ponte, 2005), the new meanings attributed to these curricular topics of mathematics (Canavarro, 2011; Teles, 2007) and especially through the mobilization and articulation of distinct mathematical representations in the process of formalization of the procedures used to conduct the calculations required.

Considering **perimeter as a measure,** one of the aspects revealed by the exploratory approach concerns the *process of measuring the length of the line that defines the outline* (Albuquerque & Carvalho, 1990; Serrazina & Matos, 1996; Zils, 2018) of the mosaic artwork created by the pairs with Tangram pieces. The perspective of **area as a measure** was explored through the use of geometric representations, which led the students to express them as the *measure of the internal region circumscribed by the outline* of the artwork. In relation to perimeter, a peculiar aspect stood out in the procedure adopted by one pair, which expressed perimeter as the *unit measure corresponding to the total length of the outline,* while the other pairs expressed it as the *measure obtained by the sum of the parts of the outline.* Both aspects were explained in the procedures of the pairs, which, even though they were distinct in terms of the mathematical representations and resources adopted, used the strategy of *planning mosaic art*, to represent the idea of perimeter, designing only the outline of the artwork.

In addition, the exploratory approach allowed the students to expand their understanding of the concept of perimeter as a measure (Baltar, 1996; Teles, 2007; Ventura, 2013), because they could perceive that this process requires the use of a measuring tool and in this way supposes previously determining a unit of measure to express it (Teles, 2007). The activity about area, which suggested the use of a square grid as a measuring tool (Pessoa, 2010), allowed the students to conclude that it is possible to use other resources as measuring tools, implying new units of measurement. This aspect therefore suggests the possibility to overcome some difficulties the students have in the *use and conversion* of measuring units (Facco, 2003; Teles, 2007) and in the differentiation between these concepts (D'Amore & Pinilla, 2006; Zils, 2018), difficulties that are considered by the teachers who participated in the study of the lesson in the definition of the curriculum topic to be addressed in the research lesson. They thus concluded that the estimates of area and perimeter of the mosaic artwork were measurement processes with their own specificities, which depend on the dimensions of each piece that composes them.

The perspective that associates **area and perimeter as a mathematical operation**, predominant in the solutions developed by the pairs, was mobilized in the procedures adopted and in the explanations the students gave for how they solved the problems and their conclusions. The pairs, in general, used addition and multiplication to determine a real number (result), which corresponded to the measure of length of the contour (perimeter) and of the area outlined by the design (Albuquerque & Carvalho, 1990; Lima, 1991; Pessoa, 2010). These operations were at times presented as the starting point for resolving the activities, and at other times were used to confirm or validate the result obtained. The task, specifically developed to create a significant context for exploring the distinct understandings of area and perimeter, encouraged the students to use prior mathematical knowledge (Gravemeijer, 2005; Ventura, 2013; Zils, 2018), and through different representations and strategies, expand and formalize this knowledge (Fujii, 2013), turning at times to algebraic representation to generalize their conclusions. In this sense, the exploratory approach to area and perimeter of the flat figures, involved in the context of creation of art, led the students to better understand these concepts, overcoming the confusion between them (Facco, 2003; French, 2004; Lopes, Salinas, & Palhares 2008; Ventura, 2013). However, the solutions based on the immediate application of formulas for calculating area and perimeter reveal the students' difficulty in correctly using units of measure (Teles 2007), given that they focused attention on the amounts indicated in the representations proposed without focusing on the units that they represent. This aspect reveals the importance of careful preparation of the exploratory task (Ponte 2005; Ponte & Quaresma, 2011; Serrazina & Matos, 1996), which must lead students to understand the relations and distinctions between these elements (French, 2004; Ventura, 2013), and above all, to encourage them to identify and explore the different elements considered in these measuring processes, and to analyze the nature of these elements. For example, when estimating perimeter, the elements considered were all straight segments, one dimensional elements whose dimension is given by units of length (cm, m, for example). When estimating area, which involves polygons and flat geometric shapes, the procedure most used by the pairs was the multiplication of measures of the base and height, so that the unit of measure must consider the two dimensions of the geometric shapes. This characteristic led them to perceive the difference between unit of measure of length and unit of measure of area, and the mathematical notations in each case. However, this process supposes the attentive and qualified intervention of the teacher, revealing the importance of this professional in preparing the rich and challenging tasks (Canavarro, 2011; Ponte; 2005; Ponte & Quaresma, 2011), and in leading the group discussion in an effort to stimulate the communication of mathematical ideas and conclusions (Ponte, *et al*., 2014; Richit, 2020) and in this way, the formalization of knowledge (Fujii, 2013).

Area and perimeter as properties of geometric forms is a perspective that was explored in less depth in the discussions among the students and in the explanations of how they resolved the task, an aspect that suggests a limitation of the task itself and the students' lack of familiarity with this understanding. The predominant trend in all the pairs, to explain their understandings by repeating the definitions of area and perimeter present in many didactic materials (perimeter is the sum of all the sides; area is the product of the base and the height), reveals their difficulty in differentiating the elements that define these topics (French, 2004; Lima, 1991; Ventura, 2013), such as, when one of the pairs mistakenly multiplied the measure of the sides of the figure to obtain the perimeter. This aspect was particularly expressed by the difficulty of distinguishing area from perimeter (Facco, 2003; Melo, 2003; Ventura, 2013), which are concepts of the same nature (measurements of quantities), through specific properties of elements that define them and that are considered in the measurements. Finally, this aspect indicates that the students recognize the measures of geometric shapes of the Tangram as elements that compose them (Baltar, 1996).

The analysis revealed the opportunities created by an exploratory approach (Estevam, Cyrino, & Oliveira, 2015), in which the students are involved in a mathematical task that challenges them, mobilizes them, and allows them to explore and confront concepts from different representations utilized in a more open task (Ponte, 2005; Ponte & Quaresma, 2011; Richit & Tomkelski, 2020; Serrazina & Matos, 1996). The development of a specific task to address the students' difficulties in distinguishing area and perimeter, which was the starting point for promoting the exploratory approach in the lesson study, encouraged growth by the students (Fujii, 2013) to the degree to which they explored different forms of conceiving these concepts. It also supported the development of the teachers by helping them to better understand some of the students' difficulties (Ponte, *et al*., 2014; Richit & Tomkelski, 2020), and to identify and understand the reasons for these difficulties and to find strategies to assist the students in these difficulties. Similarly, the collaborative and reflexive planning of the research lesson, permeated by a negotiation of ideas and decisions, sharing of experiences and teaching materials, reflection on the practice and on teaching mathematics, created a context for realizing a professional activity that overcame the individualism that is predominant in school routines (Richit, Ponte, & Tomasi, 2021). Finally, the planning of the research lesson focused on an open task allowed the teachers to develop the ability to promote and encourage communication in exploratory teaching (Rodrigues, Cyrino, & Oliveira, 2018).

Thus, more open tasks, such as those involving the concepts of area and perimeter, provide important learning opportunities, because they encourage a negotiation of meanings, the construction of concepts and articulation of representations (Quaresma & Ponte, 2012) and mathematical communication with verbal, written, numeric, algebraic or pictorial language. The tasks are also propitious for the students to develop their understandings of mathematical concepts and particularly to establish relations and distinctions between them (Canavarro, 2011; Serrazina & Matos, 1996), as in the experience that we conducted in the exploratory approach to area and perimeter.

Therefore, the characteristics and structure of the task that was the basis for the approach to area and perimeter were propitious to realizing a positive experience about these topics, in synergy with the principles of an research lesson of a lesson study (Fujii, 2013; Richit, 2020). The exploratory approach emphasized autonomous work of students around a task specifically developed to deepen understanding of these concepts, and the group discussion of strategies, resolutions and points of view of the students, contributing to expanding and deepening the understandings of these topics and to overcoming some of the difficulties related to distinguishing them and to their use in problem situations. The group discussion, in which the students explain how they resolved the task, present and discuss their conjectures and conclusions, present their explanations and question each other, is an instigating and positive learning context that favors the growth of all those involved (students, teachers and researchers).

CONTRIBUTION OF THE AUTHORS

AR (first author) designed the idea presented. AR developed the theory. AR and MLT adapted the methodology to this context, created the models, executed the activities and developed the data. AR analyzed the data. All the authors discussed the results and contributed to the final version of the article.

DECLARATION OF AVAILABILITY OF THE DATA

The data for this study will be provided by the authors upon a reasonable request.

ACKNOWLEDGEMENTS

We are especially grateful to the teachers participating in the lesson study and to the 8th grade students who participated in the research lesson. We are also grateful to CNPq for the financial support (Process: 305476/2020-3).

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