Aspects Considered by a Prospective Teacher When Reflecting on a Virtual Classroom

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ABSTRACT

Background: Reflection is one of the fundamental skills in initial teacher training, and curricula need to consider activities in a variety of educational contexts and situations that allow or encourage its development. Objectives: This investigation is intended to characterise the pedagogical, mathematical, and technological criteria that a prospective mathematics teacher establishes and uses when reflecting on events occurring in a virtual classroom. Design: The investigation was guided by a qualitative methodology of an exploratory-descriptive nature. Setting and Participants: the participant was a fifth-year licentiate student, and the data were collected during the second semester of 2022. Data collection and analysis: The content analysis method was used, and information was collected through questionnaires. Results: The results show that this student was able to adapt predefined schemes or proposals from the Technological, Pedagogical Content Knowledge (TPACK) model to create indicators that allowed her to carry out a descriptive but not particularly analytical reflection, however, although such adaptation was in agreement with expectations, there was little creation of new aspects. Conclusions: It is concluded that to provide an opportunity for prospective teachers to reflect on a lesson, previous aspects or other models or theories can be studied since starting directly from the TPACK domains and sub-domains may not be a practical methodological tool for student reflection, given that this framework does not have analysis criteria for improvement.

Keywords: Mathematics education; prospective teacher training; structure of knowledge; TPACK; virtual class.

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Aspectos que considera una docente en formación cuando reflexiona sobre una clase virtual

RESUMEN

Contexto: La reflexión es una de las capacidades fundamentales en la formación inicial del profesorado y se requiere que dentro de los currículos se consideren actividades en diversos contextos y situaciones educativas que permitan o propicien su desarrollo. Objetivos: Esta investigación tiene por objetivo caracterizar los aspectos pedagógicos, matemáticos y tecnológicos que establece y utiliza una profesora de matemáticas en formación cuando reflexiona sobre los eventos ocurridos en una clase virtual. Diseño: La investigación fue de tipo cualitativa con carácter exploratorio-descriptivo. Contexto y participantes: la participante fue una estudiante de licenciatura de quinto año de formación y los datos fueron recolectados durante el segundo semestre de 2022. Recolección y análisis de datos: Se utilizó el método de análisis de contenido y se recolectó la información a través de cuestionarios. Resultados: Los resultados muestran que esta estudiante es capaz de utilizar esquemas o propuestas predefinidas desde el modelo de conocimientos tecnológicos, pedagógicos y de contenido (TPACK) para adaptar indicadores que le permiten realizar una reflexión de carácter descriptivo, pero poco analítico; sin embargo, aunque tal adaptación es congruente con lo esperado, la creación de nuevos aspectos es escasa. Conclusiones: Se concluye que para ofrecer oportunidad de generar una reflexión de docentes en formación sobre una clase se pueden estudiar aspectos previos u otros modelos o teorías, ya que partir directamente de los dominios y subdominios del TPACK puede no ser una herramienta metodológica práctica para la reflexión del estudiantado, pues este modelo no posee criterios de análisis para la mejora.

Palabras clave: Enseñanza de las matemáticas; Formación de docentes; Estructura del conocimiento; TPACK; clases virtuales.

INTRODUCTION

Reflecting on what happens in the classroom is one of the most relevant skills that should be emphasised in teacher training curricula. Teachers must be able to think about what happens before, during, and after a class and be able to describe, analyse and explain events that occur during these periods. If they cannot do so, it is difficult for them to identify points for improvement in instruction (Mason, 2002). Care must be taken to bear in mind that there are two different conceptual planes in these activities: on the one hand, the activity of knowing or identifying can be linked to more circumstantial knowledge of recording and explaining events (first plane) through a set of characteristics or aspects, while, on the other hand, there is a more complex type of competence that is linked to reflection on and assessment of situations for improvement
(second plane) and identification of criteria on what should be done to achieve that improvement.

In the case of Mathematics Education, it has been shown that for prospective teachers to develop this capacity, there must be systematic support and activities that help them recognize situations of interest (Santagata, 2011; Santagata, & Guarino; 2011; Star & Strickland, 2008), and even authors such as Breda et al., (2017), Husu et al. (2008) and Williams (2020) suggest that there should be explicit guides that allow elements of description and analysis to emerge from the assessment of class events.

Along with the above, living in a post-pandemic era has presented great challenges and has allowed the exploration of emerging strategies. Although many of the activities carried out during the pandemic were reactive, teachers were also exposed to new planning, educational management, and technological requirements for which they were not prepared (Breda et al., 2020; Ledezma et al. 2023). For example, in the case of virtual classes, few were previously familiar with the details of this type of instruction, even though virtual education had already been consolidated as an educational model for several decades and, particularly in mathematics, can clearly be beneficial (Borba et al., 2018).

This justifies increased interest in investigating the capacity for reflection that prospective teachers have when they analyse class situations occurring in a virtual environment. Specifically, it becomes relevant to understand the strategies that a prospective secondary mathematics teacher (PSMT) uses when she must interpret these situations. Therefore, the objective of this investigation was to characterise the technological, pedagogical, and mathematical aspects that a PSMT established and used when she analysed the events that occurred in a virtual mathematics class on the topic of functions.

For this purpose, the PSMT received a series of training sessions on knowledge organization systems, especially on the Technological Pedagogical Content Knowledge (TPACK) model. Two batteries of indicators were selected from the international literature which contain aspects for the study of different elements of the TPACK model and were explained to the participant. Subsequently, the PSMT was shown a video recording of a virtual class on the topic of functions and was asked to select her own battery of indicators that would allow her to identify or evaluate what happened in the class. These aspects could be completely original or could be adaptations of, or identical to, those studied in the two batteries previously examined.
Part of the investigation carried out here was intended to learn more about whether the student was able to use only aspects of the first plane (knowing or identifying situations) or if this activity and the TPACK domains allowed her to generate evaluative criteria for improvement. She was, therefore, asked to write a reflection on her actions using as a reference the rubric that she had constructed with the aim of verifying the way in which she used the aspects that she herself considered.

The following sections introduce the TPACK model and its domains, as well as the use of this model in teacher training, and specifically, on internationally discussed aspects of the use of the TPACK model.

THEORETICAL BACKGROUND

The TPACK model

For several decades, improving our understanding of teachers’ activities has been on research agendas in multiple countries, such as the United States, where many efforts have been made to better understand teaching as a professional activity. The early works of Lee Shulman (1986, 1987) opened the door to the development of schemes or systems for organizing teachers’ knowledge. He considered that along with pedagogical knowledge (PK) and content knowledge (CK), there should be professional knowledge of the educator that would allow him or her to understand the action of teaching a subject in a professional manner. He called this knowledge pedagogical content knowledge (PCK). In the following years, this knowledge organization system allowed the creation of more complex models adapted to different disciplines.

On the other hand, due to the growing need to study the role of technology in education, many efforts have been made to understand the possible impacts of this resource. Mishra and Koehler (2006) emphasized the knowledge of teachers because they believe that this is the factor that has the greatest influence on the appropriate use of technologies in the classroom. They specified the existence of three domains of knowledge in this area: technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK). There are also three subdomains that are defined by the intersections of pairs of the principal domains: technological pedagogical knowledge (TPK), technological content knowledge (TCK), and pedagogical content knowledge, which coincides with Shulman’s PCK (1986; 1987). Finally, there is a subdomain that occurs from the intersection of all three principal
domains, which is called technological pedagogical content knowledge (TPCK, later renamed TPACK) (Figure 1).

Figure 1

Representation of the TPACK model and the domains and subdomains in the knowledge structuring scheme (http://www.tpack.org/ [rights free])

Each of these domains and subdomains are briefly explained below (Mishra & Koehler, 2006):

1. Technological knowledge (TK) is all general knowledge about technology, which goes beyond simply using a computer, the Internet, or a whiteboard and allows this knowledge to be used in different situations (not only educational). This knowledge is related to processes associated with information, communication, and problem-solving, among others, placing special emphasis on the different forms of interaction with technologies.

2. Pedagogical knowledge (PK) corresponds to knowledge about how to learn and teach. It is linked to general theories about instruction and how knowledge is developed and created in educational activities and the different learning theories.
3. Content knowledge (CK) refers to one’s own knowledge of the subject or content. It corresponds to basic and expert knowledge about the subject matter and its links with other areas of knowledge. In the context of this investigation, it concerns mathematics in all its possible expressions and its relationships and intersections with other disciplines.

4. Technological pedagogical knowledge (TPK) has to do with the links between the appropriate use of technological resources and education. More specifically, it has to do with theories of learning with and through technologies (for example, virtual, bimodal education), and with the use of technological resources that can serve different disciplines. In terms of the use of resources, it can be exemplified by the use of platforms such as Moodle, Google Classroom, massive open online courses (MOOCs), tools such as scientific calculators, virtual whiteboards, and other resources that can be applied transversally in many areas of education.

5. Technological content knowledge (TCK) is what allows the use of technology to generate knowledge in a specific field, in this case, mathematics. Normally, this knowledge allows the improvement of the discipline and, in turn, the creation of cutting-edge technological resources for use in the field.

6. Pedagogical content knowledge (PCK) is what allows a professional to teach about a particular type of content. This is the knowledge that relates and transforms the knowledge to be taught (Shulman, 1986). It plays a fundamental role because it differentiates between knowing about an area and knowing about how such knowledge is taught and learned. In the 1980s and 1990s, this distinction led to a re-emphasis on the difference between areas of education as professions and areas of knowledge. For example, this perspective makes it clear that understanding mathematics does not allow a person to carry out adequate instruction on the subject, adapted to various curricula and educational contexts.

7. Technological pedagogical content knowledge (TPACK) is the knowledge that emerges from the interaction between the three principal domains and allows decisions to be made about appropriate moments and ways of using technological resources in the development of specific content for effective learning and development of abilities. Thus, this knowledge is required not only...
when resources are used, but in practically all phases of the educational process. It allows us to make decisions about when, how, and why, as well as the implications and effects of the use of resources in the construction of knowledge in students. Figure 1 is a graphical representation of the TPACK knowledge organization system.

TPACK thus offers a system for the organization of knowledge that teachers have (or should have) about the use of technologies in processes of instruction (Morales-López et al., 2021). To accomplish this, the TPACK model requires the use of strategies that are appropriate for each type of content. It is also suggested that the teacher considers the three main elements (Content, Pedagogy, and Technology) together, rather than separately. Although TPACK is considered complex knowledge by some authors such as Chai et al. (2013), since it includes several elements that must be considered together when planning the class, it can still be considered a structure with great potential for student learning. In particular, this framework makes it possible to establish an order for the study of each of its domains and subdomains, and in the specific context of this investigation, it makes it possible to organize aspects of technology, pedagogy and mathematical content (functions) in a virtual class.

**Guided reflection in teachers in training**

There are many situations that occur in the classroom that could be interpreted as either relevant or irrelevant, depending on the judgment of the observer. Thus, teachers must be trained to develop their abilities to observe and identify, as well as high-order capabilities such as reflecting on and identifying opportunities for improvement and executing changes based on solid knowledge of the subject and the way in which it is taught (Mason 2002; Schön, 1983). However, reflection involves several actions beyond observing the environment, and it is not enough to simply describe what happens (first plane). Prospective teaching staff must be taught to make decisions derived from their reflective practice and to make use of this tool to improve situations that arise in the class (second plane) (Husu et al., 2008; Wei et al., 2023).

However, it is difficult for prospective teachers to determine what may or may not be relevant in classroom situations and it may also be difficult for them to organize their ideas; even if they have knowledge, this does not ensure that they can take it into account when carrying out a reflective practice or
action. To avoid this, authors such as Risko et al. (2019), Santagata et al. (2011), Schön (1983), Morales-López, & Araya-Román (2020), Seckel & Font (2015), Font & Breda (2015) and others propose working with prospective teachers through practices that are mainly designed to guide reflections through not only aspects but criteria, so that eventually, they can have a better idea of what is happening in the classroom.

In findings that are relevant to this investigation, works such as that of Baran et al. (2019) show that teaching strategies based on reflection on the use of technology in education are highly correlated with the TPACK knowledge of prospective teachers. Furthermore, research such as that of Wang & Zhao (2021) has found relevant evidence that, through the use of knowledge models such as TPACK, prospective teachers can better perceive the importance and power of technologies.

Furthermore, studies such as those by Borko et al. (2008), Kleinknecht et al. (2013), and Rosaen et al. (2008) have found evidence that the use of video recordings can be an effective tool in reflective practices and that, through different dynamics, it allows improving the capacity for reflection through the study of situations and scenes that may be of interest. This resource provides an opportunity for prospective teaching staff to organize their ideas and describe situations (first plane), analyze them, and propose alternatives for improvement (second plane) (Carrillo et al., 2011; Karsenty et al., 2017). This is still valid in the COVID-19 pandemic and post-pandemic contexts and is evident in works such as those of Ester et al. (2023).

**Batteries of indicators based on the TPACK model.**

There are already an enormous number of efforts included in the international literature concerning the creation of methodological and practical indicators to operationalize the TPACK model. Works such as those of Abbitt (2011) and Koehler et al. (2011) have noted the need to create groups of indicators in different types of studies (including longitudinal, descriptive, exploratory, and statistical) with multiple instruments (such as questionnaires, surveys, semi-open interviews, and observations) that may provide a valid (although perhaps not complete) representation of the knowledge of prospective teachers about the use of technologies in different situations. More recently, investigations have been carried out using specific instruments such as surveys in which problems with scales, quantity and types of items, and
adaptation of the instrument to a particular context, among others, are addressed (Njiku et al., 2020).

In the specific case of PSMT, in works such as those of Wahyuni et al. (2021), Lyublinskaya et al. (2022), Schmidt et al. (2009) and Chacón and Vargas (2021), specific batteries have been created related to teaching different mathematical topics and the levels with which the TPACK model is associated.

For this investigation, the work and batteries of Schmidt et al. (2009) and Chacón and Vargas (2021) were selected. The first of these authors developed one of the most detailed investigations related to the creation of aspects and scales, contextualization, validation (Cronbach’s alpha and other techniques), correlations and other methods to analyse TPACK in prospective teachers (especially TK, PK and PCK). The work of Chacón and Vargas (2021) was selected, because based on an exhaustive literature review, they adapted and created multiple aspects to recognize and identify key elements of the function topic in the context of TPACK with PSMT (most strongly linked to the content: CK, TCK and TPACK). Both studies used the domains and subdomains of the TPACK model to organize their indicators. Further information about specific batteries may be found in Schmidt et al. (2009) and Chacón and Vargas (2021).

It should be noted that these batteries are a mixture of indicators. They can be descriptive and explanatory, which are considered as aspects and are in the first plane of competencies mentioned above, and, on the other hand, as evaluative indicators, which belong to the second plane and are focused on analysis, reflection, and construction of improvement proposals; the latter are called criteria.

**METHODOLOGY**

For this investigation, a qualitative methodology was applied, based on an exploratory-descriptive approach (Hernández, Fernández, and Baptista, 2014), which was used to analyse the pedagogical, technological and mathematical aspects used by a prospective mathematics teacher in initial training at the time of her study of a mathematics class on the topic of functions that was offered virtually by three high school teachers. The ultimate goal of this study was to characterize the type of activity carried out by the PSMT faced with a series of predefined aspects and, if she used them directly, whether she adapted them, or created new ones, transforming them from aspects to be identified to analytical criteria to be used in reflection about practice.
Participant

The participant was a female PSMT\(^1\), 22 years old, who was in the second semester of the fifth year of training in the Bachelor’s and Licentiate’s degree program in Mathematics at the Universidad Nacional de Costa Rica. Her selection was made based on convenience and availability to participate in the study. The PSMT digitally signed an informed consent regarding the scope, limitations, and rights of the participant in the investigation. In this document, she will be referred to as “Patricia” (name-coded to avoid her identification and to protect her rights as a participant). The research data was collected virtually during the second half of 2022.

Recorded video material for participant analysis.

The recorded video class used lasted 120 minutes and was planned and executed synchronously by three secondary school teachers. The video recording was obtained with written permission signed by the director of the Mathematics Education Reform project in Costa Rica. There were 13 organizational blocks that considered administrative and class management elements up to the development of content. In the class, the software Zoom (for creating the online session), Awwapp (as a whiteboard), Nearpod, and GeoGebra were used for the development of mathematical content.

During the class, a mathematical problem on the topic of functions was established; space was provided for independent work and interactive discussion; a video about the problem was presented to those who participated; an explanation of concepts and a closing ceremony was carried out, as well as a final evaluation. Two practices were later carried out with students, and there was a final solution to the problem. Finally, an evaluation of practices was carried out with a meta-closing of the entire video-recorded class, and an attempt was made to relate it to current situations regarding COVID-19. A news story from a Costa Rican newspaper was presented (Figure 2a), and information on active cases was extracted to work on the concepts of functions (Figure 2b).

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\(^1\) The subjects signed an Informed Consent Form (ICF) but there was no approval by the Ethics Committee. Therefore, the authors assume and exempt Acta Scientiae from any consequences arising, including full assistance and possible compensation for any damage to any research participants, per Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.
Figure 2.

Image capture of the video-recorded class: Class closures with respect to active cases of COVID-19 in the first days of the pandemic (Morales-López & Poveda-Vásquez, 2022)

Protocol and information collection in the investigation

In the first instance, the concept of a knowledge organization system was explained to the participant in a two-hour session in which several systems were addressed; special emphasis was placed on the Shulman (1986) model and some of its most well-known derivatives.

In a second session (3 days later) an explanation of the TPACK model was provided, explaining its foundation, its origin, and its domains and subdomains. A written document was shared with the participant with a summary of the information, and she was requested to study it.

In a third session (seven days later) the work of Schmidt et al. (2009) and Chacón and Vargas (2021) was explained to the participants as ways to operationalise the TPACK model (and its quantitative and qualitative visions, respectively) was discussed. In this session, the participant was instructed to observe the video-recorded class and develop indicators that allowed her to highlight the most important elements of this class on functions in a virtual context. The main instruction was that the use and adaptation of indicators from Schmidt et al. (2009) and Chacón and Vargas (2021) was permitted and that new indicators could be created if she wished to do so. The result of this was a rubric with indicators (Activity 1).
Finally, in the last session, she was asked to reflect on what happened in the recorded video class based on the indicators she constructed (whether they were aspects or criteria) (Activity 2). She was instructed that, after this reflection, she could modify her original rubric if she considered it pertinent. Table 1 summarises the characteristics of the sessions.

Table 1.

*Distribution of time and contents of the sessions related to the investigation.*

<table>
<thead>
<tr>
<th>Session</th>
<th>Face-to-face / outside the classroom</th>
<th>The topic discussed or the activity carried out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 hours / 3 days</td>
<td>Face-to-face: Explanation of the concept of knowledge system. Outside the classroom: the study of the face-to-face session and documentation. Face-to-face: Explanation of the TPACK model, its domains and subdomains and some relevant research.</td>
</tr>
<tr>
<td>2</td>
<td>3 hours / 7 days</td>
<td>Outside the classroom: 1) study of the face-to-face session and documentation Face-to-face: Explanation of external battery indicators and assignment of Activity 1.</td>
</tr>
<tr>
<td>3</td>
<td>3 hours / 7 days</td>
<td>Outside the classroom: 1) study of the face-to-face session and documentation; 2) observation of the videotaped class; 3) construction of the indicator’s rubric (Activity 1) Face-to-face: Explanation of activity 2.</td>
</tr>
<tr>
<td>4</td>
<td>1 hour / 7 days</td>
<td>Outside the classroom: Construction of the reflection and possible redesign of the battery.</td>
</tr>
</tbody>
</table>

**RESULTS AND ANALYSES**

The results of the application of Activities 1 and 2 were then examined. Activity 1 consisted of asking the participant to create or adapt indicators for the analysis of the video-recorded virtual class on functions. Patricia initially decided to use the TPACK domains and subdomains to organise the indicators. In each domain and subdomain, she specified what she considered relevant to demonstrate the existence of knowledge (see Appendix 1 for the complete battery).

For the PK domain, she proposed eight indicators, of which she classified five as her own and three as adaptations or copies from the batteries of Schmidt *et al.* (2009) and Chacón and Vargas (2021). In this domain, she
emphasised learning styles, methodologies, evaluation, prior knowledge, handling of student errors, and student participation. In this section, Patricia also specified aspects linked to the teaching of mathematics, which should be classified as PCK, but the indicator itself is correct, even though it does not correspond to that domain. This would be a point of interest for recognising if the participant fully understood the PK. She also tries to address the issue that classes are taught virtually by mentioning student organisation and participation.

In the case of the TK domain, she selected five indicators (three of her own and two adopted). She highlighted the use of technology, teachers’ ease with the use of technology, benefits and difficulties prior to using technologies, handling of technical problems and how up-to-date teachers stay with new technologies. This last indicator has the peculiarity that although it is selected from a pre-existing battery, it cannot be applied directly to a battery that evaluates a video-recorded class. The presence of this indicator shows that Patricia considered it important but was unable to adapt it to the particular situation being studied.

For the CK domain, the participant proposed five indicators (two of her own and three adopted). She emphasised the existence of sufficient knowledge of the subject, the relationship between the content knowledge of the teaching staff and the cognitive level of the students, and knowledge of concepts related to content and methodological strategies. The PSMT once again includes an indicator that is difficult to assess in the execution of the class since it proposes that the teachers list the concepts that they are going to explain, which pertains more to a planning stage than instruction itself. This has another possible interpretation, which is that she is literally waiting for the teaching staff to list the concepts to their students prior to the implementation of the class.

In the TPK subdomain, Patricia suggested five indicators (three of her own and two adopted), which are related to the appropriate selection of technologies and software for teaching content, the proposal of real problems that promote the use of technologies, the application of concepts studied in which technology is involved, and the proposal of tools that promote content analysis. Even though the last two indicators are somewhat broad and could be codependent (closer to a criterion than an aspect), all the proposed indicators coincide with the notion of teachers’ pedagogical technological knowledge and finally reduce to aspect-type indicators.

The PSMT highlighted seven indicators for the PCK subdomain (two of her own and five adapted) in which she referred to the interpretation of the
concepts with respect to context, design of activities, complexity of tasks, links with the curriculum, and suggested strategy, proposal of real problems and knowledge about errors. In the case of the TCK subdomain, four indicators were defined (one of her own and three adapted) related to knowledge of technologies for working with functions, ability to interpret concepts of functions using different computer programs, use of graphic resources and animations, and use of software to represent functions. In the cases of PCK and TCK, the indicators correspond to the definition of these subdomains.

In the final category, TPACK, Patricia developed five indicators (three of her own and two adapted) in which she considered the effective integration of technologies, mathematics, and teaching, the use of symbols and presentations to explain the concept of a function with technologies, use of technologies to measure students’ preliminary knowledge, identify difficulties and design lessons with technologies and contextualised problems that are related to other areas, and the topic of functions. In this case, the PSMT adopted certain indicators literally without making any adaptations, which resulted in a lack of concordance with the video recording since, for example, the quadratic function was not included.

One of the main deductions obtained from the evidence is that while Patricia managed to integrate content, technological, and pedagogical indicators in the different domains and subdomains, the enormous absence of indicators linked to the development of a virtual class is clear. In only one of them, did she manage to integrate this type of education into topics on how to organise the class to motivate participation (1/39)? There are at least two possible explanations for this lack of synchronisation between the battery and the observed video recording, which are addressed in the final section of this document.

A final note regarding Patricia’s battery is that several of the indicators that she claims are her own correspond directly to indicators in one of the two pre-existing batteries. This does not necessarily indicate that she copied them and reported them as her own since there is a possibility that she genuinely included them with the ideas in mind from her previous readings. What is certain is that several of these indicators were already in the batteries that were previously studied.

Next, the observations that the PSMT made when she was asked to use her own battery to analyse the video recording were examined (Activity 2). For this exercise, she was allowed to observe the class as many times as she considered necessary.
Patricia presented Activity 2 in two separate parts. In the first part, she limited herself to describing what she considered to be the most relevant thing that she observed during the class. In the second section, she wrote a reflection about what happened.

In the first descriptive part, she explained the introductory problem that was posed and the technological tools that were used and emphasised that the teaching staff could see the answers of all the students in the software in order to obtain a consensual answer that could be linked to the theme.

[Patricia]: After solving the selection questions, the teacher analysed, asked and reasoned about them by asking the students questions to reach a solution together. After reviewing the introductory problem, the teacher presented the student with a video in which these answers were summarised, observing the analysis and some important concepts related to the functions. (Activity 2: reflection)

The PSMT highlighted the use of GeoGebra to present and explain the concepts of functions and the manipulations that can be performed to show different cases. She summarised the concepts that were studied in the class, which coincided with what she intended to do when designing her battery. Patricia then described the following activity, the exercises, and the time needed to perform it, and highlighted that at some moments, errors or missing elements may be noticed that can cause confusion, exemplifying this by using a screen capture of the video recording (using the indicators as criteria). Figure 3 shows the comments she made and the exercise to which she refers.

It should be noted that she is implicitly observing that it is not appropriate to encourage this type of ambiguity in classes. Here she is moving from observation to assessment (turning aspects into criteria).

In the second part of this virtual class exercise, Patricia pointed out problems with the use of technological resources that could be solved by the teacher and also pointed out that the activity allowed the teacher to see student errors when trying to sketch a graph of the situation that was explained to them. She highlighted three situations that she observed in the video where a correct drawing appears on the problem posed (Figure 4a) and two in which, according to her, there are errors with the relationship between a preimage and its image (Figure 4b), and the interval in which the domain is defined (Figure 4c), respectively.
Figure 3

Patricia’s comment on the second activity proposed in the virtual class and the screen capture she used when explaining her statement.

Patricia: […] However, in the first part, the graphs presented are not clear, because they lack closing points or continuity arrows (Activity 2).

Translations: Graphic representations of the exercises

Nota: Tomadas del video

Note: Taken from the video

Figure 4

Drawings of students that the PSMT captured from the video recording and used to show that the mathematical task allows errors to be observed and addressed.

Figure 4a. Figure 4b. Figure 4c.

Note: The problem posed was based on sketching a graph of a function whose domain was [-3, 6].

At the end of this description, Patricia pointed out that information from newspaper articles was used to connect the concepts with current problems related to COVID-19, and she interpreted this as allowing the group of students to see the usefulness of such an exercise and explain possible errors when reading the graphic. She highlighted that the graphic by points (active
cases) could confuse the students if they are working in a real domain and that the teacher did not explain this (turning this into a criterion more related to the issue of ambiguities).

In the section that she called class analysis (the second part of Activity 2), the PSMT organized her analysis directly using the three domains and four subdomains of the TPACK model and, unlike the previous section, directly used many of the indicators that she defined in her battery (in the entire descriptive section she did not refer to those indicators directly, although many of them can be indirectly linked). This can be explained because TPACK itself does not have tools for descriptive analysis.

**Figure 5**

*Comparison between the indicators previously constructed by Patricia and her written analysis of pedagogical content knowledge.*
Due to the reasons discussed above, the analysis that Patricia constructed is basically the sum of the indicators along with grammatical connectors of addition, logic, and exemplification, with few justifications (cause-effect, contrasts, explanations, or possible explanations). If what she wrote is analysed separately from her battery, one can assume coherence and reasonableness in what was written, and it is clear that she used the indicators in her battery to construct paragraphs in each domain and subdomain (Figure 5).

This also occurred in all remaining domains and subdomains of the TPACK in this section of Activity 2. Finally, although optional, the PSMT did not consider it necessary to present an updated version of her battery after having completed Activities 1 and 2.

CONCLUSIONS

The activity that Patricia carried out has provided evidence of multiple elements linked to the ability to describe (first plane) and analyze what has happened (second plane). Regarding the objective of characterizing the technological, pedagogical, and mathematical aspects that the PSMT had established and used when reflecting on the events that occurred in the virtual mathematics class on the topic of functions, several findings of great interest were encountered. In the first place, she managed to effectively interpret the TPACK indicators in a general way, but she also highlighted the limitations of these indicators, including the fact that they are focused on inferring types of knowledge and not necessarily actions, skills, and competences in teachers or students.

Although she took several of the indicators, she found in the batteries literally and did not create many of her own, they helped her understand which elements might be of interest in her analysis. A finding of extreme relevance is that a very high dependence on the indicators of the batteries studied was evident, which helped her, while in a certain way restricting what she found of interest. This is because practically all of these indicators were aspects, and many were not used as criteria.

The clearest example of this situation is that the indicators in the batteries used included very little about the activity of learning and teaching mathematics in a virtual education model. It cannot be said that Patricia knows or does not know about this type of education, but if this knowledge indeed existed, she was not able to make use of it adequately with the use of batteries.
or with the domains or subdomains of TPACK, especially with the TPK, a result that agrees with findings by Silva (2022). Thus, a possible hypothesis is that, for prospective teachers, batteries of indicators and organizational systems such as the TPACK can be appropriate guides to what is relevant to study; however, using a model such as the TPACK without adequate assessment criteria can cause inconsistencies such as situations in which, although the teacher manages to identify ambiguities and errors, when evaluating them she does so incorrectly. Even so, prospective teachers must have prior contact with this type of situation to be able to understand what they will experience in their classrooms and to be able to determine possible solutions to educational problems.

Secondly, in the first part of Activity 2, some of the indicators that she defined in her battery emerged, but she did not use them as organizers. She decided to describe the teachers’ role, the exercises, and what she considered to be the objective of each one. It is thus possible to conclude that in most of her work, she focused on describing and justifying what she believed was happening. This coincides with the results of research such as Peguera-Carré et al. (2023) and Santagata et al. (2011), which shows that these are the elements in which prospective teachers are most interested.

Regarding what she considered as her reflection (the second part of Activity 2), it is evident that she used the criteria as elements of evaluation of presence or absence and did not attempt to justify, suggest, or propose improvements to what was happening. Despite this, as mentioned, her discussion is coherent and incorporates many of the elements that she had already selected as of interest. It is clear that she fails to note contrasts or propose modifications, considered more complex activities in a reflective practice (Schön, 1984).

The implications of this study also allow us to suggest that use of the domains and subdomains for the TPACK model by themselves is not sufficient for a PSMT to be able to organize, describe, and reflect on a mathematics class. The batteries of indicators that are derived from a model of knowledge organization whose structure is based on domains and subdomains are highly limited in their capacity to include evaluation and reflection criteria for all the activities carried out in the class, which undoubtedly requires attempts to adapt indicators of the first plane (aspects), and to complement them in some way, with some external theoretical model which provides greater knowledge about evaluative and reflection criteria. Thus, the TPACK model has a concrete nature.
and helps to infer types of knowledge, but to observe and evaluate a mathematics class, other theoretical and methodological tools are required.

It has been shown that stimulating prospective teachers to understand and experience previous batteries can help them better understand a knowledge organization system and can allow them to operationalize it. Even so, evidence also shows that this operationalization does not solve theoretical deficiencies that the PSMT may have because, although she may be able to describe and justify several elements of interest that occur in the class, this leads to little reflection. To address these deficiencies, it is necessary for the PSMT to have theoretical and methodological knowledge in the field of mathematics education that would allow her to adequately explain what is happening. In this sense, not only is solid training necessary during the teachers’ careers, but they also require adequate support to stimulate capacity for reflection through joint work with their peers, other secondary school teachers, and with their career teachers (Cheng, 2017).

The findings of this research may have important implications for teacher training faculty and the authorities that define training curricula since when implementing guidelines linked to the use of technologies, teaching staff must be able to reflect on what happens when they are introduced. The results of this investigation show that it is not enough to study the domains and subdomains of the TPACK model, but that there must be a detailed study of mathematical education models that consistently complement what the PSMT can interpret and the way she can link her perceptions with explicit evidence (Amador, 2022).

Some open questions arise from this investigation that will need to be addressed in future studies, as it appears that this type of activity can create links and connections between research and practice. It will be necessary to investigate, for example, what happens to the capacity for reflection when the PSMT has a broader and more robust theoretical or conceptual framework for mathematics instruction that can be integrated into models such as the TPACK to recognize the importance of technologies in education.

Regarding the limitations of this study, it is important to mention that the results are not intended to be generalized, but rather to be used as inputs for the understanding and improvement of teacher training and their ability to reflect on the use of technologies.
AUTHORS’ CONTRIBUTIONS STATEMENTS

YML conceived the presented idea, developed the theory, adapted the methodology to this context, created the models, performed the activities, and collected the data. YML analysed the data. All authors actively discussed the results and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, YML, upon reasonable request.

ACKNOWLEDGEMENTS

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Sukiman, Haningsih, S., & Rohmi, P. (2022). The pattern of hybrid learning to maintain learning effectiveness at the higher education level post-


**APPENDIX**

**Table 3**

*Indicators defined by the participant for the study of technological, pedagogical and content knowledge related to the topic of functions in a virtual class with technological resources, 2022.*

<table>
<thead>
<tr>
<th>Domain</th>
<th>Indicator</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedagogical knowledge (PK)</td>
<td>Considers student characteristics such as cognitive and learning styles that influence the teaching process</td>
<td>Adopted from Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td></td>
<td>Uses active methodological strategies for teaching and learning mathematics using “learning-by-doing”</td>
<td>Participant</td>
</tr>
<tr>
<td></td>
<td>Carries out an appropriate final evaluation of learning achieved</td>
<td>Participant</td>
</tr>
<tr>
<td></td>
<td>Ability to adapt teaching according to the students' previous knowledge</td>
<td>Participant</td>
</tr>
</tbody>
</table>
Knows procedures and common student misconceptions
Handles errors appropriately to generate a new starting point for learning (SIC).
Promotes student participation by generating an environment of security and trust
Adequately organizes group participation during the virtual session
Uses technology appropriately
Implements new technologies with ease
Knows the benefits and difficulties of new technologies before implementing them
Is aware of the types of technical problems students have and knows how to solve them
Keeps up to date with important new technologies
Has sufficient knowledge about the topic of the quadratic function
Uses a way of thinking mathematically that is related to students’ cognitive level when explaining the quadratic function
Makes a list of the concepts to be explained that are related to the quadratic function
Provides adequate definitions of the concepts related to the quadratic function
Has several forms and methodological strategies to develop understanding of the quadratic function
Selects technologies that can facilitate teaching and learning of the quadratic function
Analyzes whether the software used promotes students’ teaching and learning of the quadratic function
Represents real situations in which quadratic functions and software can be used
Proposes learning activities in which students manipulate technological tools and apply the concepts presented about the quadratic function
Proposes technological tools that promote the analysis of content related to the quadratic function
Correctly interprets the elements of the quadratic function according to the context
Designs activities about the quadratic function that promote the conceptual and procedural development of students
Plans activities in chronological order of complexity
Learns about the specific skills proposed by the Mathematics Curriculum for teaching and learning the quadratic function in secondary school
Proposes problems for the quadratic function that present a meaningful and real context for students’ teaching and learning
<table>
<thead>
<tr>
<th>Technological content knowledge (TCK)</th>
<th>Chacón &amp; Vargas, (2021)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addresses the four moments established by the MEP study program with the quadratic function</td>
<td></td>
</tr>
<tr>
<td>Knows about student errors in understanding the quadratic function</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Knows about technological resources that can be used to work with quadratic functions</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Interprets the basic concepts of the quadratic function using different technological programs</td>
<td>Adopted from Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Uses animations and graphic drawings to enrich proposed mathematical activity related to the quadratic function</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Knows how to use programs appropriately to represent quadratic functions and generate variations when organizing an analysis of the topic</td>
<td>Participant</td>
</tr>
<tr>
<td>Effectively integrates mathematics, technology and pedagogy in planning an activity for teaching about the quadratic function.</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Uses symbols and simple representations to explain aspects related to the quadratic function using a technological tool</td>
<td>Participant</td>
</tr>
<tr>
<td>Has the ability to use technological devices to measure students’ preliminary knowledge of mathematical content</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Knows how to use technological devices that assist in identifying the difficulties that students have with learning quadratic functions</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Designs lessons related to the quadratic function using technologies and contextualized problems that are related to other areas (such as science and arts)</td>
<td>Participant</td>
</tr>
<tr>
<td>Technological pedagogical content knowledge (TPACK)</td>
<td>Chacón &amp; Vargas, (2021)</td>
</tr>
<tr>
<td>Has the ability to use technological devices to measure students’ preliminary knowledge of mathematical content.</td>
<td>Participant</td>
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<td>Knows how to use technological devices that assist in identifying the difficulties that students have with learning quadratic functions</td>
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